

References

- Abeyratne, M.K., Freeden, W., Mayer, C.: Multiscale deformation analysis by Cauchy-Navier wavelets. *J. Appl. Math.* **2003**(12), 605–645 (2003)
- Abramowitz, M., Stegun, I.A.: *Handbook of Mathematical Functions*. Dover Publications, New York (1972)
- Anger, G., Gorenflo, R., Jochmann, H., Moritz, H., Webers, W.: *Inverse Problems: Principles and Applications in Geophysics, Technology, and Medicine*. Mathematical Research, vol. 74. Akademie Verlag, Berlin (1993)
- Ansonge, R., Sonar, T.: *Mathematical Models of Fluid Dynamics*. Wiley-VCH, Weinheim (2009)
- Aronszajn, N.: Theory of reproducing kernels. *Trans. Am. Math. Soc.* **68**, 337–404 (1950)
- Aronszajn, N., Creese, T.M., Lipkin, L.J.: *Polyharmonic Functions*. Clarendon, Oxford (1983)
- Artin, E.: *The Gamma Function*. Holt, Rinehart and Winston, New York (1964)
- Atkinson, K., Han, W.: *Spherical Harmonics and Approximations on the Unit Sphere: An Introduction*. Lecture Notes in Mathematics, vol. 2044. Springer, Heidelberg (2012)
- Backus, G.E.: Converting vector and tensor equations to scalar equations in spherical coordinates. *Geophys. J. R. Astron. Soc.* **13**, 61–101 (1967)
- Backus, G.E.: Poloidal and toroidal fields in geomagnetic field modelling. *Rev. Geophys.* **24**, 75–109 (1986)
- Backus, G.E., Parker, R., Constable, C.: *Foundations of Geomagnetism*. Cambridge University Press, Cambridge (1996)
- Ballani, L., Engels, J., Grafarend, E.W.: Global base functions for the mass density in the interior of a massive body (Earth). *Manuscr. Geod.* **18**, 99–114 (1993)
- Bayer, M., Freeden, W., Maier, T.: A vector wavelet approach in iono- and magnetospheric geomagnetic satellite data. *J. Atmos. Sol. Terr. Phys.* **63**, 581–597 (2001)
- Bauch, H.: *Approximationssätze für die Lösung der Grundgleichung der Elastostatik*. Ph.D. thesis, Rheinisch-Westfälische Technische Hochschule (RWTH) Aachen (1981)
- Beatson, R.K., Greengard, L.: A short course on fast multipole methods. In: Light, W., Ainsworth, M., Levesley, J., Marletta, M. (eds.) *Wavelets, Multilevel Methods, and Elliptic PDEs*, pp. 1–37. Oxford University Press, Oxford (1997)
- Benedetto, J.J.: Frame decompositions, sampling, and uncertainty principle inequalities. In: Benedetto, J.J., Frazier, M.W. (eds.) *Wavelets: Mathematics and Applications*, pp. 247–304. CRC, Boca Raton (1994)
- Benedetto, J.J.: *Harmonic Analysis and Applications*. CRC, Boca Raton (1996)
- Benedetto, J.J., Zayed, A.I. (eds.): *Sampling, Wavelets, and Tomography*. Birkhäuser, Boston (2004)
- Berman, C.L., Greengard, L.: A renormalization method for the evaluation of lattice sums. *J. Math. Phys.* **35**(11), 6036–6048 (1994)

- Bessel, F.W.: Untersuchung des Theils der planetarischen Störungen, welcher aus der Bewegung der Sonne entsteht. *Berliner Abh.*, pp. 1–52 (1824)
- Biedenharn, L.C., Louck, J.D.: *Angular Momentum in Quantum Physics (Theory and Application)*. Encyclopedia of Mathematics and Its Applications. Addison-Wesley, Reading (1981)
- Blakely, R.J.: *Potential Theory in Gravity and Magnetic Applications*. Cambridge University Press, Cambridge (1996)
- Blatt, J., Weisskopf, V.: *Theoretical Nuclear Physics*. Wiley, New York (1952)
- Borwein, D., Borwein, J.M., Shail, R.: Analysis of certain lattice sums. *J. Math. Anal. Appl.* **143**, 126–137 (1989)
- Brackx, F., Delanghe, R.: On harmonic potential fields and the structure of monogenic functions. *J. Anal. Appl.* **22**, 261–273 (2003)
- Brackx, F., Delanghe, R., Sommen, F.: *Clifford Analysis*. Research Notes in Mathematics, vol. 76. Pitman, Boston (1982)
- Braun, M.: Laguerre polynomials and the vibrations of a multiple pendulum. *SIAM OP-SF Newsl.* **7**(3), 17–20 (1997)
- Brink, D.M., Satchler, G.R.: *Angular Momentum*. Oxford Library of the Physical Sciences. Clarendon, Oxford (1968)
- Butzer, P.L., Nessel, R.: *Fourier Analysis and Approximation Theory*. Birkhäuser, Basel (1971)
- Cassels, J.W.S.: *An Introduction to the Geometry of Numbers*. Springer, Berlin/Heidelberg/New York (1968)
- Cheng, H., Greengard, L., Rokhlin, V.: A fast adaptive multipole algorithm in three dimensions. *J. Comput. Phys.* **155**, 468–498 (1999)
- Cherrie, J.B., Beatson, R.K., Newsam, G.N.: Fast evaluation of radial basis functions: methods for generalised multiquadrics in \mathbb{R}^n . *SIAM J. Sci. Comput.* **23**(5), 1549–1571 (2002)
- Choi, C.H., Ivancic, J., Gordon, M.S., Ruedenberg, K.: Rapid and staple determination of rotation matrices between spherical harmonics by direct recursion. *J. Chem. Phys.* **111**(19), 8825–8831 (1999)
- Ciarlet, P.G.: *Mathematical Elasticity: Volume I: Three-Dimensional Elasticity*. Studies in Mathematics and Its Applications. North-Holland, Amsterdam (1994)
- Clenshaw, C.W.: A note on the summation of Chebyshev series. *Math. Table Wash.* **9**, 118–120 (1955)
- Clifford, W.K.: Applications of Grassmann's extensive algebra. *Am. J. Math.* **1**, 350–358 (1878)
- Cohen, L.: *Time Frequency Analysis: Theory and Applications*. Prentice Hall, Englewood Cliffs (1995)
- Colton, D., Kress, R.: *Inverse Acoustic and Electromagnetic Scattering Theory*. Applied Mathematical Sciences, vol. 93, 2nd edn. Springer, Berlin/Heidelberg/New York (1998)
- Cooley, J.W., Tukey, O.W.: An algorithm for the machine calculation of complex Fourier series. *Math. Comput.* **19**, 297–301 (1965)
- Courant, R., Hilbert, D.: *Methoden der Mathematischen Physik I, II*, 2nd edn. Springer, Berlin (1968)
- Cui, J., Freeden, W.: Equidistribution on the sphere. *SIAM J. Sci. Stat. Comput.* **18**(2), 595–609 (1997)
- Davis, P.J.: *Interpolation and Approximation*. Blaisdell Publishing Company, Waltham (1963)
- Davis, P.J., Rabinowitz, P.: *Numerical Integration*. Blaisdell, Toronto/London (1967)
- De Bie, H., Sommen, F.: Spherical harmonics and integration in superspace. *J. Phys. A* **40**(26), 7193–7212 (2007)
- De Bie, H., Eelbode, D., Sommen, F.: Spherical harmonics and integration in superspace: II. *J. Phys. A* **42**(24), 245204 (2009)
- Delanghe, R.: On regular-analytic functions with values in a Clifford algebra. *Math. Ann.* **185**, 91–111 (1970)
- Delanghe, R.: Clifford analysis: history and perspective. *Comput. Method Funct. Theory* **1**, 107–153 (2001)
- Deuffhard, P.: On algorithms for the summation of certain special functions. *Computing* **17**, 37–48 (1976)

