Appendix A
Riemann Curvature Tensor

We will often use the notation $(...)_{,\,\mu}$ and $(...)_{;\,\mu}$ for a partial and covariant derivative respectively, and the [anti]-symmetrization brackets defined here by

$$T_{[ab]} := \frac{1}{2}\,(T_{ab} - T_{ba}) \quad \text{and} \quad T_{(ab)} := \frac{1}{2}\,(T_{ab} + T_{ba})\,, \tag{A.1}$$

for a tensor of second rank T_{ab}, but easily generalized for tensors of arbitrary rank.

We can define the Riemannian curvature tensor in coordinate representation by the action of the commutator of two covariant derivatives on a vector field v^α

$$[\nabla_\mu, \nabla_\nu]\, v^\rho = R^\rho{}_{\sigma\mu\nu}\, v^\sigma, \tag{A.2}$$

with the explicit formula in terms of the symmetric Christoffel symbols $\Gamma^\rho_{\mu\nu}$

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\sigma\nu} - \partial_\nu \Gamma^\rho_{\sigma\mu} + \Gamma^\rho_{\alpha\mu}\Gamma^\alpha_{\sigma\nu} - \Gamma^\rho_{\alpha\nu}\Gamma^\alpha_{\sigma\mu}. \tag{A.3}$$

From this definition it is obvious that $R^\rho{}_{\sigma\mu\nu}$ possesses the following symmetries

$$R_{\alpha\beta\gamma\delta} = -R_{\beta\alpha\gamma\delta} = -R_{\alpha\beta\delta\gamma} = R_{\gamma\delta\alpha\beta}. \tag{A.4}$$

In addition, there is an identity for the cyclic permutation of the last three indices

$$R_{\alpha[\beta\gamma\delta]} = \frac{1}{3}(R_{\alpha\beta\gamma\delta} + R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\delta\beta}) = 0. \tag{A.5}$$

An arbitrary tensor of fourth rank in d dimension has d^4 independent components. Since a tensor is built from tensor products, we can think of $R_{\alpha\beta\gamma\delta}$ as being composed of two $(d \times d)$ matrices $A^1_{\alpha\beta}$ and $A^2_{\gamma\delta}$. From (A.4), it follows that A^1 and A^2 are antisymmetric, each having $n(n-1)/2$ independent components. Writing $R_{\alpha\beta\gamma\delta} \equiv R_{A^1 A^2}$, where we have collected the index pairs $(\alpha\beta) \to A^1$ and $(\gamma\delta) \to A^2$, this corresponds to a matrix, in which each index A^1 and A^2 labels $d(d-1)/2$

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components. Taking into account the symmetry under pairwise exchange of $(\alpha\beta)$ and $(\gamma\delta)$, we can consider $R_{A^1 A^2}$ as a symmetric matrix in A^1 and A^2, having $A^1(A^1+1)/2$ independent components. Altogether, we find

$$\frac{1}{2}\left[\frac{1}{2}\,d\,(d-1)\right]\left\{\left[\frac{1}{2}d\,(d-1)\right]+1\right\}=\frac{1}{8}\,d^4-\frac{1}{4}\,d^3+\frac{3}{8}\,d^2-\frac{1}{4}\,d \qquad \text{(A.6)}$$

independent components. We still need to figure out how many components are related by the cyclic identity (A.5) in d dimensions. We can artificially write

$$R_{\alpha\beta\gamma\delta}=\frac{1}{8}\left[R_{\alpha\beta\gamma\delta}-R_{\alpha\beta\delta\gamma}+R_{\beta\alpha\delta\gamma}-R_{\beta\alpha\gamma\delta}+(\alpha\beta)\leftrightarrow(\gamma\delta)\right] \qquad \text{(A.7)}$$

and similar expressions for $R_{\alpha\gamma\delta\beta}$ and $R_{\alpha\delta\beta\gamma}$. Inserting these expressions into (A.5) leads to the condition $R_{[\alpha\beta\gamma\delta]}=0$. This totally antisymmetric object is identical zero for two identical indices but gives one additional constraint for each choice of four distinct, orderless indices, reducing the number of independent components by one respectively. In d dimensions, the number of additional constraints corresponds to "choose 4 out of d". For integers d, n there exists a product formula to calculate the binomial

$$\binom{d}{n}=\prod_{k=1}^{n}\frac{d-n+k}{k}. \qquad \text{(A.8)}$$

Inserting $n=4$, we obtain

$$\binom{d}{4}=\frac{1}{24}\left[d\,(d-1)\,(d-2)\,(d-3)\right]=\frac{1}{24}\,d^4-\frac{1}{4}\,d^3+\frac{11}{24}\,d^2-\frac{1}{4}\,d. \qquad \text{(A.9)}$$

Thus, the number of independent components $I_{\text{Riem}}(d)$ of $R_{\alpha\beta\gamma\delta}$ is given by

$$I_{\text{Riem}}(d):=\frac{1}{4}\,d\,(d-1)\left[\frac{1}{2}\,d\,(d-1)+1\right]-\binom{d}{4}=\frac{1}{12}\,d^2\,(d^2-1). \qquad \text{(A.10)}$$

We consider $I_{\text{Riem}}(d)$ for different dimensions d:

$$I_{\text{Riem}}(1)=0,\quad I_{\text{Riem}}(2)=1,\quad I_{\text{Riem}}(3)=6,\quad I_{\text{Riem}}(4)=20. \qquad \text{(A.11)}$$

This shows that gravity in one dimension is trivial, since there is no dynamical degree of freedom. It further shows that gravity in two dimensions can be described by the Ricci scalar R and in three dimensions by $R^{(3)}_{\mu\nu}=R^{(3)}_{\nu\mu}$. In general, the d-dimensional Ricci tensor $R_{\mu\nu}:=g^{\gamma\delta}R_{\gamma\mu\delta\nu}$ has $I_{\text{Ric}}(d):=\frac{1}{2}\,d\,(d+1)$ independent components. In particular $I_{\text{Ric}}(4)=10$. Thus, ten components of $R_{\alpha\beta\gamma\delta}$ not contained in $R_{\mu\nu}$ still remain. They are contained in the Weyl tensor

$$C_{\alpha\beta\gamma\delta} := R_{\alpha\beta\gamma\delta} - \frac{2}{d-2}\left(g_{\alpha[\gamma} R_{\delta]\beta} - g_{\beta[\gamma} R_{\delta]\alpha}\right) + \frac{2}{(d-1)(d-2)}\, g_{\alpha[\gamma} g_{\delta]\beta}\, R. \tag{A.12}$$

It follows that the Weyl tensor has (for $d > 3$)

$$I_{\mathrm{Weyl}}(d) = I_{\mathrm{Riem}}(d) - I_{\mathrm{Ric}}(d) = \frac{1}{12}\, d\,(d+1)\,(d+2)\,(d-3) \tag{A.13}$$

independent components, and in particular $I_{\mathrm{Weyl}}(4) = 10$.

Appendix B
Variations and Derivatives

B.1 Covariant Differentiation in General Relativity

The action of the metric compatible covariant derivative $\nabla_\mu\, g_{\alpha\beta} = 0$ with respect to the Christoffel symbol $\Gamma^{\rho}_{\mu\nu}$ on a general tensor is defined as

$$\nabla_\mu\, T^{\alpha\ldots\gamma}_{\quad\beta\ldots\delta} = \partial_\mu\, T^{\alpha\ldots\gamma}_{\quad\beta\ldots\delta} + \Gamma^{\alpha}_{\mu\rho}\, T^{\rho\ldots\gamma}_{\quad\beta\ldots\delta} + \ldots + \Gamma^{\gamma}_{\mu\rho}\, T^{\alpha\ldots\rho}_{\quad\beta\ldots\delta} - \Gamma^{\rho}_{\mu\beta}\, T^{\alpha\ldots\gamma}_{\quad\rho\ldots\delta} - \ldots - \Gamma^{\rho}_{\mu\delta}\, T^{\alpha\ldots\gamma}_{\quad\beta\ldots\rho}, \tag{B.1}$$

with the symmetric Christoffel symbol

$$\Gamma^{\alpha}_{\mu\nu}(g) := \frac{1}{2}\, g^{\alpha\rho}\left(\partial_\nu\, g_{\mu\rho} + \partial_\mu\, g_{\nu\rho} - \partial_\rho\, g_{\mu\nu}\right) = \Gamma^{\alpha}_{\nu\mu}(g). \tag{B.2}$$

For scalar functions $\nabla_\mu\varphi = \partial_\mu\varphi$. For vector fields v^μ, a useful formula is

$$\nabla_\mu\, v^\mu = \frac{1}{\sqrt{g}}\,\partial_\mu\left(\sqrt{g}\, v^\mu\right) \quad \text{or} \quad \sqrt{g}\,\nabla_\mu\, v^\mu = \partial_\mu\left(\sqrt{g}\, v^\mu\right). \tag{B.3}$$

B.2 Functional Derivative

We use again the condensed DeWitt notation for a generalized field $\phi^i = \phi^A(x)$, introduced in Sect. 4.4. We want to emphasize that an object with two DeWitt indices like $\Omega_{ij'}$ corresponds to a *two-point function* or a generalized *bi-tensor* $\Omega_{AB}(x, x')$ in conventional notation. The primed indices like j' refer to the space-time point x'. This means we can construct objects which behave as tensors of different rank at different space-time points. A particular case of a generalized bi-tensor is a bi-scalar which has no indices $A, B, ..$ at all. A special bi-scalar, in turn, is the Dirac Delta-Distribution $\delta(x,\ x')$ which is defined in a 2ω-dimensional curved space-time with metric $g_{\mu\nu}(x)$ for a general test field $\Psi^A(x)$ by the equation

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$$\Psi^A(x) = \int \mathrm{d}^{2\omega}x' \, |g(x')|^{1/2} \, \delta(x, x') \, \Psi^A(x'), \tag{B.4}$$

For simplicity, we specify $2\omega = 4$. In the condensed notation (B.4) takes the form $\Psi^i = \delta^i{}_{j'} \Psi^{j'}$ with

$$\delta^i_{j'} = |g(x')|^{1/2} \, \delta^A{}_B \, \delta(x, x') = \delta^A{}_B \, \tilde{\delta}(x, x') \quad . \tag{B.5}$$

The quantity $\tilde{\delta}(x, x') := |g(x')|^{1/2} \delta(x, x')$ is no longer a bi-scalar, since it transforms as a scalar at the point x, but as a scalar density at the point x'. The expansion of $Z[\Psi]$ up to linear order in condensed notation is defined by

$$Z[\Psi + \zeta] =: Z[\Psi] + Z_{,\,i}[\Psi] \, \zeta^i, \tag{B.6}$$

with $\zeta^i := \delta\psi^i$ and

$$Z_{,\,i}[\Psi] \, \zeta^i = \int d^4x' \, \frac{\delta Z[\Psi(x)]}{\delta \Psi^A(x')} \, \zeta^A(x') \tag{B.7}$$

in conventional notation. Therefore, it seems obvious to define the functional derivative $Z_{,\,i} = \delta Z[\Psi(x)]/\delta\Psi^A(x')$ by the variation [3]

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon} \left(Z[\Psi + \epsilon\zeta] - Z[\Psi] \right) =: \int d^4x' \, \frac{\delta Z[\Psi(x)]}{\delta \Psi^A(x')} \, \zeta^A(x'). \tag{B.8}$$

For the special case $Z = \mathrm{Id}$, we obtain from (B.8)

$$\zeta^A = \int d^4x' \, \frac{\delta \Psi^A(x)}{\delta \Psi^B(x')} \, \zeta^B(x') \quad \text{or} \quad \frac{\delta \Psi^A(x)}{\delta \Psi^B(x')} = \delta^A{}_B \, \tilde{\delta}(x, x'). \tag{B.9}$$

B.3 Lie Derivative

The Lie derivative $\mathcal{L}_\xi$ of an arbitrary tensor $T^{\gamma\ldots\delta}_{\alpha\ldots\beta}$ in the direction of the vector ξ^μ is a measure of the difference between $T^{\gamma'\ldots\delta'}_{\alpha'\ldots\beta'}(x')$ "dragged" along the integral curve γ_ξ of ξ^μ compared to the tensor $T^{\gamma\ldots\delta}_{\alpha\ldots\beta}(x')$ at this point. Calculating the infinitesimal flow of $T^{\gamma'\ldots\delta'}_{\alpha'\ldots\beta'}(x')$ along γ_ξ back from x' to x with the infinitesimal coordinate transformation matrix $\partial x^{\mu'}/\partial x^\nu = \delta^\mu_\nu + \epsilon\, \xi^\mu_{,\,\nu}$ yields

$$T^{\gamma'\ldots\delta'}_{\alpha'\ldots\beta'}(x') = T^{\gamma\ldots\delta}_{\alpha\ldots\beta} + \epsilon \left(\xi^\gamma_{,\rho} T^{\rho\ldots\delta}_{\alpha\ldots\beta} + \ldots + \xi^\delta_{,\rho} T^{\gamma\ldots\rho}_{\alpha\ldots\beta} - \xi_{\alpha,}{}^{\rho} T^{\gamma\ldots\delta}_{\rho\ldots\beta} - \ldots - \xi_{\beta,}{}^{\rho} T^{\gamma\ldots\delta}_{\alpha\ldots\rho} \right). \tag{B.10}$$

Taylor expansion of $T^{\gamma\ldots\delta}_{\alpha\ldots\beta}(x')$ around x yields

$$T^{\gamma\ldots\delta}_{\alpha\ldots\beta}(x+\epsilon\,\xi) = T^{\gamma\ldots\delta}_{\alpha\ldots\beta}(x) + \epsilon\, T^{\gamma\ldots\delta}_{\alpha\ldots\beta,\,\rho}\,\xi^{\rho}. \tag{B.11}$$

Since the right-hand sides of (B.10) and (B.11) involve only quantities at x, we can compare these two objects at x and define the Lie derivative as the difference

$$\begin{aligned}\left(\mathcal{L}_{\xi}\,T\right)^{\gamma\ldots\delta}_{\alpha\ldots\beta}(x) &:= \lim_{\epsilon\to 0}\frac{T^{\gamma\ldots\delta}_{\alpha\ldots\beta}(x') - T^{\gamma'\ldots\delta'}_{\alpha'\ldots\beta'}(x')}{\epsilon}\\ &= T^{\gamma\ldots\delta}_{\alpha\ldots\beta,\,\rho}\,\xi^{\rho} - T^{\rho\ldots\delta}_{\alpha\ldots\beta}\,\xi^{\gamma}_{,\rho} - \ldots - T^{\gamma\ldots\rho}_{\alpha\ldots\beta}\,\xi^{\delta}_{,\rho}\\ &\quad + T^{\gamma\ldots\delta}_{\rho\ldots\beta}\,\xi_{\alpha,}^{\ \rho} + \ldots + T^{\gamma\ldots\delta}_{\alpha\ldots\rho}\,\xi_{\beta,}^{\ \rho}.\end{aligned} \tag{B.12}$$

A special case is the Lie derivative of the metric tensor. The vanishing of the Lie derivative $\left(\mathcal{L}_{\xi}\,g\right)_{\mu\nu} = 0$ in direction ξ^{μ} signalizes a symmetry of the space-time manifold and means that the vector ξ^{μ} is a generator of an isometry

$$\left(\mathcal{L}_{\xi}\,g\right)_{\mu\nu} = g_{\mu\nu,\,\rho}\,\xi^{\rho} + g_{\rho\nu}\,\xi^{\rho}_{,\,\mu} + g_{\nu\rho}\,\xi^{\rho}_{,\,\nu} = \xi_{\mu;\,\nu} + \xi_{\nu;\,\mu} = 0. \tag{B.13}$$

The vector fields ξ^{μ} obeying (B.13) are called Killing vector fields.

