

Appendix A

Massless Graviton Implementation

Here, we present in detail the implementation of massless graviton routines within MADGRAPH/MADEVENT. Initially, we alter helicity routines (HELAS) for spin-2 particles in order to handle massless particles. We subsequently create a model file for the particular theory described on Chap. 3. All information characterizing implementation can be found in Ref. [1] and is reproduced in what follows.

A.1 HELAS Routines Modifications

The HELAS routines for massive spin-2 particles have been implemented by K. Hagiwara et al. [2] in 2008. A summary of the routines can be found on Table A.1. In order to have massless spin-2 particles in MADGRAPH/MADEVENT, we have modified the tensor wave function subroutine TXXXXXX, and four subroutines for the off-shell tensor were added to the former implementation.

A.1.1 TXXXXXX

This subroutine computes the graviton tensor wave function, namely $\epsilon^{\mu\nu}(p, \lambda)$ and $\epsilon^{\mu\nu}(p, \lambda)^*$ in terms of the graviton four-momentum p and its helicity λ . The helicity states of the graviton can be expressed by

$$\begin{aligned} \epsilon^{\mu\nu}(p, \pm 2) &= \epsilon^\mu(p, \pm) \epsilon^\nu(p, \pm) \\ \epsilon^{\mu\nu}(p, \pm 1) &= \frac{1}{\sqrt{2}} \left[\epsilon^\mu(p, \pm) \epsilon^\nu(p, 0) + \epsilon^\mu(p, 0) \epsilon^\nu(p, \pm) \right] \\ \epsilon^{\mu\nu}(p, 0) &= \frac{1}{\sqrt{6}} \left[\epsilon^\mu(p, +) \epsilon^\nu(p, -) + \epsilon^\mu(p, -) \epsilon^\nu(p, +) + 2\epsilon^\mu(p, 0) \epsilon^\nu(p, 0) \right], \end{aligned} \tag{A.1}$$

Table A.1 Summary of Spin-2 HELAS routines from [2]

Vertex	Inputs	Output	Subroutine
SST	SST	Amplitude	SSTXXX
	ST	S	HSTXXX
	SS	T	USSXXX
FFT	FFT	Amplitude	IOTXXX
	FT	F	FTIXXX, FTTOXXX
	FF	T	UIOXXX
VVT	VVT	Amplitude	VVTXXX
	VT	V	JVTXXX
	VV	T	UVVXXX
FFVT	FFVT	Amplitude	IOVXX
	FVT	F	FVTIXX, FVTOXX
	FFT	V	JIoTXX
	FFV	T	UIOVXX
VVVT	VVVT	Amplitude	VVVTXX
	VVT	V	JVVTXX
	VVV	T	UVVVXX
VVVVT	GGGGT	Amplitude	GGGGTX
	GGGT	G	JGGGTX
	GGGG	T	UGGGGX

where $\epsilon^\mu(p, \lambda)$ is the vector boson wave function, already pre-defined on the HELAS convention. For completeness it is important to state here that the spin-2 wave functions are traceless, transverse, orthogonal and symmetric, obeying therefore the following relations:

$$\begin{aligned}
\epsilon^\mu{}_\mu(p, \lambda) &= 0, & \epsilon^{\mu\nu}(p, \lambda)\epsilon_{\mu\nu}(p, \lambda')^* &= \delta_{\lambda\lambda'} \\
\epsilon^{\mu\nu}(p, \lambda) &= \epsilon^{\nu\mu}(p, \lambda) & p_\mu\epsilon^{\mu\nu}(p, \lambda) &= p_\nu\epsilon^{\mu\nu}(p, \lambda) = 0
\end{aligned} \tag{A.2}$$

The sum of helicity states for the graviton is introduced on the routines to be given by

$$\sum_{\lambda=\pm 2} \epsilon_{\mu\nu}(p, \lambda)\epsilon_{\alpha\beta}(p, \lambda)^* = \tilde{\mathcal{P}}_{\mu\nu, \alpha\beta}(p), \tag{A.3}$$

where $\tilde{\mathcal{P}}_{\mu\nu\alpha\beta}$ is the massless graviton propagator defined on Eq.(3.73). While the massive graviton has five helicity states of polarization, the massless spin-2 particles will only have two physical states. The sum of polarization for the massless graviton is therefore modified on the routines to set $\lambda = \pm 1, 0$ to zero allowing only $\lambda = \pm 2$ to contribute.

A.1.2 UIOXXX, UVVXXX, UIOVXX, UVVVXX

These subroutines compute off-shell tensor currents U by the FFT, VVT, FFVT, and VVVT vertices respectively. The main modification here was the inclusion of the zero mass graviton propagator to be the one showed in Eq. (3.73) for the massless case.

The UIOXXX routine computes the bi-spinor spin-2 current from the flowing-in and flowing-out fermions calculated by the FFT vertex.

The flowing-in FI and flowing-out FO fermions are defined to be arrays which contain their four-momenta:

$$\text{FI} = \begin{pmatrix} p_I^0 \\ p_I^1 \\ p_I^2 \\ p_I^3 \end{pmatrix}, \quad \text{FO} = \begin{pmatrix} p_o^0 & p_o^1 & p_o^2 & p_o^3 \end{pmatrix}. \quad (\text{A.4})$$

The output is a complex array defined by

$$T^{\alpha\beta} = g_t \frac{\tilde{\mathcal{P}}^{\mu\nu\alpha\beta}}{q^2} \text{FO} \left[-\eta_{\mu\nu} (\not{p}_I + \not{p}_o - 2m_f) + \gamma_\mu (p_I + p_o)_\nu + \gamma_\nu (p_I + p_o)_\mu \right] \text{FI}$$

where q is the four-momentum related to the massless propagator, $g_t = \frac{1}{4\Lambda}$ and Λ is the effective scale of the model. For the MGM theory, $\Lambda = \overline{M}_{pl}(\mu_\star) \sim \mu_\star$.

The UVVXXX is the subroutine that computes an off-shell spin-2 current from the two flowing-out vector bosons calculated by the VVT vertex. The graviton output is the complex array:

$$T^{\alpha\beta} = g_t \frac{\tilde{\mathcal{P}}^{\mu\nu\alpha\beta}}{q^2} \left[(m_V^2 + p_I \cdot p_o) C_{\mu\nu\rho\sigma} + D_{\mu\nu\rho\sigma}(p_I, p_o) + \xi^{-1} E_{\mu\nu\rho\sigma}(p_I, p_o) \right] V_I^\rho V_o^\sigma \quad (\text{A.5})$$

where where q is the four-momentum related to the massless propagator, $g_t = \frac{1}{\Lambda}$, Λ is the effective scale of the model, and the parameters $C_{\mu\nu\alpha\beta}$, $D_{\mu\nu\alpha\beta}$ and $E_{\mu\nu\alpha\beta}$ are defined on Table 3.1.