- Deuffhard, P., Hohmann, A.: *Numerische Mathematik*. de Gruyter, Berlin/New York (1991)
- Dressler, A.: Über die ungleichförmige Verteilung von Gitterpunkten in ebenen Bereichen. *Math. Nachr.* **52**, 1–20 (1972)
- Dufour, H.M.: Fonctions Orthogonales dans la Sphère—Résolution Théoretique du Problème du Potentiel Terrestre. *B. Geod.* **51**, 227–237 (1977)
- Edmonds, A.R.: *Drehimpulse in der Quantenmechanik*. Bibliographisches Institut, Mannheim (1964)
- Engl, H.W., Louis, A.K., Rundell, W. (eds.): *Inverse Problems in Geophysical Applications*. SIAM, Philadelphia (1997)
- Engl, H.W., Hanke, M., Neubauer, A.: *Regularization of Inverse Problems*. Kluwer Academic, Dordrecht (2000)
- Epstein, P.S.: Zur Theorie allgemeiner Zetafunktionen I. *Math. Ann.* **56**, 615–644 (1903)
- Epstein, P.S.: Zur Theorie allgemeiner Zetafunktionen II. *Math. Ann.* **63**, 205–216 (1907)
- Epton, M.A., Dembart, B.: Multipole translation theory for the three-dimensional Laplace and Helmholtz equations. *SIAM J. Sci. Comput.* **16**(4), 865–897 (1995)
- Erdős, P., Gruber, M., Hammer, J.: *Lattice Points. Pitman Monographs and Surveys in Pure and Applied Mathematics*, vol. 39. Longman Scientific-Technical/Wiley, New York (1989)
- Euler, L.: Methodus universalis serierum convergentium summas quam proxime inveniendi. *Commentarii Academiae Scientiarum Petropolitanae* **8**, 3–9, *Opera Omnia* (XIV), 101–107 (1736a)
- Euler, L.: Methodus universalis series summandi ulterius promota. *Commentarii Academiae Scientiarum Petropolitanae* **8**, 147–158; *Opera Omnia* (XIV): 124–137 (1736b)
- Evans, M., Hastings, N., Peacock, B.: *Statistical Distributions*, 3rd edn. Wiley, New York (2000)
- Ewald, P.P.: Die Berechnung optischer und elektrostatischer Gitterpotentiale. *Ann. Phys.* **64**, 253–287 (1921)
- Fengler, M.J.: *Vector spherical harmonic and vector wavelet based non-linear Galerkin schemes for solving the incompressible Navier–Stokes equation on the sphere*. Ph.D. thesis, Geomatics Group, TU Kaiserslautern, Shaker, Aachen, 2005
- Fengler, M.J., Freeden, W.: A non-linear Galerkin scheme involving vector and tensor spherical Harmonics for solving the incompressible Navier–Stokes equation on the sphere. *SIAM J. Sci. Comput.* **27**, 967–994 (2005)
- Fengler, M.J., Freeden, W., Gutting M.: Darstellung des Gravitationsfelds und seiner Funktionale mit Multiskalentechniken. *Zeitschrift für Geodäsie, Geoinformation und Landmanagement (ZfV)* **129**(5), 323–334 (2004)
- Fengler, M.J., Freeden, W., Gutting, M.: The spherical Bernstein wavelet. *Int. J. Pure Appl. Math.* **31**(2), 209–230 (2006)
- Freeden, W.: Über eine Klasse von Integralformeln der Mathematischen Geodäsie. Veröffentlichung des Geodätischen Instituts der Rheinisch-Westfälischen Technischen Hochschule (RWTH) Aachen, vol. 27. Aachen (1979)
- Freeden, W.: On integral formulas of the (unit) sphere and their application to numerical computation of integrals. *Computing* **25**, 131–146 (1980a)
- Freeden, W.: Über die Gaußsche Methode zur angenäherten Berechnung von Integralen. *Math. Method Appl. Sci.* **2**, 397–409 (1980b)
- Freeden, W.: On spherical spline interpolation and approximation. *Math. Method Appl. Sci.* **3**, 551–575 (1981)
- Freeden, W.: Multidimensional Euler summation formulas and numerical cubature. *Int. Ser. Num. Monogr.* **57**, 77–88 (1982)
- Freeden, W.: Spherical spline interpolation: basic theory and computational aspects. *J. Comput. Appl. Math.* **11**, 367–375 (1984)
- Freeden, W.: Interpolation by multidimensional periodic splines. *J. Approx. Theory* **55**, 104–117 (1988)
- Freeden, W.: Some applications of approximation theory to the first boundary value problem of elastostatics. *Int. Ser. Num. Monogr.* **94**, 121–129 (1990)
- Freeden, W.: *Multiscale Modelling of Spaceborne Geodata*. B.G. Teubner, Leipzig (1999)

- Freeden, W.: *Metaharmonic Lattice Point Theory*. Chapman & Hall/CRC, Boca Raton (2011)
- Freeden, W., Fleck, J.: Numerical integration by means of adapted Euler summation formulas. *Numer. Math.* **51**, 37–64 (1987)
- Freeden, W., Gerhards, C.: *Geomathematically Oriented Potential Theory*. Chapman & Hall/CRC, Boca Raton (2012)
- Freeden, W., Gutting, M.: On the completeness and closure of vector and tensor spherical harmonics. *Integral Transform. Spec. Funct.* **19**, 713–734 (2008)
- Freeden, W., Hermann, P.: Uniform approximation by spherical spline interpolation. *Math. Z.* **193**, 265–275 (1986)
- Freeden, W., Hesse, K.: On the multiscale solution of satellite problems by use of locally supported kernel functions corresponding to equidistributed data on spherical orbits. *Stud. Sci. Math. Hung.* **39**, 37–74 (2002)
- Freeden, W., Michel, V.: *Multiscale Potential Theory (with Applications to Geoscience)*. Birkhäuser, Boston/Basel/Berlin (2004)
- Freeden, W., Michel, V.: Wavelet deformation analysis for spherical bodies. *Int. J. Wavelet Multi.* **3**, 523–558 (2005)
- Freeden, W., Nutz, H.: Satellite gravity gradiometry as tensorial inverse problem. *Int. J. Geomath.* **2**, 177–218 (2011)
- Freeden, W., Reuter, R.: A class of multidimensional periodic splines. *Manuscr. Math.* **35**, 371–386 (1981)
- Freeden, W., Reuter, R.: Remainder terms in numerical integration formulas of the sphere. *Int. Ser. Num. Monogr.* **61**, 151–170 (1982)
- Freeden, W., Reuter, R.: Exact computation of spherical harmonics. *Computing* **32**, 365–378 (1984)
- Freeden, W., Reuter, R.: An efficient algorithm for the generation of homogeneous harmonic polynomials. In: Cox, M.G., Mason, J.C. (eds.) *Scientific Software Systems*, pp. 166–180. Chapman & Hall, London (1990)
- Freeden, W., Schreiner, M.: Multiresolution analysis by spherical up functions. *Constr. Approx.* **23**, 241–259 (2006)
- Freeden, W., Schreiner, M.: *Spherical Functions of Mathematical Geosciences, A Scalar, Vectorial, and Tensorial Setup*. Springer, Berlin/Heidelberg (2009)
- Freeden, W., Gervens, T., Mason, J.C.: A minimum norm interpolation method for determining the displacement field of a homogeneous isotropic elastic body from discrete data. *IMA J. Appl. Math.* **44**, 55–76 (1990)
- Freeden, W., Schreiner, M., Franke, R.: A survey on spherical spline approximation. *Surv. Math. Ind.* **7**, 29–85 (1997)
- Freeden, W., Gervens, T., Schreiner, M.: *Constructive Approximation on the Sphere (with Applications to Geomathematics)*. Oxford Science Publications/Clarendon, Oxford (1998)
- Freeden, W., Nashed, M.Z., Sonar, T. (eds.): *Handbook of Geomathematics*, vol. 1+2. Springer, Berlin/Heidelberg (2010)
- Freund, R.W., Hoppe, R.H.W.: *Stoer/Bulirsch: Numerische Mathematik 1*, 10th edn. Springer, Berlin/Heidelberg (2007)
- Fricker, F.: Geschichte des Kreisproblems. *Mitt. Math. Sem. Giessen* **111**, 1–34 (1975)
- Fricker, F.: Einführung in die Gitterpunktlehre. Birkhäuser, Basel (1982)
- Funk, H.: Beiträge zur Theorie der Kugelfunktionen. *Math. Ann.* **77**, 136–152 (1916)
- Gauß, C.F.: *Disquisitiones Arithmetica*. Leipzig (1801)
- Gauß, C.F.: De nexu inter multitudinem classicum, in quas formae binariae secundi grauds distribuuntur, earumque determinanem. *Werke* **2**, 269–291 (1826)
- Gautschi, W.: *Orthogonal Polynomials, Computation and Approximation*. Oxford University Press, Oxford (2004)
- Gerhards, C.: Spherical multiscale methods in terms of locally supported wavelets: theory and application to geomagnetic modeling. Ph.D. thesis, Geomathematics Group, TU Kaiserslautern (2011)

