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Bibliography

- [1] R. P. Agarwal and D. O'Regan. *Infinite Interval Problems for Differential, Difference and Integral Equations*. Kluwer Academic Publishers, 2001.
- [2] U. M. Ascher, R. M. M. Mattheij, and R. D. Russell. *Numerical Solution of Boundary Value Problems for Ordinary Differential Equations*. Prentice-Hall Series in Computational Mathematics. Prentice-Hall, Englewood Cliffs, 1988.
- [3] A. Ashyralyev and P.E. Sobolevskii. *New Difference Schemes for Partial Differential Equations*. Birkhauser Verlag, Basel-Boston-Berlin, 2006.
- [4] W. Auzinger, O. Koch, J. Petrickovic, and E. Weinmüller. Numerical solution of boundary value problems with an essential singularity. Technical Report 3/03, TU Wien, Institute for Applied Mathematics, 2003.
- [5] W. Auzinger, O. Koch, D. Praetorius, and E. Weinmüller. New a posteriori error estimates for singular boundary value problems. *Numerical Algorithms*, 40:79–100, 2005.
- [6] G. Bateman and A. Erday. *Higher Transcendental Functions [Russian translation]*, volume Vol. 2. Nauka, Moscow, 1974.
- [7] J. C. Butcher. On Runge-Kutta processes of high order. *Journal Austral. Math. Soc.*, 4:179–194, 1964.
- [8] J. C. Butcher. *Numerical Methods for Ordinary Differential Equations*. John Wiley & Sons, Ltd., Chichester, West Sussex, 2003.
- [9] C. Carathéodory. *Vorlesungen über reelle Funktionen*. Dover (reprint), 1948.
- [10] E. A. Coddington and N. Levinson. *Theory of Ordinary Differential Equations*. McGraw-Hill, New York, 1955.
- [11] L. Collatz. *The Numerical Treatment of Differential Equations*. Springer Verlag, Berlin, 1960.
- [12] S. D. Conte and C. deBoor. *Elementary Numerical Analysis*. McGraw-Hill, New York, 1972.

- [13] E. J. Doedel. The constructions to ordinary differential equations. *SIAM J. Numer. Analysis*, 15:450–465, 1978.
- [14] J. R. Dormand, E. A. El-Mikkawy, and P. J. Prince. Families of Runge-Kutta-Nystrom formulae. *IMA J. of Numer. Analysis*, 7:235–250, 1987.
- [15] Yu. Eidelman, V. Milman, and A. Tzolomitis. *Functional Analysis - An Introduction*. American Mathematical Society, Providence, Rhode Island, 2004.
- [16] R. Fazio. A novel approach to the numerical solution of boundary value problems on infinite intervals. *SIAM J. Numer. Anal.*, 33:1473–1483, 1996.
- [17] L. Fox. *Numerical Solution of Two-Point Boundary Value Problems in Ordinary Differential Equations*. Clarendon Press, Oxford, 1957.
- [18] S. Fučík and A. Kufner. *Nonlinear Differential Equations*. Elsevier, Amsterdam, Oxford, New York, 1980.
- [19] E. C. Jr. Gartland. Strong stability of compact discrete boundary value problems via exact discretizations. *SIAM J. Numer. Analysis*, 25:111–123, 1988.
- [20] I. P. Gavriljuk. *Grid schemes with exact and explicit spectra (in Russian)*. PhD thesis, Taras Shevchenko National University of Kiev, 1977.
- [21] I. P. Gavriljuk. Algorithm for solving a class of one-dimensional variational inequalities. *J. Sov. Math.*, 66:2250–2255, 1993.
- [22] I. P. Gavriljuk. A class of one-dimensional variational inequalities in difference schemes of arbitrary given degree of accuracy. *Z. Anal. Anwend.*, 12:751–758, 1993.
- [23] I. P. Gavriljuk. Exact difference schemes and difference schemes of arbitrary given degree of accuracy for generalized one-dimensional third boundary value problems. *Z. Anal. Anwend.*, 12:549–566, 1993.
- [24] I. P. Gavriljuk, M. Hermann, M. Kutniv, and I. L. Makarov. Two-point difference schemes of an arbitrary given order of accuracy for nonlinear BVPs. *Applied Mathematics Letters*, 23(5):585–590, 2010.
- [25] I. P. Gavriljuk, M. Hermann, M. V. Kutniv, and V.L. Makarov. New methods for nonlinear BVPs on the half-axis using Runge-Kutta IVP-solvers. Technical Report 05-18, Friedrich Schiller University Jena, Department of Mathematics and Computer Science, 2005.
- [26] I. P. Gavriljuk, M. Hermann, M. V. Kutniv, and V.L. Makarov. Difference schemes for nonlinear BVPs on the semiaxis. *Comput. Methods Appl. Math.*, 7:25–47, 2007.