The UIOVXX subroutine computes an off-shell spin-2 current calculated by the FFVT vertex. The graviton output complex array is given by:

$$T^{\alpha\beta} = g_t \frac{\tilde{\mathcal{P}}^{\mu\nu\alpha\beta}}{q^2} (\eta_{\mu\nu} \eta_{\rho\sigma} - C_{\mu\nu\rho\sigma}) \text{FO} \left[V^\rho \gamma^\sigma \left(g_f^{(1)} P_L + g_f^{(2)} P_R \right) \right] \text{FI}, \quad (\text{A.6})$$

where q is the four-momentum related to the massless propagator, $g_t = \frac{1}{2\Lambda}$, Λ is the effective scale of the model, the parameter $C_{\mu\nu\alpha\beta}$ is defined on Table 3.1 and $g_f^{(1)}$ and

$g_f^{(2)}$ are the SM FFV couplings. For example, if we have a quark-anti-quark-gluon-graviton vertex, $g_f^{(1)} = g_f^{(2)} = g_S$.

Finally, the `UVVVXX` subroutine computes an off-shell spin-2 current calculated by the `VVVT` vertex. The graviton tensor is given by

$$T^{\alpha\beta} = \frac{\tilde{\mathcal{P}}^{\mu\nu\alpha\beta}}{q^2} g_V [C_{\mu\nu\rho\sigma}(p_a - p_b)_\lambda + C_{\mu\nu\sigma\lambda}(p_b - p_c)_\rho + C_{\mu\nu\lambda\rho}(p_c - p_a)_\sigma + F_{\mu\nu\rho\sigma\lambda}] V_a^\rho V_b^\sigma V_c^\lambda, \quad (\text{A.7})$$

where q is the four-momentum related to the massless propagator, $g_t = \frac{1}{\Lambda}$, Λ is the effective scale of the model, the parameters $C_{\mu\nu\alpha\beta}$ and $F_{\mu\nu\rho\sigma\lambda}$ are defined on Table 3.1 and g_V is the SM `VVV` coupling.

A.2 MGM Model File

The model file was created for MADGRAPH/MADEVENT version 4. It contains all the information regarding particles, couplings and interactions and follows the structure presented on [2] for theories containing a massive spin-2 particle. A summary of the model content is presented below.

The particle content of the mode is similar to the SM, in addition to a massless tensor:

```
# This is a special data file which contains particles of the
# model. The format for entering new particles is:
# Name anti_Name Spin Linetype Mass Width Color Label PDG
y y T D ZERO ZERO S G 39
```

The graviton is represented by the letter `y`, and its PDG number is 39.

The new coupling associated to the graviton particles are:

```
# The ordering of particles is very important.
# For FFV interactions, must use particles, not anti_particles.
#
# 3-particle vertices are entered as follows:
# particle1 particle2 particle3 coupling_name coupling_order
#
g g y GTV QTD
a a y GTV QTD
z z y GTV QTD
w- w+ y GTV QTD
q q y GTF QTD
l- l- y GTF QTD
h h y GTS QTD
#
# 4-particle vertices are entered as follows:
```

```
# part1 part2 part3 part4 2xcoupling_names 2xcoupling_orders
#
g  g  g  y  G  GTV  QCD  QTD
q  q  g  y  GG  GTFV  QCD  QTD
qd  qd  a  y  GAD  GTFV  QED  QTD
qu  qu  a  y  GAU  GTFV  QED  QTD
l-  l-  a  y  GAL  GTFV  QED  QTD
qd  qd  z  y  GZD  GTFV  QED  QTD
qu  qu  z  y  GZU  GTFV  QED  QTD
l-  l-  z  y  GZL  GTFV  QED  QTD
vl  vl  z  y  GZN  GTFV  QED  QTD
qd  qu  w-  y  GWF  GTFV  QED  QTD
qu  qd  w+  y  GWF  GTFV  QED  QTD
vl  l-  w+  y  GWF  GTFV  QED  QTD
l-  vl  w-  y  GWF  GTFV  QED  QTD
```

Here, all types of quarks are represented by q , up-quark family is qu and down-quark family is qd . For the leptons, electron, muon and tau are defined as l^- , and the three types of neutrinos are represented by νl . The interaction order is the proportionality of the vertex with the g_s coupling for QCD, g_e coupling for QED, and g_t coupling for QTD, which is the new interaction defined for spin-2 particles. The couplings depend on the particles of the vertex.

The new couplings related to the spin-2 particle are GTV , GTF and GTS . They are defined to be:

$$GTV = \frac{1}{\Lambda}, \quad GTS = \frac{1}{2\Lambda}, \quad GTF = \frac{1}{4\Lambda} \quad (\text{A.8})$$

where Λ is the effective scale of the theory.

References

1. P. de Aquino, K. Hagiwara, Q. Li, F. Maltoni, Simulating graviton production at hadron colliders. *Journal of High Energy Physics* **2011**(6) (2011)
2. K. Hagiwara, J. Kanzaki, Q. Li, K. Mawatari, HELAS and MadGraph/MadEvent with spin-2 particles. *Eur. Phys. J. C* **56**(3), pp. 435–447 (2008, 0805.2554)

Appendix B

New Routines for ADD Theories in MadGraph

B.1 5-Point Vertices

In Chap. 3 we have obtained the Feynman rules and vertices for a theory with a general spin-2 particle coupled to the SM theory. These are exhibited on Fig. 3.1.

We can check the existence of 5-point vertices, such as the ggggG vertex or the qqggG. Previously, MADGRAPH/MADEVENT version 4 [1] did not have the automatic generation of 5-point diagrams, which was necessary for the correct evaluation of processes with more than 1 jet in the final state. Routines to automatic generate 5-point vertices have been implemented in collaboration with Q. Li [2]. Here we reproduce details of such implementation.¹

Recall that the ggggG vertex is described on Fig. 3.1. It can be re-organized and written as:

$$\mathcal{L}_{\text{ggggT}} = -\text{GT GC}^2 f^{abe} f^{cde} T^{\mu\nu*} \times \left[\frac{1}{4} \eta_{\mu\nu} g^{a,\rho*} g^{b,\sigma*} g_\rho^{c*} g_\sigma^{d*} - g^{b,\rho*} g_\mu^{a*} g_\nu^{c*} g_\rho^{d*} \right], \tag{B.1}$$

with the coupling constants $\text{GT} = -\frac{\kappa}{2}$ and $\text{GC} = g_s$. In HELAS such vertex can be encoded as GGGGTx computing the portion of the amplitude of the ggggT vertex from four gluon (G) polarization vectors and a tensor (T) boson wavefunction corresponding to the color structure $f^{abe} f^{cde}$, as it is illustrated on Fig. B.1a.