- Gervens, T.: Vektorkugelfunktionen mit Anwendungen in der Theorie der elastischen Verformungen für die Kugel. Ph.D. thesis, Rheinisch-Westfälische Technische Hochschule (RWTH) Aachen (1989)
- Gill, A.E.: *Atmosphere-Ocean Dynamics*. Academic, New York (1982)
- Górski, K.M., Hivon, E., Banday, A.J., Wandelt, B.D., Hansen, F.K., Reinecke, M., Bartelmann, M.: HEALPix: a framework for high-resolution discretization and fast analysis of data distributed on the sphere. *Astrophys. J.* **622**, 759–771 (2005)
- Grafarend, E.W.: Three-dimensional deformation analysis: global vector spherical harmonic and local finite element representation. *Tectonophysics* **130**, 337–359 (1986)
- Grafarend, E.W., Klapp, M., Martinec, Z.: Spacetime modeling of the Earth's gravity field by ellipsoidal harmonics. In: Freedon, W., Nashed, M.Z., Sonar, T. (eds.) *Handbook of Geomathematics*, vol. 1, pp. 159–252. Springer, Berlin/Heidelberg (2010)
- Greengard, L.: *The Rapid Evaluation of Potential Fields in Particle Systems*. MIT, Cambridge (1988)
- Greengard, L., Rokhlin, V.: A fast algorithm for particle simulations. *J. Comput. Phys.* **73**(1), 325–348 (1987)
- Greengard, L., Rokhlin, V.: Rapid Evaluation of Potential Fields in Three Dimensions. In: Anderson, C., Greengard, L. (eds.) *Vortex Methods*, pp. 121–141. Springer, Berlin (1988)
- Greengard, L., Rokhlin, V.: A new version of the fast multipole method for the Laplace equation in three dimensions. *Acta Numer.* **6**, 229–269 (1997)
- Greville, T.N.E.: Introduction to spline functions. In: Greville, T.N.E. (ed.) *Theory and Applications of Spline Functions*, pp. 1–35. Academic, New York (1969)
- Groten, E.: *Geodesy and the Earth's Gravity Field*, vol. I and II. Dümmler, Bonn (1979)
- Gürlebeck, K., Sprößig, W.: *Quaternionic Analysis and Elliptic Boundary Value Problems*. Mathematical Research, vol. 56. Akademie Verlag, Berlin (1989)
- Gürlebeck, K., Habetha, K., Sprößig, W.: *Holomorphic Functions in the Plane and n -Dimensional Space*. Birkhäuser, Basel/Boston/Berlin (2008)
- Gurtin, M.E.: *The Linear Theory of Elasticity*. *Handbuch der Physik*, vol. 6, 2nd edn. Springer, Heidelberg (1972)
- Gutting, M.: Fast multipole methods for oblique derivative problems. Ph.D. thesis, Geomathematics Group, TU Kaiserslautern, Shaker, Aachen (2008)
- Gutting, M.: Fast multipole accelerated solution of the oblique derivative boundary value problem. *Int. J. Geomath.* **3**(2), 223–252 (2012)
- Haar, A.: Zur Theorie der orthogonalen Funktionensysteme. *Math. Ann.* **69**, 331–371 (1910)
- Hämmerlin, G., Hoffmann, K.-H.: *Numerische Mathematik*. Springer, Berlin/Heidelberg/New York (1992)
- Hamilton, W.R.: *Elements of Quaternions*. Longmans Green, London (1866)
- Hardy, G.H.: On the expression of a number as the sum of two squares. *Q. J. Math. (Oxford)* **46**, 263–283 (1915)
- Hardy, G.H., Landau, E.: The lattice points of a circle. *Proc. R. Soc. A* **105**, 244–258 (1924)
- Hartman, P., Wilcox, C.: On solutions of the Helmholtz equation in exterior domains. *Math. Z.* **75**, 228–255 (1961)
- Hecke, E.: Über orthogonal-invariante Integralgleichungen. *Math. Ann.* **78**, 398–404 (1918)
- Heisenberg, W.: Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. *Z. Phys. A Hadron. Nucl.* **43**(3), 172–198 (1927)
- Heiskanen, W.A., Moritz, H.: *Physical Geodesy*. Freeman, San Francisco (1967)
- Helms, L.L.: *Introduction to Potential Theory*. Wiley-Interscience, New York (1969)
- Hesse, K., Sloan, I.H., Womersley, R.S.: Numerical integration on the sphere. In: Freedon, W., Nashed, M.Z., Sonar, T. (eds.) *Handbook of Geomathematics*, vol. 2, pp. 1187–1220. Springer, Berlin/Heidelberg (2010)
- Hielscher, R., Mainprice, D., Schaeben, H.: Material behavior: texture and anisotropy. In: Freedon, W., Nashed, M.Z., Sonar, T. (eds.) *Handbook of Geomathematics*, vol. 2, pp. 973–1003. Springer, Berlin/Heidelberg (2010)

