# Appendix A
# Additional Information

## A.1 Basic Properties of the Rate Based Optimizations

This appendix shows that the optimum of the QoS problem (1.9) and the RB problem (1.12) is strictly positive and monotonic in a joint scaling of the rate targets and the power limitations, respectively. Furthermore, it presents the proof for the inversion property between (1.9) and (1.12).

### *A.1.1 Positivity and Monotonicity for the Optimum of the QoS Problem*

The optimum of the QoS optimization (1.9) is strictly *positive* because any feasible precoder is $\boldsymbol{t} \neq \mathbf{0}$ and $\mathcal{P}$ is compact, i.e., closed and bounded. The rate point $\boldsymbol{r}(\mathbf{0}) = \mathbf{0}$ was excluded from the target region $\mathcal{R}$ such that $\boldsymbol{t} = \mathbf{0}$ is infeasible.

Furthermore, the optimum of (1.9) is *monotonic* in a joint scaling of the target rates, e.g., $(1-\epsilon)^{-1}\boldsymbol{r} \in \mathcal{R}$ with $\epsilon \in (0, 1)$.[1] To show this, let $p$ and $\boldsymbol{t}$ be the solution of (1.9), that is, $p^{-1}\boldsymbol{t} \in \mathcal{P}$ and $\boldsymbol{r}(\boldsymbol{t}) \in \mathcal{R}$. Then, the loosened requirement $(1-\epsilon)^{-1}\boldsymbol{r}(\boldsymbol{t}') \in \mathcal{R}$ is also achieved by a $\boldsymbol{t}' = (1+\delta)^{-1}\boldsymbol{t}$ with a sufficiently small $\delta > 0$. This follows from the monotonicity of the data rates $r_k(\boldsymbol{t}')$ in $(1+\delta)$. Using $\boldsymbol{t}'$ instead of $\boldsymbol{t}$, the objective can be decreased to $p' = (1+\delta)^{-1}p$ such that $p'^{-1}\boldsymbol{t}' \in \mathcal{P}$. Hence, the optimum of (1.9) decreases when increasing the target region by downscaling the rates.

[1]The scaling of the rates results in the weakened rate requirements $r_k \geq (1-\epsilon)\rho_k$, $k = 1, \ldots, K$, for the exemplary per-user constrained target region (1.10).



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### A.1.2 *Positivity and Monotonicity for the Optimum of the Rate Balancing Problem*

The optimum of the balancing problem (1.12) is also *positive* for regular channel conditions. The optimum would only be zero if at least one of the rates (1.3) was zero for all $\boldsymbol{t} \in \mathcal{P}$. Since $\mathcal{P} \in \mathbb{C}^{NK}$ is convex and has non-empty interior, $r_k(\boldsymbol{t}) = 0$ for all $\boldsymbol{t} \in \mathcal{P}$ only if $\boldsymbol{h}_k = \mathbf{0}$. For non-zero rates $\boldsymbol{r}(\boldsymbol{t})$, the positive but unbounded scaling $\rho^{-1} \in (0, \infty)$ allows us to reach any point on the line segment $\boldsymbol{z} = \rho^{-1}\boldsymbol{r}(\boldsymbol{t})$. Hence, (1.12) is feasible and has a strictly positive optimum.

To show that the maximum of (1.12) is *monotonic* in a scaling of the precoder $\boldsymbol{t} \in \mathcal{P}$, let $\rho$ and $\boldsymbol{t}$ be the solution of (1.12), i.e., $\rho^{-1}\boldsymbol{r}(\boldsymbol{t}) \in \mathcal{R}$ and $\boldsymbol{t} \in \mathcal{P}$. When we loosen the power constraint set by the joint scaling $(1+\delta)^{-1}\boldsymbol{t}' \in \mathcal{P}$ with $\delta > 0$ and replace $\boldsymbol{t}$ by $\boldsymbol{t}' = (1+\delta)\boldsymbol{t}$, the rates are increased. There is a $\epsilon \in (0, 1)$ that satisfies $\rho^{-1}(1-\epsilon)\boldsymbol{r}(\boldsymbol{t}') \in \mathcal{R}$, which increases the objective to $\rho' = (1-\epsilon)^{-1}\rho$.

### A.1.3 *Relation Between the QoS Optimization and the Rate Balancing Problem*

The QoS problem (1.9) and the RB problem (1.12) are inverse to each other in that

$$p(\rho(p_0)) = p_0$$
$$\rho(p(\rho_0)) = \rho_0$$

holds if $p(\rho)$ denotes the QoS optimum (1.13) and $\rho(p)$ the RB optimum (1.14).