B.4 Variation of Metric Quantities

In the case of pure gravity, $\Psi^{i} = g_{\mu\nu}(x)$ and $\zeta^{i} = h_{\mu\nu}(x)$, we find from (B.9)

$$\frac{\delta g_{\mu\nu}(x)}{\delta g_{\alpha\beta}(x')} = \delta^{\alpha\beta}_{\mu\nu}\,\tilde{\delta}(x,x') \quad \text{with} \quad \delta^{\alpha\beta}_{\mu\nu} := \delta^{\alpha}_{(\mu}\delta^{\beta}_{\nu)} = \frac{1}{2}\Big(\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu} + \delta^{\alpha}_{\nu}\delta^{\beta}_{\mu}\Big). \tag{B.14}$$

By using (B.8) with $F(\Psi) = F(g) = (g_{\mu\nu})^{-1}$ and $g^{\mu\nu} := (g_{\mu\nu})^{-1}$, it follows

$$\delta_{g}\,g^{\mu\nu} = -g^{\mu(\alpha}\,g^{\beta)\nu}\,h_{\alpha\beta}. \tag{B.15}$$

By using (B.8) with $F(g) = g^{1/2}$ and the identity $\det(g) = \exp\big\{\,\mathrm{tr}[\log(g_{\mu\nu})]\,\big\}$, we find

$$\delta_{g}\,g^{1/2} = \frac{1}{2}\,g^{1/2}\,g^{\alpha\beta}\,h_{\alpha\beta}. \tag{B.16}$$

By using (B.4) with an *arbitrary* test function $t(x) \neq 0$ that does not depend on $g_{\mu\nu}$ and differentiating both sides, we find

$$\int \mathrm{d}^4x\,\left[\frac{\delta}{\delta g_{\mu\nu}(y)}\,\tilde{\delta}(x,x')\right]t(x') = 0 \quad \text{or} \quad \frac{\delta}{\delta g_{\mu\nu}(y)}\,\tilde{\delta}(x,x') = 0. \tag{B.17}$$

Combining (B.14) with (B.17) leads to

$$\frac{\delta^2 g_{\mu\nu}(x)}{\delta g_{\alpha\beta}(y)\,\delta g_{\gamma\delta}(z)} = 0 \quad \text{or} \quad \delta_g^2\, g_{\mu\nu} = \delta_g\, h_{\mu\nu} = 0. \tag{B.18}$$

With the basic results (B.15), (B.16) and (B.18), we can construct the variation of more complicated objects. Using the fact that the operations of variation and partial differentiation commute, we calculate the variation of the Christoffel symbol

$$\begin{aligned}\delta_g \Gamma^{\rho}_{\mu\nu} = &-\frac{1}{2} g^{\rho\alpha} g^{\delta\beta} \left(\partial_\nu\, g_{\mu\delta} + \partial_\mu\, g_{\nu\delta} - \partial_\delta\, g_{\mu\nu}\right)\, h_{\alpha\beta} \\ &+ \frac{1}{2}\, g^{\rho\delta} \,\left(\partial_\nu\, h_{\mu\delta} + \partial_\mu\, h_{\nu\delta} - \partial_\delta\, h_{\mu\nu}\right).\end{aligned} \tag{B.19}$$

Using (B.1), we rewrite the ∂_μ's acting on the $h_{\mu\nu}$'s in terms of the ∇_μ's

$$\begin{aligned}\delta_g \Gamma^{\rho}_{\mu\nu} = &-g^{\rho\alpha}\, \Gamma^{\beta}_{\mu\nu}\, h_{\alpha\beta} + \frac{1}{2}\, g^{\rho\delta}\, \Big(\nabla_\nu\, h_{\mu\delta} + \nabla_\mu\, h_{\nu\delta} - \nabla_\delta\, h_{\mu\nu} \\ &+ \Gamma^{\epsilon}_{\nu\mu} h_{\epsilon\delta} + \Gamma^{\epsilon}_{\nu\delta} h_{\mu\epsilon} + \Gamma^{\epsilon}_{\mu\nu} h_{\epsilon\delta} + \Gamma^{\epsilon}_{\mu\rho} h_{\nu\epsilon} - \Gamma^{\epsilon}_{\mu\rho} h_{\epsilon\nu} - \Gamma^{\epsilon}_{\nu\rho} h_{\mu\epsilon}\Big) \\ = &-g^{\rho\alpha}\, \Gamma^{\beta}_{\mu\nu}\, h_{\alpha\beta} + \frac{1}{2}\, g^{\rho\delta}\, \Big(\nabla_\nu\, h_{\mu\delta} + \nabla_\mu\, h_{\nu\delta} - \nabla_\delta\, h_{\mu\nu} + 2\Gamma^{\epsilon}_{\nu\mu} h_{\epsilon\delta}\Big) \\ = &\frac{1}{2}\, g^{\rho\delta}\, \left(\nabla_\nu\, h_{\mu\delta} + \nabla_\mu\, h_{\nu\delta} - \nabla_\delta\, h_{\mu\nu}\right) = \frac{1}{2}\, g^{\rho\delta}\, \left(h_{\mu\delta;\,\nu} + h_{\nu\delta;\,\mu} - h_{\mu\nu;\,\delta}\right).\end{aligned} \tag{B.20}$$

Then, the variation of the Riemann tensor yields

$$\begin{aligned}\delta_g R^{\rho}{}_{\sigma\mu\nu} &= \delta_g\, \Big(\partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\alpha}\Gamma^{\alpha}_{\nu\sigma} - \Gamma^{\rho}_{\nu\alpha}\Gamma^{\alpha}_{\mu\sigma}\Big) \\ &= (\delta_g \Gamma^{\rho}_{\nu\sigma})_{,\,\mu} - (\delta_g \Gamma^{\rho}_{\mu\sigma})_{,\,\nu} + (\delta_g \Gamma^{\rho}_{\mu\alpha})\Gamma^{\alpha}_{\nu\sigma} + (\delta_g \Gamma^{\alpha}_{\nu\sigma})\Gamma^{\rho}_{\mu\alpha} \\ &\quad - (\delta_g \Gamma^{\rho}_{\nu\alpha})\Gamma^{\alpha}_{\mu\sigma} - (\delta_g \Gamma^{\alpha}_{\mu\sigma})\Gamma^{\rho}_{\nu\alpha}.\end{aligned} \tag{B.21}$$

Using (B.1), we can again rewrite the partial derivative in terms of a covariant derivative plus terms proportional to the connection

$$(\delta_g \Gamma^{\rho}_{\nu\sigma})_{,\,\mu} = (\delta_g \Gamma^{\rho}_{\nu\sigma})_{;\,\mu} - (\delta_g \Gamma^{\alpha}_{\nu\sigma})\Gamma^{\rho}_{\alpha\mu} + (\delta_g \Gamma^{\rho}_{\alpha\sigma})\Gamma^{\alpha}_{\nu\mu} + (\delta_g \Gamma^{\rho}_{\nu\alpha})\Gamma^{\alpha}_{\sigma\mu}. \tag{B.22}$$

Substituting this expression (and the same term interchanging μ and ν) in (B.21), we find that all terms proportional to $(\delta_g \Gamma)\Gamma$ cancel identically. Thus, the variation of the Riemann tensor yields

$$\delta_g R^{\rho}{}_{\sigma\mu\nu} = (\delta_g \Gamma^{\rho}_{\nu\sigma})_{;\,\mu} - (\delta_g \Gamma^{\rho}_{\mu\sigma})_{;\,\nu}. \tag{B.23}$$

For the variation of the Ricci tensor, we contract (B.23) over ρ and μ

$$\delta^{\mu}_{\rho}(\delta_g R^{\rho}{}_{\sigma\mu\nu}) = \delta_g(\delta^{\mu}_{\rho} R^{\rho}{}_{\sigma\mu\nu}) = \delta_g R^{\rho}{}_{\sigma\rho\nu} = (\delta_g \Gamma^{\rho}_{\nu\sigma})_{;\,\rho} - (\delta_g \Gamma^{\rho}_{\rho\sigma})_{;\,\nu}. \tag{B.24}$$

Substituting the variation (B.20), we obtain

$$\delta_g R_{\mu\nu} = (\delta_g \Gamma^{\rho}_{\nu\mu})_{;\,\rho} - (\delta_g \Gamma^{\rho}_{\mu\rho})_{;\,\nu} = \frac{1}{2} g^{\rho\delta} \left(h_{\mu\rho;\,\nu\delta} + h_{\nu\delta;\,\mu\rho} - h_{\mu\nu;\,\rho\delta} - h_{\rho\delta;\,\mu\nu}\right). \tag{B.25}$$

Finally, the variation of the Ricci scalar can ultimately be obtained from (B.15) and (B.25) by the relation

$$\delta_g R = (\delta_g g^{\mu\nu}) R_{\mu\nu} + g^{\mu\nu} (\delta_g R_{\mu\nu}). \tag{B.26}$$

Appendix C
Young Tableaux for $SU(3)$

The most efficient way to calculate direct products of different representations and their dimensionality is to use the technique of Young tableaux. This is an abstract symbolical notation for projecting out the symmetric and antisymmetric components of a tensor. A linear representation ρg of a group element $g \in G$ can be written as a multilinear form (a tensor). Each index runs over the dimension of the group parameters from 1, ..., N. Everything we will work out can easily be generalized to $SU(N)$, but we are mainly interested in $SU(3)$. We have to specify several conditions and rules which define a valid diagram and its combination with other diagrams. The fundamental representation **3** is represented as a single box

$$\mathbf{3} = \square. \tag{C.1}$$

This corresponds to a tensor A^i with one contravariant index (for $SU(3)$ all indices run from 1, ..., 3). The conjugated representation $\bar{\mathbf{3}}$ corresponds to a tensor A_i with one covariant index. To obtain the corresponding diagram, we have to raise this index with the three-dimensional epsilon tensor ϵ^{ijk}. This gives a completely antisymmetric contravariant tensor A^{jk} of second rank. Anti-symmetrization is diagrammatically indicated by vertical boxes, one box for each index and symmetrization by boxes in a row. Thus, we obtain the diagram for $\bar{\mathbf{3}}$

$$\bar{\mathbf{3}} = \begin{array}{c}\square\\[-4pt]\square\end{array}. \tag{C.2}$$

For a valid Young diagram The numbers of boxes in a corresponding row, starting from the first row, has to be smaller or equal to the number of boxes in the previous row. For $SU(N)$ the number of vertical boxes can never exceed N because otherwise the procedure of anti-symmetrization of $N + 1$ indices, each index running from 1, ..., N would result in zero. We write tensor product between two fundamental representations **3** as

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$$\mathbf{3} \otimes \mathbf{3} = \square \otimes \boxed{a} = \begin{array}{|c|c|}\hline & a \\ \hline\end{array} \oplus \begin{array}{|c|}\hline \\ \hline a \\ \hline\end{array} = \mathbf{6} \oplus \bar{\mathbf{3}}. \tag{C.3}$$

The algorithm of the calculation can be summarized in different steps:

1. First, we fill all boxes in the first row of the right diagram with a, all boxes in the second row of the right diagram with b etc., until we have filled all rows of the second diagram in that way.
2. Second, we take all boxes filled with a and attach them to the first diagram in all possible ways to construct a valid Young diagram. After having attached the blocks containing the a's, we do the same procedure successively for the b's, the c's etc., until all boxes of the original right diagram are attached to the left diagram. This will result in a big number of different diagrams (two diagrams are considered equal only if they have the same shape and the same a's, b', etc. in the same boxes).
3. Third, there is one more rule which guarantees the avoidance of double counting. We will write a sequence of letters for each diagram in the following way. We start with the last row and write down all a's, b's etc. in the order they appear reading the row from left to right. We continue writing the sequence generated by the letters from the last row by attaching a sequence of letters generated from the above row in the same way. We do this for all rows ending with the first row. We finally have a string of letters for each diagram. We keep only those diagrams in which for an arbitrary position in the string the number of a's right to that position is bigger or equal than the number of b's right to this position. We require the same for the number of b's with respect to the number of c's and so on. We throw away all diagrams which do not satisfy this constraint. The resulting diagrams correspond to the decomposition into a direct sum of irreducible representations.

We finally have to estimate the dimensionality of the representation corresponding to a diagram. We do this again in several steps.

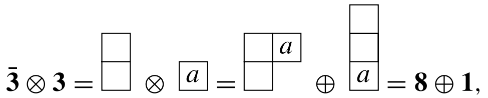
1. First, we duplicate the corresponding diagram.
2. In one diagram we fill the boxes according to the following rule: Begin with the box of the first row in the right corner. Insert the dimension of the Group (in case of SU(3), a 3). Fill the remaining boxes in the same row by the value of the foregoing box and add 1. Fill the first box of the second row with the value of the box above it and subtract one. Fill the remaining boxes in the same row according to the same rule like for the first row. Proceed in this way until you have filled all boxes. Calculate the product of the numbers contained in all boxes of the diagram. Take the result as the value of the numerator of the number for the dimensionality.
3. Fill the copy of the diagram accordingly to the following rule: For each box count the number of boxes to the right and the number of boxes below and add one. Fill this value into the box. Fill all boxes in this way and calculate again the product of the numbers of all boxes. The result is the denominator of the dimensionality.

4. The fraction of these two numbers gives the dimensionality of the representation corresponding to each diagram in the direct sum.

The dimensionality for the simple example above can be calculated by

$$\frac{\begin{array}{|c|c|}\hline 3 & 4 \\ \hline \end{array}}{\begin{array}{|c|c|}\hline 2 & 1 \\ \hline \end{array}} = \frac{3 \cdot 4}{2 \cdot 1} = \frac{12}{2} = 6. \tag{C.4}$$

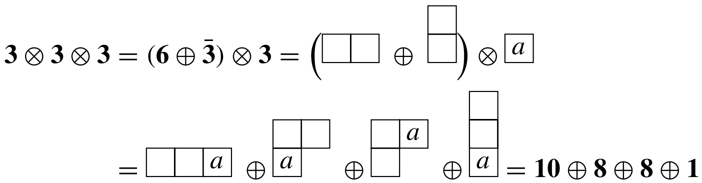
Now we calculate the product between the fundamental representation and its conjugated. It is easier to reverse the order of multiplication because the right-hand side has fewer boxes, reducing the amount of calculations.


(C.5)

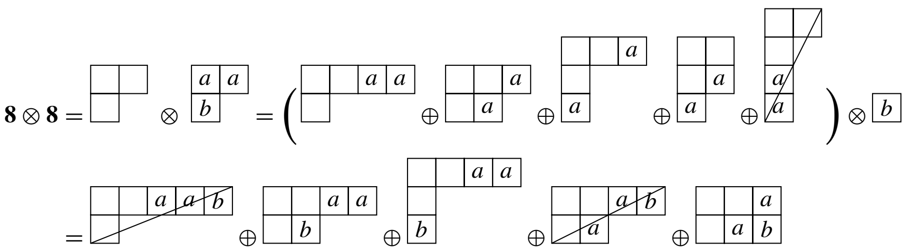
where we have calculated the dimensionality by

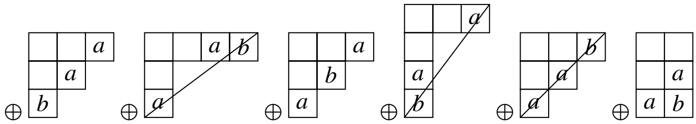
$$\frac{\begin{array}{|c|c|}\hline 3 & 4 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array}}{\begin{array}{|c|c|}\hline 3 & 1 \\ \hline 1 & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array}} = \frac{4 \cdot 3 \cdot 2}{3 \cdot 1 \cdot 1} = 8, \qquad \frac{\begin{array}{|c|}\hline 3 \\ \hline 2 \\ \hline 1 \\ \hline \end{array}}{\begin{array}{|c|}\hline 3 \\ \hline 2 \\ \hline 1 \\ \hline \end{array}} = 1. \tag{C.6}$$

The product of three quarks can be calculated by


(C.7)

Finally, we have to figure out how the product of two gluons transforms. Since the gluon itself transforms in the adjoint representation **8** we have to calculate





$$= \mathbf{27} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1} \tag{C.8}$$

We have calculated all relevant products of representations of $SU(3)$ needed for the application in the Standard Model.

Appendix D
Synge's World Function

The notation "world function" dates back to the Irish relativist Synge [7] and describes a non-local extension $\sigma(x,\, x')$ of the space-time metric $g_{\mu\nu}(x)$. It is defined by connecting the two space-time points $\{x = z(s_1),\, x' = z(s_0)\} \in \mathcal{M}, \quad s \in \mathbb{R}$ by the unique geodesic $\gamma \equiv z(s)$ parametrized through s

$$\sigma(x,\, x') := \frac{1}{2}(s_1 - s_0) \int_{s_0}^{s_1} g_{\mu\nu}(z(s))\, t^\mu t^\nu \, \mathrm{d}s. \tag{D.1}$$

The restriction to unique geodesics γ excludes conjugate points on γ. This is always satisfied for sufficient small distances between x and x' and it is necessary in order to define the derivative, the normalized tangential vector

$$t^\mu := \frac{\mathrm{d}z^\mu(s)}{\mathrm{d}s} \in \mathrm{T}\mathcal{M}. \tag{D.2}$$

The world function is a *bi-scalar* that transforms *independently* at x and x' as a scalar. The tangential vectors are parallelly transported along the geodesic γ

$$\frac{\mathrm{D}t^\mu}{\mathrm{D}s} := \frac{\mathrm{d}^2 z^\mu(s)}{\mathrm{d}s^2} + \Gamma^\mu_{\alpha\beta} \frac{\mathrm{d}z^\alpha(s)}{\mathrm{d}s} \frac{\mathrm{d}z^\beta(s)}{\mathrm{d}s} = 0. \tag{D.3}$$

Thus, the scalar product and in particular the norm are conserved on γ

$$g_{\mu\nu}(z)\, t^\mu t^\nu := N = \text{const.} \tag{D.4}$$

This allows us to write the world function with $\Delta s := (s_1 - s_0)$ as

$$\sigma(x,\, x') = \frac{1}{2}(s_1 - s_0)^2\, g_{\mu\nu}(z)\, t^\mu t^\nu = \frac{N}{2}\,(\Delta s)^2. \tag{D.5}$$

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The sign of N fixes the causal nature of the geodesic, i.e. $N = (-1, +1, 0)$ corresponds to γ =(time-like, space-like, light-like). From (D.5) it is clear that $\sigma(x, x') = \sigma(x', x)$ is symmetric under the exchange $x \leftrightarrow x'$ For Minkowski space-time ($g_{\mu\nu} = \eta_{\mu\nu}$), geodesics are straight lines and (D.1) reduces to

$$\sigma(x,\, x') = \frac{1}{2}\eta_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu). \tag{D.6}$$

