Appendix A. Support for modules over commutative rings

Let A be a commutative noetherian ring. We consider the category $\mathsf{Mod}\,A$ of A-modules and its full subcategory $\mathsf{mod}\,A$ which is formed by all finitely generated A-modules. Note that an A-module is finitely generated if and only if it is noetherian.

The *spectrum* $\operatorname{Spec} A$ of A is the set of prime ideals in it. A subset of $\operatorname{Spec} A$ is *Zariski closed* if it is of the form

$$\mathcal{V}(\mathfrak{a}) = \{\mathfrak{p} \in \operatorname{Spec} A \mid \mathfrak{a} \subseteq \mathfrak{p}\}$$

for some ideal $\mathfrak{a}$ of A. A subset $\mathcal{V}$ of $\operatorname{Spec} A$ is *specialisation closed* if for any pair $\mathfrak{p} \subseteq \mathfrak{q}$ of prime ideals, $\mathfrak{p} \in \mathcal{V}$ implies $\mathfrak{q} \in \mathcal{V}$. The *specialisation closure* of a subset $\mathcal{U} \subseteq \operatorname{Spec} A$ is the subset

$$\operatorname{cl}\mathcal{U} = \{\mathfrak{p} \in \operatorname{Spec} A \mid \text{there exists } \mathfrak{q} \in \mathcal{U} \text{ with } \mathfrak{q} \subseteq \mathfrak{p}\}\,.$$

This is the smallest specialisation closed subset containing $\mathcal{U}$.

A.1 Big support

The *big support* of an A-module M is the subset

$$\operatorname{Supp}_A M = \{\mathfrak{p} \in \operatorname{Spec} A \mid M_{\mathfrak{p}} \neq 0\}\,.$$

Observe that this is a specialisation closed subset of $\operatorname{Spec} A$.

Lemma A.1. *One has* $\operatorname{Supp}_A A/\mathfrak{a} = \mathcal{V}(\mathfrak{a})$ *for each ideal* $\mathfrak{a}$ *of* A.

Proof. Fix $\mathfrak{p} \in \operatorname{Spec} A$ and let $S = A \setminus \mathfrak{p}$. Recall that for any A-module M, an element x/s in $S^{-1}M = M_{\mathfrak{p}}$ is zero iff there exists $t \in S$ such that $tx = 0$. Thus we have $(A/\mathfrak{a})_{\mathfrak{p}} = 0$ iff there exists $t \in S$ with $t(1+\mathfrak{a}) = t + \mathfrak{a} = 0$ iff $\mathfrak{a} \not\subseteq \mathfrak{p}$. □

Lemma A.2. *If* $0 \to M' \to M \to M'' \to 0$ *is an exact sequence of* A*-modules, then* $\operatorname{Supp}_A M = \operatorname{Supp}_A M' \cup \operatorname{Supp}_A M''$.

D.J. Benson et al., *Representations of Finite Groups: Local Cohomology and Support*, Oberwolfach Seminars 43, DOI 10.1007/978-3-0348-0260-4, © Springer Basel AG 2012

Proof. The sequence $0 \to M'_{\mathfrak{p}} \to M_{\mathfrak{p}} \to M''_{\mathfrak{p}} \to 0$ is exact for each $\mathfrak{p}$ in $\operatorname{Spec} A$. □

Lemma A.3. *Let $M = \sum_i M_i$ be an A-module, written as a sum of submodules M_i. Then $\operatorname{Supp}_A M = \bigcup_i \operatorname{Supp}_A M_i$.*

Proof. The assertion is clear if the sum $\sum_i M_i$ is direct, since

$$\bigoplus_i (M_i)_{\mathfrak{p}} = (\bigoplus_i M_i)_{\mathfrak{p}}.$$

As $M_i \subseteq M$ for all i one gets $\bigcup_i \operatorname{Supp}_A M_i \subseteq \operatorname{Supp}_A M$, from Lemma A.2. On the other hand, $M = \sum_i M_i$ is a factor of $\bigoplus_i M_i$, so $\operatorname{Supp}_A M \subseteq \bigcup_i \operatorname{Supp}_A M_i$. □

We write $\operatorname{ann}_A M$ for the ideal of elements in A that annihilate M.