In order to implement this vertex in MADGRAPH, which supports at most 4-point vertices, we reinterpret the scattering in the t , u and s channels of the non-propagating octet tensor boson t_A as an auxiliary particle. With t_A we can decompose the 5-point vertex into two 3-point vertices: ggt_A and $t_A t_A T$, which can then be generated by MADGRAPH automatically. The color structure of the vertex $f^{abe} f^{cde}$ is main-

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Fig. B.1 Illustration of the decomposition of the 5-point vertex into two 3-point vertices to be automatically treated in MADGRAPH version 4

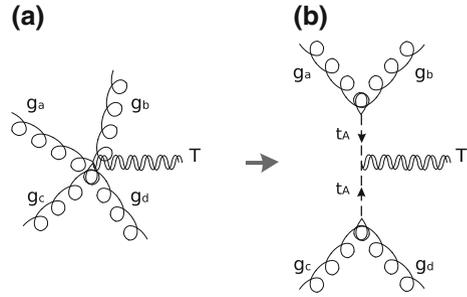


Table B.1 Summary of the HELAS sub-routines introduced for the implementation of the 5-point vertex in MADGRAPH extracted from [2]

Sub-routine	Summary
VVTCXX	computes the amplitude of the $gg t_A$ vertex
JVTCXX	computes the off-shell vector current J made from the interactions of a Vector gluon and an auxiliary Tensor boson by the $gg t_A$ vertex
UVVCXX	computes the off-shell tensor current U for the auxiliary tensor t_A , made from two flowing-out Vector gluons by the $gg t_A$ vertex
TTTXXX	computes the amplitude of the $t_A t_A T$ vertex
UTTAXX	computes the off-shell non-propagating tensor current U , made from the flowing-out graviton Tensor and the auxiliary Tensor by the $t_A t_A T$ vertex
UTTBXX	computes the off-shell graviton tensor current U , made from the two flowing-out auxiliary Tensors by the $t_A t_A T$ vertex

tained. Figure B.1 illustrates the decomposition. Observe that we assign a flow to t_A , i.e. we treat t_A different from its antiparticle in MADGRAPH. The idea is to forbid the interaction of t_A and its antiparticle to avoid appearance of additional diagrams for $gg \rightarrow gg$.

In order to introduce the auxiliary particle t_A , several HELAS sub-routines have been modified and some have been created. Table B.1 presents a summary of all modifications. We follow with a full description of each routine.

B.1.1 GGGGTX

The subroutine GGGGTX computes the portion of the amplitude of the $ggggT$ vertex from four Gluon polarization vectors and a Tensor boson wavefunction corresponding to the color structure $f^{abe} f^{cde}$, which should be called as

$$\text{CALL GGGGTX}(VA, VB, VC, VD, TC, GC, GT, VERTEX) \quad (\text{B.2})$$

The inputs $\text{VA}(6)$, $\text{VB}(6)$ and $\text{VC}(6)$ are complex six-dimensional arrays which contain the Vector boson wavefunctions, and their momenta as

$$\begin{aligned} p_a^\mu &= (\Re\text{eVA}(5), \Re\text{eVA}(6), \Im\text{mVA}(6), \Im\text{mVA}(5)), \\ p_b^\mu &= (\Re\text{eVB}(5), \Re\text{eVB}(6), \Im\text{mVB}(6), \Im\text{mVB}(5)), \\ p_c^\mu &= (\Re\text{eVC}(5), \Re\text{eVC}(6), \Im\text{mVC}(6), \Im\text{mVC}(5)). \end{aligned}$$

The input $\text{TC}(18)$ is a complex 18-dimensional array which contains the wavefunction of the Tensor boson:

$$T^{\mu+1, \nu+1} = \text{TC}(4\mu + \nu + 1), \quad (\text{B.3})$$

and its four-momentum as

$$p^\mu = (\Re\text{eTC}(17), \Re\text{eTC}(18), \Im\text{mTC}(17), \Im\text{mTC}(17)),$$

The output VERTEX is a complex number in units of GeV:

$$\text{VERTEX} = -\text{GT GC}^2 T^{\mu\nu} G_{\mu\nu, \rho\lambda\sigma\delta} V_A^\rho V_B^\sigma V_C^\lambda V_D^\delta \quad (\text{B.4})$$

with $G_{\mu\nu, \rho\sigma\lambda\delta}$ given on Table 3.1, and we use the notation

$$\begin{aligned} V_A^\mu &= \text{VA}(\mu+1), \quad V_B^\mu = \text{VB}(\mu+1), \\ V_C^\mu &= \text{VC}(\mu+1), \quad V_D^\mu = \text{VD}(\mu+1). \end{aligned} \quad (\text{B.5})$$

In order to insert automatically 5-point vertices in **MADGRAPH**, we introduce non-propagating colourless tensor boson t_A as an auxiliary particle. With t_A the portion of the 5-point vertex with color structure $f^{abe} f^{cde}$ can be reduced automatically. After introducing particle t_A , several new **HELAS** subroutines were added. They are summarized on Table B.1, and can be called on the following way:

B.1.2 VVTCXX

This subroutine computes the amplitude of the $gg t_A$ vertex,

$$\text{CALL VVTCXX}(\text{VA}, \text{VB}, \text{TC}, \text{GC}, \text{VMASS}, \text{VERTEX})$$

VMASS represents the vector gluon mass which is zero (although it does not play any role here, we keep it as an input argument in accordance with the subroutine **VVTXXX**, for convenience of **MADGRAPH**). What we compute here is

$$\text{VERTEX} = \text{GC} \eta_{\mu\rho} \eta_{\nu\sigma} T^{\mu\nu} V_A^\rho V_B^\sigma. \quad (\text{B.6})$$

B.1.3 JVTCXX

This subroutine computes an off-shell vector current \mathcal{J} made from the interactions of a Vector gluon and an auxiliary Tensor boson by the $gg t_A$ vertex, and should be called as

CALL JVTCXX(VC, TC, GC, VMASS, VWIDTH, JVTC)

The input $VC(6)$ is the wavefunction and momentum of the gluon. The output $JVTC(6)$ gives the off-shell vector current multiplied by the gluon propagator, which is expressed as a complex six-dimensional array:

$$JVTC(+1) = -\frac{GC}{q^2} \eta_{\alpha\mu} V^\alpha T^{\mu\nu}, \quad (\text{B.7})$$

and

$$JVTC(5) = V(5) + TC(17), \quad (\text{B.8})$$

$$JVTC(6) = V(6) + TC(18). \quad (\text{B.9})$$

Here the momenta q are

$$q^\mu = (\Re JVTC(5), \Re JVTC(6), \Im JVTC(6), \Im JVTC(5)).$$

B.1.4 UVVCXX

This subroutine computes an off-shell tensor current \mathcal{U} for the auxiliary tensor t_A , made from two flowing-out Vector gluons by the $gg t_A$ vertex, and should be called as