To prove this statement, we assume the opposite. Let $p_0 = p(\rho_0)$ with optimizer $\boldsymbol{t}$ be the solutions for the QoS problem. Furthermore, assume that $\rho(p_0) < \rho_0$ with $\boldsymbol{t}'$ were the solutions for the balancing problem. This contradicts the optimality of $\boldsymbol{t}'$ for the balancing problem since $\boldsymbol{t}$ is feasible and achieves $\rho_0$. Otherwise, assume that $\rho(p_0) > \rho_0$ with $\boldsymbol{t}'$ were the solutions for the balancing problem. This contradicts the optimality of $\boldsymbol{t}$ for the QoS problem since we can find a scaling $(1+\delta)^{-1}\boldsymbol{t}'$, with sufficiently small $\delta > 0$, that satisfies $\rho_0^{-1}\boldsymbol{r}(\boldsymbol{t}') \in \mathcal{R}$ and achieves the objective value $p' = (1+\delta)^{-1}p_0$.

## A.2 Interference Functions and Property Preserving Transforms

According to [75], *positivity*, *monotonicity*, and *sublinearity* have to be verified to show that $I_k : \mathbb{R}_+^K \to \mathbb{R}_{++}$ , $k = 1, \ldots, K$ form a standard interference function

$$\boldsymbol{I} : \mathbb{R}_+^K \to \mathbb{R}_+^K, \quad \boldsymbol{I}(\boldsymbol{p}) = [I_1(\boldsymbol{p}), \ldots, \mathbf{I}_K(\boldsymbol{p})]^{\mathrm{T}}. \tag{A.1}$$

Alternatively, one can exploit the structure of $I_k$ and show that it is based on functions that satisfy the properties, e.g., affine maps $I_k(\boldsymbol{p}) = \boldsymbol{a}_k^{\mathrm{T}}\boldsymbol{p} + b_k$ with $\boldsymbol{a} \geq \boldsymbol{0}$ and $b_k > 0$ [75] or concave increasing functions, or combinations of these functions and property preserving transforms.

**Lemma A.1** *Let* $I_k : \mathbb{R}_+^K \rightarrow \mathbb{R}_{++}$, $k = 1, \ldots, K$ *be concave increasing. Then,* (A.1) *defines a standard interference function* [323, Proposition 1].

*Proof* Since positivity and monotonicity are assumed, it remains to show the sublinearity property. Let $I_k$ be concave and $\alpha = 1/\lambda$ for $\lambda \in (0, 1)$. Then,

$$\alpha I_k(\boldsymbol{p}) = \frac{1}{\lambda} I_k\Big(\lambda \frac{1}{\lambda}\boldsymbol{p} + (1-\lambda)\boldsymbol{0}\Big) \geq I_k\Big(\frac{1}{\lambda}\boldsymbol{p}\Big) + (1-\lambda)I_k(\boldsymbol{0}) > I_k(\alpha\boldsymbol{p}).$$

The inequalities are due to concavity of $I_k$ and $I_k(\boldsymbol{0}) > 0$. □

There is a class of operations that preserve the interference function properties of the above basic blocks (cf. [75, 323]). Namely, the standard interference property is preserved for (1) non-negative affine mappings, (2) pointwise minimum (or maximum) with respect to a parameter of a closed set, (3) composition with a sublinear non-decreasing function, and (4) composition of standard interference functions. We also identify a concept like the perspective function in convex optimization [102, Section 3.2.6, p. 89].[2]

**Lemma A.2** *Let* $f_k : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $x \mapsto f(x)$ *be sublinear increasing and* $I_k' : \mathbb{R}_+^K \rightarrow \mathbb{R}_+$ *with* $\boldsymbol{p} \mapsto I_k'(\boldsymbol{p})$ *be an interference function. The perspective* $I_k : \mathbb{R}_+^{K+1} \rightarrow \mathbb{R}_+$,

$$I_k([t, \boldsymbol{p}^{\mathrm{T}}]^{\mathrm{T}}) = t f_k(I_k'(\boldsymbol{p})/t), \quad t > 0$$

*is an interference function. This even holds if $t$ is any entry of the vector $\boldsymbol{p}$, e.g.,* $I_k : \mathbb{R}_+^K \rightarrow \mathbb{R}_+$ *with*

$$I_k(\boldsymbol{p}) = p_k f_k(I_k'(\boldsymbol{p})/p_k), \quad p_k > 0.$$

*Proof* Positivity of $I_k$ and monotonicity in $\boldsymbol{p}$ are by definition of $f_k$ and $I_k'$. Monotonicity in $t$ follows with sublinearity of $f_k$. Let $\alpha > 1$, then the inequality $t' = \alpha t > t$ implies

$$I_k([\alpha t, \boldsymbol{p}^{\mathrm{T}}]^{\mathrm{T}}) = \alpha t f_k(I_k'(\boldsymbol{p})/(\alpha t)) > t f_k(\alpha I_k'(\boldsymbol{p})/(\alpha t)) = I_k([t, \boldsymbol{p}^{\mathrm{T}}]^{\mathrm{T}}). \tag{A.2}$$