- Hilbert, D.: Grundzüge einer allgemeinen Theorie der linearen Integralgleichungen. Teubner, Leipzig (1912)
- Hill, E.H.: The theory of vector spherical harmonics. *Am. J. Phys.* **22**, 211–214 (1954)
- Hlawka, E.: Gleichverteilung auf Produkten von Sphären. *J. Reine Angew. Math.* **330**, 1–43 (1982)
- Hlawka, E.: Näherungslösungen der Wellengleichung und verwandter Gleichungen durch zahlen-theoretische Methoden. *Öst. Akad. Wiss. Sber. II* **193**(8–10), 359–442 (1984)
- Hobson, E.W.: The Theory of Spherical and Ellipsoidal Harmonics. Reprint Chelsea Publishing Company, New York (1955)
- Hochstadt, H.: The Functions of Mathematical Physics. Wiley-Intersciences, New York (1971)
- Hofmann-Wellenhof, B., Moritz, H.: Physical Geodesy. Springer, Vienna/New York (2005)
- Ivanow, V.K.: Higher-dimensional generalization of the Euler summation formula (Russian). *Izv. Vuz. Mat.* **6**(37), 72–80 (1963)
- Jackson, J.D.: Classical Electrodynamics, 3rd edn. Wiley, New York (1998)
- James, R.W.: The Adams and Elsasser dynamo integrals. *Proc. R. Soc. Lond. A* **331**, 469 (1973)
- James, R.W.: The spectral form of the magnetic induction equation. *Proc. R. Soc. Lond. A* **340**, 287 (1974)
- James, R.W.: New tensor spherical harmonics, for application to the partial differential equations of mathematical physics. *Philos. Trans. R. Soc. Lond. A* **281**, 195–221 (1976)
- Jänich, K.: Vektoranalysis, 5th edn. Springer, Berlin/Heidelberg (2004)
- Jaswon, M.A., Symm, G.T.: Integral Equation Methods in Potential Theory and Elastostatics. Academic, London (1977)
- Jones, N.M.: Spherical Harmonics and Tensors for Classical Field Theory. Research Studies Press/Wiley, New York (1985)
- Kaula, W.M.: Theory of Satellite Geodesy. Blaisdell Company, Waltham (1966)
- Kellogg, O.D.: Foundations of Potential Theory. Frederick Ungar Publishing Company, New York (1929)
- Kirsch, A.: An Introduction to the Mathematical Theory of Inverse Problems. Springer, New York (1996)
- Knopp, K.: Funktionentheorie II. Sammlung Göschen, Bd. 703. Walter de Gruyter & Co, Berlin (1971)
- Knops, R.J., Payne, L.E.: Uniqueness Theorems in Linear Elasticity. Springer, Berlin (1971)
- Kotsiaris, S., Olsen, N.: The geomagnetic field gradient tensor. *Int. J. Geomath.* **3**(2), 297–314 (2012)
- Krätzel, E.: Lattice Points. Kluwer Academic, Dordrecht/Boston/London (1988)
- Kuipers, L., Niederreiter, H.: Uniform Distribution of Sequences. Wiley, New York (1974)
- Kupradze, V.D.: Potential Methods in the Theory of Elasticity. Israel Program for Scientific Translations, Jerusalem (1965)
- Lai, M., Krempf, E., Ruben, D.: Introduction to Continuum Mechanics, 4th edn. Elsevier, Amsterdam (2010)
- Lafín Fernández, N., Prestin, J.: Localization of the spherical Gauß-Weierstrass kernel. In: Bojanov, B.D. (ed.) Constructive Theory of Functions, pp. 267–274. DA2BA, Sofia (2003)
- Lamp, U., Schleicher, K.-T., Wendland, W.L.: The fast Fourier transform and the numerical solution of one-dimensional boundary integral equations. *Numer. Math.* **47**, 15–38 (1985)
- Landau, E.: Über die Gitterpunkte in einem Kreise. *I. Nachr. v. d. Gesellschaft d. Wiss. zu Göttingen, Math.-Phys. Klasse* 148–160 (1915)
- Landau, E.: Vorlesungen über Zahlentheorie. Chelsea Publishing Compagny, New York (1969) (reprint from the original version published by S. Hirzel, Leipzig (1927))
- Landau, E.: Ausgewählte Abhandlungen zur Gitterlehre. VEB, Berlin (1962)
- Landau, L.D., Lifshitz, L.M.: Quantum Mechanics. Course of Theoretical Physics, vol. 3, 3rd edn. Elsevier, Amsterdam (2004)
- Lebedev, N.N.: Spezielle Funktionen und ihre Anwendungen. Bibliographisches Institut, Mannheim (1973)
- Leis, R.: Vorlesungen über partielle Differentialgleichungen zweiter Ordnung. BI-Hochschultaschenbücher, 165/165a, Bibliographisches Institut, Mannheim (1967)

- Lekkerkerker, C.G.: *Geometry of Numbers*. North Holland, Amsterdam/London (1969)
- Lense, J.: *Kugelfunktionen*. *Mathematik und ihre Anwendungen in Physik und Technik*, Reihe A, vol. 23. Akademie Verlagsgesellschaft, Leipzig (1954)
- Lesieur, M.: *Turbulence in Fluids*, Third Revised and Enlarged Edition. Kluwer Academic, Dordrecht/Boston/London (1997)
- Liu, H., Ryan, J.: Clifford analysis techniques for spherical PDE. *J. Fourier Anal. Appl.* **8**(6), 535–563 (2002)
- Louis, A.K.: *Inverse und schlecht gestellte Probleme*. Teubner, Stuttgart (1989)
- Lurje, A.: *Räumliche Probleme der Elastizitätstheorie*. Akademie Verlag, Berlin (1963)
- Maclaurin, C.: *A Treatise of Fluxions*. Edinburgh (1742)
- Magnus, W., Oberhettinger, F., Soni, R.P.: *Formulas and Theorems for the Special Functions of Mathematical Physics*. *Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen*, Bd. 52, 3rd edn. Springer, Berlin (1966)
- Maier, T.: *Multiscale geomagnetic field modeling from satellite data*. Ph.D. thesis, Geomathematics Group, TU Kaiserslautern (2003)
- Maier, T.: Wavelet Mie representations for solenoidal fields with applications to ionospheric geomagnetic data. *SIAM J. Appl. Math.* **65**(6), 1888–1912 (2005)
- Marion, M., Teman, R.: Non-linear Galerkin methods. *SIAM J. Numer. Anal.* **26**(5), 1139–1157 (1989)
- Marsden, J.E., Hughes, T.J.R.: *Mathematical Foundations of Elasticity*. Dover Publications, New York (1994)
- Martensen, E.: *Potentialtheorie*. *Leitfäden der Angewandten Mathematik und Mechanik*, Bd. 12. Teubner, Leipzig (1968)
- Mayer, C.: *Wavelet modelling of ionospheric currents and induced magnetic fields from satellite data*. Ph.D. thesis, Geomathematics Group, TU Kaiserslautern (2003)
- Messiah, A.: *Quantenmechanik*. Walter de Gruyter, Berlin/New York (1990)
- Michel, V.: *A multiscale method for the gravimetry Problem—theoretical and numerical aspects of harmonic and anharmonic modelling*. Ph.D. thesis, Geomathematics Group, TU Kaiserslautern, Shaker, Aachen, 1999
- Michel, V.: *Tomography: Problems and Multiscale Solutions*. In: Freeden, W., Nashed, M.Z., Sonar, T. (eds.) *Handbook of Geomathematics*, vol. 2, pp. 949–972. Springer, Berlin/Heidelberg (2010)
- Michel, V.: *Lectures on Constructive Approximation—Fourier, Spline, and Wavelet Methods on the Real Line, the Sphere, and the Ball*. Birkhäuser, Boston (2012)
- Michlin, S.G.: *Mathematical Physics, an Advanced Course*. North Holland, Amsterdam/London (1970)
- Michlin, S.G.: *Lehrgang der Mathematischen Physik*, 2nd edn. Akademie Verlag, Berlin (1975)
- Miranda, C.: *Partial Differential Equations of Elliptic Type*. Springer, Berlin (1970)
- Misner, C.W., Thorne, K.S., Wheeler, J.A.: *Gravitation*. W.H. Freeman, San Francisco (1973)
- Mochizuki, E.: Spherical harmonic development of an elastic tensor. *Geophys. J. Int.* **93**(3), 521–526 (1988)
- Moisil, G.C., Teodorescu, N.: Fonctions holomorphes dans l'espace. *Mathematica (Cluj)* **5**, 142–159 (1931)
- Mordell, L.J.: Poisson's summation formula in several variables and some applications to the theory of numbers. *Math. Proc. Camb.* **25**, 412–420 (1928)
- Mordell, L.J.: Poisson's summation formula and the Riemann Zeta function. *J. Lond. Math. Soc.* **4**, 285–296 (1929)
- Morse, P.M., Feshbach, H.: *Methods of Theoretical Physics*. McGraw-Hill, New York (1953)
- Müller, C.: Über die ganzen Lösungen der Wellengleichung (nach einem Vortrag von G. Herglotz). *Math. Ann.* **124**, 235–264 (1952)
- Müller, C.: Eine Verallgemeinerung der Eulerschen Summenformel und ihre Anwendung auf Fragen der analytischen Zahlentheorie. *Abh. Math. Sem. Univ. Hamburg* **19**, 41–61 (1954)
- Müller, C.: *Spherical Harmonics*. *Lecture Notes in Mathematics*, vol. 17. Springer, Berlin (1966)
- Müller, C.: *Foundations of the Mathematical Theory of Electromagnetic Waves*. Springer, Berlin (1969)