Without loss of generality we can fix one point, say $x' = 0$ and regard $\sigma(x,\, 0)$ as an ordinary scalar function at x. The light cone structure emerges from

$$\sigma(x, 0) = -\frac{(x^0)^2}{(\sqrt{2})^2} + \frac{(x^1)^2}{(\sqrt{2})^2} + \frac{(x^2)^2}{(\sqrt{2})^2} + \frac{(x^3)^2}{(\sqrt{2})^2} = 0. \tag{D.7}$$

D.1 Derivatives of the World Function

By repeated differentiation of the world function, we can generate *bi-tensors* of different rank at x and x'. The covariant derivative of $\sigma(x,\, x')$ at the point x is defined as $\nabla_\alpha\sigma := \sigma_{;\,\alpha} = \sigma_{,\,\alpha}$. Correspondingly, the derivative of σ at the point x' is defined as $\sigma_{;\,\alpha'}$. The world function and its derivatives transform independently at x and x'. For example, this means $\sigma_{;\,\alpha'\beta'\gamma}$ transforms like a tensor of second rank at x' and like a covariant vector at x. This implies that the order of primed and unprimed indices can be changed arbitrarily for any general bi-tensor $A_{\dots;\,\beta'\alpha} = A_{\dots;\,\alpha\,\beta'}$. In particular, we can freely permute primed and unprimed derivatives of σ as long as the order among the primed and the order among the unprimed indices remain maintained, e.g.

$$\sigma_{;\,\alpha'\beta'\gamma} = \sigma_{;\,\alpha'\gamma\beta'} = \sigma_{;\,\gamma\alpha'\beta'} \neq \sigma_{;\,\beta'\alpha'\gamma}. \tag{D.8}$$

For an explicit expression of the first derivative of σ at x, we fix the point x' and vary x, i.e. $\delta z(s_0) := \delta x' = 0$ and $\delta z(s_1) := \delta x$, see [4]

$$\delta\sigma = \sigma(x + \delta x,\, x') - \sigma(x,\, x') = \Delta s\, g_{\mu\nu}\, t^\mu\, \delta x^\nu. \tag{D.9}$$

Comparison with the result of the chain rule $\delta\sigma = \sigma_{,\,\mu}\delta x^\mu$ yields

$$\sigma_{;\,\mu} = \Delta s\; (g_{\mu\nu}\, t^\nu)|_{s=s_1} \quad \text{or} \quad \sigma^{;\,\mu} = \Delta s\; t^\mu, \tag{D.10}$$

which shows that $\sigma_{;\,\mu}$ is proportional to the tangential vector t_μ with length of the geodesic distance between x and x' and pointing in direction $x' \to x$. The derivative of σ at x' can be obtained in an analogue manner

$$\sigma_{;\,\mu'} = -\Delta s\; (g_{\mu'\nu'}\, t^{\nu'})|_{s=s_0} \quad \text{or} \quad \sigma^{;\,\mu'} = -\Delta s\; t^{\mu'}. \tag{D.11}$$

It has the same properties as (D.10) but points in the opposite direction $x \to x'$. With (D.10) we can derive an important identity for the norm of $\sigma_{;\,\mu}$

$$g^{\mu\nu}\,\sigma_{,\mu}\sigma_{;\,\nu} = g_{\mu\nu}\,(\Delta s)^2\,t^\mu t^\nu = (\Delta s)^2\,N = 2\sigma \quad \text{or} \quad \sigma = \frac{1}{2}\sigma_{;\,\mu}\sigma^{;\,\mu}. \tag{D.12}$$

D.2 Densities and Determinants

For any ordinary tensor $A^{b_1\ldots b_n}_{a_1\ldots a_n}$ a tensor density $\mathcal{A}^{b_1\ldots b_n}_{a_1\ldots a_n}$ of weight w is defined by its transformation behaviour under general coordinate transformations $x^\mu \to \tilde{x}^\mu$

$$\tilde{\mathcal{A}}^{a_1\ldots a_n}_{b_1\ldots b_n} = \det\left(\frac{\partial x_c}{\partial \tilde{x}^d}\right)^w \frac{\partial \tilde{x}^{a_1}}{\partial x^{a'_1}}\cdots\frac{\partial \tilde{x}^{a_n}}{\partial x^{a'_n}}\frac{\partial x^{b'_1}}{\partial \tilde{x}^{b_1}}\cdots\frac{\partial x^{b'_n}}{\partial \tilde{x}^{b_n}}\,\mathcal{A}^{a'_1\ldots a'_n}_{b'_1\ldots b'_n}, \tag{D.13}$$

where $g = \det(g_{\mu\nu}(x)) = \frac{\partial(\tilde{x}_\mu)}{\partial(x^\nu)}$ is the Jacobian that takes care of a coordinate transformation from x^μ to $\tilde{x}^\nu$. Since we deal with bi-tensors, we can generalize this for the transformation of the two variables (x^μ, x'^ρ) that characterize the geodesic γ by its endpoints to the variables $(\sigma_{;\,\nu'}, x'_\sigma)$ that characterize γ by one endpoint and the tangent at this point. This Jacobian is given by

$$\left|\frac{\partial\,(\sigma_{;\,\nu'},\, x'_\sigma)}{\partial\,(x^\mu,\, x'^\rho)}\right| = \left|\frac{\partial\,(\sigma_{;\,\nu'})}{\partial\,(x^\mu)}\right| = \det(\sigma_{;\,\mu\nu'}) = \det(-D_{\mu\nu'}). \tag{D.14}$$

Following [2], we have defined the matrix $D_{\mu\nu'} := -\sigma_{;\,\mu\nu'}$ and its determinant as $D(x,\,x') := -\det(D_{\mu\nu'})$. The convention of the minus sign takes care of the Lorentzian signature of space-time. The square root of this determinant can be thought of as the generalize bi-volume factor $D^{1/2}(x,\,x')$ replacing $g^{1/2}(x)$. Differentiating (D.12) twice with respect to x^μ and x'^ν, we obtain the relation

$$D_{\mu\nu'} = \sigma_{;\ \ \mu}^{\ \rho}\,D_{\rho\,\nu'} + \sigma_{;}^{\ \rho}\,D_{\mu\nu';\,\rho}. \tag{D.15}$$

Multiplication with $(D^{-1})^{\mu\nu'}$, and using Jacobi's formula yields

$$D^{-1}\,(D\,\sigma_{;}^{\ \rho})_{;\,\rho} = d. \tag{D.16}$$

with $d = 4$ in four space-time dimensions. Finally, we introduce the Van-Fleck determinate $\Delta(x,\,x')$ by

$$\Delta := g^{-1/2}\,D\,g'^{-1/2}. \tag{D.17}$$

Since $(g^{1/2})_{;\,\mu} = 0$, we obtain relation (D.16) in terms of Δ

$$\Delta^{-1}\,(\Delta\,\sigma_{;}^{\ \rho})_{;\,\rho} = d. \tag{D.18}$$

D.3 The Parallel Propagator

We further need the geometrical concept of parallel transport. Following [4], we define the parallel propagator as the tensor product of two tetrads. We introduce an orthonormal tetrad $e_I^\mu(z)$ which satisfies

$$g_{\mu\nu}\, e_I^\mu\, e_J^\nu = \eta_{IJ}, \tag{D.19}$$

with $(I, J, ... = 0, 1, 2, 3)$ being Lorentz-indices of the frame with the Minkowski metric $\eta_{IJ} = \mathrm{diag}(-1, 1, 1, 1)$. We define the dual triad

$$e_\mu^I := \eta^{IJ}\, g_{\mu\nu}\, e_J^\nu \tag{D.20}$$

and via the completeness relation

$$g^{\mu\nu} = \eta^{IJ}\, e_I^\mu e_J^\nu \quad \text{it follows} \quad e_I^\mu e_\nu^I = \delta_\nu^\mu \quad \text{and} \quad e_I^\mu e_\mu^J = \delta_I^J. \tag{D.21}$$

The tetrad basis $e_I^\mu(z)$ and its dual $e_\mu^I(z)$ are parallelly transported along $\gamma = z(s)$

$$\frac{\mathrm{D}e_I^\mu}{\mathrm{D}s} = 0 \quad \text{and} \quad \frac{\mathrm{D}e_\mu^I}{\mathrm{D}s} = 0. \tag{D.22}$$

We can expand each contravariant vector field $v^\nu(z)$ into this orthonormal basis

$$v^\mu = v^I e_I^\mu, \tag{D.23}$$

whereas the Lorentz components of v can be expanded in the dual basis

$$v^I = v^\nu e_\nu^I. \tag{D.24}$$

If the vector v^μ is parallelly transported along γ, then the frame coefficients v^I are constant since e_I^μ is also parallelly transported along γ. Since the v^I are constant, we can express them as $v^I = v^{\nu'} e_{\nu'}^I$ where the vector $v^{\nu'}$ and the tetrad $e_{\nu'}^I$ are evaluated at x'. Putting all this together, v^μ at x can be expressed by

$$v^\mu(x) = (v^{\nu'}\, e_{\nu'}^I)\, e_I^\mu \quad \text{or} \quad v^\mu = g^\mu{}_{\mu'}\, v^{\mu'}, \tag{D.25}$$

with $g^\mu{}_{\mu'}$ being the parallel propagator $g^\mu{}_{\mu'} := e_I^\mu\, e_{\mu'}^I$. It is a bi-vector that transports the vector $v^{\mu'}$ from the point $x' = z(s_1)$ to the vector v^μ at the point $x = z(s_0)$ along the unique geodesic $\gamma = z(s)$. The inverse parallel transporter from x to x' is given by $g^{\mu'}{}_\mu = e_I^{\mu'}\, e_\mu^I$. It is clear that the parallel transporter $g_{\mu\nu'}$ satisfies the boundary condition

$$\lim_{x' \to x} g^\mu{}_{\nu'} = \delta_\nu^\mu \quad \text{or} \quad \lim_{x' \to x} g_{\mu\nu'} = g_{\mu\nu}. \tag{D.26}$$

The parallel transport easily generalizes to tensors of arbitrary rank $T^{\mu\dots\nu}_{\rho\dots\sigma}$. Since a tensor is a multilinear form we need one (inverse) parallel propagator for each (covariant) contravariant index to transport it along γ from x' to x.

$$T^{\mu\dots\nu}_{\rho\dots\sigma} = g^{\mu}{}_{\mu'} \cdots g^{\nu}{}_{\nu'}\, g^{\rho'}{}_{\rho} \cdots g^{\sigma'}{}_{\sigma}\; T^{\mu'\dots\nu'}_{\rho'\dots\sigma'}\,. \tag{D.27}$$

Since the tetrads e^{μ}_{I} are parallelly propagated along γ this means

$$\frac{\mathrm{D}e^{\mu}_{I}(z(s))}{\mathrm{D}s} = e^{\mu}_{I\,;\,\nu}\frac{\mathrm{d}z^{\nu}(s)}{\mathrm{d}s} = \Delta s\; e^{\mu}_{I\,;\,\nu}\,\sigma^{\nu} = 0. \tag{D.28}$$

From (D.28) it follows $e^{\mu}_{I\,;\,\nu}\,\sigma_{;}{}^{\nu} = 0$ and $e^{I}_{\mu'\,;\,\nu}\,\sigma_{;}{}^{\nu} = 0$. This, in turn, implies

$$g^{\mu}{}_{\mu'\,;\,\nu}\,\sigma_{;}{}^{\nu} = 0 \quad \text{and} \quad g^{\mu}{}_{\mu'\,;\,\nu'}\,\sigma_{;}{}^{\nu'} = 0. \tag{D.29}$$

Since $\sigma_{;\,\mu} \propto -t_{\mu}$, it is automatically parallelly transported along γ and thus

$$\sigma_{;\,\mu} = -g^{\mu'}{}_{\mu}\,\sigma_{;\,\mu'} \quad \text{and} \quad \sigma_{;\,\mu'} = -g^{\mu}{}_{\mu'}\,\sigma_{;\,\mu}. \tag{D.30}$$

From the determinants of the tetrad and its dual

$$\det(e^{\mu}_{I}) = g^{-1/2} \quad \text{and} \quad \det(e^{I}_{\mu}) = g^{1/2}, \tag{D.31}$$

we find for the determinant of the parallel propagator

$$\begin{aligned}\det(-g_{\mu\nu'}) &= \det(-g_{\mu\nu}g^{\nu}{}_{\nu'}) = \det(-g_{\mu\nu}e^{\nu}_{I}e^{I}_{\nu'}) \\ &= \det(-g_{\mu\nu})\det(e^{\nu}_{I})\det(e^{I}_{\nu'}) = g^{1/2}g'^{1/2}.\end{aligned} \tag{D.32}$$

D.4 Field Curvature

We assume that we have a set of fields collected in the generalized field $\phi^{A}(x)$. Each component of the generalized field consists of some fundamental field present in the action of the theory together forming the field space $\mathcal{C}$. If we consider $\mathcal{C}$ as a manifold, $\phi^{A}(x)$ corresponds to a coordinate of a point in this manifold. The fields collected in $\phi^{A}(x)$ do have their own symmetries. In order to define invariant differentiation, we need to introduce a covariant derivative. We introduce an abbreviation by suppressing the capital index and write instead $\phi := \phi^{A}$ and a hat for operators $\hat{O} := O_{A}{}^{B}$. The law of covariant differentiation is not commutative but defines the field (bundle) curvature $\hat{\mathcal{R}}_{\mu\nu}$

$$[\hat{\nabla}_{\mu},\ \hat{\nabla}_{\nu}]\,\phi =: \hat{\mathcal{R}}_{\mu\nu}\,\phi. \tag{D.33}$$

To clarify the role of ϕ, we will consider two examples. We assume $\phi = \phi^A \equiv q^a$ consists of a single field q^a with an internal gauge symmetry described by the Lie group G and the group index a. Covariant differentiation of q^a means

$$\hat{\nabla}_\mu q^a \equiv \partial_\mu q^a + A_\mu q^a \tag{D.34}$$

with the gauge vector field $A_\mu := A^a_\mu G_a$ and the generators G_a of the group G. The commutator of covariant derivatives then yields

$$[\hat{\nabla}_\mu, \hat{\nabla}_\nu]\,\phi \equiv (A^a_{\mu,\,\nu} - A^a_{\nu,\,\mu} + c^a_{\;bc}\, A^b_\mu\, A^c_\nu)\,\phi_a = F^a_{\mu\nu}\,\phi_a \tag{D.35}$$

with the field strength tensor

$$\hat{\mathcal{R}}_{\mu\nu} \equiv F^a_{\mu\nu} = A^a_{\mu,\,\nu} - A^a_{\nu,\,\mu} + c^a_{bc}\, A^b_\mu\, A^c_\nu \tag{D.36}$$

and the structure constants c^a_{bc} of the group G. If the field ϕ instead only consists of a symmetric tensor field $\phi \equiv q_{\mu\nu} = q_{\nu\mu}$ with ordinary space-time indices, the covariant derivative is defined with respect to the usual Christoffel symbol $\hat{\nabla}_\mu q_{\alpha\beta} \equiv \nabla_\mu q_{\alpha\beta} = \partial_\mu q_{\alpha\beta} - \Gamma^{\gamma}_{\;\alpha\mu} q_{\gamma\beta} - \Gamma^{\gamma}_{\;\beta\mu} q_{\alpha\gamma}$ and we obtain

$$[\hat{\nabla}_\mu, \hat{\nabla}_\nu]\,\phi \equiv (\nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu)\,q_{\alpha\beta} = -R^{\gamma}_{\;\alpha\mu\nu} q_{\gamma\beta} - R^{\gamma}_{\;\beta\mu\nu} q_{\alpha\gamma} \tag{D.37}$$

where we have used $u_{\alpha;\,\beta\gamma} - u_{\alpha;\,\gamma\beta} = R^{\delta}_{\;\alpha\beta\gamma} u_\delta$ and have written $q_{\alpha\beta}$ without loss of generality as the tensor product $q_{\alpha\beta} := u_\alpha \otimes v_\beta$. This can also be written as

$$[\hat{\nabla}_\mu, \hat{\nabla}_\nu]\,\phi \equiv (\nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu)\,q_{\alpha\beta} = -2\,\delta^{(\gamma}_{(\alpha}\, R^{\delta)}_{\;\beta)\,\mu\nu}\, q_{\gamma\delta} \tag{D.38}$$

and shows that in this case the field curvature is given by the operator

$$\hat{\mathcal{R}}_{\mu\nu} \equiv -2\,\delta^{(\gamma}_{(\alpha}\, R^{\delta)}_{\;\beta)\,\mu\nu}, \tag{D.39}$$

acting on $q_{\gamma\delta}$. In general, ϕ is a collection of many different fields. We can therefore define a parallel displacement bi-matrix $\hat{I}(x,\, x')$, which parallelly transports $\phi(x)$ to $\phi(x')$ along the geodesic γ. It is a matrix in field space and from the general properties of a parallel propagator, it follows as in (D.29) and (D.26) that $\hat{I}(x,\, x')$ must satisfy the relations

$$\sigma^{;\,\mu}\, \hat{I}_{;\,\mu} = 0 \quad , \quad \sigma^{;\,\mu'}\, \hat{I}_{;\,\mu'} = 0 \quad \text{and} \quad \lim_{x \to x'} \hat{I}(x,\, x') = \delta^A_B(x) := \hat{\mathbf{1}}, \tag{D.40}$$

where $\hat{\mathbf{1}}$ is the unit matrix at the point x.