For $t = p_k$, let $\boldsymbol{p}' = [p_1, \ldots, \alpha p_k, \ldots, p_K]^{\mathrm{T}}$ with $\alpha > 1$. Then, (A.2) becomes

$$I_k(\boldsymbol{p}') > p_k f_k(I_k'(\boldsymbol{p}')/p_k) \geq p_k f_k(I_k'(\boldsymbol{p})/p_k),$$

[2] The standard perspective $t I_k(\boldsymbol{p}_{\backslash\{k\}}/t)$ fails ($sublinearity$) as it is *scale invariant*.

where the first inequality is as in (A.2) and the second is due to monotonicity of $I_k$ and $f_k$. Finally, $I_k$ is sublinear as the interference function $I'_k$ is sublinear and $f_k$ is increasing, i.e., $I'_k(\boldsymbol{p}) > I'_k(\alpha\boldsymbol{p})/\alpha$ imposes

$$\alpha I_k([t, \boldsymbol{p}]) = \alpha t f_k(I'_k(\boldsymbol{p})/t) > \alpha t f_k(I'_k(\alpha\boldsymbol{p})/\alpha t) = \alpha I_k(\alpha[t, \boldsymbol{p}]). \qquad \square$$

Besides exploiting concavity, also this transforms can be used to prove that the following data rate like function results in a standard interference function for (A.1):

$$r_k = \log_2\left(1 + p_k(\boldsymbol{a}_k^{\mathrm{T}}\boldsymbol{p} + b_k)^{-1}\right), \quad \boldsymbol{a}_k \geq 0, \quad b_k > 0, \quad k = 1, \ldots, K. \tag{A.3}$$

**Lemma A.3** *The function* $I_k(\boldsymbol{p}) = p_k/r_k$ *with* $r_k$ *in* (A.3) *is concave increasing.*

*Proof* First, the function $f : \mathbb{R}_+ \to \mathbb{R}_+$ with $x \mapsto f(x)$ and

$$f(x) = \frac{1}{\ln\left(1 + \frac{1}{x}\right)} \tag{A.4}$$

is concave increasing. Its first and second order derivatives read as

$$f'(x) = f^2(x)\frac{1}{x^2 + x} \geq 0$$
$$f''(x) = 2f'(x)\Big(f(x) - x - \frac{1}{2}\Big) \leq 0,$$

respectively, where positivity of $f'$ is due to $x > 0$. To show that $f''(x) \leq 0$ for $x \geq 0$, we substitute $x = 1/y$ and remark that $g : \mathbb{R}_+ \to \mathbb{R}_+$ with $y \mapsto g(y)$ and

$$g(y) = \frac{1}{\ln(1 + y)} - \frac{1}{y}$$

has the limited function values $g(y) \in (0, 1/2]$ for $y \geq 0$.

Therewith, $I_k$ can be constructed with the perspective $h([t, x]^{\mathrm{T}}) = t/f(x/t)$ and a composition with an affine map in $\boldsymbol{p}$. This proves concavity also for $I_k(\boldsymbol{p})$. $\square$

**Theorem A.1** *The function* $\boldsymbol{I} : \mathbb{R}_+^K \to \mathbb{R}_{++}^K$ *with* $\boldsymbol{p} \mapsto \boldsymbol{I}(\boldsymbol{p}) = [I_1(\boldsymbol{p}), \ldots, I_K(\boldsymbol{p})]^{\mathrm{T}}$ *and* $I_k$ *from Lemma A.3 is a standard interference function.*

*Proof* It suffices to note that any positive concave increasing functions $I_k$, $k = 1, \ldots, K$ yield a standard interference function (see Lemma A.1).[3] An alternative proof can be based on the perspective of an interference function. As $f$ in (A.4) is sublinear increasing, we obtain a standard interference function with Lemma A.2:

[3]Also Cavalcante et al. [323, Proposition 1] mentioned that strictly concave functions are standard interference functions, but a proof is missing in this work.

$$I_k(\boldsymbol{p}) = p_k f(I'_k(\boldsymbol{p})/p_k), \quad k = 1, \ldots, K,$$

where $I'_k$ is affine in $\boldsymbol{p}$ for $f$ from (A.4). □

A mapping that does not preserve the standard interference function properties is

$$\boldsymbol{I}(\boldsymbol{p}) = \boldsymbol{\Gamma}\boldsymbol{I}'(\boldsymbol{p}), \quad \boldsymbol{\Gamma} = \text{diag}(\vartheta_1, \ldots, \vartheta_K) \in \mathbb{R}_+^{K \times K} \tag{A.5}$$

