Appendix A: Derivation for M/G/1 Queue

In this appendix, we apply the method of z-transform or generating functions to find the waiting time of the M/G/1 model.

The probability of having $k$ arrivals during the service time $t$ is

$$p_k = \int_0^\infty p(k) dH(t) = \int_0^\infty \frac{(\lambda t)^k}{k!} e^{-\lambda t} dH(t) \quad (A.1)$$

where $H(t)$ is the service time distribution.

Let $N$ be the number of customers present in the system and $Q$ be the number of customers in the queue. Let the probability that an arriving customer finds $j$ other customers present be

$$\Pi_j = \text{Prob}(N = j), \quad j = 0, 1, 2, \cdots \quad (A.2)$$

It can be shown using the theorem of total probability and the equilibrium imbedded-Markov-chain that

$$\Pi_j = p_j \Pi_0 + \sum_{i=1}^{j+1} p_{j-i+1} \Pi_i, \quad j = 0, 1, 2, \cdots \quad (A.3)$$

We define the probability-generating functions

$$g(z) = \sum_{j=0}^\infty \Pi_j z^j \quad (A.4a)$$

$$h(z) = \sum_{j=0}^\infty p_j z^j \quad (A.4b)$$
Substituting (Eq. A.4a) into (Eq. A.3) results in

\[ g(z) = \frac{(z - 1)h(z)}{z - h(z)} \pi_0 \]  
(E.5)

The normalization equation

\[ \sum_{j=0}^{\infty} \pi_j = 1 \]  
(E.6)

implies that \( g(1) = 1 \). With a single application of L’Hopital’s rule, we find

\[ \pi_0 = 1 - \rho \]  
(E.7)

where \( \rho = \lambda/\mu = \lambda \tau \). If we define \( \eta(s) \) as the Laplace-Stieltjes transform of the service-time distribution function \( H(t) \),

\[ \eta(s) = \int_0^\infty e^{-st}dH(t) \]  
(E.8)

Substitution of (Eq. A.1) into (Eq. A.4b) yields

\[ h(z) = \eta(\lambda - \lambda z) \]  
(E.9)

and substitution of (Eq. A.7) and (Eq. A.9) into (Eq. A.5) leads to

\[ g(z) = \frac{(z - 1)\eta(\lambda - \lambda z)}{z - \eta(\lambda - \lambda z)} (1 - \rho) \]  
(E.10)

Differentiating this and applying L’Hopital rule twice, we obtain

\[ g'(1) = \frac{\rho^2}{2(1 - \rho)} \left( 1 + \frac{\sigma^2}{\tau^2} \right) + \rho \]  
(E.11)

The mean values of the number of customers in the system and queue are respectively given by

\[ E(N) = \sum_{j=0}^{\infty} j\pi_j = g'(1) \]  
(E.12a)

\[ E(Q) = E(N) - \rho \]  
(E.12b)
By applying Little’s theorem, the mean value of the response time is

\[
E(T) = \frac{E(N)}{\lambda} = \frac{\rho \tau}{2(1 - \rho)} \left(1 + \frac{\sigma^2}{\tau^2}\right) + \tau
\]

(A.13)

Thus we obtain the mean waiting time as

\[
E(W) = \frac{E(Q)}{\lambda} = \frac{\rho \tau}{2(1 - \rho)} \left(1 + \frac{\sigma^2}{\tau^2}\right)
\]

which is Pollaczek-Khintchine formula.
Appendix B: Useful Formulas

\[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \]

\[ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \]

\[ \sum_{i=1}^{n} i^3 = \left[ \sum_{i=1}^{n} i \right]^2 = \frac{n^2}{4} (n+1)^2 \]

\[ \sum_{n=1}^{\infty} x^n = \frac{1}{1-x}, \quad |x| < 1 \]

\[ \sum_{n=k}^{\infty} x^n = \frac{x^k}{1-x}, \quad |x| < 1 \]

\[ \sum_{n=1}^{k} x^n = \frac{x - x^{k+1}}{1-x}, \quad x \neq 1 \]

\[ \sum_{n=0}^{k} x^n = \frac{1 - x^{k+1}}{1-x}, \quad x \neq 1 \]

\[ \sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2}, \quad |x| < 1 \]

\[ \sum_{n=1}^{k} nx^n = x \left( \frac{1-x^k}{(1-x)^2} - kx^k(1-x) \right), \quad x \neq 1 \]
\[
\sum_{n=1}^{\infty} n^2 x^n = \frac{x(1+x)}{(1-x)^3}, \quad |x| < 1
\]

\[
\sum_{n=1}^{\infty} n(n+1)x^n = \frac{2x}{(1-x)^3}, \quad |x| < 1
\]

\[
\sum_{n=0}^{\infty} \frac{(n+k)!}{n!} x^n = \frac{k!}{(1-x)^{k+1}}, \quad |x| < 1, k \geq 0
\]

\[
\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x, \quad -\infty < x < \infty
\]

\[
\sum_{n=0}^{\infty} \frac{x^n}{(n+1)!} = \frac{e^x - 1}{x}, \quad -\infty < x < \infty
\]

\[
\sum_{n=1}^{\infty} \frac{x^n}{n} = \ln \left( \frac{1}{1-x} \right), \quad |x| < 1
\]

\[
\sum_{n=1}^{\infty} \frac{x^{(2n-1)}}{(2n-1)!} = \frac{e^x - e^{-x}}{2}, \quad -\infty < x < \infty
\]

\[
\sum_{n=0}^{\infty} \binom{N+n-1}{n} x^{-n} = \left( \frac{x}{x-1} \right)^N, \quad |x| < 1
\]

\[
\sum_{k=1}^{n} \binom{n}{k} x^k = (1+x)^n
\]
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