D.5 Coincidence Limits

Since $\sigma(x,\ x')$ is a bi-scalar and derivatives of σ are bi-tensors, we are interested in the value of these objects in the coincidence limit $x \to x'$. For any bi-tensor $A^{\nu\cdots}_{\mu\ldots}(x,\ x')$ we define the limit $x \to x'$ by the symbolic bracket-notation

$$[\, A^{\cdots}_{\cdots}(x,\ x')\,] := \lim_{x' \to x} A^{\cdots}_{\cdots}(x,\ x'). \tag{D.41}$$

The result of this limit is an ordinary tensor at x'. There is a relation between coincidence limits of primed and unprimed indices, denoted Synge's rule [4]

$$[A_{\ldots\alpha'}] = [A_{\ldots}]_{;\,\alpha'} - [A_{\ldots\alpha}]. \tag{D.42}$$

By repeated differentiation of (D.12) we have a recursive algorithm to systematically calculate the coincidence limit of higher derivatives $[\sigma_{;\,\alpha_1,\ldots,\alpha_N}]$.

$$\sigma = \frac{1}{2}\sigma^{;\,\lambda}\,\sigma_{,\lambda}, \tag{D.43}$$

$$\sigma_{;\,\alpha} = \sigma_{;}{}^{\lambda}{}_{\alpha}\,\sigma_{\lambda}, \tag{D.44}$$

$$\sigma_{;\,\alpha\beta} = \sigma_{;}{}^{\lambda}{}_{\alpha\beta}\,\sigma_{;\,\lambda} + \sigma_{;}{}^{\lambda}{}_{\alpha}\,\sigma_{;\,\lambda\beta}, \tag{D.45}$$

$$\sigma_{;\,\alpha\beta\gamma} = \sigma_{;}{}^{\lambda}{}_{\alpha\beta\gamma}\,\sigma_{;\,\lambda} + \sigma_{;}{}^{\lambda}{}_{\alpha\beta}\,\sigma_{;\,\lambda\gamma} + \sigma_{;}{}^{\lambda}{}_{\alpha\gamma}\,\sigma_{;\,\lambda\beta} + \sigma_{;}{}^{\lambda}{}_{\alpha}\,\sigma_{;\,\lambda\beta\gamma}, \tag{D.46}$$

$$\begin{aligned}\sigma_{;\,\alpha\beta\gamma\delta} &= \sigma_{;}{}^{\lambda}{}_{\alpha\beta\gamma\delta}\,\sigma_{;\,\lambda} + \sigma_{;}{}^{\lambda}{}_{\alpha\beta\gamma}\,\sigma_{;\,\lambda\delta} + \sigma_{;}{}^{\lambda}{}_{\alpha\beta\delta}\,\sigma_{;\,\lambda\gamma} + \sigma_{;}{}^{\lambda}{}_{\alpha\beta}\,\sigma_{;\,\lambda\gamma\delta},\\ &\quad + \sigma_{;}{}^{\lambda}{}_{\alpha\gamma\delta}\,\sigma_{;\,\lambda\beta} + \sigma_{;}{}^{\lambda}{}_{\alpha\gamma}\,\sigma_{;\,\lambda\beta\delta} + \sigma_{;}{}^{\lambda}{}_{\alpha\delta}\,\sigma_{;\,\lambda\beta\gamma} + \sigma_{;}{}^{\lambda}{}_{\alpha}\,\sigma_{;\,\lambda\beta\gamma\delta}\\ &\quad\vdots\end{aligned} \tag{D.47}$$

From (D.5) and (D.10), it follows directly that $[\sigma] = 0$ and $[\sigma_{;\,\mu}] = 0$. From (D.45) it is clear that $[\sigma_{;\,\mu\nu}] = g_{\mu\nu}$. The remaining coincidence limits for (D.46) and (D.47) are obtained recursively. By commuting covariant derivatives we generate curvature expressions. Using the symmetries of the Riemann tensor, we obtain

$$[\,\sigma_{;\,\alpha\beta\gamma}\,] = 0 \quad \text{and} \quad [\,\sigma_{;\,\alpha\beta\gamma\delta}\,] = \frac{1}{3}\left(R_{\alpha\gamma\beta\delta} + R_{\alpha\delta\beta\gamma}\right). \tag{D.48}$$

For further application we will also need two additional coincidence limits. Contracting indices in (D.47) and differentiating twice, we obtain after some algebra

$$\left[\sigma_{;}{}^{\mu}{}_{\mu}{}^{\nu}{}_{\nu\alpha}\right] = R_{;\,\alpha}, \tag{D.49}$$

$$\left[\sigma_{;}{}^{\mu}{}_{\mu}{}^{\nu}{}_{\nu}{}^{\alpha}{}_{\alpha}\right] = \frac{8}{5}\,R_{;}{}^{\alpha}{}_{\alpha} + \frac{4}{15}\,R^{\alpha\beta}R_{\alpha\beta} - \frac{4}{15}\,R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}. \tag{D.50}$$

We are ultimately interested in the coincidence limit of the coefficient matrix $\hat{a}_2$. As shown in Chap. 4, $\hat{a}_2$ can be calculated by the recursion relation (4.109). The coincidence limits can then be again obtained recursively. In order to calculate $[\,\hat{a}_1\,]$ and $[\,\hat{a}_2\,]$, we must calculate several further coincidence limits. It is easy to see that

$$[\,D_{\mu\nu'}\,] = g_{\mu\nu}, \quad [\,D\,] = g \quad \text{and} \quad [\,\Delta\,] = 1. \tag{D.51}$$

For the coincidence limits of higher derivatives of the field parallel propagator matrix $\hat{I}$, we differentiate (D.40) and use (D.33) in order to obtain

$$[\,\hat{I}_{;\,\mu}\,] = 0, \quad [\,\hat{I}_{;\,\mu\nu}\,] = -\frac{1}{2}\,\hat{\mathcal{R}}_{\mu\nu}, \tag{D.52}$$

$$[\,\hat{I}_{;\,\mu\ \nu}^{\ \ \mu}\,] = \frac{1}{3}\,\hat{\mathcal{R}}_{\mu\nu;}{}^{\mu}, \quad [\,\hat{I}_{;\,\mu\ \nu}^{\ \ \mu\ \nu}\,] = \frac{1}{2}\,\hat{\mathcal{R}}_{\mu\nu}\hat{\mathcal{R}}^{\mu\nu}. \tag{D.53}$$

Proceeding in a similar manner, we differentiate (D.18) in order to obtain recursively the coincidence limits of the derivatives of the Van-Fleck determinant

$$[\,\Delta^{1/2}_{;\,\mu}\,] = 0, \quad [\,\Delta^{1/2}_{;\,\mu\nu}\,] = -\frac{1}{6}\,R_{\mu\nu}, \tag{D.54}$$

$$[\,\Delta^{-1/2}_{;\,\mu\nu}\,] = \frac{1}{6}\,R_{\mu\nu}, \quad [\,\Delta^{1/2}{}_{;\,\mu\ \nu}^{\ \ \mu}\,] = -\frac{1}{6}\,R_{;\,\nu} \tag{D.55}$$

$$[\,\Delta^{1/2}{}_{;\,\mu\ \nu}^{\ \ \mu\ \nu}\,] = -\frac{1}{5}\,R_{;\ \alpha}^{\ \alpha} + \frac{1}{36}\,R^2 - \frac{1}{30}\,R^{\alpha\beta}R_{\alpha\beta} + \frac{1}{30}\,R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}. \tag{D.56}$$

Appendix E
Corrections to the Initial Values of λ and y_t

The pole mass matching scheme used to correct the initial values for λ and y_{t} was developed in [6, 8]. We have used these results at the one-loop level. The correction function for the Higgs mass is

$$\Delta_{\mathrm{H}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \frac{M_{\mathrm{Z}}^2}{16\,\pi^2} \left[\zeta\, f_1(\zeta) + f_0(\zeta) + \zeta^{-1}\, f_{-1}(\zeta)\right] \tag{E.1}$$

with

$$f_1(\zeta) = 6 \ln \frac{M_{\mathrm{t}}^2}{M_{\mathrm{H}}^2} + \frac{3}{2} \ln \zeta - \frac{1}{2} Z\left[\frac{1}{\zeta}\right] - Z\left[\frac{c_{\mathrm{W}}^2}{\zeta}\right] - \ln c_{\mathrm{W}}^2 + \frac{9}{2}\left(\frac{25}{9} - \frac{\pi}{\sqrt{3}}\right), \tag{E.2}$$

$$\begin{aligned} f_0(\zeta) = &- 6 \ln \frac{M_{\mathrm{t}}^2}{M_{\mathrm{Z}}^2}\left(1 + 2\,c_{\mathrm{W}}^2 - 2\,\frac{M_{\mathrm{t}}^2}{M_{\mathrm{Z}}^2}\right) + \frac{3\,c_{\mathrm{W}}^2\,\zeta}{\zeta - c_{\mathrm{W}}^2} \ln \frac{\zeta}{c_{\mathrm{W}}^2} + 2\,Z\left[\frac{1}{\zeta}\right] \\ &+ 4 c_{\mathrm{W}}^2\, Z\left[\frac{c_{\mathrm{W}}^2}{\zeta}\right] + \left(\frac{3\,c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2} + 12\,c_{\mathrm{W}}^2\right) \ln c_{\mathrm{W}}^2 - \frac{15}{2}\left(1 + 2\,c_{\mathrm{W}}^2\right) \\ &- 3\,\frac{M_{\mathrm{t}}^2}{M_{\mathrm{Z}}^2}\left(2\,Z\left[\frac{M_{\mathrm{t}}^2}{M_{\mathrm{Z}}^2\,\zeta}\right] + 4 \ln \frac{M_{\mathrm{t}}^2}{M_{\mathrm{Z}}^2} - 5\right), \end{aligned} \tag{E.3}$$

$$\begin{aligned} f_{-1}(\zeta) = &\, 6 \ln \frac{M_{\mathrm{t}}^2}{M_{\mathrm{Z}}^2}\left(1 + 2\,c_{\mathrm{W}}^2 - 4\,\frac{M_{\mathrm{t}}^2}{M_{\mathrm{Z}}^2}\right) - 6\,Z\left[\frac{1}{\zeta}\right] - 12\,c_{\mathrm{W}}^4\, Z\left[\frac{c_{\mathrm{W}}^2}{\zeta}\right] - 12\,c_{\mathrm{W}}^4 \ln c_{\mathrm{W}}^2 \\ &+ 8\left(1 + 2\,c_{\mathrm{W}}^4\right) + 24\,\frac{M_{\mathrm{t}}^4}{M_{\mathrm{Z}}^4}\left(\ln \frac{M_{\mathrm{t}}^2}{M_{\mathrm{Z}}^2} - 2 + Z\left[\frac{M_{\mathrm{t}}^2}{M_{\mathrm{Z}}^2\,\zeta}\right]\right), \end{aligned} \tag{E.4}$$

the Fermi constant G_{F} and $\zeta := M_{\mathrm{H}}^2/M_{\mathrm{Z}}^2$, $s_{\mathrm{W}}^2 := \sin^2\theta_{\mathrm{W}}$, $c_{\mathrm{W}}^2 := \cos^2\theta_{\mathrm{W}}$. The function $Z[z]$ is defined with respect to its domains depending on z

$$Z[z] := \begin{cases} 2\,A\,\arctan\frac{1}{A}, & (z > 1/4), \\ A\,\ln\frac{1+A}{1-A}, & (z < 1/4), \end{cases} \qquad A := \sqrt{|1 - 4\,z|}. \tag{E.5}$$

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The correction for the top-quark mass is $\Delta_t(M_H)$ and reads

$$\Delta_t(M_H) = \delta_t^{QCD} + \delta_t^{W} + \delta_t^{QED}, \quad \delta_t^{QCD} = -\frac{4\,\alpha_s(M_t)}{3\pi}, \quad \delta_t^{QED} + \delta_t^{W} = -\frac{4\,\alpha(M_t)}{9\,\pi}. \tag{E.6}$$

We have neglected two-loop corrections. The numerical values of α and α_s can be obtained from [1].

Appendix F
Transfer Equations

Not all structures appearing in the divergent part of the one-loop effective action are independent. Neglecting surface terms and making use of the Bianchi identities, we can convert certain structures into others via an integration by parts. In this way, we can reduce the number of different structures in (5.102) to a minimum. The "transfer equations" below describe explicitly how the contributions of the dependent structures are distributed among the minimal set of independent structures:

$$F\,\Phi^{a\ \ \mu}_{;\,\mu}\,n_a \to -F'\,\Gamma_4 - \frac{F}{\varphi}\,(\Gamma_3 - \Gamma_4), \tag{F.1}$$

$$\begin{aligned} F\,\Phi^{a}_{;\,\mu\nu}\Phi_a^{;\,\mu\nu} \to\ & F'\,\Gamma_{21} - F(\Gamma_7 - \Gamma_{17}) + \frac{F'}{\varphi}\,\Gamma_{15} \\ & - \frac{1}{2}\left(\frac{F'}{\varphi} - F''\right)\Gamma_{14} + \frac{1}{2}F'\,\Gamma_{19} - \frac{1}{2}\frac{F'}{\varphi}\,\Gamma_{12}, \end{aligned} \tag{F.2}$$

$$\begin{aligned} F\,\Phi^{a}_{;\,\mu\nu}\,n_a\Phi^{b;\,\mu\nu}\,n_b \to\ & \left(4\frac{F}{\varphi^2} - \frac{5}{2}\frac{F'}{\varphi} + \frac{1}{2}F''\right)\Gamma_{13} + \left(2\,\frac{F'}{\varphi} - 4\,\frac{F}{\varphi^2}\right)\Gamma_{16} \\ & + \left(\frac{3}{2}F' - 3\,\frac{F}{\varphi}\right)\Gamma_{20} + \left(\frac{1}{2}\frac{F'}{\varphi} - \frac{F}{\varphi^2}\right)\Gamma_{14} \\ & + 2\,\frac{F}{\varphi}\,\Gamma_{21} - F\Gamma_8 + \frac{F}{\varphi}\,\Gamma_{19} + F\,\Gamma_{18} + \frac{F}{\varphi^2}\,\Gamma_{15}, \end{aligned} \tag{F.3}$$

$$\begin{aligned} F\,\Phi^{a}_{;\,\mu\nu}\,n_a\,\Phi^{b,\,\mu}\,\Phi_b^{;\,\nu} \to\ & -\frac{1}{2}\left(\frac{F}{\varphi} - F'\right)\Gamma_{14} + \left(\frac{F}{\varphi} - F'\right)\Gamma_{16} \\ & - F\,\Gamma_{21} + \frac{1}{2}F\,\Gamma_{19} + \frac{1}{2}\frac{F}{\varphi}\Gamma_{12} - \frac{F}{\varphi}\Gamma_{15}, \end{aligned} \tag{F.4}$$

$$F\,\Phi^{a}_{;\,\mu\nu}\,\Phi_a^{;\,\mu}\,\Phi_b^{;\,\nu}\,n^b \to \frac{1}{2}\left(\frac{F}{\varphi} - F'\right)\Gamma_{14} - \frac{F}{2}\,\Gamma_{19} - \frac{1}{2}\frac{F}{\varphi}\,\Gamma_{12}, \tag{F.5}$$

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$$F\,\Phi^a_{;\,\mu\nu}\,n_a\,\Phi_b^{;\,\mu}\,n^b\,\Phi_c^{;\,\nu}\,n^c \to \frac{1}{2}\left(3\,\frac{F}{\varphi}-F'\right)\Gamma_{13}-\frac{F}{\varphi}\,\Gamma_{16} - \frac{1}{2}\,F\,\Gamma_{20}-\frac{1}{2}\,\frac{F}{\varphi}\,\Gamma_{14}, \tag{F.6}$$

$$F\,R^{\mu\nu}\,\Phi^a_{;\,\mu\nu}\,n_a \to \left(\frac{F}{\varphi}-F'\right)\Gamma_8-\frac{1}{2}\left(\frac{F}{\varphi}-F'\right)\Gamma_{10} - \frac{F}{\varphi}\,\Gamma_7+\frac{1}{2}\,F\,\Gamma_{11}+\frac{1}{2}\,\frac{F}{\varphi}\,\Gamma_9, \tag{F.7}$$

$$F\,R^{\mu\nu\rho\sigma}\,R_{\mu\nu\rho\sigma} \to 4\,F\,\Gamma_5-F\,\Gamma_6. \tag{F.8}$$

The last equation is a topological invariant, the Gauss–Bonnet identity. In the single field limit, cf. Table 5.1, the other seven transfer equations reduce to the three reduction formulas given in [5]. In principle, even more scalar invariants composed of different scalar contractions between derivatives ∇_μ and field variables $(g_{\mu\nu},\ \Phi^a)$, containing up to four derivatives, can appear at the one-loop level, but they do not occur in our calculations.

Appendix G
Gradient Structures

As already explained in Sect. 4.5.2, the coefficients α_i, $i = 12, ..., 21$ do all correspond to gradient structures, symbolically denoted as $\partial^4\Phi\Phi\Phi\Phi$. In the cosmological model of Higgs inflation, discussed in Chap. 6, these structures are additionally suppressed compared to (5.108)–(5.118) and thus less important from the viewpoint of an effective field theory. For the sake of completeness we list their coefficients in a closed form.