if $\vartheta_i > 0$ for only a subset of the entries $i \in \mathcal{I} \subset \{1, \ldots, K\}$, with $|\mathcal{I}| < K$. The resulting function $\boldsymbol{I}$ is non-standard because it violates strict positivity with $\vartheta_k I_k(\boldsymbol{p}) = 0$ for $k \notin \mathcal{I}$. Nevertheless, the iteration $\boldsymbol{p}^{(n+1)} = \boldsymbol{I}(\boldsymbol{p}^{(n)})$ shares the monotonicity properties of (1.18) and, moreover, either converges to the unique point $\boldsymbol{p} = \boldsymbol{I}(\boldsymbol{p})$, which minimizes $\mathbf{1}^{\mathrm{T}}\boldsymbol{p}$, or diverges (cf. Theorem 1.1) in some entries of $\boldsymbol{p}$. The reason is that those entries $p_k$ with $k \notin \mathcal{I}$ are immediately zero and the reduced interference function $\boldsymbol{J}(\boldsymbol{s}) = \boldsymbol{\Gamma}_{\mathcal{I}}\boldsymbol{I}_{\mathcal{I}}(\boldsymbol{p}_{\mathcal{I}}(\boldsymbol{s}))$ is standard if it comprises the entries $i \in \mathcal{I} = \{j(1), \ldots, j(|\mathcal{I}|)\}$ and $\boldsymbol{p}_{\mathcal{I}} : \mathbb{R}_+^{|\mathcal{I}|} \to \mathbb{R}_+^K$ defines the mapping $p_{\mathcal{I},j(i)}(\boldsymbol{s}) = s_i$ and $p_{\mathcal{I},k}(\boldsymbol{s}) = 0$ for $k \notin \mathcal{I}$.

The non-negative linear map (A.5) is amongst others important for balancing optimizations, where $\vartheta_k$, $k = 1, \ldots, K$ defines monotonically increasing or decreasing functions of a common balancing level that become zero beyond some lower or upper bound, respectively. Then, the following statement holds.[4]

**Proposition A.1** *Let* $\boldsymbol{I} : \mathbb{R}_+^K \to \mathbb{R}_+^K$ *be a standard interference function and* $\boldsymbol{\Gamma}(\rho) \in \mathbb{R}^{K \times K}$, $\boldsymbol{\Gamma}(\rho) = \text{diag}(\vartheta_1(\rho), \ldots, \vartheta_K(\rho))$ *comprises the values of the monotonically increasing functions* $\vartheta_k : \mathbb{R}_+ \to \mathbb{R}_+$, $k = 1, \ldots, K$, *with* $\vartheta_i(\rho') > \vartheta_i(\rho)$ *if* $\rho' > \rho$ *and* $\vartheta_i(\rho) > 0$. *Furthermore, consider existence of* $\hat{\rho} > \check{\rho}$ *and* $\hat{\boldsymbol{\lambda}}, \check{\boldsymbol{\lambda}} \in \mathbb{R}_+^K$ *such that*

(i) $\vartheta_i(\check{\rho}) > 0$ *for* $i \in \check{\mathcal{I}} \neq \emptyset$, *and* $\check{\boldsymbol{\lambda}} = \boldsymbol{\Gamma}(\check{\rho})\boldsymbol{I}(\check{\boldsymbol{\lambda}})$ *with* $\mathbf{1}^{\mathrm{T}}\check{\boldsymbol{\lambda}} < c$[5]*;*
(ii) $\vartheta_i(\hat{\rho}) > 0$ *for* $i \in \hat{\mathcal{I}} \neq \emptyset$ *and* $\hat{\boldsymbol{\lambda}} = \boldsymbol{\Gamma}(\hat{\rho})\boldsymbol{I}(\hat{\boldsymbol{\lambda}})$ *with* $\mathbf{1}^{\mathrm{T}}\hat{\boldsymbol{\lambda}} > c$.

*Then, there is a unique* $(\rho^\star, \boldsymbol{\lambda}^\star)$ *that satisfies* $\boldsymbol{\lambda}^\star = \boldsymbol{\Gamma}(\rho^\star)\boldsymbol{I}(\boldsymbol{\lambda}^\star)$ *and* $c = \mathbf{1}^{\mathrm{T}}\boldsymbol{\lambda}^\star$.

*Proof* The relation $\hat{\rho} > \check{\rho}$ and monotonically increasing functions $\vartheta_k$ imply $\check{\mathcal{I}} \subseteq \hat{\mathcal{I}}$. Moreover, it imposes $\hat{\boldsymbol{\lambda}} \geq \boldsymbol{\Gamma}(\hat{\rho})\boldsymbol{I}(\check{\boldsymbol{\lambda}}) \geq \boldsymbol{\Gamma}(\check{\rho})\boldsymbol{I}(\check{\boldsymbol{\lambda}}) = \check{\boldsymbol{\lambda}}$, where the second inequality only holds with equality for entries $k \notin \hat{\mathcal{I}}$ and is strict for $i \in \hat{\mathcal{I}}$.