- Müller, C.: Analysis of Spherical Symmetries in Euclidean Spaces. Springer, New York/Berlin/Heidelberg (1998)
- Müller, C., Dressler, A.: Über eine gewichtete Mittelung der Gitterpunkte in der Ebene. *J. Reine Angew. Math.* **252**, 82–87 (1972)
- Nashed, M.Z.: On moment-discretization and least-squares solutions of linear integral equations of the first kind. *J. Math. Anal. Appl.* **53**, 359–366 (1976a)
- Nashed, M.Z.: Generalized Inverses and Applications. Academic, New York (1976b)
- Nashed, M.Z., Whaba, G.: Generalized inverses in reproducing kernel spaces: an approach to regularization of linear operator equations. *SIAM J. Math. Anal.* **5**, 974–987 (1974)
- Nashed, M.Z.: Operator-theoretic and computational approaches to ill-posed problems with applications to antenna theory. *IEEE Trans. Antenn. Propag.* **29**, 220–231 (1981)
- Newton, I.: *Philosophiæ Naturalis Principia Mathematica*, vol. 3, De Munde Systemate (1687)
- Niederreiter, H.: Random Number Generation and Quasi-Monte Carlo Method. SIAM, Philadelphia (1992)
- Nielsen, N.: Handbuch der Theorie der Gammafunktion. Teubner, Leipzig (1906)
- Niemeyer, H.: Lokale und asymptotische Eigenschaften der Lösung der Helmholtzschen Schwingungsgleichung. *Jahresbericht d. DMV* **65**, 1–44 (1962)
- Nijboer, B.R.A., de Wette, F.W.: On the calculation of lattice sums. *Physica* **23**, 309–321 (1957)
- Norbury, J., Roulstone, I.: Large-Scale Atmosphere-Ocean Dynamics I. Analytic Methods and Numerical Methods. Cambridge University Press, Cambridge (2002a)
- Norbury, J., Roulstone, I.: Large-Scale Atmosphere-Ocean Dynamics II. Geometric Methods and Models. Cambridge University Press, Cambridge (2002b)
- Nutz, H.: A unified setup of gravitational field observables. Ph.D. thesis, Geomathematics Group, TU Kaiserslautern, Shaker, Aachen, 2002
- Ostermann, I.: Modeling heat transport in deep geothermal systems by radial basis functions. Ph.D. thesis, Geomathematics Group, TU Kaiserslautern (2011)
- Pail, R., Plank, G.: Assessment of three numerical solution strategies for gravity field recovery from GOCE satellite gravity gradiometry implemented on a parallel platform. *J. Geod.* **76**, 462–474 (2002)
- Papoulis, A., Pillai, S.U.: Probability, Random Variables, and Stochastic Processes, 4th edn. McGraw-Hill, New York (2002)
- Pedlowsky, J.: Geophysical Fluid Dynamics. Springer, New York/Heidelberg/Berlin (1979)
- Qian, T., Hempfling, T., McIntosh, A., Sommen, F. (eds.): Advances in Analysis and Geometry: New Developments Using Clifford Algebras. Trends in Mathematics. Birkhäuser, Basel (2004)
- Rademacher, H.: Topics in Analytic Number Theory. Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, Bd. 169. Springer, Berlin/Heidelberg/New York (1973)
- Rakhmanov, E.A., Saff, E.B., Zhou Y.M.: Minimal discrete energy on the sphere. *Math. Res. Lett.* **1**, 647–662 (1994)
- Reed, M., Simon, B.: Functional Analysis I. Academic, New York (1972)
- Reuter, R.: Über Integralformeln der Einheitssphäre und harmonische Splinefunktionen. Ph.D. thesis, Veröff. Geod. Inst. RWTH Aachen, Report No. 33, 1982
- Richter, L.: Über die Inversion einer Legendreschen Integraltransformation und ihre Anwendung. Ph.D. thesis, Rheinisch-Westfälische Technische Hochschule (RWTH), Aachen, 1971
- Rieder, A.: Keine Probleme mit inversen Problemen. Vieweg, Wiesbaden (2003)
- Rivlin, T.J.: Chebychev Polynomials, 2nd edn. Wiley, New York (1990)
- Rokhlin, V.: Rapid solution of integral equations of classical potential theory. *J. Comput. Phys.* **60**, 187–207 (1985)
- Rose, M.E.: Elementary Theory of Angular Momentum. Wiley, New York/London/Sydney (1957)
- Rudin, W.: Functional Analysis. McGraw-Hill, Boston (1991)
- Rummel, R., van Gelderen, M.: Spectral analysis of the full gravity tensor. *Geophys. J. Int.* **111**, 159–169 (1992)
- Rvachev, V.A.: Compactly supported solutions of functional-differential equations and their applications. *Russ. Math. Surv.* **45**, 87–120 (1990)
- Saff, E.B., Kuijlaars, A.B.J.: Distributing many points on a sphere. *Math. Intell.* **19**(1), 5–11 (1997)

- Sard, A.: Best approximate integration formulas. *Am. J. Math.* **71**, 80–91 (1949)
- Sauter, S.A.: Der Aufwand der Panel-Clustering-Methode für Integralgleichungen. Technical Report 9115, Institute for Computer Science and Applied Mathematics, University of Kiel (1991)
- Schaeben, H., van den Boogaart, K.G.: Spherical harmonics in texture analysis. *Tectonophysics* **370**, 253–268 (2003)
- Schmidt, K.E., Lee, M.A.: Implementing the fast multipole method in three dimensions. *J. Stat. Phys.* **63** 1223–1235 (1991)
- Schoenberg, I.J.: On trigonometric spline interpolation. *J. Math. Mech.* **13**, 795–825 (1964)
- Schreiner, M.: Tensor spherical harmonics and their application in satellite gradiometry. Ph.D. thesis, Geomathematics Group, TU Kaiserslautern (1994)
- Schulten, K., Gordon, R.G.: Exact recursive evaluation of $3j$ - and $6j$ -coefficients for quantum-mechanical coupling of angular momentum. *J. Math. Phys.* **16**(10), 1961–1970 (1975)
- Schulten, K., Gordon, R.G.: Recursive evaluation of $3j$ and $6j$ coefficients. *Comput. Phys. Commun.* **11**, 269–278 (1976)
- Schumaker, L.L.: *Spline Functions: Basic Theory*. Wiley, New York (1981)
- Shore, B.W., Menzel, D.H.: *Principles of Atomic Spectra*. Wiley, New York/London/Sydney (1968)
- Sieber, N., Sebastian, H.-J.: *Spezielle Funktionen*, 3rd edn. B.G. Teubner, Leipzig (1988)
- Sneddon, I.N.: *Special Functions of Mathematical Physics and Chemistry*, 3rd edn. Longman, New York (1980)
- Sneeuw, N.: A semi-analytical approach to gravity field analysis from satellite observations. Ph.D. thesis, TU Munich, Deutsche Geodätische Kommission, Reihe A, 527 (2000)
- Sommerfeld, A.: *Partielle Differentialgleichungen der Physik*, 6th edn. Akademische Verlagsgesellschaft, Leipzig (1966)
- Stein, E.M., Weiss, G.: *Introduction to Fourier Analysis on Euclidean Spaces*. Princeton University Press, Princeton (1971)
- Strubecker, K.: *Differentialgeometrie I, II*. Sammlung Götschen, de Gruyter, Berlin (1964)
- Szegő, G.: *Orthogonal Polynomials*. American Mathematical Society, Rhode Island (1967)
- Teman, R.: *Navier–Stokes Equations: Theory and Numerical Analysis*. North-Holland, Amsterdam/New York/Oxford (1979)
- Teman, R.: *Navier–Stokes Equations and Non-Linear Functional Analysis*. SIAM, Philadelphia (1983)
- Thomson, K.: Generalized Spiral Points: Further Improvement. <https://groups.google.com/d/topic/sci.math/CYMQX7HO1Cw/discussion> (2007). Cited 17 Jul 2012.
- Tichy, R.F.: Ein Approximationsverfahren zur Lösung spezieller partieller Differentialgleichungen. *ZAMM*. **68**, 187–188 (1988)
- Tichy, R.F.: Random points in the cube and on the sphere with applications to numerical analysis. *J. Comput. Appl. Math.* **31**(1), 191–197 (1990)
- Titchmarsh, E.C.: *The Theory of Riemann Zeta-Function*. Clarendon, Oxford (1951)
- Torge, W.: *Geodesy*, 3rd edn. de Gruyter, Berlin (2001)
- Tscherning, C.C.: Computation of the second-order derivatives of the normal potential based on the representation by a Legendre series. *Manusc. Geodæt.* **1**, 71–92 (1976)
- Tscherning, C.C.: Isotropic reproducing kernels for the inner of a sphere or spherical shell and their use as density covariance functions. *Math. Geol.* **28**, 161–168 (1996)
- van der Corput, B.L.: Verteilungsfunktionen I. *Proc. Nederl. Akad. Wetensch.* **38**, 813–821 (1935a)
- van der Corput, B.L.: Verteilungsfunktionen II. *Proc. Nederl. Akad. Wetensch.* **38**, 1058–1066 (1935b)
- van der Waerden, B.L.: *Mathematical Statistics*. Springer, Heidelberg (1969)
- Varšalovič, D.A., Moskalev, A.N., Chersonskij, V.K.: *Quantum Theory of Angular Momentum*. World Scientific, Singapore (1988)
- Vilenkin, N.J.: *Special Functions and the Theory of Group Representations*. Translations of Mathematical Monographs, vol. 22. American Mathematical Society, Providence (1968)