- [27] I. P. Gavrilyuk, M. Hermann, M.V. Kutniv, and V.L. Makarov. Difference schemes for nonlinear BVPs using Runge-Kutta IVP-solvers. *Advances in Difference Equations*, 2006:1–29, 2006. Article ID 12167.
- [28] L. B. Gnativ and M. V. Kutniv. Modified three-point difference schemes of high-order accuracy for systems of the second-order ordinary differential equations with monotone operator(in Ukrainian). *Mathematical Methods and Physicomechanical Fields*, 47:32–42, 2004.
- [29] L. B. Gnativ, M. V. Kutniv, and V. L. Makarov. Generalized three-point difference schemes of high-order accuracy for systems of second order nonlinear ordinary differential equations. *Differential Equations*, 45(7):998–1019, 2009.
- [30] G. H. Golub and C. F. Van Loan. *Matrix Computations*. The John Hopkins University Press, Baltimore and London, 1996.
- [31] E. Hairer, S. P. Nørsett, and G. Wanner. *Solving Ordinary Differential Equations I, Nonstiff Problems*. Springer Verlag, Berlin, Heidelberg, New York, 1993.
- [32] E. Hairer and G. Wanner. *Solving Ordinary Differential Equations II, Stiff and Differential-Algebraic Problems*. Springer Verlag, Berlin, Heidelberg, New York, 1996.
- [33] Ph. Hartman. *Ordinary Differential Equations*. Birkhäuser Verlag, Boston, Basel, Stuttgart, 1982.
- [34] M. Hermann and D. Kaiser. Shooting methods for two-point BVPs with partially separated endconditions. *ZAMM*, 75:651–668, 1995.
- [35] Martin Hermann. *Numerik gewöhnlicher Differentialgleichungen, Anfangs- und Randwertprobleme*. Oldenbourg Verlag, München, Wien, 2004.
- [36] F. R. de Hoog and R. Weiss. The numerical solution of boundary value problems with an essential singularity. *SIAM J. Numer. Anal.*, 16:637–669, 1979.
- [37] F. R. de Hoog and R. Weiss. An approximation theory for boundary value problems on infinite intervals. *Computing*, 24:227–239, 1980.
- [38] S. R. K. Iyengar and A. C. R. Pillai. Difference schemes of polynomial and exponential orders. *Applied Mathematical Modelling*, 13:58–62, 1989.
- [39] H. B. Keller. *Numerical Methods for Two-Point Boundary-Value Problems*. Blaisdell Publishing Company, Waltham, Massachusetts, Toronto, London, 1968.
- [40] H. B. Keller. *Numerical Solution of Two-Point Boundary Value Problems*. SIAM, Philadelphia, 1976.