Based on these relations, let $\rho > 0$ reside between the bounds, i.e., $\hat{\rho} > \rho > \check{\rho}$. Then, $\hat{\boldsymbol{\lambda}} \in \mathbb{R}_+^K$ is feasible, i.e., $\hat{\boldsymbol{\lambda}} \geq \boldsymbol{\Gamma}(\rho)\boldsymbol{I}(\hat{\boldsymbol{\lambda}})$, and the sequence $\boldsymbol{\lambda}^{(n)}$ monotonically decreases when starting with $\boldsymbol{\lambda}^{(0)} = \hat{\boldsymbol{\lambda}}$. Similarly, the sequence $\boldsymbol{\lambda}^{(n)}$ monotonically increases when $\boldsymbol{\lambda}^{(0)} = \check{\boldsymbol{\lambda}}$ because $\check{\boldsymbol{\lambda}} \leq \boldsymbol{\Gamma}(\rho)\boldsymbol{I}(\check{\boldsymbol{\lambda}})$. The support set $\mathcal{I}$, which is defined by $\rho$, is the same in both sequences after $n = 1$ iterations, that is, $\lambda_i^{(1)} > 0$ for $i \in \mathcal{I}$

[4] A similar statement can be employed for the reversed monotonicity, i.e., when the function $\vartheta_k(\rho)$ is decreasing and becomes zero if $\rho$ exceeds some value $\rho_{\max,k}$ for each $k = 1, \ldots, K$.

[5] This condition can be relaxed to $\check{\boldsymbol{\lambda}} \geq \boldsymbol{\Gamma}(\check{\rho})\boldsymbol{I}(\check{\boldsymbol{\lambda}})$ without changing the statement of the lemma.

and $\lambda_i^{(1)} = 0$ for $i \notin \mathcal{I}$. Moreover, the convergence point of the reduced iterations, i.e., based on the non-zero entries of $\boldsymbol{\lambda}^{(n)}$, is equal and unique because $\mathcal{J}(\cdot;\rho)$ is standard. Hence, there is a unique $\boldsymbol{\lambda}^\star$ for any $\rho \in [\check{\rho}, \hat{\rho}]$, with strictly increasing entries $\lambda_i$, $i \in \mathcal{I}$ for increasing balancing level $\rho$.

This shows in turn that there is only one unique $\rho^\star$ and the corresponding solution $\boldsymbol{\lambda}^\star$ that satisfies the requirement $c = 1^{\mathrm{T}}\boldsymbol{\lambda}^\star$ for given $c \in \mathbb{R}_+$. □

## A.3 Ergodic Rate Bounds for Multiplicative Fading

This appendix details the derivation of the ergodic rate bounds in Sect. 3.2.2, with the structure of (3.11) and the noise and the offset parameters from Table 3.1. The basis is Jensen's inequality (e.g., [102, p. 77]): If $f : \mathcal{X} \to \mathbb{R}$ is convex and the random variable $z$ is in the domain of $f$, i.e., $\Pr(z \in \mathcal{X}) = 1$, the mean of the function values $\mathrm{E}[f(z)]$ is bounded as

$$\mathrm{E}[f(z)] \geq f(\mathrm{E}[z]).$$

The inequality is strict if the function $f$ is strictly convex, and it is reversed for a (strictly) concave function $f$.

### A.3.1 Derivation of Lower and Upper Bounds on the Ergodic Rate

The ergodic rate is lower and upper bounded by the bounds LB1 and UB1, that is,

$$R_k^{(\mathrm{LB1})}(\boldsymbol{t}) \leq R_k(\boldsymbol{t}) \leq R_k^{(\mathrm{UB1})}(\boldsymbol{t}), \tag{A.6}$$

where the two bounds have the structure in (3.11) and the parameters from Table 3.1. The lower bound LB1 follows directly with Jensen's inequality and the fact that $r_k(\boldsymbol{t}, \zeta_k^{-1/2}\bar{\boldsymbol{h}}_k)$ is convex decreasing in $\zeta_k$ [cf. (3.11)].[6] The derivation of $R_k^{(\mathrm{UB1})}$ additionally exploits the structure of $A_k : \mathbb{R}_+ \to \mathbb{R}_+$ (see Footnote 5 of Chap. 3):

$$A_k(x) = \mathrm{E}[a_k(\zeta_k, x)], \quad a_k(\zeta_k, x) = \log_2(\zeta_k + x).$$

If the expectation $\mathrm{E}[\zeta_k]$ of the random variable $\zeta_k \in \mathbb{R}_+$ exists and is finite, then $A_k(x) \leq a_k(\mathrm{E}[\zeta_k], x)$ follows from Jensen's inequality since $a_k(\zeta_k, x)$ is concave

[6] The function $\log_2(1 + \frac{a}{\zeta_k + b})$, with $a, b \in \mathbb{R}_+$ and $\zeta_k \in \mathbb{R}_+$, is the composition of the convex function $\log_2(1 + z^{-1})$ with the affine mapping $z = \frac{\zeta_k + b}{a}$, and thus convex in $\zeta_k$ (cf. [102, p.$\tilde{7}$9]).