- Wahba, G.: Spline interpolation and smoothing on the sphere. *SIAM J. Sci. Stat. Comp.* **2**, 5–16 (1981) (also errata: *SIAM J. Sci. Stat. Comp.* **3**, 385–386 (1982))
- Walfisz, A.: Gitterpunkte in mehrdimensionalen Kugeln. *Acta Arith.* **6**, 115–136, 193–215 (1960)
- Wangerin, A.: *Theorie des Potentials und der Kugelfunktionen (I,II)*. de Gruyter, Leipzig (1921)
- Watson, G.N.: *A Treatise on the Theory of Bessel Functions*, 2nd edn. Cambridge University Press, Cambridge (1944)
- Weyl, H.: *The Theory of Groups and Quantum Mechanics*. E.P. Dutton, New York (1931)
- Wienholtz, E., Kalf, H., Kriecherbauer, T.: *Elliptische Differentialgleichungen zweiter Ordnung*. Springer, Heidelberg (2009)
- Wienkamp, R.: *Über eine Klasse verallgemeinerter Zetafunktionen*. Ph.D. thesis, Rheinisch-Westfälische Technische Hochschule (RWTH) Aachen (1958)
- White, C.A., Head-Gordon, M.: Rotating around the quartic angular momentum barrier in fast multipole method calculations. *J. Chem. Phys.* **105**(12), 5061–5067 (1996)
- Whittaker, E.T., Watson, G.N.: *A Course of Modern Analysis*. Cambridge University Press, Cambridge (1948)
- Xu, C., Sneeuw, N., Sideris, M.G.: The torus approach in spaceborne gravimetry. In: Xu, P., Liu, J., Dermanis, A. (eds.) *IAG Symposium*, vol. 132, pp. 23–28. Springer, Heidelberg (2008)
- Yoshida, K.: *Functional Analysis*. Springer, Berlin (1980)
- Zare, R.N.: *Angular Momentum*. Wiley-Interscience, New York (1988)

Index

- Addition theorem
 - 2D homogeneous harmonic polynomials, 315
 - 2D spherical harmonics, 314, 315
 - 3D harmonic polynomials, 135, 138
 - 3D homogeneous polynomials, 128
 - 3D spherical harmonics, 140
 - complex-valued spherical harmonics, 142
 - homogeneous polynomials satisfying the Navier equation, 261
 - qD Bessel functions, 365
 - qD harmonic polynomials, 307
 - qD homogeneous polynomials, 304, 306
 - qD spherical harmonics, 312
 - vector spherical harmonics, 239, 240
- Angular function, 289
- Anharmonic, 184
- Approximate identity, 176
- Area
 - 2D sphere, 31
 - 3D sphere, 31
 - qD sphere, 32
- Associated Legendre function, 142, 323

- Beltrami integral formula, 162, 164, 300
 - Helmholtz, 335
 - iterated, 301
 - iterated Helmholtz, 336
- Beltrami operator, 116, 166, 289
 - 3D eigenspectrum, 140
 - Helmholtz, 331
 - Laplace, 298, 331
 - qD eigenspectrum, 337
 - vectorial, 236, 238
- Bernoulli function, 397, 411
 - of degree 1, 403
 - of degree 2, 401
- Bernoulli numbers, 396, 415
 - representations, 411
- Bernoulli polynomials, 396
 - of degree 1,2,3,4, 396
 - recurrence relations, 395
- Bernstein kernel, 148, 224, 270, 318
- Bernstein summability, 148, 224, 228
- Bessel functions, 13, 347
 - 2D half odd integer order, 370
 - 2D theory, 370
 - differential equation, 349
 - generating function, 355
 - Hankel functions, 352
 - integral representations, 357
 - Neumann functions, 352
 - of first kind, 350
 - of the second kind, 352
 - of the third kind, 352
 - orthogonality relations, 354
 - qD addition theorem, 365
 - qD asymptotic behavior, 368, 375
 - qD differential equation, 367, 369
 - qD generating function, 365, 369
 - qD Hankel functions, 373
 - qD integral representation, 364
 - qD Kelvin function, 379
 - qD modified, 371
 - qD modified Hankel function, 379
 - qD Neumann functions, 378
 - qD recurrence relations, 367, 369
 - qD regular, 364
 - qD series representation, 366
 - qD theory, 363
 - recurrence relations, 358
 - with integer index, 355
- Beta distribution, 46

- Cauchy–Riemann operator, 204
- Christoffel–Darboux formula
 - monic orthogonal polynomials, 64
 - orthonormal polynomials, 63
- Clenshaw algorithm, 104
 - modified, 105
- Clifford algebra, 202
- Closure, 50
 - 3D spherical harmonics, 148, 152
 - qD periodic polynomials, 431
 - qD spherical harmonics, 320
 - vector spherical harmonics, 230, 235
- Completeness, 50
 - 3D spherical harmonics, 153
 - qD periodic polynomials, 431
 - qD spherical harmonics, 321
 - vector spherical harmonics, 231, 235
- Convergence theorems
 - ID Poisson summation formula, 418
 - qD Poisson summation formula, 454
- Coordinates
 - polar, 114, 288
- Coupling integrals, 275
 - advection related terms, 279, 283
 - Coriolis related terms, 279, 282
- Differential equation
 - Bessel, 349
 - Legendre, 93
 - of the Helmholtz operator, 12, 348, 363, 385
 - of the Laplace–Beltrami operator, 165
 - Poisson, 2, 90, 296, 333, 435
 - qD Bessel, 367
 - qD Legendre, 309, 337
- Dirac operator, 204
- Dirichlet problem, 327
- Discontinuous integrals of Weber–Schlafheitlin type, 360
- Discrepancy, 171
- Duplication formula, 36
- Eigenspectrum
 - ID Laplace operator, 399
 - 3D Beltrami operator, 140
 - qD Beltrami operator, 332
 - qD Laplace operator, 431
- Elasticity, 16, 254
 - Cauchy stress tensor, 18
 - Cauchy–Navier equation, 20, 254
 - elasticity tensor, 18
 - Lamé parameters, 19, 254
 - Navier equation, 20, 254
 - Poisson’s ratio, 20
 - vector spherical harmonics, 262
- Equidistribution, 171
- Error functions, 44
- Euler angles, 191
- Euler summation formula
 - ID Laplace operator, 404, 407
 - ID ordinary, 404
 - ID shifted, 407
 - qD extended, 438
 - qD for the operator $D(a) + \lambda$, 479
 - qD for the operator $D(a) + \lambda$, iterated, 480
 - qD Laplace operator, 8, 436, 437
- Euler’s Beta function, 29
 - Beta distribution, 46
 - incomplete, 44
 - relation to the Gamma function, 30
- Euler’s constant, 40, 406, 407
- Expectation value
 - frequency domain, 265
 - space domain, 264
- Fast multipole method, 109, 186, 191, 196
- Fourier coefficients, 48, 147, 228, 432
 - Bessel’s equality, 49
 - Bessel’s inequality, 49
- Functional equation
 - ID Riemann Zeta function, 414
 - ID Theta function, 419, 420
 - qD Theta function, 458
 - qD Zeta function, 442
- Fundamental cell
 - ID lattice, 398
 - qD inverse lattice, 429
 - qD lattice, 428
- Fundamental solution
 - Laplace operator, 292, 382
- Funk–Hecke formula, 157, 246, 249, 250, 316
- Gamma distribution, 45
- Gamma function, 25
 - definition, 26
 - derivative, 27
 - duplication formula, 36
 - extended Stirling’s formula, 405
 - extension to the complex plane, 38
 - functional equation, 27
 - Gamma distribution, 45
 - incomplete, 43
 - multiplication formula, 38
 - product representation, 41