Lemma A.4. *One has $\operatorname{Supp}_A M \subseteq \mathcal{V}(\operatorname{ann}_A M)$, with equality when M is in $\operatorname{mod} A$.*

Proof. Write $M = \sum_i M_i$ as a sum of cyclic modules $M_i \cong A/\mathfrak{a}_i$. Then

$$\operatorname{Supp}_A M = \bigcup_i \operatorname{Supp}_A M_i = \bigcup_i \mathcal{V}(\mathfrak{a}_i) \subseteq \mathcal{V}(\bigcap_i \mathfrak{a}_i) = \mathcal{V}(\operatorname{ann}_A M),$$

and equality holds if the sum is finite. □

Lemma A.5. *Let $M \neq 0$ be an A-module. If $\mathfrak{p}$ is maximal in the set of ideals which annihilate a non-zero element of M, then $\mathfrak{p}$ is prime.*

Proof. Suppose $0 \neq x \in M$ and $\mathfrak{p}x = 0$. Let $a, b \in A$ with $ab \in \mathfrak{p}$ and $a \notin \mathfrak{p}$. Then $(\mathfrak{p}, b)$ annihilates $ax \neq 0$, so the maximality of $\mathfrak{p}$ implies $b \in \mathfrak{p}$. Thus $\mathfrak{p}$ is prime. □

Lemma A.6. *Let $M \neq 0$ be an A-module. There exists a submodule of M which is isomorphic to $A/\mathfrak{p}$ for some prime ideal $\mathfrak{p}$.*

Proof. The set of ideals annihilating a non-zero element has a maximal element, since A is noetherian. Now apply Lemma A.5. □

Lemma A.7. *For each M in $\operatorname{mod} A$ there exists a finite filtration*

$$0 = M_0 \subseteq M_1 \subseteq \ldots \subseteq M_n = M$$

such that each factor M_i/M_{i-1} is isomorphic to $A/\mathfrak{p}_i$ for some prime ideal $\mathfrak{p}_i$. In that case one has $\operatorname{Supp}_A M = \bigcup_i \mathcal{V}(\mathfrak{p}_i)$.

Proof. Repeated application of Lemma A.6 yields a chain of submodules $0 = M_0 \subseteq M_1 \subseteq M_2 \subseteq \ldots$ of M such that each M_i/M_{i-1} is isomorphic to $A/\mathfrak{p}_i$ for some $\mathfrak{p}_i$. This chain stabilises since M is noetherian, and therefore $\bigcup_i M_i = M$.

The last assertion follows from Lemmas A.2 and A.1. □

A.2 Serre subcategories

A full subcategory C of A-modules is called *Serre subcategory* if for every exact sequence $0 \to M' \to M \to M'' \to 0$ of A-modules, M belongs to C if and only if M' and M'' belong to C. We set

$$\operatorname{Supp}_A \mathsf{C} = \bigcup_{M \in \mathsf{C}} \operatorname{Supp}_A M.$$

Proposition A.8. *The assignment* $\mathsf{C} \mapsto \operatorname{Supp}_A \mathsf{C}$ *induces a bijection between*

(1) *the set of Serre subcategories of* $\operatorname{\mathsf{mod}} A$, *and*

(2) *the set of specialisation closed subsets of* $\operatorname{Spec} A$.

Its inverse takes $\mathcal{V} \subseteq \operatorname{Spec} A$ *to* $\{M \in \operatorname{\mathsf{mod}} A \mid \operatorname{Supp} M \subseteq \mathcal{V}\}$.

Proof. Both maps are well defined by Lemmas A.2 and A.4. If $\mathcal{V} \subseteq \operatorname{Spec} A$ is a specialisation closed subset, let $\mathsf{C}_\mathcal{V}$ denote the smallest Serre subcategory containing $\{A/\mathfrak{p} \mid \mathfrak{p} \in \mathcal{V}\}$. Then we have $\operatorname{Supp} \mathsf{C}_\mathcal{V} = \mathcal{V}$, by Lemmas A.1 and A.2. Now let C be a Serre subcategory of $\operatorname{\mathsf{mod}} A$. Then

$$\operatorname{Supp} \mathsf{C} = \{\mathfrak{p} \in \operatorname{Spec} A \mid A/\mathfrak{p} \in \mathsf{C}\}$$

by Lemma A.7. It follows that $\mathsf{C} = \mathsf{C}_\mathcal{V}$ for each Serre subcategory C, where $\mathcal{V} = \operatorname{Supp} \mathsf{C}$. Thus $\operatorname{Supp} \mathsf{C}_1 = \operatorname{Supp} \mathsf{C}_2$ implies $\mathsf{C}_1 = \mathsf{C}_2$ for each pair $\mathsf{C}_1, \mathsf{C}_2$ of Serre subcategories. □