in $\zeta_k$. The difference $a_k(\mathrm{E}[\zeta_k], x) - A_k(x)$ decreases with increasing $x \in \mathbb{R}_+$.[7] Its maximum is $\mathrm{E}[\log_2(\mathrm{E}[\zeta_k]/\zeta_k)]$ at $x = 0$ and becomes zero for $x \to \infty$. Therefore,

$$\begin{aligned} a_k(\mathrm{E}[\zeta_k], x+y) - A_k(x+y) + \mathrm{E}[\log_2(\mathrm{E}[\zeta_k]/\zeta_k)] \\ \geq a_k(\mathrm{E}[\zeta_k], x) - A_k(x) \\ \geq a_k(\mathrm{E}[\zeta_k], x+y) - A_k(x+y) \end{aligned} \tag{A.7}$$

holds for all $x, y \in \mathbb{R}_+$. The first inequality provides the rate bound $R_k^{(\mathrm{UB1})}(\boldsymbol{t}) \geq R_k(\boldsymbol{t})$ when adding $A_k(x+y)$ and subtracting $a_k(\mathrm{E}[\zeta_k], x)$ at both sides of (A.7) and replacing $y$ and $x$ with the useful signal power $\mathtt{S}_k$ (3.14) and the interference $\mathtt{I}_k$ (3.15), respectively. The second inequality of (A.7) in turn proves validness of the lower bound LB1 in (A.6). This bound is tight when the second inequality in (A.7) becomes an equality, i.e., if either $y = 0$ or $x \to \infty$. In contrast, the first inequality of (A.7) and thus UB1 are tight only if both $x = 0$ and $y \to \infty$ hold.

Similar derivation steps show that UB2 and LB2 bound the ergodic rate as

$$R_k^{(\mathrm{UB2})}(\boldsymbol{t}) \geq R_k(\boldsymbol{t}) \geq R_k^{(\mathrm{LB2})}(\boldsymbol{t}). \tag{A.8}$$

Now, Jensen's inequality and concavity of $r_k(\boldsymbol{t}, \zeta_k^{-1/2}\bar{\boldsymbol{h}})$ in $\zeta_k^{-1}$ are sufficient to find UB2.[8] To obtain LB2, Jensen's inequality is applied to the function [cf. (3.5)]

$$B_k(x) = \mathrm{E}[b_k(\zeta_k^{-1}, x)], \quad b_k(\zeta_k^{-1}, x) = \log_2(1 + \zeta_k^{-1}x).$$

As a result $b_k(\mathrm{E}[\zeta_k^{-1}], x) - B_k(x) \geq 0$, because $b_k(\zeta_k^{-1}, x)$ is concave in $\zeta_k^{-1}$. This difference is zero for $x = 0$ and increases with $x \in \mathbb{R}_+$ up to $\mathrm{E}[\log_2(\zeta_k^{-1}/\,\mathrm{E}[\zeta_k^{-1}])]$ for $x \to \infty$.[9] Therewith, the following series of inequalities are valid for $x, y \in \mathbb{R}_+$:

$$\begin{aligned} b_k(\mathrm{E}[\zeta_k^{-1}], x) - B_k(x) &\leq b_k(\mathrm{E}[\zeta_k^{-1}], x+y) - B_k(x+y) \\ &\leq b_k(\mathrm{E}[\zeta_k^{-1}], x) - B_k(x) + \mathrm{E}[\log_2(\zeta_k^{-1}/\,\mathrm{E}[\zeta_k^{-1}])]. \end{aligned} \tag{A.9}$$

Here, the second inequality results in the lower bound LB2 when adding $B_k(x+y)$, subtracting $b_k(\mathrm{E}[\zeta_k^{-1}], x)$ and $\mathrm{E}[\log_2(\zeta_k^{-1}/\,\mathrm{E}[\zeta_k^{-1}])]$ at both sides of the inequality, and replacing $y$ and $x$ with the signal power $\mathtt{S}_k$ and the interference $\mathtt{I}_k$, respectively.

[7] The derivatives for both sides of $\mathrm{E}[\ln(\zeta_k + x)] \leq \ln(\mathrm{E}[\zeta_k] + x)$ satisfy $\mathrm{E}\left[\frac{1}{\zeta_k + x}\right] \geq \frac{1}{\mathrm{E}[\zeta_k] + x}$.

[8] The expression $\log_2(1 + sa/(1 + sb))$ is composed of the two concave increasing functions $\log_2(1 + z)$ and $z = \frac{a}{1/s + b}$ in $s$. Thus, it is also concave increasing in $s = \zeta_k^{-1}$ (cf. [102, p. 79]).