- reciprocal, 38
- Stirling's formula, 33, 405
- Gauß theorem, 290
- surface, 116
- Gegenbauer polynomials, 72, 80, 308
 - derivative, 81
 - differential equation, 81
 - estimates, 107
 - explicit representation, 81, 84
 - generating function, 86
 - hypergeometric representation, 81–83
 - L^2 -norm, 82
 - recurrence relations, 89
 - Rodrigues' representation, 81
 - three-term recurrence, 82
- Geomagnetic field, 9
- Gravitational field, 1
- Green's function
 - 1D \mathbb{Z} -lattice Laplace operator, 400
 - 1D \mathbb{Z} -periodic, 400
 - 1D Legendre operator, 101
 - 3D sphere, iterated Laplace–Beltrami, 163
 - 3D sphere, Laplace–Beltrami, 159, 220, 224
 - qD iterated Laplace operator, 435
 - qD sphere, Helmholtz–Beltrami, 331
 - qD sphere, iterated Helmholtz–Beltrami, 335
 - qD sphere, iterated Laplace–Beltrami, 301
 - qD sphere, Laplace–Beltrami, 298
 - qD \mathbb{Z} -lattice Laplace operator, 432
 - qD \mathbb{Z} -periodic, 432
- Green's surface theorem
 - first, 117, 298
 - second, 118, 298
 - third, 161, 162, 164, 300
- Green's theorem
 - extended second, 291
 - first, 290
 - second, 290
 - third, 295
- Haar kernel, 176, 269
- Hankel functions, 352
 - 2D theory, 384
 - qD asymptotic behavior, 375, 376
 - qD characteristic singularity, 380
 - qD definition, 373
 - qD modified, 379
 - qD theory, 372
- Hankel transform, 359
- Hardy–Landau identity
 - 1D interval, 417
 - 2D ball, 474, 477
- Harmonic, 131, 291
- Helmholtz decomposition theorem, 220
- Helmholtz equation
 - entire solutions, 390
 - expansion for the inner space of a ball, 386
 - expansion for the outer space of a ball, 388
- Helmholtz–Beltrami operator, 331
- Hermite polynomials, 95, 98
 - differential equation, 96
 - explicit representation, 95
 - Rodrigues' representation, 95
 - three-term recurrence, 95
- Holomorphic
 - \mathbb{H} -holomorphic, 205
 - \mathbb{H} -left holomorphic, 205
 - \mathbb{H} -right holomorphic, 205
- Homogeneous harmonic polynomials
 - 2D addition theorem, 315
 - 2D theory, 313, 314
 - 3D addition theorem, 135
 - 3D theory, 131
 - qD addition theorem, 307
 - qD theory, 302
 - vectorial, 231
- Homogeneous polynomials, 125, 302
 - satisfying the Navier equation, 256, 257
- Hydrogen atom, 165
- Hypergeometric function, 76, 82, 83, 106
- Incomplete Beta function, 44
- Incomplete Gamma function, 43
 - relation to error functions, 44
- Inner harmonics, 187
 - \mathbb{H} -holomorphic, 209
 - translation theorem, 189
- Integral formula
 - 3D Laplace–Beltrami, 162, 164
 - qD Helmholtz–Beltrami, 335
 - qD iterated Helmholtz–Beltrami, 336
 - qD iterated Laplace–Beltrami, 301
 - qD Laplace–Beltrami, 300
- Invariance
 - orthogonal, 119, 120, 247, 258, 260, 315
 - with respect to reflections, 121
 - with respect to rotations, 119, 121, 156, 247
- Inverse lattice, 429
- Irreducible, 121, 122
 - Harm_n , 156
 - harm_n , 247

- Jacobi polynomials, 70
 - derivative, 77
 - differential equation, 73, 74
 - explicit representation, 76
 - hypergeometric representation, 76
 - L^2 -norm, 78
 - Rodrigues' formula, 77
 - special cases, 72
 - three-term recurrence, 79
- Kelvin functions
 - 2D theory, 384
 - differential equation, 383
 - qD asymptotic behavior, 379, 382
 - qD characteristic singularity, 380
 - qD definition, 379
 - qD theory, 378
 - recurrence relations, 383
- Laguerre polynomials, 98, 100, 168
 - differential equation, 99
 - explicit representation, 98
 - Rodrigues' representation, 98
 - three-term recurrence, 99
- Laplace–Beltrami operator, 116, 289, 298, 331
- Lattice
 - 1D integer, 398
 - qD inverse, 429
 - qD periodic, 5, 427
- Lattice balls
 - inside balls, 472
- Lattice function
 - 1D bilinear expansion, 402
 - 1D explicit representation, 402
 - 1D Fourier expansion, 402
 - 1D iterated Laplacian, 408, 410, 420, 421
 - 1D Laplace operator, 400
 - 1D \mathbb{Z} -periodic, 400
 - 2D Helmholtz operator, 474
 - integral asymptotics, 443, 448
 - qD for the operator $D(a) + \lambda$, 479
 - qD for the operator $D(a) + \lambda$, iterated, 480
 - qD iterated Laplace operator, 435
 - qD Laplace operator, 432
 - qD \mathbb{Z} -periodic, 432
 - uniqueness, 401
- Lattice points
 - inside balls, 472
 - inside circles, 474
 - non-uniform distribution in the plane, 478
- Legendre coefficient, 157
- Legendre differential equation, 93
- Legendre functions
 - 3D associated, 142
 - qD associated, 323
- Legendre harmonic, 154
- Legendre operator, 93, 116
 - Green's function, 101
 - qD, 309, 337
- Legendre polynomials, 73, 93, 135
 - 3D Maxwell representation, 144
 - Laplace's integral representation, 316
 - qD, 90, 308
 - qD asymptotic estimates, 317
 - qD differential equation, 309, 337
 - qD generating function, 311
 - qD integral relations, 310
 - qD Maxwell representation, 311
 - qD orthogonality, 308
 - qD recurrence relations, 309
 - qD Rodrigues formula, 309
 - qD theory, 308, 315
 - vectorial counterpart, 251
- Legendre rank-2 tensor kernel
 - of type (i, k) with respect to $o^{(i)}, O^{(i)}$, 241, 243, 251
 - with respect to $o^{(i)}, O^{(i)}$, 241
- Legendre vector field, 253
- Legendre vector field of type i , 253
- Lipschitz continuity, 176, 297
- Low discrepancy method, 170
- Maxwell representation, 144, 311
- Maxwell's equations
 - full system, 9
 - pre-Maxwell equations, 11, 90, 204
- Metaharmonic, 292
- Metric tensor, 274
- Modulus of continuity, 297
- Moving frame, 114, 123
- Multi-dimensional angle, 445
- Multi-indices, 287
- Multiplication formula, 38
- Multiresolution analysis, 182
- Navier–Stokes equation, 13, 15
 - incompressible, 15
 - incompressible, spherical, 16
- Neumann functions, 352
 - qD definition, 378
- Newton integral, 5
- Newton potential, 2
- Normal vector field, 213
- Numerical integration