The coefficient α_{12} belonging to the structure $(\Phi^a_{,\mu}\Phi_a^{;\mu})^2$:

$$\begin{aligned}
\alpha_{12} = s^2\Bigg(& \frac{25\,(U')^8}{48G^2\varphi^4U^4} + \frac{3\,(U')^8}{16U^6} - \frac{(U')^7}{4G\varphi^3U^4} - \frac{3\,(U')^7}{8\varphi U^5} + \frac{5G'\,(U')^6}{4G^2\varphi^3U^3} - \frac{11\,(U')^6}{72G\varphi^4U^3} \\
& - \frac{9\,(U')^6}{8\varphi^2U^4} - \frac{13G'\,(U')^5}{12G\varphi^2U^3} - \frac{G'\,(U')^5}{2U^4} - \frac{3U''\,(U')^5}{4G\varphi^3U^3} + \frac{35\,(U')^5}{12\varphi^3U^3} \\
& + \frac{13\,(G')^2\,(U')^4}{24G^2\varphi^2U^2} - \frac{G'\,(U')^4}{G\varphi^3U^2} + \frac{15G'\,(U')^4}{8\varphi U^3} + \frac{3G''\,(U')^4}{8U^3} - \frac{695\,(U')^4}{432\varphi^4U^2} \\
& + \frac{(G')^2\,(U')^3}{8G\varphi U^2} - \frac{47G'\,(U')^3}{18\varphi^2U^2} - \frac{3G''\,(U')^3}{4\varphi U^2} + \frac{3G'U''\,(U')^3}{4G\varphi^2U^2} - \frac{U''\,(U')^3}{4\varphi^3U^2} \\
& - \frac{(G')^3\,(U')^2}{4G^2\varphi U} + \frac{139\,(G')^2\,(U')^2}{72G\varphi^2U} + \frac{(G')^2\,(U')^2}{8U^2} + \frac{19G'\,(U')^2}{36\varphi^3U} \\
& + \frac{3G'G''\,(U')^2}{4G\varphi U} + \frac{(G')^3\,U'}{6GU} - \frac{(G')^2\,U'}{12\varphi U} - \frac{G'G''U'}{4U} + \frac{G'U''U'}{4\varphi^2U} \\
& + \frac{(G')^4}{48G^2} - \frac{7\,(G')^3}{24G\varphi} + \frac{11\,(G')^2}{24\varphi^2} + \frac{(G'')^2}{8} - \frac{(G')^2\,G''}{8G} + \frac{G'G''}{2\varphi}\Bigg) \\
& + s\Bigg(- \frac{25\,(U')^6}{72G^2\varphi^4U^3} - \frac{(U')^6}{8U^5} + \frac{(U')^5}{6G\varphi^3U^3} - \frac{37\,(U')^5}{8\varphi U^4} + \frac{5G'\,(U')^4}{12G^2\varphi^3U^2} \\
& - \frac{43\,(U')^4}{216G\varphi^4U^2} + \frac{21\,(U')^4}{4\varphi^2U^3} + \frac{13G'\,(U')^3}{36G\varphi^2U^2} + \frac{11G'\,(U')^3}{12U^3} + \frac{U''\,(U')^3}{2G\varphi^3U^2}
\end{aligned}$$

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$$+\frac{3U''(U')^3}{2\varphi U^3}-\frac{107(U')^3}{36\varphi^3U^2}+\frac{77(G')^2(U')^2}{72G^2\varphi^2U}+\frac{15G'(U')^2}{4G\varphi^3U}-\frac{15G'(U')^2}{8\varphi U^2}$$
$$-\frac{G''(U')^2}{8U^2}-\frac{3U''(U')^2}{\varphi^2U^2}-\frac{(U')^2}{12\varphi^4U}+\frac{5(G')^2U'}{6G\varphi U}+\frac{13G'U'}{6\varphi^2U}-\frac{G'U''U'}{4G\varphi^2U}$$
$$+\frac{U''U'}{12\varphi^3U}-\frac{(G')^3}{6G^2\varphi}-\frac{13(G')^2}{12G\varphi^2}-\frac{G'G''}{4G\varphi}+\frac{G'U''}{2\varphi U}\Bigg)+\frac{25(U')^4}{432G^2\varphi^4U^2}$$
$$+\frac{(U')^4}{48U^4}+\frac{59(U')^3}{36G\varphi^3U^2}+\frac{19(U')^3}{12\varphi U^3}-\frac{G'(U')^2}{9G^2\varphi^3U}+\frac{5(U')^2}{12G\varphi^4U}+\frac{17(U')^2}{3\varphi^2U^2}$$
$$-\frac{4G'U'}{3G\varphi^2U}-\frac{G'U'}{4U^2}-\frac{5U''U'}{12G\varphi^3U}-\frac{U''U'}{2\varphi U^2}-\frac{5U'}{3\varphi^3U}+\frac{13GU'}{6\varphi U^2}$$
$$+\frac{(N-1)(G')^2}{2G^2\varphi^2}-\frac{5(G')^2}{12G^2\varphi^2}-\frac{7G'}{6\varphi U}+\frac{5U''}{3\varphi^2U}-\frac{G^2}{4U^2} \tag{G.1}$$

The coefficient α_{13} belonging to the structure $(\Phi^a_{\ ,\mu}n_a\Phi^{b\,,\,\mu}n_b)^2$:

$$\alpha_{13}=s^4\Bigg(\frac{81(U')^{12}}{32U^8}-\frac{81U''(U')^{10}}{4U^7}-\frac{27G'(U')^9}{8U^6}+\frac{243(U'')^2(U')^8}{4U^6}$$
$$+\frac{81G'U''(U')^7}{4U^5}-\frac{81(U'')^3(U')^6}{U^5}+\frac{27(G')^2(U')^6}{16U^4}-\frac{81G'(U'')^2(U')^5}{2U^4}$$
$$+\frac{81(U'')^4(U')^4}{2U^4}-\frac{27(G')^2U''(U')^4}{4U^3}-\frac{3(G')^3(U')^3}{8U^2}$$
$$+\frac{27G'(U'')^3(U')^3}{U^3}+\frac{27(G')^2(U'')^2(U')^2}{4U^2}+\frac{3(G')^3U''U'}{4U}+\frac{(G')^4}{32}\Bigg)$$
$$+s^3\Bigg(-\frac{3(U')^{10}}{2G\varphi^2U^6}+\frac{45(U')^{10}}{8U^7}-\frac{81(U')^9}{8\varphi U^6}-\frac{9G'(U')^8}{4G\varphi U^5}+\frac{19U''(U')^8}{4G\varphi^2U^5}$$
$$+\frac{27U''(U')^8}{8U^6}+\frac{25(U')^8}{4\varphi^2U^5}+\frac{G'(U')^7}{G\varphi^2U^4}-\frac{15G'(U')^7}{4U^5}+\frac{81U''(U')^7}{2\varphi U^5}$$
$$+\frac{5(G')^2(U')^6}{8GU^4}+\frac{155(U'')^2(U')^6}{8G\varphi^2U^4}-\frac{351(U'')^2(U')^6}{4U^5}+\frac{39G'(U')^6}{8\varphi U^4}$$
$$+\frac{9G''(U')^6}{8U^4}+\frac{3G'U''(U')^6}{G\varphi U^4}-\frac{305U''(U')^6}{12\varphi^2U^4}+\frac{3(G')^2(U')^5}{2G\varphi U^3}$$
$$-\frac{81(U'')^2(U')^5}{2\varphi U^4}-\frac{25G'(U')^5}{6\varphi^2U^3}+\frac{15G'U''(U')^5}{4G\varphi^2U^3}-\frac{39G'U''(U')^5}{4U^4}$$
$$+\frac{261(U'')^3(U')^4}{2U^4}+\frac{29(G')^2(U')^4}{24U^3}+\frac{93G'(U'')^2(U')^4}{4G\varphi U^3}+\frac{803(U'')^2(U')^4}{24\varphi^2U^3}$$
$$+\frac{(G')^2U''(U')^4}{GU^3}-\frac{8G'U''(U')^4}{\varphi U^3}-\frac{9G''U''(U')^4}{2U^3}+\frac{(G')^2(U')^3}{8\varphi U^2}$$
$$+\frac{39G'(U'')^2(U')^3}{U^3}-\frac{3G'G''(U')^3}{4U^2}+\frac{9(G')^2U''(U')^3}{G\varphi U^2}+\frac{41G'U''(U')^3}{4\varphi^2U^2}$$
$$-\frac{27(U'')^4(U')^2}{U^3}+\frac{3(G')^3(U')^2}{4G\varphi U}+\frac{3(G')^2(U')^2}{4\varphi^2U}-\frac{31(G')^2(U'')^2(U')^2}{8GU^2}$$

$$+\frac{13G'\left(U''\right)^2\left(U'\right)^2}{4\varphi U^2}+\frac{9G''\left(U''\right)^2\left(U'\right)^2}{2U^2}+\frac{41\left(G'\right)^2U''\left(U'\right)^2}{24U^2}-\frac{\left(G'\right)^3U'}{4U}$$
$$-\frac{9G'\left(U''\right)^3U'}{U^2}-\frac{3\left(G'\right)^3U''U'}{2GU}+\frac{3\left(G'\right)^2U''U'}{2\varphi U}+\frac{3G'G''U''U'}{2U}-\frac{\left(G'\right)^4}{8G}+\frac{\left(G'\right)^3}{8\varphi}$$
$$-\frac{13\left(G'\right)^2\left(U''\right)^2}{24U}+\frac{1}{8}\left(G'\right)^2G''\Bigg)+s^2\Bigg(\frac{25\left(U'\right)^8}{16G^2\varphi^4U^4}-\frac{19\left(U'\right)^8}{6G\varphi^2U^5}-\frac{189\left(U'\right)^8}{16U^6}$$
$$-\frac{5\left(U'\right)^7}{4G\varphi^3U^4}-\frac{123\left(U'\right)^7}{8\varphi U^5}+\frac{15G'\left(U'\right)^6}{4G^2\varphi^3U^3}+\frac{G'\left(U'\right)^6}{4G\varphi U^4}-\frac{U''\left(U'\right)^6}{6G\varphi^2U^4}+\frac{273U''\left(U'\right)^6}{8U^5}$$
$$+\frac{133\left(U'\right)^6}{24G\varphi^4U^3}-\frac{223\left(U'\right)^6}{72\varphi^2U^4}-\frac{25G'\left(U'\right)^5}{24G\varphi^2U^3}+\frac{85G'\left(U'\right)^5}{8U^4}-\frac{5G''\left(U'\right)^5}{4G\varphi U^3}-\frac{15G'U''\left(U'\right)^5}{4G^2\varphi^2U^3}$$
$$-\frac{33U''\left(U'\right)^5}{4G\varphi^3U^3}+\frac{87U''\left(U'\right)^5}{4\varphi U^4}+\frac{3U'''\left(U'\right)^5}{4G\varphi^2U^3}-\frac{3U'''\left(U'\right)^5}{2U^4}-\frac{113\left(U'\right)^5}{12\varphi^3U^3}+\frac{\left(G'\right)^2\left(U'\right)^4}{G^2\varphi^2U^2}$$
$$-\frac{4\left(G'\right)^2\left(U'\right)^4}{3GU^3}-\frac{55\left(U''\right)^2\left(U'\right)^4}{6G\varphi^2U^3}-\frac{81\left(U''\right)^2\left(U'\right)^4}{8U^4}+\frac{5G'\left(U'\right)^4}{2G\varphi^3U^2}+\frac{167G'\left(U'\right)^4}{24\varphi U^3}$$
$$-\frac{7G''\left(U'\right)^4}{8U^3}+\frac{45G'U''\left(U'\right)^4}{4G\varphi U^3}+\frac{881U''\left(U'\right)^4}{36\varphi^2U^3}+\frac{889\left(U'\right)^4}{144\varphi^4U^2}-\frac{7\left(G'\right)^2\left(U'\right)^3}{8G\varphi U^2}$$
$$+\frac{9\left(U''\right)^2\left(U'\right)^3}{2\varphi U^3}+\frac{325G'\left(U'\right)^3}{72\varphi^2U^2}-\frac{8G''\left(U'\right)^3}{3\varphi U^2}-\frac{9\left(G'\right)^2U''\left(U'\right)^3}{2G^2\varphi U^2}-\frac{31G'U''\left(U'\right)^3}{2G\varphi^2U^2}$$
$$-\frac{25G'U''\left(U'\right)^3}{U^3}-\frac{13G''U''\left(U'\right)^3}{4G\varphi U^2}-\frac{11U''\left(U'\right)^3}{4\varphi^3U^2}-\frac{3G'U'''\left(U'\right)^3}{4G\varphi U^2}-\frac{6U''U'''\left(U'\right)^3}{U^3}$$
$$+\frac{U'''\left(U'\right)^3}{4\varphi^2U^2}-\frac{3\left(G'\right)^3\left(U'\right)^2}{2G^2\varphi U}-\frac{39\left(U''\right)^3\left(U'\right)^2}{2U^3}-\frac{8\left(G'\right)^2\left(U'\right)^2}{3G\varphi^2U}-\frac{19\left(G'\right)^2\left(U'\right)^2}{8U^2}$$
$$-\frac{43G'\left(U''\right)^2\left(U'\right)^2}{4G\varphi U^2}-\frac{713\left(U''\right)^2\left(U'\right)^2}{72\varphi^2U^2}-\frac{31G'\left(U'\right)^2}{12\varphi^3U}-\frac{125\left(G'\right)^2U''\left(U'\right)^2}{24GU^2}$$
$$-\frac{2G'U''\left(U'\right)^2}{3\varphi U^2}+\frac{7G''U''\left(U'\right)^2}{4U^2}-\frac{G'U'''\left(U'\right)^2}{2U^2}-\frac{\left(G'\right)^3U'}{8GU}-\frac{23\left(G'\right)^2U'}{12\varphi U}$$
$$+\frac{17G'\left(U''\right)^2U'}{2U^2}-\frac{G'G''U'}{4U}+\frac{3\left(G'\right)^3U''U'}{4G^2U}-\frac{4G'U''U'}{3\varphi^2U}+\frac{3G'G''U''U'}{2GU}$$
$$+\frac{23G''U''U'}{12\varphi U}-\frac{G'U'''U'}{4\varphi U}+\frac{3\left(G'\right)^4}{16G^2}+\frac{9\left(U''\right)^4}{2U^2}+\frac{\left(G'\right)^3}{8G\varphi}+\frac{5\left(G'\right)^2}{8\varphi^2}+\frac{\left(G''\right)^2}{8}$$
$$+\frac{13\left(G'\right)^2\left(U''\right)^2}{24GU}-\frac{5G'\left(U''\right)^2}{2\varphi U}-\frac{3G''\left(U''\right)^2}{2U}+\frac{\left(G'\right)^2G''}{8G}+\frac{3G'G''}{4\varphi}+\frac{\left(G'\right)^2U''}{2U}\Bigg)$$
$$+s\Bigg(-\frac{25\left(U'\right)^6}{24G^2\varphi^4U^3}-\frac{35\left(U'\right)^6}{9G\varphi^2U^4}+\frac{61\left(U'\right)^6}{8U^5}+\frac{7G'\left(U'\right)^5}{8G^2\varphi^2U^3}-\frac{37\left(U'\right)^5}{6G\varphi^3U^3}+\frac{189\left(U'\right)^5}{8\varphi U^4}$$
$$+\frac{5\left(G'\right)^2\left(U'\right)^4}{8G^3\varphi^2U^2}-\frac{3G'\left(U'\right)^4}{8G^2\varphi^3U^2}-\frac{11G'\left(U'\right)^4}{12G\varphi U^3}-\frac{5G''\left(U'\right)^4}{8G^2\varphi^2U^2}+\frac{16U''\left(U'\right)^4}{9G\varphi^2U^3}-\frac{227U''\left(U'\right)^4}{8U^4}$$
$$-\frac{79\left(U'\right)^4}{72G\varphi^4U^2}+\frac{521\left(U'\right)^4}{27\varphi^2U^3}-\frac{5\left(G'\right)^2\left(U'\right)^3}{2G^2\varphi U^2}-\frac{56G'\left(U'\right)^3}{9G\varphi^2U^2}-\frac{13G'\left(U'\right)^3}{2U^3}-\frac{19G''\left(U'\right)^3}{12G\varphi U^2}$$
$$+\frac{G'U''\left(U'\right)^3}{G^2\varphi^2U^2}+\frac{3U''\left(U'\right)^3}{2G\varphi^3U^2}-\frac{95U''\left(U'\right)^3}{2\varphi U^3}-\frac{U'''\left(U'\right)^3}{2G\varphi^2U^2}+\frac{3U'''\left(U'\right)^3}{U^3}+\frac{101\left(U'\right)^3}{36\varphi^3U^2}$$
$$+\frac{3\left(G'\right)^3\left(U'\right)^2}{4G^3\varphi U}+\frac{17\left(G'\right)^2\left(U'\right)^2}{8G^2\varphi^2U}+\frac{\left(G'\right)^2\left(U'\right)^2}{4GU^2}+\frac{101\left(U''\right)^2\left(U'\right)^2}{72G\varphi^2U^2}+\frac{53\left(U''\right)^2\left(U'\right)^2}{2U^3}$$