[9] The derivatives of $B_k(x)$ and $b_k(\zeta_k^{-1}, x)$ with respect to $x$ satisfy $\mathrm{E}\left[\frac{1}{1/\zeta_k^{-1} + x}\right] \leq \frac{1}{1/\,\mathrm{E}\left[\zeta_k^{-1}\right] + x}$.

For the alternative bounds ALB and AUB, we substitute $\zeta_k = \mathrm{e}^{z_k}$ into $A_k$:

$$A_k(x) = \mathrm{E}[a_k(\mathrm{e}^{z_k}, x)] \geq a_k(\mathrm{e}^{\mathrm{E}[z_k]}, x) = \tilde{A}_k(x), \tag{A.10}$$

where the inequality is due to convexity of $\log_2(\mathrm{e}^{z_k} + x)$ in $z_k$. Equality holds in (A.10) for $x = 0$ and $x \to \infty$, but the first order derivatives of $A_k$ and $\tilde{A}_k$,

$$A_k'(x) = \log_2(\mathrm{e})\,\mathrm{E}\left[\frac{1}{\mathrm{e}^{z_k} + x}\right] \qquad \tilde{A}_k'(x) = \log_2(\mathrm{e})\frac{1}{\mathrm{e}^{\mathrm{E}[z_k]} + x},$$

respectively, are only equal to zero for $x \to \infty$, while

$$\tilde{A}_k'(0) = \log_2(\mathrm{e})\mathrm{e}^{-\mathrm{E}[z_k]} \leq \log_2(\mathrm{e})\,\mathrm{E}[\mathrm{e}^{-z_k}] = A_k'(0).$$

Since both functions furthermore smoothly increase with $x \in \mathbb{R}_+$, we expect the difference $A_k(x) - \tilde{A}_k(x)$ to be pseudo-concave. This allows a numerical computation of the distance

$$d_k = \max\{d \in \mathbb{R}_+ : A_k(x) - \tilde{A}_k(x),\ x \in \mathbb{R}_+\}$$

via a golden section search [324, Chapter 8] and bounding $A_k(x + y) - A_k(y)$, $x, y \geq 0$ with the inequalities

$$\tilde{A}_k(x+y) - \tilde{A}_k(x) - d_k \leq A_k(x+y) - A_k(x) \leq \tilde{A}_k(x+y) - \tilde{A}_k(x) + d_k. \tag{A.11}$$

The bounds $R_k^{(\mathrm{ALB})}$ and $R_k^{(\mathrm{AUB})}$ with parameters in Table 3.1 follow again from replacing $y$ and $x$ with $\mathtt{S}_k$ and $\mathtt{I}_k$, respectively, and $z_k = \ln(\zeta_k)$ within (A.11).

## A.4 Feasible QoS Region with Ergodic Rate Bounds

The proof of Lemma 3.1 exploits the MMSE feasible region from Lemma 1.1. Therewith, it is sufficient to rewrite the rate constraints of (3.16) as uplink MMSE requirements. Since the ergodic rate bounds (3.11) have the same logarithmic structure as the perfect CSI rates (1.3), the standard (SINR) dual formulation by[10]

$$\begin{aligned} R_k^{(\mathrm{B}),(\mathrm{ul})} &= \log_2\left(1 + \mathrm{SINR}_k^{(\mathrm{B}),(\mathrm{ul})}\right) + \mu_k^{(\mathrm{B})}, \\ \mathrm{SINR}_k^{(\mathrm{B}),(\mathrm{ul})} &= \frac{\nu_k^{(\mathrm{B}),-1} |\bar{\boldsymbol{h}}_k^{\mathrm{H}} \boldsymbol{u}_k|^2 \lambda_k}{1 + \sum_{i=1}^{K} \nu_i^{(\mathrm{B}),-1} |\bar{\boldsymbol{h}}_i^{\mathrm{H}} \boldsymbol{u}_k|^2 \lambda_i}. \end{aligned} \tag{A.12}$$

[10]The duality and feasibility test are without loss of generality based on a sum power restriction.

Rewriting (A.12) as MSEs and inserting the equalizers, the MMSEs read as

$$\mathrm{MMSE}_k^{(\mathrm{B}),(\mathrm{ul})} = 1 - \nu_k^{(\mathrm{B})} \bar{\boldsymbol{h}}_k^{\mathrm{H}} \Big( \mathbf{I} + \sum_{i=1}^{K} \bar{\boldsymbol{h}}_i \lambda_i \nu_i^{(\mathrm{B})} \bar{\boldsymbol{h}}_i^{\mathrm{H}} \Big) \bar{\boldsymbol{h}}_k \tag{A.13}$$

and the uplink MMSE constraints for the rate constraints of (3.16) are[11]

$$\mathrm{MMSE}_k^{(\mathrm{B}),(\mathrm{ul})} \leq 2^{-\max\left\{\rho_k - \mu_k^{(\mathrm{B})}, 0\right\}} = \varepsilon_k. \tag{A.14}$$

With $\rho_k \in \mathbb{R}_+$, feasible targets $\varepsilon_k$ still satisfy the box constraint $0 < \varepsilon_k \leq 1$. The proof for $\sum_{k=1}^{K} \varepsilon_k \geq K - N$ is equivalent to [65, Proof of Theorem 1].