CALL UVVCXX(VA, VB, GC, VMASS, XM, XW, UVVC)

The inputs XM and XW are two dummy arguments for which we keep them in accordance with the subroutine $UVVXXX$. The output $UVVC(18)$ is a complex 18-dimensional array:

$$T^{\alpha\beta} = GC V_A^\alpha V_B^\beta \quad (\text{B.10})$$

for the first 16 components of $UVVC$, and

$$UVVC(17) = VA(5) + VB(5), \quad (\text{B.11})$$

$$UVVC(18) = VA(6) + VB(6). \quad (\text{B.12})$$

B.1.5 TTTXXX

This subroutine computes the amplitude of the $t_A t_{AT}$ vertex, called as

CALL TTTXXX(TC, T1C, T2C, GT, VERTEX)

The inputs T1C (18) and T2C (18) are complex 18-dimensional arrays which contain the wavefunction and momenta for the auxiliary tensors. What we compute here is

$$\text{VERTEX} = \text{GT} T^{\mu\nu} T_1^{\rho\lambda} T_2^{\sigma\delta} G_{\mu\nu,\rho\sigma\lambda\delta}, \quad (\text{B.13})$$

with $T_{1,2}$ defined from T1C (18) and T2C (18) as in Eq. (B.3).

B.1.6 UTTAXX

This subroutine computes an off-shell non-propagating tensor current U, made from the flowing-out graviton Tensor and the auxiliary Tensor by the $t_A t_{AT}$ vertex, and should be called as

CALL UTTAXX(TC, T1C, GT, UTTA)

The output UTTA (18) is a complex 18-dimensional array:

$$T_A^{\alpha\beta} = \text{GT} \eta^{\sigma\alpha} \eta^{\delta\beta} T^{\mu\nu} T_1^{\rho\lambda} G_{\mu\nu,\rho\sigma\lambda\delta} \quad (\text{B.14})$$

for the first 16 components of UTTA, and

$$\text{UTTA}(17) = \text{T}(17) + \text{T1}(17), \quad (\text{B.15})$$

$$\text{UTTA}(18) = \text{T}(18) + \text{T1}(18). \quad (\text{B.16})$$

B.1.7 UTTBXX

This subroutine computes an off-shell graviton tensor current U, made from the two flowing-out auxiliary Tensors by the $t_A t_{AT}$ vertex, and should be called as

CALL UTTBXX(T1C, T2C, GT, TMASS, TWIDTH, UTTB)

The inputs TMASS and TWIDTH are the graviton mass and width, m_T and Γ_T . The output UTTA (18) is a complex 18-dimensional array:

$$T^{\alpha\beta} = \text{GT} \frac{-\mathcal{P}_{\mu\alpha,\nu\beta}}{q^2 - m_T^2 + im_T\Gamma_T} T_1^{\rho\lambda} T_2^{\sigma\delta} G_{\mu\nu,\rho\sigma\lambda\delta} \quad (\text{B.17})$$

for the first 16 components of UTTB, and

$$\text{UTTB}(17) = \text{T1}(17) + \text{T2}(17), \quad (\text{B.18})$$

$$\text{UTTB}(18) = \text{T1}(17) + \text{T2}(18). \quad (\text{B.19})$$

Here q is the momentum of the off-shell tensor boson given in Eqs. (B.18) and (B.19) as

$$q^\mu = (\Re e \text{UTTB}(17), \Re e \text{UTTB}(18), \Im m \text{UTTB}(18), \Im m \text{UTTB}(17)).$$

And $\mathcal{P}_{\mu\alpha,\nu\beta}$ is the polarization summation tensor (3.77) [2].

B.2 Summation Over the Mass Spectrum

On Chap. 2 it has been shown that for the ADD theory we expect to have an infinite number of spin-2 particles couples to the SM. Because their masses lie very close to each other experimental searches would hope to detect signs of gravity through the sum of such states.

The MC simulation would therefore have to perform an extra integral (accounting for all the KK modes of the graviton) for each point of the phase-space. The amplitude integration over the mass density function is specified on Eq. (3.40).

For virtual graviton exchange, the tower of KK graviton modes leads to the summation of their propagators, which is divergent since ADD is only an effective theory. It is possible to introduce a cutoff for the highest KK modes and replace the summation a factor on the effective scale, as it was first proposed by Han et al. [3]. This was the method chosen for the implementation in MADGRAPH, which can be easily performed by setting the graviton mass to zero and the extra dimensional scale to be the same as the effective scale. Note however, that the propagator of the massive graviton is still employed here, and setting the mass of the graviton to zero has only been accomplished locally aiming on executing such trick.

Nevertheless, it is more complicated for graviton emission, because it is actually necessary to perform the integration described by Eq. (3.40). We have implemented the integration through the introduction of new vertices and two auxiliary dummy particles x_1 and x_2 . These are massless colourless particles, in which the first is

Table B.2 Summary of new HELAS sub-routines introduced for the implementation of the KK mass integration in MADGRAPH/MADEVENT within the framework of the ADD model. Table extracted from [2]

Sub-routine	Summary
PXXXXX	stores the helicity and momentum of the auxiliary particles x_1 and x_2
TPSXXX	computes the amplitude of x_1x_2T vertex from a set of parameters specified by the theory
UPSXXX	computes an off-shell graviton tensor current U , made from the flowing-out auxiliary Scalar and Pseudo-particles, by the x_1x_2T vertex

defined as a scalar, and the second a pseudo-tensor. MADGRAPH is subsequently modified to include a new particle type P , which is only involved in the interaction with graviton: x_1x_2T .

For generate a process (exclusively for the ADD theory) in which a spin-2 particle is emitted, one should request for example $pp \rightarrow x_1x_2 + X$, instead of $pp \rightarrow T + X$. The phase spaces are related by

$$d\Phi(x_1x_2 + X) = \frac{dm_T^2}{16\pi^2} d\Phi(T + X). \quad (\text{B.20})$$

We can thus perform the mass integration with the help of the new HELAS subroutines for the x_1x_2T vertex summarized in Table B.2.

B.2.1 PXXXXX

This subroutine stores the helicity and momentum of the auxiliary spin-2 Pseudo-particle for further usage, and should be called as

```
CALL PXXXXX(P, XM, NHEL, NST, PC)
```

The input $P(0:3)$ is a real four-dimensional array which contains the four-momentum p^μ of the pseudo particle, $NHEL$ ($= \pm 2, \pm 1, 0$) specifies its helicity λ , NST specifies whether the boson is in the final state ($NST = 1$) or in the initial state ($NST = -1$). XM is its mass but does not play any role here.