$$
\begin{aligned}
&+\frac{11G'\left(U'\right)^2}{8G\varphi^3U}-\frac{433G'\left(U'\right)^2}{36\varphi U^2}-\frac{41G''\left(U'\right)^2}{24G\varphi^2U}+\frac{G''\left(U'\right)^2}{U^2}-\frac{17G'U''\left(U'\right)^2}{6G\varphi U^2}-\frac{21U''\left(U'\right)^2}{4\varphi^2U^2}\\
&+\frac{3U'''\left(U'\right)^2}{\varphi U^2}+\frac{\left(U'\right)^2}{4\varphi^4U}+\frac{3\left(G'\right)^3U'}{8G^2U}+\frac{17\left(G'\right)^2U'}{12G\varphi U}+\frac{3\left(U''\right)^2U'}{2\varphi U^2}-\frac{17G'U'}{12\varphi^2U}+\frac{G'G''U'}{4GU}\\
&-\frac{G''U'}{2\varphi U}+\frac{3\left(G'\right)^2U''U'}{2G^2\varphi U}+\frac{17G'U''U'}{4G\varphi^2U}+\frac{27G'U''U'}{2U^2}+\frac{13G''U''U'}{12G\varphi U}-\frac{5U''U'}{12\varphi^3U}\\
&+\frac{G'U'''U'}{4G\varphi U}-\frac{U''U'''U'}{U^2}-\frac{U'''U'}{12\varphi^2U}-\frac{\left(G'\right)^4}{8G^3}-\frac{\left(G'\right)^3}{8G^2\varphi}-\frac{\left(U''\right)^3}{2U^2}+\frac{23\left(G'\right)^2}{24U}-\frac{\left(G'\right)^2}{4G\varphi^2}\\
&-\frac{\left(G''\right)^2}{4G}+\frac{G'\left(U''\right)^2}{G\varphi U}+\frac{\left(U''\right)^2}{6\varphi^2U}-\frac{\left(G'\right)^2G''}{8G^2}-\frac{G'G''}{2G\varphi}+\frac{\left(G'\right)^2U''}{2GU}+\frac{25G'U''}{12\varphi U}-\frac{G'U'''}{U}\Bigg)\\
&+\frac{25\left(U'\right)^4}{144G^2\varphi^4U^2}+\frac{46\left(U'\right)^4}{27G\varphi^2U^3}+\frac{85\left(U'\right)^4}{8U^4}-\frac{\left(G'\right)^3}{4G^3\varphi}-\frac{7G'\left(U'\right)^3}{24G^2\varphi^2U^2}+\frac{133\left(U'\right)^3}{36G\varphi^3U^2}+\frac{3\left(U'\right)^3}{4\varphi U^3}\\
&-\frac{23\left(G'\right)^2}{24GU}-\frac{\left(G'\right)^2}{2G^2\varphi^2}-\frac{5\left(G'\right)^2\left(U'\right)^2}{24G^3\varphi^2U}-\frac{7G'\left(U'\right)^2}{24G^2\varphi^3U}-\frac{89G'\left(U'\right)^2}{36G\varphi U^2}-\frac{\left(U'\right)^2}{4G\varphi^4U}+\frac{3\left(U'\right)^2}{2\varphi^2U^2}\\
&-\frac{\left(U''\right)^2}{6G\varphi^2U}+\frac{9\left(U''\right)^2}{2U^2}-\frac{\left(G'\right)^2U'}{2G^2\varphi U}-\frac{19G'U'}{12G\varphi^2U}+\frac{5\left(U'\right)^2G''}{24G^2\varphi^2U}+\frac{U'G''}{2G\varphi U}-\frac{9\left(U'\right)^2U''}{4G\varphi^2U^2}\\
&-\frac{3\left(U'\right)^2U''}{4U^3}+\frac{11G'U''}{12G\varphi U}+\frac{G'U'U''}{12G^2\varphi^2U}+\frac{5U'U''}{12G\varphi^3U}-\frac{6U'U''}{\varphi U^2}+\frac{U'U'''}{12G\varphi^2U}-\frac{U'U'''}{U^2}\\
&+\frac{(N-1)\left(G'\right)^2}{8G^2\varphi^2}+\frac{(N-1)\left(G'\right)^3}{8G^3\varphi}+\frac{(N-1)\left(G'\right)^4}{32G^4}-\frac{(N-1)G'G''}{4G^2\varphi}\\
&-\frac{(N-1)\left(G'\right)^2G''}{8G^3}+\frac{(N-1)\left(G''\right)^2}{8G^2}
\end{aligned}
\tag{G.2}
$$

The coefficient α_{14} belonging to the structure $(\Phi^a_{\ ,\mu}\Phi_a^{;\mu})\,(\Phi^c_{\ ,\nu}n_c\Phi^{d\,,\nu}n_d)$:

$$
\begin{aligned}
\alpha_{14}=s^3\Bigg(&\frac{\left(U'\right)^{10}}{2G\varphi^2U^6}-\frac{9\left(U'\right)^{10}}{8U^7}+\frac{9\left(U'\right)^9}{8\varphi U^6}+\frac{3G'\left(U'\right)^8}{4G\varphi U^5}-\frac{19U''\left(U'\right)^8}{12G\varphi^2U^5}+\frac{9U''\left(U'\right)^8}{2U^6}\\
&+\frac{29\left(U'\right)^8}{12\varphi^2U^5}-\frac{G'\left(U'\right)^7}{3G\varphi^2U^4}+\frac{9G'\left(U'\right)^7}{4U^5}-\frac{9U''\left(U'\right)^7}{2\varphi U^5}-\frac{5\left(G'\right)^2\left(U'\right)^6}{24GU^4}\\
&-\frac{155\left(U''\right)^2\left(U'\right)^6}{24G\varphi^2U^4}-\frac{9\left(U''\right)^2\left(U'\right)^6}{2U^5}-\frac{19G'\left(U'\right)^6}{8\varphi U^4}-\frac{9G''\left(U'\right)^6}{8U^4}-\frac{G'U''\left(U'\right)^6}{G\varphi U^4}\\
&-\frac{343U''\left(U'\right)^6}{36\varphi^2U^4}-\frac{\left(G'\right)^2\left(U'\right)^5}{2G\varphi U^3}+\frac{9\left(U''\right)^2\left(U'\right)^5}{2\varphi U^4}-\frac{29G'\left(U'\right)^5}{18\varphi^2U^3}-\frac{5G'U''\left(U'\right)^5}{4G\varphi^2U^3}\\
&-\frac{15G'U''\left(U'\right)^5}{2U^4}-\frac{95\left(G'\right)^2\left(U'\right)^4}{72U^3}-\frac{31G'\left(U''\right)^2\left(U'\right)^4}{4G\varphi U^3}+\frac{493\left(U''\right)^2\left(U'\right)^4}{72\varphi^2U^3}\\
&-\frac{\left(G'\right)^2U''\left(U'\right)^4}{3GU^3}+\frac{26G'U''\left(U'\right)^4}{3\varphi U^3}+\frac{9G''U''\left(U'\right)^4}{2U^3}+\frac{29\left(G'\right)^2\left(U'\right)^3}{24\varphi U^2}\\
&+\frac{6G'\left(U''\right)^2\left(U'\right)^3}{U^3}+\frac{3G'G''\left(U'\right)^3}{4U^2}-\frac{3\left(G'\right)^2U''\left(U'\right)^3}{G\varphi U^2}+\frac{31G'U''\left(U'\right)^3}{12\varphi^2U^2}\\
&-\frac{\left(G'\right)^3\left(U'\right)^2}{4G\varphi U}+\frac{\left(G'\right)^2\left(U'\right)^2}{4\varphi^2U}+\frac{31\left(G'\right)^2\left(U''\right)^2\left(U'\right)^2}{24GU^2}-\frac{121G'\left(U''\right)^2\left(U'\right)^2}{12\varphi U^2}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{9G''\left(U''\right)^2\left(U'\right)^2}{2U^2}+\frac{43\left(G'\right)^2U''\left(U'\right)^2}{18U^2}+\frac{\left(G'\right)^3U'}{4U}+\frac{\left(G'\right)^3U''U'}{2GU}\\
&-\frac{7\left(G'\right)^2U''U'}{2\varphi U}-\frac{3G'G''U''U'}{2U}+\frac{\left(G'\right)^4}{24G}-\frac{7\left(G'\right)^3}{24\varphi}-\frac{5\left(G'\right)^2\left(U''\right)^2}{72U}\\
&-\frac{1}{8}\left(G'\right)^2G''\Bigg)+s^2\Bigg(-\frac{25\left(U'\right)^8}{24G^2\varphi^4U^4}-\frac{13\left(U'\right)^8}{9G\varphi^2U^5}-\frac{15\left(U'\right)^8}{2U^6}+\frac{\left(U'\right)^7}{2G\varphi^3U^4}\\
&+\frac{39\left(U'\right)^7}{4\varphi U^5}-\frac{5G'\left(U'\right)^6}{2G^2\varphi^3U^3}+\frac{7G'\left(U'\right)^6}{12G\varphi U^4}-\frac{7U''\left(U'\right)^6}{9G\varphi^2U^4}+\frac{141U''\left(U'\right)^6}{8U^5}\\
&+\frac{11\left(U'\right)^6}{36G\varphi^4U^3}-\frac{185\left(U'\right)^6}{54\varphi^2U^4}+\frac{127G'\left(U'\right)^5}{72G\varphi^2U^3}+\frac{11G'\left(U'\right)^5}{8U^4}-\frac{5G''\left(U'\right)^5}{12G\varphi U^3}\\
&+\frac{5G'U''\left(U'\right)^5}{4G^2\varphi^2U^3}+\frac{3U''\left(U'\right)^5}{2G\varphi^3U^3}-\frac{111U''\left(U'\right)^5}{4\varphi U^4}-\frac{3U'''\left(U'\right)^5}{4G\varphi^2U^3}-\frac{35\left(U'\right)^5}{6\varphi^3U^3}\\
&-\frac{7\left(G'\right)^2\left(U'\right)^4}{8G^2\varphi^2U^2}+\frac{17\left(G'\right)^2\left(U'\right)^4}{18GU^3}+\frac{55\left(U''\right)^2\left(U'\right)^4}{18G\varphi^2U^3}-\frac{27\left(U''\right)^2\left(U'\right)^4}{4U^4}\\
&+\frac{2G'\left(U'\right)^4}{G\varphi^3U^2}-\frac{47G'\left(U'\right)^4}{9\varphi U^3}+\frac{G''\left(U'\right)^4}{U^3}-\frac{131G'U''\left(U'\right)^4}{12G\varphi U^3}+\frac{1249U''\left(U'\right)^4}{108\varphi^2U^3}\\
&+\frac{695\left(U'\right)^4}{216\varphi^4U^2}-\frac{\left(G'\right)^2\left(U'\right)^3}{4G\varphi U^2}+\frac{39\left(U''\right)^2\left(U'\right)^3}{2\varphi U^3}+\frac{1307G'\left(U'\right)^3}{216\varphi^2U^2}+\frac{31G''\left(U'\right)^3}{36\varphi U^2}\\
&+\frac{3\left(G'\right)^2U''\left(U'\right)^3}{2G^2\varphi U^2}-\frac{11G'U''\left(U'\right)^3}{12G\varphi^2U^2}+\frac{29G'U''\left(U'\right)^3}{4U^3}-\frac{13G''U''\left(U'\right)^3}{12G\varphi U^2}\\
&+\frac{U''\left(U'\right)^3}{2\varphi^3U^2}+\frac{3G'U'''\left(U'\right)^3}{4G\varphi U^2}-\frac{U'''\left(U'\right)^3}{4\varphi^2U^2}+\frac{3\left(G'\right)^3\left(U'\right)^2}{4G^2\varphi U}-\frac{97\left(G'\right)^2\left(U'\right)^2}{24G\varphi^2U}\\
&+\frac{2\left(G'\right)^2\left(U'\right)^2}{3U^2}+\frac{25G'\left(U''\right)^2\left(U'\right)^2}{4G\varphi U^2}-\frac{583\left(U''\right)^2\left(U'\right)^2}{216\varphi^2U^2}-\frac{19G'\left(U'\right)^2}{18\varphi^3U}-\frac{7G'G''\left(U'\right)^2}{4G\varphi U}\\
&+\frac{143\left(G'\right)^2U''\left(U'\right)^2}{72GU^2}-\frac{6G'U''\left(U'\right)^2}{\varphi U^2}-\frac{17G''U''\left(U'\right)^2}{4U^2}-\frac{11\left(G'\right)^3U'}{24GU}-\frac{2\left(G'\right)^2U'}{9\varphi U}\\
&-\frac{13G'\left(U''\right)^2U'}{2U^2}+\frac{G'G''U'}{2U}-\frac{\left(G'\right)^3U''U'}{4G^2U}+\frac{2\left(G'\right)^2U''U'}{G\varphi U}+\frac{5G'U''U'}{12\varphi^2U}+\frac{G'G''U''U'}{2GU}\\
&+\frac{23G''U''U'}{36\varphi U}+\frac{G'U'''U'}{4\varphi U}-\frac{\left(G'\right)^4}{12G^2}+\frac{5\left(G'\right)^3}{6G\varphi}-\frac{3\left(G'\right)^2}{4\varphi^2}-\frac{\left(G''\right)^2}{4}-\frac{49\left(G'\right)^2\left(U''\right)^2}{72GU}\\
&+\frac{49G'\left(U''\right)^2}{18\varphi U}+\frac{3G''\left(U''\right)^2}{2U}+\frac{\left(G'\right)^2G''}{3G}-\frac{11G'G''}{12\varphi}-\frac{2\left(G'\right)^2U''}{3U}\Bigg)+s\Bigg(\frac{25\left(U'\right)^6}{36G^2\varphi^4U^3}\\
&+\frac{43\left(U'\right)^6}{54G\varphi^2U^4}+\frac{12\left(U'\right)^6}{U^5}+\frac{5G'\left(U'\right)^5}{24G^2\varphi^2U^3}-\frac{\left(U'\right)^5}{3G\varphi^3U^3}-\frac{7\left(U'\right)^5}{\varphi U^4}-\frac{5\left(G'\right)^2\left(U'\right)^4}{24G^3\varphi^2U^2}\\
&-\frac{5G'\left(U'\right)^4}{24G^2\varphi^3U^2}-\frac{47G'\left(U'\right)^4}{36G\varphi U^3}+\frac{5G''\left(U'\right)^4}{8G^2\varphi^2U^2}+\frac{79U''\left(U'\right)^4}{54G\varphi^2U^3}-\frac{157U''\left(U'\right)^4}{8U^4}+\frac{43\left(U'\right)^4}{108G\varphi^4U^2}\\
&-\frac{3281\left(U'\right)^4}{324\varphi^2U^3}-\frac{\left(G'\right)^2\left(U'\right)^3}{G^2\varphi U^2}+\frac{137G'\left(U'\right)^3}{108G\varphi^2U^2}-\frac{4G'\left(U'\right)^3}{3U^3}+\frac{41G''\left(U'\right)^3}{36G\varphi U^2}-\frac{5G'U''\left(U'\right)^3}{6G^2\varphi^2U^2}\\
&-\frac{U''\left(U'\right)^3}{G\varphi^3U^2}+\frac{23U''\left(U'\right)^3}{\varphi U^3}+\frac{U'''\left(U'\right)^3}{2G\varphi^2U^2}+\frac{3U'''\left(U'\right)^3}{2U^3}+\frac{107\left(U'\right)^3}{18\varphi^3U^2}-\frac{\left(G'\right)^3\left(U'\right)^2}{4G^3\varphi U}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{25\left(G'\right)^2\left(U'\right)^2}{9G^2\varphi^2U}-\frac{11\left(G'\right)^2\left(U'\right)^2}{12GU^2}-\frac{65\left(U''\right)^2\left(U'\right)^2}{216G\varphi^2U^2}+\frac{15\left(U''\right)^2\left(U'\right)^2}{4U^3}-\frac{139G'\left(U'\right)^2}{24G\varphi^3U}\\
&+\frac{1231G'\left(U'\right)^2}{216\varphi U^2}+\frac{G'G''\left(U'\right)^2}{2G^2\varphi U}+\frac{41G''\left(U'\right)^2}{24G\varphi^2U}-\frac{G''\left(U'\right)^2}{24U^2}+\frac{35G'U''\left(U'\right)^2}{6G\varphi U^2}\\
&+\frac{145U''\left(U'\right)^2}{108\varphi^2U^2}-\frac{3U'''\left(U'\right)^2}{\varphi U^2}+\frac{\left(U'\right)^2}{6\varphi^4U}-\frac{\left(G'\right)^3U'}{8G^2U}-\frac{79\left(G'\right)^2U'}{36G\varphi U}-\frac{11\left(U''\right)^2U'}{2\varphi U^2}\\
&-\frac{41G'U'}{12\varphi^2U}+\frac{G'G''U'}{4GU}+\frac{7G''U'}{6\varphi U}-\frac{\left(G'\right)^2U''U'}{2G^2\varphi U}+\frac{4G'U''U'}{9G\varphi^2U}-\frac{2G'U''U'}{U^2}+\frac{13G''U''U'}{36G\varphi U}\\
&-\frac{U''U'}{6\varphi^3U}-\frac{G'U'''U'}{4G\varphi U}+\frac{U'''U'}{12\varphi^2U}+\frac{\left(G'\right)^4}{24G^3}+\frac{11\left(G'\right)^3}{24G^2\varphi}-\frac{19\left(G'\right)^2}{72U}+\frac{3\left(G'\right)^2}{2G\varphi^2}\\
&-\frac{\left(G''\right)^2}{12G}-\frac{11G'\left(U''\right)^2}{9G\varphi U}-\frac{\left(G'\right)^2G''}{24G^2}-\frac{G'G''}{12G\varphi}-\frac{\left(G'\right)^2U''}{2GU}+\frac{7G'U''}{36\varphi U}+\frac{2G''U''}{3U}\\
&+\left.\frac{G'U'''}{2U}\right)-\frac{25\left(U'\right)^4}{216G^2\varphi^4U^2}-\frac{10\left(U'\right)^4}{81G\varphi^2U^3}-\frac{25\left(U'\right)^4}{8U^4}-\frac{5G'\left(U'\right)^3}{72G^2\varphi^2U^2}-\frac{53\left(U'\right)^3}{18G\varphi^3U^2}\\
&-\frac{6\left(U'\right)^3}{\varphi U^3}+\frac{5\left(G'\right)^2\left(U'\right)^2}{72G^3\varphi^2U}+\frac{25G'\left(U'\right)^2}{72G^2\varphi^3U}+\frac{55G'\left(U'\right)^2}{108G\varphi U^2}-\frac{5G''\left(U'\right)^2}{24G^2\varphi^2U};+\frac{125U''\left(U'\right)^2}{108G\varphi^2U^2}\\
&+\frac{53U''\left(U'\right)^2}{12U^3}-\frac{\left(U'\right)^2}{6G\varphi^4U}-\frac{27\left(U'\right)^2}{2\varphi^2U^2}-\frac{3G\left(U'\right)^2}{U^3}+\frac{\left(G'\right)^2U'}{2G^2\varphi U}+\frac{35G'U'}{12G\varphi^2U}\\
&+\frac{15G'U'}{4U^2}-\frac{G''U'}{2G\varphi U}+\frac{5G'U''U'}{36G^2\varphi^2U}+\frac{U''U'}{6G\varphi^3U}+\frac{83U''U'}{6\varphi U^2}-\frac{U'''U'}{12G\varphi^2U}-\frac{U'''U'}{2U^2}\\
&-\frac{2GU'}{\varphi U^2}+\frac{\left(G'\right)^3}{12G^3\varphi}+\frac{55\left(G'\right)^2}{72GU}+\frac{3\left(G'\right)^2}{4G^2\varphi^2}-\frac{\left(U''\right)^2}{3U^2}+\frac{G'}{\varphi U}-\frac{G'G''}{6G^2\varphi}-\frac{G''}{U}\\
&-\frac{37G'U''}{36G\varphi U}+\frac{2GU''}{U^2}-\frac{(N-1)\left(G'\right)^2}{2G^2\varphi^2}-\frac{(N-1)\left(G'\right)^3}{4G^3\varphi}+\frac{(N-1)G'G''}{2G^2\varphi}
\end{aligned}
\tag{G.3}
$$

