Symbols

a – outlet eccentricity [m]

$2a$ – silo width [m]

a_c – micro-polar constant [–]

a_i – micro-polar coefficients [–]

A – cross-section area [m^2]

$2b$ – silo depth [m]

h – height [m]

b – width [m]

c – cohesion [kPa]

C_c – compression index [–]

C_s – swelling index [–]

d – silo diameter [m]

d_{kl}^{c} – polar rate of deformation [–]

d_{50} – mean particle diameter [m]

e – void ratio [–]

e_{i0} – maximum void ratio at pressure equal to zero [–]

e_{d0} – minimum void ratio at pressure equal to zero [–]

e_{c0} – critical void ratio at pressure equal to zero [–]

e_f – filling eccentricity [m]

e_o – outlet eccentricity [m]

E – modulus of elasticity [kPa]

f – yield function [kPa]

f_d – density factor [–]

f_s – stiffness factor [–]

g – potential function, gravitational acceleration [m/s^2]

h_s – granular hardness [kPa]

k_k – rate-of-curvatures [1/m]

K – lateral pressure coefficient [–]

K_0 – pressure coefficient at rest [–]

$\overline{K}$ – mean pressure coefficient [–]

l – depth [m]

l_c – characteristic length of micro-structure [m]

m – silo constant, non-locality parameter [–]

m_i – couple stress vector [kN/m]

m_1 – horizontal couple stress [kN/m]

m_2 – vertical couple stress [kN/m]

M – pressure parameter [–]
N – resultant normal force [kN]
p – mean stress [kPa]
p_b – vertical pressure on horizontal silo bottom [kPa]
p_h – horizontal pressure [kPa]
p_n – pressure perpendicular to wall surface [kPa]
p_v – vertical pressure [kPa]
p_w – vertical friction traction [kPa]
p_L – overpressure [kPa]
P – resultant vertical force in solid [kN]
r – silo radius [m]
r_w – wall roughness [m]
s – height of local patch zone [m]
s_{ij} – non-symmetric deviatoric stress tensor [kPa]
t – wall thickness [m]
T – resultant wall friction force [kN]
u – internal circumference [m]
u – vertical bottom displacement [m]
u_1 – horizontal displacement [m]
u_2 – vertical displacement [m]
U – internal circumference of cross-section [m]
w – vertical displacement of upper free boundary [m]
w^c – rate of Cosserat rotation [1/s]
x_1 – horizontal co-ordinate [m]
x_2 – vertical co-ordinate [m]
z – vertical co-ordinate in silos [m]
α – wall inclination to horizontal in hoppers [°]
β – angle of dilatancy [^{o}], non-uniformity coefficient [–]
δ – effective internal friction angle, solid slope inclination [°]
ε_{ij} – deformation tensor [–]
$\dot{\varepsilon}_{ij}$ – rate of deformation [1/s]
λ – pressure coefficient [–]
ϕ – internal friction angle [°]
φ_w – wall friction angle [°]
ϕ – angle of internal friction [°]
ϕ_i – angle of repose [°]
γ – unit weight [kN/m^3]
ω – non-polar rotation [–], weighting function [–]
ω^c – Cosserat rotation [–]
σ_{ij} – stress tensor [kPa]
σ_{11} – horizontal normal stress [kPa]
σ_{22} – vertical normal stress [kPa]
σ_{12} – horizontal shear stress [kPa]
σ_{21} – vertical normal stress [kPa]
σ_{33} – normal stress perpendicular to deformation plane [kPa]

σ_z – mean vertical normal stress [kPa]
σ_n – horizontal wall normal stress [kPa]
σ_s – compressive stress in arch [kPa]
σ_2^o – unconfined yield strength [kPa]
σ_i – principal stresses [kPa]
τ – vertical shear wall stress, second invariant of deviatoric stress tensor [kPa]
θ_w – wall inclination to vertical in hoppers [°]
μ – mobilised friction factor, wall friction coefficient [–]
χ – silo coefficient [–]
ν – Poisson's ratio [–]
γ^p – plastic shear deformation [–]

Summary
Confined Granular Flow in Silos Experimental and Numerical Investigations

During confined flow of bulk solids in silos some characteristic phenomena can be created, such as:

- sudden and significant increase of wall stresses,
- different flow patterns,
- formation and propagation of wall and interior shear zones,
- fluctuation of pressures and,
- strong autogeneous coupled dynamic-acoustic effects.

These phenomena have not been described or explained in detail yet. The main intention of the experimental and theoretical research presented in the book is to explain the above mentioned phenomena in granular bulk solids and to describe them with numerical FE models verified by experimental results.

Laboratory and full-scale silo tests were carried out in order to study the behaviour of bulk solids during quasi-static and dynamic silo problems. Different technical methods were proposed to suppress strong dynamic effects. Deformations in bulk solids were experimentally measured using 3 different non-invasive methods: particle image velocimetry PIV (digital image correlation DIC), electrical capacitance tomography (ECT) and X-ray tomography.

To describe the behaviour of granular material during incipient and advanced silo flow, a micro-polar hypoplastic, non-local hypoplastic and micro-polar elasto-plastic constitutive model were applied using two different descriptions of motion: Updated Lagrangian approach and Arbitrary Lagrangian-Eulerian approach. Quasi-static and dynamic simulations were carried out for model and large silos. Silo flow with controlled outflow and free outlet velocity were studied. The effect of silo inserts on the solid behaviour was analyzed. Attention was paid to the propagation of shear zones along silo walls and in the interior of the bulk solid. The numerical FE results were compared with the corresponding tests. In addition, flow patterns in silos were also analyzed using cellular automata.