Corollary A.9. *Let M and N be in* $\operatorname{\mathsf{mod}} A$. *Then* $\operatorname{Supp}_A N \subseteq \operatorname{Supp}_A M$ *if and only if N belongs to the smallest Serre subcategory containing M.*

Proof. With C denoting the smallest Serre subcategory containing M, there is an equality $\operatorname{Supp}_A \mathsf{C} = \operatorname{Supp}_A M$ by Lemma A.2. Now apply Proposition A.8. □

A.3 Localising subcategories

A full subcategory C of A-modules is said to be *localising* if it is a Serre subcategory and if for any family of A-modules $M_i \in \mathsf{C}$ the sum $\bigoplus_i M_i$ is in C. The result below is from [31, p. 425].

Corollary A.10. *The assignment* $\mathsf{C} \mapsto \operatorname{Supp}_A \mathsf{C}$ *gives a bijection between*

(1) *the set of localising subcategories of* $\operatorname{\mathsf{Mod}} A$, *and*

(2) *the set of specialisation closed subsets of* $\operatorname{Spec} A$.

Its inverse takes $\mathcal{V} \subseteq \operatorname{Spec} A$ *to* $\{M \in \operatorname{\mathsf{Mod}} A \mid \operatorname{Supp}_A M \subseteq \mathcal{V}\}$.

Proof. The proof is essentially the same as the one of Proposition A.8 if we observe that any A-module M is the sum $M = \sum_i M_i$ of its finitely generated submodules. Note that M belongs to a localising subcategory C if and only if all M_i belong to C. In addition, we use that $\operatorname{Supp}_A M = \bigcup_i \operatorname{Supp}_A M_i$; see Lemma A.3. □

A.4 Injective modules

The following proposition collects the basic properties of injective modules over a commutative noetherian ring; for a proof see [43, §18]. For each $\mathfrak{p} \in \operatorname{Spec} A$ we denote $E(A/\mathfrak{p})$ the injective hull of $A/\mathfrak{p}$.

Proposition A.11. (1) *An arbitrary direct sum of injective modules is injective.*

(2) *Every injective module decomposes essentially uniquely as a direct sum of injective indecomposables.*

(3) $E(A/\mathfrak{p})$ *is indecomposable for each* $\mathfrak{p}$ *in* $\operatorname{Spec} A$.

(4) *Each injective indecomposable is isomorphic to* $E(A/\mathfrak{p})$ *for a unique prime ideal* $\mathfrak{p}$. □

Let $\mathfrak{p}$ a prime ideal in A and let M be an A-module. The module M is said to be $\mathfrak{p}$-*torsion* if each element of M is annihilated by a power of $\mathfrak{p}$; equivalently:

$$M = \{x \in M \mid \text{there exists an integer } n \geq 0 \text{ such that } \mathfrak{p}^n \cdot x = 0\}.$$

The module M is $\mathfrak{p}$-*local* if the natural map $M \to M_{\mathfrak{p}}$ is bijective.

For example, $A/\mathfrak{p}$ is $\mathfrak{p}$-torsion, but it is $\mathfrak{p}$-local only if $\mathfrak{p}$ is a maximal ideal, while $A_{\mathfrak{p}}$ is $\mathfrak{p}$-local, but it is $\mathfrak{p}$-torsion only if $\mathfrak{p}$ is a minimal prime ideal. The A-module $E(A/\mathfrak{p})$ is both $\mathfrak{p}$-torsion and $\mathfrak{p}$-local. Using this observation the following is easy to prove.