The coefficient α_{15} belonging to the structure $\Phi^a_{,\mu}\Phi_{a,\nu}\Phi^{b,\mu}\Phi_b^{;\nu}$:

$$
\begin{aligned}
\alpha_{15}=s^2&\left(\frac{25\left(U'\right)^8}{24G^2\varphi^4U^4}+\frac{3\left(U'\right)^8}{8U^6}-\frac{3\left(U'\right)^7}{\varphi U^5}+\frac{5G'\left(U'\right)^6}{2G^2\varphi^3U^3}+\frac{205\left(U'\right)^6}{36G\varphi^4U^3}+\frac{9\left(U'\right)^6}{\varphi^2U^4}\right.\\
&+\frac{5G'\left(U'\right)^5}{6G\varphi^2U^3}+\frac{G'\left(U'\right)^5}{2U^4}-\frac{12\left(U'\right)^5}{\varphi^3U^3}+\frac{13\left(G'\right)^2\left(U'\right)^4}{12G^2\varphi^2U^2}+\frac{6G'\left(U'\right)^4}{G\varphi^3U^2}-\frac{3G'\left(U'\right)^4}{\varphi U^3}\\
&+\frac{1681\left(U'\right)^4}{216\varphi^4U^2}+\frac{\left(G'\right)^2\left(U'\right)^3}{G\varphi U^2}+\frac{113G'\left(U'\right)^3}{18\varphi^2U^2}-\frac{\left(G'\right)^3\left(U'\right)^2}{2G^2\varphi U}-\frac{77\left(G'\right)^2\left(U'\right)^2}{36G\varphi^2U}\\
&+\left.\frac{\left(G'\right)^2\left(U'\right)^2}{4U^2}-\frac{41G'\left(U'\right)^2}{18\varphi^3U}-\frac{\left(G'\right)^3U'}{6GU}-\frac{2\left(G'\right)^2U'}{3\varphi U}+\frac{\left(G'\right)^4}{24G^2}+\frac{\left(G'\right)^3}{6G\varphi}+\frac{\left(G'\right)^2}{6\varphi^2}\right)\\
&+s\left(-\frac{25\left(U'\right)^6}{36G^2\varphi^4U^3}+\frac{2\left(U'\right)^6}{U^5}-\frac{5\left(U'\right)^5}{2G\varphi^3U^3}+\frac{\left(U'\right)^5}{2\varphi U^4}-\frac{5G'\left(U'\right)^4}{3G^2\varphi^3U^2}-\frac{205\left(U'\right)^4}{108G\varphi^4U^2}\right.\\
&+\frac{7\left(U'\right)^4}{\varphi^2U^3}-\frac{16G'\left(U'\right)^3}{9G\varphi^2U^2}-\frac{G'\left(U'\right)^3}{6U^3}+\frac{19\left(U'\right)^3}{6\varphi^3U^2}-\frac{31\left(G'\right)^2\left(U'\right)^2}{36G^2\varphi^2U}-\frac{2G'\left(U'\right)^2}{G\varphi^3U}\\
&-\left.\frac{5G'\left(U'\right)^2}{2\varphi U^2}-\frac{\left(G'\right)^2U'}{3G\varphi U}-\frac{11G'U'}{6\varphi^2U}+\frac{\left(G'\right)^3}{6G^2\varphi}+\frac{\left(G'\right)^2}{4U}+\frac{\left(G'\right)^2}{3G\varphi^2}\right)+\frac{25\left(U'\right)^4}{216G^2\varphi^4U^2}
\end{aligned}
$$

$$
-\frac{17\left(U'\right)^4}{24U^4}+\frac{2\left(U'\right)^3}{3G\varphi^3U^2}+\frac{\left(U'\right)^3}{6\varphi U^3}+\frac{G'\left(U'\right)^2}{9G^2\varphi^3U}-\frac{\left(U'\right)^2}{3G\varphi^4U}-\frac{5\left(U'\right)^2}{3\varphi^2U^2}+\frac{G\left(U'\right)^2}{4U^3}
$$
$$
+\frac{G'U'}{2G\varphi^2U}+\frac{U''U'}{3G\varphi^3U}+\frac{5U'}{3\varphi^3U}-\frac{GU'}{6\varphi U^2}+\frac{\left(G'\right)^2}{6G^2\varphi^2}+\frac{7G'}{6\varphi U}-\frac{5U''}{3\varphi^2U}+\frac{3G^2}{2U^2} \quad (G.4)
$$

The coefficient α_{16} belonging to the structure $\Phi^a_{,\mu}n_a\Phi^b_{,\nu}n_b\Phi^{c,\mu}\Phi_c^{;\nu}$:

$$
\begin{aligned}
\alpha_{16}=s^3\Bigg(&\frac{\left(U'\right)^{10}}{G\varphi^2U^6}-\frac{9\left(U'\right)^{10}}{4U^7}+\frac{9\left(U'\right)^9}{\varphi U^6}+\frac{3G'\left(U'\right)^8}{2G\varphi U^5}-\frac{19U''\left(U'\right)^8}{6G\varphi^2U^5}+\frac{9U''\left(U'\right)^8}{U^6}\\
&-\frac{26\left(U'\right)^8}{3\varphi^2U^5}-\frac{2G'\left(U'\right)^7}{3G\varphi^2U^4}-\frac{36U''\left(U'\right)^7}{\varphi U^5}-\frac{5\left(G'\right)^2\left(U'\right)^6}{12GU^4}-\frac{155\left(U''\right)^2\left(U'\right)^6}{12G\varphi^2U^4}\\
&-\frac{9\left(U''\right)^2\left(U'\right)^6}{U^5}-\frac{5G'\left(U'\right)^6}{2\varphi U^4}-\frac{2G'U''\left(U'\right)^6}{G\varphi U^4}+\frac{629U''\left(U'\right)^6}{18\varphi^2U^4}-\frac{\left(G'\right)^2\left(U'\right)^5}{G\varphi U^3}\\
&+\frac{36\left(U''\right)^2\left(U'\right)^5}{\varphi U^4}+\frac{52G'\left(U'\right)^5}{9\varphi^2U^3}-\frac{5G'U''\left(U'\right)^5}{2G\varphi^2U^3}+\frac{3G'U''\left(U'\right)^5}{U^4}+\frac{13\left(G'\right)^2\left(U'\right)^4}{36U^3}\\
&-\frac{31G'\left(U''\right)^2\left(U'\right)^4}{2G\varphi U^3}-\frac{1451\left(U''\right)^2\left(U'\right)^4}{36\varphi^2U^3}-\frac{2\left(G'\right)^2U''\left(U'\right)^4}{3GU^3}-\frac{2G'U''\left(U'\right)^4}{3\varphi U^3}\\
&-\frac{4\left(G'\right)^2\left(U'\right)^3}{3\varphi U^2}-\frac{6G'\left(U''\right)^2\left(U'\right)^3}{U^3}-\frac{6\left(G'\right)^2U''\left(U'\right)^3}{G\varphi U^2}-\frac{77G'U''\left(U'\right)^3}{6\varphi^2U^2}\\
&-\frac{\left(G'\right)^3\left(U'\right)^2}{2G\varphi U}-\frac{\left(G'\right)^2\left(U'\right)^2}{\varphi^2U}+\frac{31\left(G'\right)^2\left(U''\right)^2\left(U'\right)^2}{12GU^2}+\frac{41G'\left(U''\right)^2\left(U'\right)^2}{6\varphi U^2}\\
&-\frac{11\left(G'\right)^2U''\left(U'\right)^2}{9U^2}+\frac{\left(G'\right)^3U''U'}{GU}+\frac{2\left(G'\right)^2U''U'}{\varphi U}+\frac{\left(G'\right)^4}{12G}+\frac{\left(G'\right)^3}{6\varphi}-\frac{5\left(G'\right)^2\left(U''\right)^2}{36U}\Bigg)\\
+s^2\Bigg(&-\frac{25\left(U'\right)^8}{12G^2\varphi^4U^4}+\frac{83\left(U'\right)^8}{18G\varphi^2U^5}+\frac{15\left(U'\right)^8}{2U^6}+\frac{\left(U'\right)^7}{G\varphi^3U^4}+\frac{9\left(U'\right)^7}{\varphi U^5}-\frac{5G'\left(U'\right)^6}{G^2\varphi^3U^3}\\
&-\frac{5G'\left(U'\right)^6}{6G\varphi U^4}+\frac{17U''\left(U'\right)^6}{18G\varphi^2U^4}-\frac{33U''\left(U'\right)^6}{2U^5}-\frac{205\left(U'\right)^6}{18G\varphi^4U^3}-\frac{73\left(U'\right)^6}{54\varphi^2U^4}-\frac{17G'\left(U'\right)^5}{36G\varphi^2U^3}\\
&-\frac{19G'\left(U'\right)^5}{4U^4}+\frac{5G''\left(U'\right)^5}{3G\varphi U^3}+\frac{5G'U''\left(U'\right)^5}{2G^2\varphi^2U^3}+\frac{15U''\left(U'\right)^5}{2G\varphi^3U^3}+\frac{6U''\left(U'\right)^5}{\varphi U^4}+\frac{73\left(U'\right)^5}{3\varphi^3U^3}\\
&-\frac{7\left(G'\right)^2\left(U'\right)^4}{4G^2\varphi^2U^2}+\frac{7\left(G'\right)^2\left(U'\right)^4}{18GU^3}+\frac{55\left(U''\right)^2\left(U'\right)^4}{9G\varphi^2U^3}-\frac{19G'\left(U'\right)^4}{2G\varphi^3U^2}-\frac{11G'\left(U'\right)^4}{18\varphi U^3}\\
&-\frac{G''\left(U'\right)^4}{U^3}-\frac{G'U''\left(U'\right)^4}{3G\varphi U^3}-\frac{973U''\left(U'\right)^4}{27\varphi^2U^3}-\frac{1681\left(U'\right)^4}{108\varphi^4U^2}-\frac{24\left(U''\right)^2\left(U'\right)^3}{\varphi U^3}\\
&-\frac{1537G'\left(U'\right)^3}{108\varphi^2U^2}+\frac{23G''\left(U'\right)^3}{9\varphi U^2}+\frac{3\left(G'\right)^2U''\left(U'\right)^3}{G^2\varphi U^2}+\frac{47G'U''\left(U'\right)^3}{3G\varphi^2U^2}+\frac{G'U''\left(U'\right)^3}{U^3}\\
&+\frac{13G''U''\left(U'\right)^3}{3G\varphi U^2}+\frac{5U''\left(U'\right)^3}{2\varphi^3U^2}+\frac{3\left(G'\right)^3\left(U'\right)^2}{2G^2\varphi U}+\frac{83\left(G'\right)^2\left(U'\right)^2}{12G\varphi^2U}+\frac{\left(G'\right)^2\left(U'\right)^2}{3U^2}\\
&+\frac{9G'\left(U''\right)^2\left(U'\right)^2}{2G\varphi U^2}+\frac{1361\left(U''\right)^2\left(U'\right)^2}{108\varphi^2U^2}+\frac{97G'\left(U'\right)^2}{18\varphi^3U}+\frac{G'G''\left(U'\right)^2}{G\varphi U}\\
&+\frac{29\left(G'\right)^2U''\left(U'\right)^2}{9GU^2}+\frac{20G'U''\left(U'\right)^2}{3\varphi U^2}+\frac{2G''U''\left(U'\right)^2}{U^2}+\frac{7\left(G'\right)^3U'}{12GU}+\frac{26\left(G'\right)^2U'}{9\varphi U}
\end{aligned}
$$