The output $PC(18)$ is a complex 18-dimensional array, among which only the following matter, namely

$$PC(1) = NHEL, \quad (\text{B.21})$$

and

$$(PC(17), PC(18)) = NST (P(0) + iP(3), P(1) + iP(2)). \quad (\text{B.22})$$

B.2.2 TPSXXX

This subroutine computes the amplitude of the $x_1 x_2 T$ vertex, and should be called as

```
CALL TPSXXX(TC0, PC, SC, GP, TMASS0, TWIDTH, VERTEX)
```

SC (3) is a complex three-dimensional array which contains the momentum for x_1 . GP(1:2) here are not coupling constant, but contain the ADD model inputs:

$$\begin{aligned} GP(1) &= \Lambda + i\delta, \\ GP(2) &= M_{low} + iM_{up}, \end{aligned} \quad (B.23)$$

M_{low} and M_{up} are the lower and upper limits for graviton mass integration. TMASS0 (m_{T0} for below) is only temporarily used and got from the MG input file `param_card.dat`, while the real graviton mass m_T is defined as the invariant mass of x_1 and x_2 , i.e.,

$$m_T \equiv |q_1 + q_2|, \quad (B.24)$$

with q_1 and q_2 are the momenta of x_1 and x_2 .

Also note the input TC0 is indeed calculated from the other side of Feynman diagram, and contains the polarization summation tensor $\mathcal{P}_{\mu\nu\rho\sigma}$, which can be used to project out the real graviton wave function, due to the relations of the 5 helicity states of the tensor boson [4]:

$$\mathcal{P}_{\mu\nu,\alpha\beta}(p) = \sum_{\lambda=\pm 2, \pm 1, 0} \epsilon_{\mu\nu}(p, \lambda) \epsilon_{\alpha\beta}(p, \lambda)^*, \quad (B.25)$$

and

$$\epsilon^{\mu\nu}(p, \lambda) \epsilon_{\mu\nu}(p, \lambda')^* = \delta_{\lambda\lambda'}, \quad (B.26)$$

with $p \equiv q_1 + q_2$, and the helicity λ is got from the pseudo-particle's wavefunction, or Eq. (B.21).

Now we can get the true graviton tensor wave function by calling the subroutine

$$\text{CALL TXXXXX}(P, m_T, \lambda, +1, TC) \quad (B.27)$$

Finally, the output VERTEX is

$$\begin{aligned} \text{VERTEX} &= \frac{8\pi^2}{m_T} \rho(m_T) (p^2 - m_{T0}^2 + im_{T0}\Gamma_T) T_{\mu\nu} T_0^{\mu\nu} \\ &\times \theta(m_T - M_{low}) \theta(M_{up} - m_T), \end{aligned} \quad (B.28)$$

where θ represents the Heaviside step function. In equation (B.28), we include the mass density factor, the inverse graviton propagator, and the compensation factor for the decay phase space.

B.2.3 UPSXXX

This subroutine computes an off-shell graviton tensor current U , made from the flowing-out auxiliary Scalar and Pseudo-particles, by the $x_1 x_2 T$ vertex, and should be called as

```
CALL UPSXXX(PC, SC, GP, TMASS0, TWIDTH, UPS)
```

The output UPS is a complex 18-dimensional array, which is indeed got simply by calling inside UPSXXX

```
CALL TXXXXX(P,  $m_T$ ,  $\lambda$ , +1, UPS). (B.29)
```

and including the step functions in Eq. (B.28).

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Appendix C

ALOHA Automatic Libraries of Helicity Amplitudes for Feynman Diagram Computations

ALOHA is a package for the automation of the HELAS (HELicity Amplitude Subroutines) library [1] corresponding to the Feynman rules of any in quantum field theory Lagrangian.

From the input given by the Universal FeynRules Output (UFO) [2], ALOHA produces the complete set of routines, wave-functions and amplitudes, that are needed for the computation of Feynman diagrams. The code is written in Python, but the output language can be chosen by the user between Python, C++ and Fortran. Here,² we present how ALOHA works, and show a sample application implemented in the MADGRAPH 5 framework for theories containing spin-2.

C.1 Introduction

We have seen that due to the numerous quantity of possible BSM theories that could show up at the LHC, extensive efforts are being made to automate the process of going from the Lagrangian of a new theory, to simulating events that can be then compared with data. ALOHA is one important piece of that process with the ability to automatically create helicity amplitude routines necessary for calculating the matrix element associated with any new model. Such an ability will be useful to many groups writing custom generators, however, at present the greatest impact is on the recently released MADGRAPH 5 suite [4].

Helicity amplitudes methods [5–11] are a powerful way to evaluate the squared matrix element of any process. Several Monte-Carlo (MC) event generators, matrix-element based, use such routines already at the initial stage of the calculation.

² This appendix is a reproduction of excerpts and tables of [3]. Reprinted from Computer Physics Communications, Vol. 183/10, Priscila de Aquino, William Link, Fabio Maltoni, Olivier Mattelaer and Tim Stelzer, ALOHA: Automatic libraries of helicity amplitudes for Feynman diagram computations, Pages No. 2254 - 2263, Licence numbers 3126460314522 and 3126460539382 (2012), with permission from Elsevier.

The HELAS [1] library has been a particularly successful implementation of an helicity amplitude method for tree-level processes. Amplitudes are generated by initializing a set of external wavefunctions using their helicity and momenta. These wavefunctions are combined based on the particle interactions in the Lagrangian to determine the wavefunctions of the internal lines (propagators). Once all of the wavefunctions are determined, they are combined to calculate the complex number corresponding to the amplitude for the diagram. These amplitudes can then be added and squared to give the required result. The set of routines needed to calculate amplitudes for the SM at tree-level was released in the original HELAS and successfully employed for many phenomenological studies thereafter.

C.2 Automation of the Method

ALOHA basically uses the same strategy as these of HELAS. These routines consists of three categories:

External particle routines: Wavefunction routines corresponding to each external leg of a Feynman diagram. They are computed for a given momentum and helicity. The type of the particle must be given as an input. The choices are: vector, scalar, tensor or incoming/outgoing fermion.

Wavefunction/Off-shell routines: The propagator wave function, which is generated when one leg of a vertex has no associated external wave-function. These routines return both the internal wavefunction and the momentum of the propagator.

Amplitude/On-shell routines: The amplitude routines, which is computed after all the legs of a diagram are associated to a wave-function. These can be used to compute the matrix-element of the diagram by the associated amplitude routines.

To illustrate how ALOHA works from the beginning, let us start with the one example of vertex produced by the UFO. The UFO output for the $e^+e^-\gamma$ vertex is:

```
FFV = Lorentz(name = 'FFV',
              spins = [ 2, 2, 3 ],
              structure = 'Gamma(3,2,1)')
```

The `spins` attribute contains the list of spins of each particle (present in the vertex) in the $2*S+1$ scheme.³ The `structure` property collects the analytical expression for the Lorentz structure in the vertex. The arguments of the Lorentz object refer to the particle associated with each index of the object. The list of available objects and the Python convention for different indices are explained in Table C.1.