Lemma A.12. *Let* $\mathfrak{p}$ *and* $\mathfrak{q}$ *be prime ideals in* A. *Then*

$$E(A/\mathfrak{p})_{\mathfrak{q}} = \begin{cases} E(A/\mathfrak{p}) & \text{if } \mathfrak{q} \in \mathcal{V}(\mathfrak{p}), \\ 0 & \text{otherwise.} \end{cases}$$ □

A.5 Support

Each A-module M admits a minimal injective resolution

$$0 \to M \to I^0 \to I^1 \to I^2 \to \cdots$$

and such a resolution is unique, up to isomorphism of complexes of A-modules. We say that $\mathfrak{p}$ *occurs* in a minimal injective resolution I of M, if for some integer $i \in \mathbb{Z}$, the module I^i has a direct summand isomorphic to $E(A/\mathfrak{p})$. We call the set

$$\operatorname{supp}_A M = \left\{ \mathfrak{p} \in \operatorname{Spec} A \;\middle|\; \begin{matrix} \mathfrak{p} \text{ occurs in a minimal} \\ \text{injective resolution of } M \end{matrix} \right\}$$

the *support of* M. In the literature, it is sometimes referred to as the 'small support' or the 'cohomological support', to distinguish it from the big support $\operatorname{Supp}_A M$.

Lemma A.13. *Let M be an A-module and $\mathfrak{p} \in \operatorname{Spec} A$. If I is a minimal injective resolution of M, then $I_{\mathfrak{p}}$ is a minimal injective resolution of $M_{\mathfrak{p}}$. Therefore*

$$\operatorname{supp}_A(M_{\mathfrak{p}}) = \operatorname{supp}_A M \cap \{\mathfrak{q} \in \operatorname{Spec} A \mid \mathfrak{q} \subseteq \mathfrak{p}\}.$$

Proof. For the first assertion, see for example Lemmas 5 and 6 in [43, §18]. The formula for the support of $M_{\mathfrak{p}}$ then follows from Lemma A.12. □

We write $k(\mathfrak{p})$ for the residue field $A_{\mathfrak{p}}/\mathfrak{p}A_{\mathfrak{p}}$ at $\mathfrak{p} \in \operatorname{Spec} A$.

Lemma A.14. *Let M be an A-module and $\mathfrak{p} \in \operatorname{Spec} A$. The following are equivalent:*

(1) $\mathfrak{p} \in \operatorname{supp}_A M$;

(2) $\operatorname{Ext}^*_{A_{\mathfrak{p}}}(k(\mathfrak{p}), M_{\mathfrak{p}}) \neq 0$;

(3) $\operatorname{Tor}_*^{A_{\mathfrak{p}}}(k(\mathfrak{p}), M_{\mathfrak{p}}) \neq 0$.

Proof. For the equivalence of (1) and (2) see [43, Theorem 18.7]. The equivalence of (2) and (3) is more involved, and was proved by Foxby [30]. □

Lemma A.15. *For each A-module M one has*

$$\operatorname{supp}_A M \subseteq \operatorname{cl}(\operatorname{supp}_A M) = \operatorname{Supp}_A M \subseteq \mathcal{V}(\operatorname{ann} M),$$

and equalities hold when M is finitely generated.

Proof. The equality follows from Lemma A.13, while the inclusions are obvious.

Suppose now M is finitely generated. Given Lemma A.4, to prove that equalities hold, it remains to verify $\operatorname{Supp}_A M \subseteq \operatorname{supp}_A M$. If $M_{\mathfrak{p}} \neq 0$ for some $\mathfrak{p} \in \operatorname{Spec} A$, then $k(\mathfrak{p}) \otimes_{A_{\mathfrak{p}}} M_{\mathfrak{p}} \neq 0$ by Nakayama's Lemma, for $M_{\mathfrak{p}}$ is a finitely generated module over the local ring $A_{\mathfrak{p}}$. In particular, $\operatorname{Tor}_*^{A_{\mathfrak{p}}}(k(\mathfrak{p}), M_{\mathfrak{p}}) \neq 0$, and hence $\mathfrak{p}$ is in $\operatorname{supp}_A M$, by Lemma A.14. □

A.6 Specialization closed sets

Given a subset $\mathcal{U} \subseteq \operatorname{Spec} A$, we consider the full subcategory

$$\mathsf{M}_{\mathcal{U}} = \{M \in \mathsf{Mod}\, A \mid \operatorname{supp}_A M \subseteq \mathcal{U}\}.$$

The next result does not hold for arbitrary subsets of $\operatorname{Spec} A$. In fact, it can be used to characterise the property that $\mathcal{V}$ is specialisation closed.