$$+\frac{2G'\left(U''\right)^2U'}{U^2}-\frac{\left(G'\right)^3U''U'}{2G^2U}-\frac{2\left(G'\right)^2U''U'}{G\varphi U}+\frac{2G'U''U'}{3\varphi^2U}-\frac{2G'G''U''U'}{GU}$$
$$-\frac{23G''U''U'}{9\varphi U}-\frac{\left(G'\right)^4}{6G^2}-\frac{5\left(G'\right)^3}{6G\varphi}-\frac{\left(G'\right)^2}{2\varphi^2}+\frac{5\left(G'\right)^2\left(U''\right)^2}{36GU}-\frac{2G'\left(U''\right)^2}{9\varphi U}-\frac{\left(G'\right)^2G''}{3G}$$
$$-\frac{G'G''}{3\varphi}+\frac{2\left(G'\right)^2U''}{3U}\Bigg)+s\Bigg(\frac{25\left(U'\right)^6}{18G^2\varphi^4U^3}+\frac{167\left(U'\right)^6}{54G\varphi^2U^4}-\frac{9\left(U'\right)^6}{U^5}-\frac{13G'\left(U'\right)^5}{12G^2\varphi^2U^3}$$
$$+\frac{53\left(U'\right)^5}{6G\varphi^3U^3}-\frac{25\left(U'\right)^5}{2\varphi U^4}-\frac{5\left(G'\right)^2\left(U'\right)^4}{12G^3\varphi^2U^2}+\frac{11G'\left(U'\right)^4}{6G^2\varphi^3U^2}+\frac{20G'\left(U'\right)^4}{9G\varphi U^3}-\frac{175U''\left(U'\right)^4}{54G\varphi^2U^3}$$
$$+\frac{13U''\left(U'\right)^4}{2U^4}+\frac{151\left(U'\right)^4}{54G\varphi^4U^2}-\frac{1735\left(U'\right)^4}{81\varphi^2U^3}+\frac{7\left(G'\right)^2\left(U'\right)^3}{2G^2\varphi U^2}+\frac{172G'\left(U'\right)^3}{27G\varphi^2U^2}+\frac{17G'\left(U'\right)^3}{6U^3}$$
$$+\frac{4G''\left(U'\right)^3}{9G\varphi U^2}-\frac{G'U''\left(U'\right)^3}{6G^2\varphi^2U^2}-\frac{U''\left(U'\right)^3}{G\varphi^3U^2}+\frac{23U''\left(U'\right)^3}{\varphi U^3}-\frac{161\left(U'\right)^3}{18\varphi^3U^2}-\frac{\left(G'\right)^3\left(U'\right)^2}{2G^3\varphi U}$$
$$+\frac{4\left(G'\right)^2\left(U'\right)^2}{9G^2\varphi^2U}+\frac{2\left(G'\right)^2\left(U'\right)^2}{3GU^2}-\frac{119\left(U''\right)^2\left(U'\right)^2}{108G\varphi^2U^2}+\frac{3\left(U''\right)^2\left(U'\right)^2}{U^3}+\frac{8G'\left(U'\right)^2}{3G\varphi^3U}$$
$$+\frac{289G'\left(U'\right)^2}{27\varphi U^2}-\frac{G'G''\left(U'\right)^2}{2G^2\varphi U}-\frac{G''\left(U'\right)^2}{3U^2}-\frac{3G'U''\left(U'\right)^2}{G\varphi U^2}+\frac{373U''\left(U'\right)^2}{54\varphi^2U^2}-\frac{\left(U'\right)^2}{3\varphi^4U}$$
$$-\frac{\left(G'\right)^3U'}{4G^2U}+\frac{5\left(G'\right)^2U'}{18G\varphi U}+\frac{4\left(U''\right)^2U'}{\varphi U^2}+\frac{9G'U'}{2\varphi^2U}-\frac{G'G''U'}{2GU}-\frac{2G''U'}{3\varphi U}-\frac{\left(G'\right)^2U''U'}{G^2\varphi U}$$
$$-\frac{40G'U''U'}{9G\varphi^2U}-\frac{9G'U''U'}{2U^2}-\frac{13G''U''U'}{9G\varphi U}+\frac{U''U'}{2\varphi^3U}+\frac{\left(G'\right)^4}{12G^3}-\frac{\left(G'\right)^3}{3G^2\varphi}-\frac{25\left(G'\right)^2}{36U}$$
$$-\frac{\left(G'\right)^2}{2G\varphi^2}+\frac{\left(G''\right)^2}{3G}+\frac{2G'\left(U''\right)^2}{9G\varphi U}-\frac{\left(U''\right)^2}{6\varphi^2U}+\frac{\left(G'\right)^2G''}{6G^2}+\frac{5G'G''}{6G\varphi}-\frac{25G'U''}{9\varphi U}-\frac{2G''U''}{3U}\Bigg)$$
$$-\frac{25\left(U'\right)^4}{108G^2\varphi^4U^2}-\frac{128\left(U'\right)^4}{81G\varphi^2U^3}+\frac{9\left(U'\right)^4}{4U^4}+\frac{13G'\left(U'\right)^3}{36G^2\varphi^2U^2}-\frac{55\left(U'\right)^3}{18G\varphi^3U^2}+\frac{7\left(U'\right)^3}{2\varphi U^3}$$
$$+\frac{5\left(G'\right)^2\left(U'\right)^2}{36G^3\varphi^2U}-\frac{G'\left(U'\right)^2}{18G^2\varphi^3U}+\frac{53G'\left(U'\right)^2}{27G\varphi U^2}+\frac{59U''\left(U'\right)^2}{54G\varphi^2U^2}-\frac{2U''\left(U'\right)^2}{3U^3}+\frac{\left(U'\right)^2}{3G\varphi^4U}$$
$$+\frac{8\left(U'\right)^2}{\varphi^2U^2}+\frac{19G\left(U'\right)^2}{2U^3}-\frac{G'U'}{2G\varphi^2U}-\frac{13G'U'}{2U^2}-\frac{2G'U''U'}{9G^2\varphi^2U}-\frac{U''U'}{2G\varphi^3U}-\frac{22U''U'}{3\varphi U^2}$$
$$+\frac{\left(G'\right)^3}{6G^3\varphi}+\frac{7\left(G'\right)^2}{36GU}+\frac{\left(U''\right)^2}{6G\varphi^2U}-\frac{2\left(U''\right)^2}{3U^2}-\frac{G'}{\varphi U}+\frac{G'G''}{6G^2\varphi}+\frac{G''}{U}+\frac{G'U''}{9G\varphi U} \tag{G.5}$$

The coefficient α_{17} belonging to the structure $\Phi^{a\ \mu}_{;\mu}\Phi_{a;\ \nu}^{\ \ \nu}$:

$$\alpha_{17}=s\left(-\frac{\left(U'\right)^4}{4GU^2\varphi^2}-\frac{G'U'}{2U}+\frac{\left(G'\right)^2}{4G}-\frac{\left(U'\right)^2}{12U\varphi^2}-\frac{3\left(U'\right)^4}{4U^3}\right)$$
$$+\frac{\left(U'\right)^2}{12GU\varphi^2}-\frac{U'}{U\varphi}+\frac{\left(U'\right)^2}{4U^2} \tag{G.6}$$

The coefficient α_{18} belonging to the structure $(\Phi^{c\ \mu}_{;\mu}n_c)^2$:

$$
\begin{aligned}
\alpha_{18} = s^2\left(\frac{9G'\left(U'\right)^3}{4U^2} + \frac{\left(G'\right)^2}{8} + \frac{81\left(U'\right)^6}{8U^4}\right) + s\left(\frac{\left(U'\right)^4}{4GU^2\varphi^2} - \frac{\left(G'\right)^2}{4G} + \frac{\left(U'\right)^2}{12U\varphi^2}\right. \\
\left. - \frac{27\left(U'\right)^4}{4U^3}\right) - \frac{\left(U'\right)^2}{12GU\varphi^2} + \frac{U'}{U\varphi} + \frac{13\left(U'\right)^2}{2U^2} - \frac{U''}{U} + \frac{(N-1)\left(G'\right)^2}{8G^2}
\end{aligned}
\tag{G.7}
$$

The coefficient α_{19} belonging to the structure $(\Phi^a_{\,,\mu}\Phi_a^{\,;\mu})(\Phi^b_{\,;\nu}{}^{\nu} n_b)$:

$$
\begin{aligned}
\alpha_{19} = s^2\bigg(& - \frac{\left(U'\right)^7}{4G\varphi^2U^4} - \frac{27\left(U'\right)^7}{8U^5} + \frac{27\left(U'\right)^6}{4\varphi U^4} - \frac{3G'\left(U'\right)^5}{4G\varphi U^3} - \frac{3U''\left(U'\right)^5}{4G\varphi^2U^3} - \frac{\left(U'\right)^5}{12\varphi^2U^3} \\
& + \frac{15G'\left(U'\right)^4}{8U^3} + \frac{3\left(G'\right)^2\left(U'\right)^3}{8GU^2} - \frac{7G'\left(U'\right)^3}{4\varphi U^2} - \frac{9G''\left(U'\right)^3}{4U^2} + \frac{3G'U''\left(U'\right)^3}{4G\varphi U^2} \\
& - \frac{U''\left(U'\right)^3}{4\varphi^2U^2} - \frac{3\left(G'\right)^2\left(U'\right)^2}{4G\varphi U} + \frac{G'U''U'}{4\varphi U} + \frac{\left(G'\right)^3}{8G} - \frac{\left(G'\right)^2}{2\varphi} - \frac{G'G''}{4}\bigg) \\
& + s\bigg(\frac{\left(U'\right)^5}{6G\varphi^2U^3} + \frac{15\left(U'\right)^5}{8U^4} + \frac{5G'\left(U'\right)^4}{8G^2\varphi^2U^2} - \frac{2\left(U'\right)^4}{\varphi U^3} + \frac{3G'\left(U'\right)^3}{4G\varphi U^2} + \frac{U''\left(U'\right)^3}{2G\varphi^2U^2} \\
& + \frac{3U''\left(U'\right)^3}{2U^3} + \frac{37\left(U'\right)^3}{36\varphi^2U^2} + \frac{3\left(G'\right)^2\left(U'\right)^2}{4G^2\varphi U} + \frac{41G'\left(U'\right)^2}{24G\varphi^2U} - \frac{3G'\left(U'\right)^2}{2U^2} \\
& - \frac{3U''\left(U'\right)^2}{\varphi U^2} + \frac{\left(G'\right)^2U'}{4GU} + \frac{G'U'}{\varphi U} + \frac{G''U'}{2U} - \frac{G'U''U'}{4G\varphi U} + \frac{U''U'}{12\varphi^2U} - \frac{\left(G'\right)^3}{8G^2} \\
& + \frac{G'U''}{2U}\bigg) + \frac{53\left(U'\right)^3}{36G\varphi^2U^2} - \frac{\left(U'\right)^3}{4U^3} - \frac{5G'\left(U'\right)^2}{24G^2\varphi^2U} + \frac{14\left(U'\right)^2}{\varphi U^2} - \frac{G'U'}{2G\varphi U} \\
& - \frac{U''U'}{12G\varphi^2U} - \frac{U''U'}{2U^2} + \frac{U'}{\varphi^2U} + \frac{2GU'}{U^2} - \frac{\left(G'\right)^2}{4G^2\varphi} - \frac{G'}{2U} - \frac{U''}{\varphi U} + \frac{(N-1)\left(G'\right)^2}{2G^2\varphi}
\end{aligned}
\tag{G.8}
$$

The coefficient α_{20} belonging to the structure $(\Phi^a_{\,,\mu}n_a\Phi^{b\,,\mu}n_b)(\Phi^c_{\,;\nu}{}^{\nu}\,n_c)$:

$$
\begin{aligned}
\alpha_{20} = s^3\bigg(& \frac{81\left(U'\right)^9}{8U^6} - \frac{81U''\left(U'\right)^7}{2U^5} - \frac{45G'\left(U'\right)^6}{8U^4} + \frac{81\left(U''\right)^2\left(U'\right)^5}{2U^4} + \frac{9G'U''\left(U'\right)^4}{U^3} \\
& + \frac{3\left(G'\right)^2\left(U'\right)^3}{8U^2} + \frac{9G'\left(U''\right)^2\left(U'\right)^2}{2U^2} + \frac{3\left(G'\right)^2U''U'}{2U} + \frac{\left(G'\right)^3}{8}\bigg) + s^2\bigg(\frac{3\left(U'\right)^7}{4G\varphi^2U^4} \\
& - \frac{45\left(U'\right)^7}{8U^5} - \frac{45\left(U'\right)^6}{4\varphi U^4} - \frac{G'\left(U'\right)^5}{2G\varphi U^3} + \frac{9U''\left(U'\right)^5}{2G\varphi^2U^3} + \frac{135U''\left(U'\right)^5}{4U^4} + \frac{\left(U'\right)^5}{4\varphi^2U^3} \\
& + \frac{5G'\left(U'\right)^4}{4G\varphi^2U^2} - \frac{21G'\left(U'\right)^4}{8U^3} + \frac{9U''\left(U'\right)^4}{\varphi U^3} - \frac{3\left(G'\right)^2\left(U'\right)^3}{8GU^2} - \frac{63\left(U''\right)^2\left(U'\right)^3}{2U^3} \\
& + \frac{4G'\left(U'\right)^3}{3\varphi U^2} + \frac{9G''\left(U'\right)^3}{4U^2} - \frac{13G'U''\left(U'\right)^3}{4G\varphi U^2} + \frac{3U''\left(U'\right)^3}{2\varphi^2U^2} + \frac{5G'\left(U'\right)^2}{12\varphi^2U}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{15G'U''\left(U'\right)^2}{4U^2}-\frac{\left(G'\right)^2U'}{2U}+\frac{3\left(G'\right)^2U''U'}{2GU}+\frac{23G'U''U'}{12\varphi U}+\frac{\left(G'\right)^3}{8G}+\frac{3\left(G'\right)^2}{4\varphi}\\
&-\frac{3G'\left(U''\right)^2}{2U}+\frac{G'G''}{4}\Bigg)+s\Bigg(\frac{7\left(U'\right)^5}{4G\varphi^2U^3}+\frac{15\left(U'\right)^5}{8U^4}-\frac{11G'\left(U'\right)^4}{8G^2\varphi^2U^2}-\frac{\left(U'\right)^4}{G\varphi^3U^2}\\
&+\frac{3\left(U'\right)^4}{\varphi U^3}-\frac{25G'\left(U'\right)^3}{12G\varphi U^2}-\frac{U''\left(U'\right)^3}{G\varphi^2U^2}-\frac{12U''\left(U'\right)^3}{U^3}-\frac{7\left(U'\right)^3}{3\varphi^2U^2}-\frac{19G'\left(U'\right)^2}{8G\varphi^2U}\\
&+\frac{3G'\left(U'\right)^2}{U^2}-\frac{3U''\left(U'\right)^2}{\varphi U^2}-\frac{\left(U'\right)^2}{3\varphi^3U}+\frac{\left(G'\right)^2U'}{2GU}+\frac{15\left(U''\right)^2U'}{2U^2}-\frac{G'U'}{4\varphi U}+\frac{13G'U''U'}{12G\varphi U}\\
&+\frac{U''U'}{6\varphi^2U}-\frac{\left(G'\right)^3}{8G^2}-\frac{\left(G'\right)^2}{2G\varphi}-\frac{G'G''}{2G}\Bigg)-\frac{13\left(U'\right)^3}{6G\varphi^2U^2}-\frac{29\left(U'\right)^3}{4U^3}+\frac{11G'\left(U'\right)^2}{24G^2\varphi^2U}\\
&+\frac{\left(U'\right)^2}{3G\varphi^3U}-\frac{33\left(U'\right)^2}{2\varphi U^2}+\frac{G'U'}{4G\varphi U}-\frac{3U'}{\varphi^2U}-\frac{U'U''}{6G\varphi^2U}+\frac{33U'U''}{2U^2}+\frac{3U''}{\varphi U}-\frac{U'''}{U}\\
&-\frac{(N-1)\left(G'\right)^2}{4G^2\varphi}-\frac{(N-1)\left(G'\right)^3}{8G^3}+\frac{(N-1)G'G''}{4G^2}
\end{aligned}
\tag{G.9}
$$

The coefficient α_{21} belonging to the structure $\Phi^{a\ \mu}_{;\mu}\Phi^{;\nu}_{a}\Phi^{b}_{,\nu}n_b$:

$$
\begin{aligned}
\alpha_{21}=s^2\Bigg(&-\frac{\left(U'\right)^7}{2G\varphi^2U^4}-\frac{9\left(U'\right)^7}{4U^5}+\frac{9\left(U'\right)^6}{2\varphi U^4}+\frac{5G'\left(U'\right)^5}{4G\varphi U^3}-\frac{15U''\left(U'\right)^5}{4G\varphi^2U^3}+\frac{9U''\left(U'\right)^5}{2U^4}\\
&-\frac{\left(U'\right)^5}{6\varphi^2U^3}-\frac{5G'\left(U'\right)^4}{4G\varphi^2U^2}-\frac{3G'\left(U'\right)^4}{4U^3}-\frac{9U''\left(U'\right)^4}{\varphi U^3}+\frac{5G'\left(U'\right)^3}{12\varphi U^2}+\frac{5G'U''\left(U'\right)^3}{2G\varphi U^2}\\
&-\frac{5U''\left(U'\right)^3}{4\varphi^2U^2}+\frac{3\left(G'\right)^2\left(U'\right)^2}{4G\varphi U}-\frac{5G'\left(U'\right)^2}{12\varphi^2U}+\frac{3G'U''\left(U'\right)^2}{U^2}+\frac{\left(G'\right)^2U'}{4U}\\
&-\frac{3\left(G'\right)^2U''U'}{2GU}-\frac{13G'U''U'}{6\varphi U}-\frac{\left(G'\right)^3}{4G}-\frac{\left(G'\right)^2}{4\varphi}\Bigg)+s\Bigg(-\frac{23\left(U'\right)^5}{12G\varphi^2U^3}+\frac{3\left(U'\right)^5}{4U^4}\\
&+\frac{3G'\left(U'\right)^4}{4G^2\varphi^2U^2}+\frac{\left(U'\right)^4}{G\varphi^3U^2}-\frac{\left(U'\right)^4}{\varphi U^3}+\frac{4G'\left(U'\right)^3}{3G\varphi U^2}+\frac{U''\left(U'\right)^3}{2G\varphi^2U^2}+\frac{3U''\left(U'\right)^3}{2U^3}\\
&+\frac{47\left(U'\right)^3}{36\varphi^2U^2}-\frac{3\left(G'\right)^2\left(U'\right)^2}{4G^2\varphi U}+\frac{2G'\left(U'\right)^2}{3G\varphi^2U}-\frac{G'\left(U'\right)^2}{2U^2}+\frac{6U''\left(U'\right)^2}{\varphi U^2}+\frac{\left(U'\right)^2}{3\varphi^3U}\\
&-\frac{3\left(G'\right)^2U'}{4GU}-\frac{3G'U'}{4\varphi U}-\frac{G''U'}{2U}-\frac{5G'U''U'}{6G\varphi U}-\frac{U''U'}{4\varphi^2U}+\frac{\left(G'\right)^3}{4G^2}+\frac{\left(G'\right)^2}{2G\varphi}\\
&+\frac{G'G''}{2G}\Bigg)+\frac{25\left(U'\right)^3}{36G\varphi^2U^2}-\frac{G'\left(U'\right)^2}{4G^2\varphi^2U}-\frac{\left(U'\right)^2}{3G\varphi^3U}+\frac{5\left(U'\right)^2}{2\varphi U^2}+\frac{G'U'}{4G\varphi U}\\
&+\frac{U''U'}{4G\varphi^2U}-\frac{U''U'}{U^2}+\frac{2U'}{\varphi^2U}-\frac{13GU'}{2U^2}+\frac{\left(G'\right)^2}{4G^2\varphi}+\frac{G'}{U}-\frac{2U''}{\varphi U}
\end{aligned}
\tag{G.10}
$$

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