The UFO convention requires that:

³ In the case of a fermion, the first particle is considered as an incoming fermion and the second as an outgoing fermion. For interactions containing more than two fermions, the convention is that the vertex should be defined for a given fermion flow. The outgoing fermion follows directly its associated incoming fermion.

Table C.1 The greek indices stands for lorentz indices while the lower case latin indices stands for spin indices. The indices N stands for the label of the particle

Elementary Lorentz structures	
Charge conjugation matrix: C_{ab}	$C(a, b)$
Epsilon matrix: $\epsilon^{\mu\nu\rho\sigma}$	$Epsilon(\mu, \nu, \rho, \sigma)$
Gamma matrices: $(\gamma^\mu)_{ab}$	$Gamma(\mu, a, b)$
Gamma5 matrix: $(\gamma^5)_{ab}$	$Gamma5(a, b)$
(Spinorial) Kronecker delta: δ_{ab}	$Identity(a, b)$
Minkowski metric: $\eta_{\mu\nu}$	$Metric(\mu, \nu)$
Momenta of particle N : p_N^μ	$P(\mu, N)$
Right chiral projector: $\left(\frac{1+\gamma^5}{2}\right)_{ab}$	$ProjP(a, b)$
Left chiral projector $\left(\frac{1-\gamma^5}{2}\right)_{ab}$	$ProjM(a, b)$
Sigma matrices: $(\sigma^{\mu\nu})_{ab}$	$Sigma(\mu, \nu, a, b)$
Scalar wavefunction: ϕ_N	$Scalar(N)$
Spinor wavefunction: ψ_N^a	$Spinor(a, N)$
Vector wavefunction: ϵ_μ^N	$Vector(\mu, N)$
Spin2 wavefunction: $T_N^{\mu, \nu}$	$Spin2(\mu, \nu, N)$

- i. positive indices are linked to the particle numbers;
- ii. all Lorentz indices should be listed before spin indices;
- iii. all repeated indices should be summed with the appropriate matrix;
- iv. and all momenta should be incoming.

From a given UFO model structure, ALOHA creates the analytical expression linked to the associated HELAS routine by contracting the expression with a set of wavefunctions.

Now, we exemplify how ALOHA works by relating a particular vertex produced by ALOHA and HELAS. The structure shown in the FFV python object introduced above is `Gamma(3, 2, 1)`.

C.2.1 Amplitude/On-shell Routine

The associated amplitude routine, which was previously named `IOV` by the HELAS convention, is now:

```
Spinor(2) * Gamma(3, 2, 1) * Spinor(1) * Vector(3)
```

which correspond to the analytical expression:

$$FFV_0(\psi_1, \psi_2, \epsilon_{3\mu}, g) = g * \bar{u}_2 \gamma^\mu v_1 \epsilon_\mu. \quad (C.1)$$

The analytical module of ALOHA expands Eq. (C.1) component by component. The result is written to a C++, Python or Fortran file, language chosen by the user. For instance, the Fortran output would be:

```

C      This File is Automatically generated by ALOHA
C      The process calculated in this file is:
C      Gamma(3,2,1)
C
      SUBROUTINE FFV1_0(F1,F2,V3,COUP,VERTEX)
      IMPLICIT NONE
      DOUBLE COMPLEX F1(*)
      DOUBLE COMPLEX F2(*)
      DOUBLE COMPLEX V3(*)
      DOUBLE COMPLEX COUP
      DOUBLE COMPLEX VERTEX

      VERTEX = COUP*((F2(1)*((F1(3)*((0,-1)*V3(1)+(0,1)*V3(4)))
      $ +(F1(4)*((0,1)*V3(2)+V3(3)))))+(F2(2)*((F1(3)*((0,1)
      $ *V3(2)-V3(3)))+(F1(4)*((0,-1)*V3(1)+(0,-1)*V3(4))))
      $ +((F2(3)*((F1(1)*((0,-1)*V3(1)+(0,-1)*V3(4)))+(F1(2)
      $ *((0,-1)*V3(2)-V3(3)))))+(F2(4)*((F1(1)*((0,-1)*V3(2)
      $ +V3(3)))+(F1(2)*((0,-1)*V3(1)+(0,1)*V3(4))))))
      END

```

The convention for the naming of ALOHA's routines is denoted by NAME_X, where NAME is the name provided by UFO and X is either 0 for an amplitude routine or the number corresponding to the off-shell particle for a wavefunction routine.

C.2.2 Wavefunction/Off-shell Routines

To produce the necessary off-shell routines, ALOHA loops over the list of particles and replaces the on-shell wavefunction with a propagator. For particle 1, the incoming spinor is replaced by a propagator, computing⁴:

Spinor(2) * Gamma(3,2,1) * SpinorPropagator(1) * Vector(3)

which is analogous to:

$$\text{FFV}_1(\psi_2, \epsilon_{3\mu}, g, m_1, \Gamma_1) = g * \bar{\psi}_2 \gamma^\mu \cdot i \frac{\not{p} + m_1}{p^2 - m_1^2 + im_1\Gamma_1} \epsilon_{3\mu}. \quad (\text{C.2})$$

FFV_1 is the wavefunction of the first spinor, and would be FOV in the original HELAS language. The momentum for the propagator is computed by using conservation of momentum at the vertex.

Similarly, for the outgoing spinor, we have:

$$\text{FFV}_2(\psi_1, \epsilon_{3\mu}, g, m_2, \Gamma_2) = i \frac{\not{p} + m_2}{p^2 - m_2^2 + im_2\Gamma_2} \cdot i \gamma^\mu \psi_2 \epsilon_{3\mu}. \quad (\text{C.3})$$

⁴ The propagator object is defined internally to ALOHA.

The syntax used by ALOHA reads:

```
SpinorPropagator(2) * Gamma(3,2,1) * Spinor(1) * Vector(3)
```

which can be related to the FIV routine in the original HELAS language.

Finally, ALOHA also creates the routine associated to the off-shell vector. This routine returns a vector wavefunction from a pair of incoming and outgoing fermion wavefunctions, and it is written by ALOHA as:

```
Spinor(2) * Gamma(3,2,1) Spinor(1) * VectorPropagator(3)
```

analogous to the analytic expression FFV_3, which would be JIO in the original HELAS language:

$$\text{FFV}_3(\psi_1, \psi_2, g, m_3, \Gamma_3) = \bar{\psi}_2 i \gamma_\mu \psi_1 \cdot -i \frac{\eta^{\mu\nu} - \frac{p^\mu p^\nu}{m_3^2}}{p^2 - m_3^2 + im_3 \Gamma_3}. \quad (\text{C.4})$$

Following the original HELAS convention, it is used the unitary gauge for massive particles and the light cone gauge for massless particles.