- [41] H. B. Keller and Jr. White, A. B. Difference methods for boundary value problems in ordinary differential equations. *SIAM J. Numer. Anal.*, (12):791–802, 1975.
- [42] M. V. Kutniv. Accurate three-point difference schemes for second-order monotone ordinary differential equations and their implementation. *Comput. Math. Math. Phys.*, 40:368–382, 2000.
- [43] M. V. Kutniv. Three-point difference schemes of high accuracy order for systems of nonlinear ordinary differential equations of second order. *Comput. Math. Math. Phys.*, 41:860–873, 2001.
- [44] M. V. Kutniv. High-order accurate three-point difference schemes for systems of second-order ordinary differential equations with a monotone operator. *Comput. Math. Math. Phys.*, 42(5):724–738, 2002.
- [45] M. V. Kutniv. Three-point difference schemes of high accuracy order for second order nonlinear ordinary differential equations with the boundary conditions of the third type (in Ukrainian). *Visnyk of Lviv University. Series Applied Mathematics and Computer Science*, (4):61–66, 2002.
- [46] M. V. Kutniv. Modified three-point difference schemes of high-accuracy order for second order nonlinear ordinary differential equations. *Comput. Methods Appl. Math.*, 3:287–312, 2003.
- [47] M. V. Kutniv. Modified three-point difference schemes of high accuracy order for the second-order monotone ordinary differential equations (in Ukrainian). *Mathematical Methods and Physicomechanical Fields*, 46:120–129, 2003.
- [48] M. V. Kutniv. Numerical solution of three-point difference schemes (in Ukrainian). *Visnyk of Lviv University. Series Applied Mathematics and Computer Science*, (6):68–73, 2003.
- [49] M. V. Kutniv, V. L. Makarov, and A. A. Samarskii. Accurate three-point difference schemes for second-order nonlinear ordinary differential equations and their implementation. *Comput. Math. Math. Phys.*, 39:45–60, 1999.
- [50] M. Laspinska-Chrzczonowicz and P. Matus. Exact difference schemes for parabolic equations. *Int. J. Numer. Anal. Mod.*, 5(2):303–319, 2008.
- [51] M. Lentini and H. B. Keller. Boundary value problems on semi-infinite intervals and their numerical solutions. *SIAM J. Numer. Anal.*, 17:577–604, 1980.
- [52] R.E. Lynch and J.R. Rice. A high-order difference method for differential equations. *Math. of Comp.*, 34:333–372, 1980.
- [53] Chawla M. M. A sixth order tri-diagonal finite difference method for general non-linear two point boundary value problems. *J. Inst. Math. Appl.*, 24:35–42, 1979.

- [54] V. L. Makarov. *Orthogonal polynomials and difference schemes with exact and explicit spectrum (in Russian)*. PhD thesis, Taras Shevchenko National University of Kiev, 1974.
- [55] V. L. Makarov, I. P. Gavriilyuk, M. V. Kutniv, and M. Hermann. A two-point difference scheme of arbitrary given accuracy order for BVPs for systems of first order nonlinear ODEs. *Comput. Methods Appl. Math.*, 4:464–493, 2004.
- [56] V. L. Makarov and S. G. Gocheva. Finite difference schemes of arbitrary accuracy for second-order differential equations on a half-line. *Differ. Equ.*, 17:367–377, 1981.
- [57] V. L. Makarov and V. V. Guminskii. A three-point difference scheme of a high order of accuracy for a system of second-order ordinary differential equations (the nonselfadjoint case). *Differ. Equ.*, 30:457–465, 1994.
- [58] V. L. Makarov, I. L. Makarov, and V. G. Prikazchikov. Exact difference schemes and schemes of any order of accuracy for systems of second-order differential equations (in Russian). *Differ. Uravn.*, 15:1194–1205, 1979.
- [59] V. L. Makarov and A. A. Samarskii. Exact three-point difference schemes for second-order nonlinear ordinary differential equations and their implementation. *Soviet Math. Dokl.*, 41:495–500, 1991.
- [60] V. L. Makarov and A. A. Samarskii. Realization of exact three-point difference schemes for second-order ordinary differential equations with piecewise smooth coefficients. *Soviet Math. Dokl.*, 41:463–467, 1991.
- [61] P. A. Markowich. A theory for the approximation of solution of boundary value problems on infinite intervals. *SIAM J. Math. Anal.*, 13:484–513, 1982.
- [62] P. A. Markowich. Analysis of boundary value problems on infinite intervals. *SIAM J. Math. Anal.*, 14:11–37, 1983.
- [63] P. A. Markowich and C. A. Ringhofer. Collocation methods for boundary value problems on "long" intervals. *Math. Comp.*, 40:123–150, 1983.
- [64] P. Matus, U. Irkhin, and M. Lapinska-Chrzczonowicz. Exact difference schemes for time-dependent problems. *Computational Methods in Applied Mathematics*, 5:422–448, 2005.
- [65] P. Matus and A. Kolodynska. Exact difference schemes for hyperbolic equations. *Comput. Meth. Appl. Math.*, 7(4):341–364, 2007.
- [66] P. P. Matus, U. Irkhin, M. Lapinska-Chrzczonowicz, and Lemeshevsky S. V. About exact difference schemes for hyperbolic and parabolic equations (in russian). *Differ. Uravn.*, 43(7):978–986, 2006.
- [67] N. S. Nedialkov and J. D. Pryce. Solving differential-algebraic equations by Taylor series (i): Computing Taylor coefficients. *BIT Numerical Mathematics*, 45:561–591, 2005.