Lemma A.16. *Let $\mathcal{V}$ be a specialisation closed subset of $\operatorname{Spec} A$. Then for each A-module M, one has*

$$\operatorname{supp}_A M \subseteq \mathcal{V} \quad \Longleftrightarrow \quad M_{\mathfrak{q}} = 0 \text{ for each } \mathfrak{q} \text{ in } \operatorname{Spec} A \setminus \mathcal{V}.$$

The subcategory $\mathsf{M}_{\mathcal{V}}$ of $\mathsf{Mod}\, A$ is closed under set-indexed direct sums, and in any exact sequence $0 \to M' \to M \to M'' \to 0$ of A-modules, M is in $\mathsf{M}_{\mathcal{V}}$ if and only if M' and M'' are in $\mathsf{M}_{\mathcal{V}}$.

Proof. Since $\mathcal{V}$ is specialisation closed, it contains $\operatorname{supp}_A M$ if and only if it contains $\operatorname{cl}(\operatorname{supp}_A M)$. Thus the first statement is a consequence of Lemma A.15. Given this the second statement follows, since for each $\mathfrak{q}$ in $\operatorname{Spec} A$, the functor taking an A-module M to $M_\mathfrak{q}$ is exact and preserves set-indexed direct sums. □

Torsion modules and local modules can be recognised from their supports.

Lemma A.17. *Let M be an A-module and $\mathfrak{p} \in \operatorname{Spec} A$. Then*

(1) *M is $\mathfrak{p}$-local if and only if $\operatorname{supp}_A M \subseteq \{\mathfrak{q} \in \operatorname{Spec} A \mid \mathfrak{q} \subseteq \mathfrak{p}\}$, and*

(2) *M is $\mathfrak{p}$-torsion if and only if $\operatorname{supp}_A M \subseteq \mathcal{V}(\mathfrak{p})$.*

Proof. Let I be a minimal injective resolution of M.

(1) Since $I_\mathfrak{p}$ is a minimal injective resolution of $M_\mathfrak{p}$, and minimal injective resolutions are unique up to isomorphism, $M \xrightarrow{\sim} M_\mathfrak{p}$ if and only if $I \xrightarrow{\sim} I_\mathfrak{p}$. This implies the desired equivalence, by Lemma A.12.

(2) When $\operatorname{supp}_A M \subseteq \mathcal{V}(\mathfrak{p})$, then, by definition of support, one has that I^0 is isomorphic to a direct sum of copies of $E(A/\mathfrak{q})$ with $\mathfrak{q} \in \mathcal{V}(\mathfrak{p})$. Since each $E(A/\mathfrak{q})$ is $\mathfrak{p}$-torsion, so is I^0, and hence the same is true of M, for it is a submodule of I^0.

Conversely, when M is $\mathfrak{p}$-torsion, $M_\mathfrak{q} = 0$ for each $\mathfrak{q}$ in $\operatorname{Spec} A$ with $\mathfrak{q} \not\supseteq \mathfrak{p}$. This implies $\operatorname{supp}_A M \subseteq \mathcal{V}(\mathfrak{p})$, by Lemma A.16. □

Lemma A.18. *Let $\mathfrak{p}$ be a prime ideal in A and set $\mathcal{U} = \{\mathfrak{q} \in \operatorname{Spec} A \mid \mathfrak{q} \subseteq \mathfrak{p}\}$.*

(1) *Restriction along the morphism $A \to A_\mathfrak{p}$ identifies $\mathsf{Mod}\, A_\mathfrak{p}$ with the subcategory $\mathsf{M}_\mathcal{U}$ of $\mathsf{Mod}\, A$. Therefore $\mathsf{M}_\mathcal{U}$ is closed under taking kernels, cokernels, extensions, direct sums, and products.*

(2) *If M, N are A-modules and one of them belongs to $\mathsf{M}_\mathcal{U}$, then $\operatorname{Hom}_A(M, N)$ is in $\mathsf{M}_\mathcal{U}$.*

Proof. (1) The objects in the subcategory $\mathsf{M}_\mathcal{U}$ are precisely the $\mathfrak{p}$-local A-modules, by Lemma A.17. Thus the inclusion functor has a left and a right adjoint. It follows that $\mathsf{M}_\mathcal{U}$ is an exact abelian and extension closed subcategory of $\mathsf{Mod}\, A$, closed under set-indexed direct sums and products.

(2) The action of A on $\operatorname{Hom}_A(M, N)$ factors via $\operatorname{End}_A(M)$ and $\operatorname{End}_A(N)$. If M or N is $\mathfrak{p}$-local, then this action factors through the map $A \to A_\mathfrak{p}$. □

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