- Gauß quadrature, 67, 101, 424
- interpolatory quadrature, 66
- low discrepancy method, 170
- partitioning the sphere, 170
- trapezoidal rule, 422

- Orthogonal group, 120
- Orthogonal invariance, 119, 120, 247, 258, 260, 315
 - scalar context, 122
 - vectorial context, 123
- Orthogonal polynomials, 47
 - monic orthogonal polynomials, 54
 - symmetry, 56
 - three-term recurrence, 59
 - weighted Hilbert spaces, 48
 - zeros, 57, 65
- Outer harmonics, 187
 - translation theorem, 189

- Peano kernel, 464, 467
- Periodic
 - Λ -lattice, 430
 - \mathbb{Z} -lattice, 398
- Periodic convolution, 408, 420, 435
- Periodic polynomials, 398
- Pochhammer factorial, 38
- Point set
 - boundary, 286
 - closure, 286
 - region, 286
- Pointwise expansion theorem
 - spherical harmonics, 324, 327, 330
- Poisson differential equation, 2, 90, 296, 333, 435
- Poisson integral formula, 145, 325
- Poisson summation formula
 - 1D, 417
 - 1D Laplace operator, 416
 - 1D interval, 416
 - qD Laplace operator, 6, 448, 454, 455
- Polar coordinates, 114, 288
- Poloidal vector field, 258
- Polynomial
 - Bernoulli, 396
 - Chebyshev, 72, 82, 314
 - Gegenbauer, 72, 80, 308
 - harmonic, 304
 - Hermite, 95
 - homogeneous, 125, 302
 - homogeneous harmonic, 131, 304
 - homogeneous harmonic vectorial, 231
 - Jacobi, 70
 - Laguerre, 98, 168
 - Legendre, 73, 308
 - monic, 54
 - monic orthogonal, 54
 - periodic, 398
 - Λ -periodic, 430
 - satisfying the Navier equation, 256, 257
 - ultraspherical, 72, 80, 308
- Potential function, 216

- Quadrature rule, 66
 - Gauß quadrature, 67, 101
 - Gauß–Lobatto rule, 70
 - Gauß–Radau rule, 70
 - interpolatory, 66
- Quantum-mechanical oscillator, 97
- Quaternions, 200
 - conjugate, 201
 - imaginary part, 201
 - real part, 201
 - real quaternionic algebra, 200

- Radial function, 289
- Recurrence relations
 - Bessel function, 358
 - cylinder functions, 378
 - Gegenbauer polynomials, 89
 - Kelvin function, 383
 - modified Bessel function, 371
 - qD Bessel function, 367, 369
 - qD Legendre polynomials, 309
 - ultraspherical polynomials, 89
- Reducible, 120
- Reflection, 120
- Region, 286
 - regular, 116, 286, 290
- Reproducing kernel, 128, 141, 251, 313
- Riemann Zeta function, 411, 422
- Rodrigues' rule, 310
- Rotation, 120, 191, 246

- Scaling function
 - Haar, 179
 - smoothed Haar, 177
 - up function, 182
- Sectorial spherical harmonics, 143
- Sinc-function, 370, 417
- Sobolev space
 - 1D periodic, 423
 - qD periodic, 472
 - qD spherical, 469

- Solid angle, 295, 437
- Special orthogonal group, 120
- Sphere function
 - 3D Laplace–Beltrami, 159, 220, 224
 - qD Helmholtz–Beltrami, 331
 - qD iterated Helmholtz–Beltrami, 335
 - qD iterated Laplace–Beltrami, 301
 - qD Laplace–Beltrami, 298
- Spherical convolution, 163, 224, 335
 - infinite, 181
- Spherical harmonics
 - 2D addition theorem, 314, 315
 - 3D addition theorem, 140
 - 3D closure, 148, 152
 - 3D completeness, 153
 - 3D complex-valued, 141
 - 3D definition, 138
 - 3D eigenfunctions, 140
 - 3D Funk–Hecke formula, 157
 - 3D real-valued, 142
 - gravitational field, 4
 - \mathbb{H} -spherical harmonics, 200, 206
 - in geomagnetism, 11
 - irreducible, 156
 - of degree n and order j , 140–142
 - qD addition theorem, 312
 - qD asymptotic relations, 328
 - qD closure, 320
 - qD completeness, 321
 - qD definition, 312
 - qD eigenfunctions, 330, 336
 - qD expansion theorem, 324, 327
 - qD Funk–Hecke formula, 315
 - qD Laplace representation, 316
 - qD orthogonal coefficients, 328
 - qD theory, 285, 312
 - quaternionic representation, 200
 - sectorial, 143
 - tesseral, 143
 - zonal, 143
- Spherical summation, 453
- Spheroidal vector field, 222
- Splines, 461
 - algebraically polynomial, 461
 - monosplines, 470
 - multi-periodic, 470
 - natural, 462
 - periodic polynomial, 466
 - spherical, 467, 468
- Stirling’s formula, 33
 - extended, 405, 407
 - limit form, 36
- Stream function, 216
- Summation formula
 - 1D Euler, 405
 - 1D Poisson, 415, 417
 - qD Euler, 436
 - qD Poisson, 455
- Surface curl, 115
- Surface curl gradient, 115, 214
- Surface divergence, 115, 289
- Surface gradient, 115, 214, 289
- Surface identity tensor field, 239
- Surface rotation tensor field, 239
- Tangential vector field, 213
- Tesseral spherical harmonics, 143
- Theta function, 420, 455
 - 1D functional equation, 419
 - classical, 460
 - qD functional equation, 458
 - qD properties, 458
 - qD theory, 455
- Three-term recurrence, 59, 79, 82, 95, 99
- Toroidal vector field, 223, 258
- Translation theorem, 189
 - for coefficients, 190
- Ultraspherical polynomials, 72, 80, 308
 - derivative, 81
 - differential equation, 81
 - estimates, 107
 - explicit representation, 81, 84
 - generating function, 86
 - hypergeometric representation, 81–83
 - L^2 -norm, 82
 - recurrence relations, 89
 - Rodrigues’ representation, 81
 - three-term recurrence, 82
- Uncertainty principle, 264
- Up function, 179, 181
- Variance
 - frequency domain, 265
 - space domain, 264
- Vector spherical harmonics, 211
 - addition theorem, 239, 240
 - alternate system, 235
 - by Edmonds, 276
 - closure, 230, 235
 - completeness, 231, 235
 - definition, 219
 - eigenfunctions, 238
 - Funk–Hecke formula, 246, 249, 250
 - in elasticity, 21, 262

in geomagnetism, 12
 in Navier–Stokes equation, 16
 irreducibility, 247
 Vectorial Beltrami operator, 236, 238
 Volume of the qD ball, 33

Wavelets

Haar, 179
 locally supported, 176
 smoothed Haar, 176, 178
 spherical, 178
 up function, 183
 Wigner matrices, 191
 Wigner symbols, 271

$3j$, 272
 $6j$, 274
 $9j$, 275

Zeta function

1D functional equation, 414
 1D Riemann, 411
 fast lattice point summation, 481
 qD Epstein, 439, 478, 481
 qD functional equation, 443
 qD integral representation, 413
 Zonal function, 118, 298
 Bernstein, 148, 151, 224, 318
 Zonal spherical harmonics, 143