- [68] J. M. Ortega and W. C. Rheinboldt. *Iterative Solution of Nonlinear Equations in Several Variables*. Academic Press, New York, London, 1970.
- [69] A. Paradzinska and P. Matus. High accuracy difference schemes for nonlinear transfer equation $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = f(u)$. *Math. Model. Anal.*, 12(4):469–482, 2007.
- [70] M. Ronto and A. M. Samoilenko. *Numerical-Analytic Methods in the Theory of Boundary-Value Problems*. Word Scientific, Singapore, 2000.
- [71] A. A. Samarskii. *Introduction to the Theory of Finite Difference Schemes*. Nauka, Moscow, 1972.
- [72] A. A. Samarskii. *The Theory of Difference Schemes*. Marcel Dekker Inc., New York and Basel, 2001.
- [73] A. A. Samarskii and V. L. Makarov. Realization of exact three-point difference schemes for second-order ordinary differential equations with piecewise-smooth coefficients. *Differ. Equ.*, 26:922–930, 1991.
- [74] A. A. Samarskii and B.S. Nikolaev. *Solution Methods for Grid Equations (in Russian)*. Nauka, Moscow, 1978.
- [75] A. A. Samarskii and E. S. Nikolaev. *Numerical Methods for Grid Equations, Vol.1*. Birkhäuser Verlag, Basel, 1989.
- [76] M. Schechter. *Principles of Functional Analysis. Second edition*. American Mathematical Society, Providence, Rhode Island, 2002.
- [77] C. Schmeiser. Approximate solution of boundary value problems on infinite intervals by collocation methods. *Math. Comp.*, 46:479–490, 1986.
- [78] M. R. Scott and H. A. Watts. A systemalized collection of codes for solving two-point boundary value problems. In A. K. Aziz, editor, *Numerical Methods for Differential Systems*, pages 197 – 227, New York and London, 1976. Academic Press.
- [79] J. Stoer and R. Bulirsch. *Introduction to Numerical Analysis*. Springer Verlag, New York, Berlin, Heidelberg, 2002.
- [80] A. N. Tihonov and A. A. Samarskii. Homogeneous difference schemes (in Russian). *Zh. Vychisl. Mat. i Mat. Fiz.*, 1:5–63, 1961.
- [81] A. N. Tihonov and A. A. Samarskii. Homogeneous difference schemes of a high order of accuracy on non-uniform nets (in Russian). *Zh. Vychisl. Mat. i Mat. Fiz.*, 1:425–440, 1961.
- [82] V.A. Trenogin. *Functional Analysis (in Russian)*. Nauka, Moscow, 1980.
- [83] B. A. Troesch. A simple approach to a sensitive two-point boundary value problem. *J. Comput. Phys.*, 21:279–290, 1976.

- [84] W. Wallisch and M. Hermann. *Schießverfahren zur Lösung von Rand- und Eigenwertaufgaben*. Teubner-Texte zur Mathematik, Bd. 75. Teubner Verlag, Leipzig, 1985.
- [85] A. I. Zadorin. Numerical solution of a boundary value problem for a system of equations with a small parameter. *Comput. Math. Math. Phys.*, 38(8):1201–1211, 1998.
- [86] E. Zeidler. *Nonlinear Functional Analysis and its Applications*, volume I. Springer-Verlag, New York et al., 1986.