C.2.3 External Particle Routines

ALOHA is not designed to create external particles routines, e.g., IXXXXX and OXXXXX for incoming and outgoing fermions, VXXXXX for vectors, SXXXXX for scalars and TXXXXX for spin-2 particles. These from HELAS are used. There is a relatively small number of such routines needed, and for this reason is reasonable to use a static library. This is incorporated into the ALOHA package, and it is available in Python, C++ and Fortran languages.

Two additional types of special routines are needed in order to cover the full range of routines required for any type of interaction (and model):

- **Conjugate Routines:** Required by the presence of majorana fermions and of fermion flow violation [12]. In the case of the previous example, the Lorentz expression would have to be replaced by:

```
C(-2,2) * Spinor(2) * Gamma(3,2,1) *
      Spinor(1) * Vector(3) * C(3,-3)
```

To distinguish conjugate routines from the standard ones, ALOHA adds to their name the letter C followed by the number of the fermion pair that is conjugated. In this case the name of the conjugate routine would be FFVC1_X with X being either 0, 1, 2 or 3 depending on the number corresponding to the amplitude or off-shell particle. Note that _1 (_2) returns the wavefunctions associated to the incoming (outcoming) fermion.

- **Multiple Couplings:** In the UFO scheme each Lorentz structure is associated to a single coupling. In practice, this means that some interactions are linked to more than one Lorentz structure. However, it is convenient for the readability and the compactness of the code to have one single routine per interaction. Therefore, ALOHA defines a series of wrapper functions which call different routines associated by their couplings. The name of these wrapper routines are the concatenation of the name of the associated routines combined by an underscore. If all of the associated routines start with the same prefix (with letter S, F, V, T) then this prefix is only kept the first time. For example, the wrapper function of `FFV1_X` and `FFV2_X` will be called `FFV1_2_X`.

In some cases, this method provides redundancy. Consider a four-Higgs-interaction. Due to the symmetry of the Lorentz structure, the four wavefunction routines will each have the same content. This type of symmetry is detected automatically by ALOHA and the computation is done only once. However, in order to simplify the work of the matrix element generator, all four routines are explicitly defined and have their own names. The first one is computed and the remaining three are written as an alias of the first one.

C.3 ALOHA Output

The major advantage of ALOHA comes from using it in combination with a MC event generator. To this aim, ALOHA offers two options. The first is to use ALOHA as a standalone program. The user can call ALOHA from the shell:

```
./bin/aloha UFO_path [ -f Fortran | Python | CPP ] [-o output_dir]
```

where `UFO_path` is the path to the UFO model directory, and the two possible arguments are

- the language in which the routine should be written (default is `Fortran`).
- the path for the output directory (which is by default `UFO_path/LANGUAGE`, where `LANGUAGE` can be either `Fortran`, `Python` or `C++`).

The second possibility is to internally link the ALOHA code to a MC event generator. This option is only possible for a generator written in `Python`, and is currently used by MADGRAPH version 5. For each process MADGRAPH requests a set of helicity routines, which is produced by ALOHA. This first implementation of ALOHA inside a matrix element generator played an essential role in fully validating the package.

C.4 Checks and Validation

To validate ALOHA, we use two different versions of MADGRAPH to compute values for the square of the matrix element, given a specific phase-space point of a known

process. We employ MADGRAPH version 4.4.44, which uses the original HELAS routines, and compare the results with those from MADGRAPH version 5.1.1.0, which uses the new ALOHA routines.

The comparison is made by specifying a point of phase space and a center-of-mass energy, after fixing properly the set of external parameters. The values for each matrix element squared are computed, and the difference is given by ΔD :

$$\Delta D = 2 \times \frac{|\mathcal{M}(\text{ALOHA})|^2 - |\mathcal{M}(\text{HELAS})|^2}{|\mathcal{M}(\text{ALOHA})|^2 + |\mathcal{M}(\text{HELAS})|^2}. \quad (\text{C.5})$$

Due to effects of numerical precision, one can not expect exact agreement between the two codes. Instead, the test is considered to pass if values of the squared matrix element computed by ALOHA and HELAS agree to at least five orders of magnitude, i.e., $\Delta D \leq 1 \times 10^{-5}$.

Such tests are carried out for different processes based on the Standard Model (SM), the Minimal Supersymmetric Model (MSSM) and the Randall-Sundrum type I (RS-I) model, the latter including spin-2 particles. Additional tests are made by checking gauge and Lorentz invariance of each process. We observe a great agreement for all processes, as exemplified in Table C.2 for the SM and the MSSM.

C.4.1 Spin-2 Validation Checks

We have seen that several models with extra dimensions have recently been proposed. Examples of such theories include the Large Extra Dimensional model (or ADD) [13–15] and Randall-Sundrum (RS) models [16, 17]. These models contain spin-2 particles allowed to propagate through higher dimensional space-time that couple to the SM particles in a very particular way. To complement the implementation of the ALOHA helicity routines, it is interesting to test and validate spin-2 routines comparing it to the HELAS implementation [18].

Using FEYNRULES, we implement a reduced version of the RS model [19]: it contains only the first mode of the spin-2 particle explicitly defined (which is denominated y), which couples to SM particles. Validation tests, similar to the ones presented above for the SM and MSSM, are performed. Approximately 500 $2 \rightarrow 2$ and 2000 $2 \rightarrow 3$ processes are tested, and some examples can be found in Table C.3. We obtain a full agreement between routines obtained with the use of HELAS and ALOHA.

C.4.2 Goldstino Validation checks

Similarly to what has been performed for the SM, MSSM and RS theories, a simplified SUSY theory that includes goldstino interactions has been implemented in MADGRAPH/MADEVENT via the FEYNRULES package [20]. Since the goldstino is the helicity $\pm 1/2$ component of the gravitino, it can be treated by ALOHA. A com-

Table C.2 Example of the results obtained by the validation tests for SM and MSSM processes

SM processes		MSSM processes	
Process	ΔD	Process	ΔD
$e^+ e^- \rightarrow \gamma \gamma$	1.512×10^{-7}	$e_R^+ W^+ \rightarrow e_R^+ W^+$	5.544×10^{-8}
$e^- \bar{\nu}_e \rightarrow \gamma W^-$	4.541×10^{-8}	$H_2 W^+ \rightarrow H_3 W^+$	1.108×10^{-8}
$\gamma e^- \rightarrow \nu_e W^-$	4.545×10^{-8}	$e_L^+ W^- \rightarrow e_L^+ H^-$	5.285×10^{-9}
$b e^- \rightarrow b e^-$	7.310×10^{-8}	$\nu_e \bar{\nu}_e \rightarrow H^+ H^-$	6.214×10^{-9}
$e^+ \bar{\nu}_e \rightarrow e^+ \bar{\nu}_e$	3.911×10^{-8}	$e_L^+ \bar{\nu}_e \rightarrow e_L^+ \bar{\nu}_e$	5.987×10^{-8}
$e^+ Z \rightarrow \gamma e^+$	5.458×10^{-8}	$e^+ e_L^- \rightarrow s \nu_e \bar{\nu}_e$	1.792×10^{-9}
$\nu_e \bar{\nu}_e \rightarrow e^+ e^-$	7.164×10^{-8}	$e^+ e_R^- \rightarrow H^- x_1^+$	6.679×10^{-9}
$\nu_e W^- \rightarrow \gamma e^-$	4.545×10^{-8}	$e_L^+ e_L^- \rightarrow \mu^+ \mu^-$	1.984×10^{-9}
$\bar{\nu}_e Z \rightarrow \bar{\nu}_e Z$	$< 1.0 \times 10^{-10}$	$e_L^- x_1^+ \rightarrow H_2 \nu_e$	3.028×10^{-8}
$\gamma g \rightarrow t t$	7.528×10^{-8}	$s \nu_e x_1^- \rightarrow \nu_e W^-$	3.714×10^{-8}
$g t \rightarrow t Z$	3.173×10^{-8}	$H^- s \nu_e \rightarrow \nu_e x_1^-$	1.039×10^{-8}
$t t \rightarrow W^+ W^-$	1.833×10^{-8}	$e_R^+ H^- \rightarrow e_R^+ W^-$	7.992×10^{-9}
$b W^- \rightarrow b W^-$	6.291×10^{-9}	$x_1^+ x_1^+ \rightarrow x_1^+ x_1^+$	2.863×10^{-8}
$W^+ W^- \rightarrow Z Z$	1.666×10^{-7}	$H_2 x_1^- \rightarrow e^- s \bar{\nu}_e$	1.527×10^{-7}
$Z Z \rightarrow e^+ e^-$	1.162×10^{-7}	$e_L^- W^+ \rightarrow \gamma e^- x_1^+$	6.045×10^{-7}
$e^- W^- \rightarrow e^- W^- Z$	9.767×10^{-8}	$u_R W^- \rightarrow g u_R W^-$	5.029×10^{-9}
$\nu_e Z \rightarrow \gamma e^- W^+$	2.692×10^{-8}	$n_1 Z \rightarrow u_R \bar{u} Z$	8.805×10^{-10}
$u \bar{\nu}_e \rightarrow \gamma u \bar{\nu}_e$	1.179×10^{-7}	$\gamma e_L^- \rightarrow e^- s \nu_e \bar{\nu}_e$	4.001×10^{-9}
$t \bar{\nu}_e \rightarrow e^+ t W^-$	8.095×10^{-8}	$e^- x_1^+ \rightarrow \nu_e W^+ x_1^-$	3.355×10^{-7}
$\bar{\nu}_e W^+ \rightarrow b b e^+$	8.430×10^{-8}	$s \bar{\nu}_e x_1^+ \rightarrow e^+ \nu_e \bar{\nu}_e$	6.548×10^{-8}
$\gamma d \rightarrow d e^+ e^-$	6.045×10^{-8}	$e^+ x_1^- \rightarrow s \bar{\nu}_e u_R \bar{u}_R$	1.116×10^{-10}
$\bar{u} Z \rightarrow e^+ e^- \bar{u}$	3.816×10^{-8}	$n_1 s \nu_e \rightarrow \nu_e \nu_e \bar{\nu}_e$	2.176×10^{-9}
$d \bar{d} \rightarrow H \nu_e \bar{\nu}_e$	2.566×10^{-8}	$g u_L \rightarrow e^- e_R^+ u$	3.677×10^{-10}
$H Z \rightarrow g t t$	2.457×10^{-8}	$g o g o \rightarrow u_R \bar{u}_R Z$	1.089×10^{-7}
$t t \rightarrow u \bar{u} Z$	3.962×10^{-8}	$g o u_R \rightarrow u Z Z$	1.526×10^{-10}
$t W^- \rightarrow e^- t \bar{\nu}_e$	4.744×10^{-8}	$\bar{u} \nu_e \rightarrow g s \nu_e \bar{u}_L$	1.233×10^{-8}
$b t \rightarrow \gamma b t$	7.409×10^{-8}	$e^+ u_L \rightarrow e^+ u_L Z$	1.542×10^{-8}
$t Z \rightarrow d \bar{d} t$	2.224×10^{-8}	$e_R^- \bar{u}_R \rightarrow e^- g \bar{u}$	2.204×10^{-7}
$b Z \rightarrow b Z Z$	8.196×10^{-8}	$e^- e_L^+ \rightarrow n_1 u \bar{u}$	2.122×10^{-8}
$W^+ W^- \rightarrow d \bar{d} H$	9.519×10^{-8}	$\bar{\nu}_e \bar{\nu}_e \rightarrow e_L^+ \bar{\nu}_e x_1^-$	4.661×10^{-8}

parison of the values of the matrix element squared obtained by ALOHA against these obtained with the use of HELAS [21, 22] has been performed. A full agreement is again obtained.

The next step is to add the treatment of spin-3/2 particles by ALOHA, which is not yet completed. However, since in Chap. 6 we have shown that goldstino particles are the relevant components of gravitino production at the LHC, for the phenomenological studies of gravitino emission (at the LHC) it is enough that MADGRAPH/MADEVENT handle goldstino particles.

Table C.3 Example of the results obtained by the validation tests for RS processes

RS processes	
Process	ΔD
$gu \rightarrow uy$	$< 1.0 \times 10^{-10}$
$by \rightarrow bg$	$< 1.0 \times 10^{-10}$
$\bar{t}y \rightarrow g\bar{t}$	$< 1.0 \times 10^{-10}$
$gy \rightarrow t\bar{t}$	$< 1.0 \times 10^{-10}$
$gt \rightarrow ty$	$< 1.0 \times 10^{-10}$
$gg \rightarrow gy$	$< 1.0 \times 10^{-10}$
$\bar{t}y \rightarrow gg\bar{t}$	9.766×10^{-10}
$gg \rightarrow ggy$	3.979×10^{-10}
$bg \rightarrow bgy$	8.009×10^{-10}
$b\bar{b} \rightarrow b\bar{b}y$	1.864×10^{-9}
$\bar{b}\bar{t} \rightarrow \bar{b}\bar{t}y$	7.411×10^{-10}
$t\bar{t} \rightarrow b\bar{b}y$	1.928×10^{-10}

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Curriculum Vitae

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Student at Universidade Estadual de Campinas (Campinas State University), Faculty of Physics. Received the Bachelor degree in Physics.

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