## A.5 On Uniqueness of the QoS Optimal Power Allocation

This appendix discusses uniqueness for the solution of the QoS optimization (3.48) with ergodic rate requirements and fixed beamformers. By definition, any solution of (3.48) resides in the intersection of the feasible set

$$\mathcal{S} = \left\{ \boldsymbol{p} \in \mathbb{R}_+^K : \boldsymbol{p} \geq \boldsymbol{I}(\boldsymbol{p}; \boldsymbol{\rho}) \right\}$$

and the objective's sub-level set, i.e., the compact convex polytope

$$\mathcal{T}(P^\star) = \left\{ \boldsymbol{p} \in \mathbb{R}_+^K : \tilde{\boldsymbol{A}} \boldsymbol{p} \leq P^\star \mathbf{1} \right\}, \tag{A.15}$$

where $P^\star$ is the minimal scaling of the polytope such that $\mathcal{T}(P^\star) \cap \mathcal{S}$ is non-empty. In other words, the solutions reside on the tangential plane, where the upper right boundary of $\mathcal{T}(P^\star)$ touches the lower left boundary of $\mathcal{S}$.

Due to the standard interference property of $\boldsymbol{I}(\cdot; \boldsymbol{\rho})$, this lower boundary of $\mathcal{S}$ is characterized by the unique fixed point

$$\boldsymbol{p}^\star = \boldsymbol{I}(\boldsymbol{p}^\star; \boldsymbol{\rho}), \tag{A.16}$$

which is the *minimum point* of $\mathcal{S}$ [230, Theorem 5.57],[12] i.e., $\boldsymbol{p}^\star \leq \boldsymbol{p}'$ for every $\boldsymbol{p}' \in \mathcal{S}$. Therefore, $\boldsymbol{p}^\star \in \mathcal{T}(P^\star) \cap \mathcal{S}$ is a solution of (3.48) and provides[13]

$$P^\star = \max_{\ell} \mathbf{e}_\ell^{\mathrm{T}} \tilde{\boldsymbol{A}} \boldsymbol{p}^\star. \tag{A.17}$$

[11] The maximum operation is due to $\mathrm{MMSE}_k^{(\mathrm{B}),(\mathrm{ul})} \leq 1$, which holds for $\lambda_k = 0$.

[12] A $\boldsymbol{x} \in \mathcal{A}$ is a *(strict) minimum point* of $\mathcal{A}$ if $\boldsymbol{x} \leq \boldsymbol{y}$ ($\boldsymbol{x} < \boldsymbol{y}$) for every $\boldsymbol{y} \neq \boldsymbol{x}$, $\boldsymbol{y} \in \mathcal{A}$.

[13] It is impossible to further reduce the objective of (3.48) as $\tilde{\boldsymbol{A}} \geq \mathbf{0}$ and $\boldsymbol{p}' \notin \mathcal{S}$ if $\exists i : p_i' < p_i^\star$.

While (3.48) can have multiple solutions in general, $\boldsymbol{p}^\star$ is the unique solution for cases where $\mathcal{S} \cap \mathcal{T}(P^\star)$ becomes a singleton. This occurs if $\tilde{\boldsymbol{A}} > \mathbf{0}$ [230, Corollary 5.58]. An example is the total sum power minimization [230, p. 182], where the objective degenerates to $\tilde{\boldsymbol{a}}^{\mathrm{T}} \boldsymbol{p}$.

The solution of (3.48) is also unique if $\boldsymbol{p}^\star$ is a *strict minimum point* of $\mathcal{S}$, i.e., $\boldsymbol{p}^\star < \boldsymbol{p}'$ for $\boldsymbol{p}' \neq \boldsymbol{p}^\star$, $\boldsymbol{p}' \in \mathcal{S}$. Then, there is no feasible $\boldsymbol{p}' \geq \boldsymbol{p}^\star$, $\boldsymbol{p}' \neq \boldsymbol{p}^\star$ that still resides in $\mathcal{T}(P^\star)$ because $\tilde{\boldsymbol{A}}\boldsymbol{p}' > \tilde{\boldsymbol{A}}\boldsymbol{p}^\star$. A strict minimum point is caused by strict (local) monotonicity of $\boldsymbol{I}(\cdot;\boldsymbol{\rho})$ at $\boldsymbol{p}^\star$. Then, forcing either element of $\boldsymbol{p}'$ to lie strictly above $\boldsymbol{p}^\star$, e.g., $p_i' = p_i^\star + \delta_i$ with $\delta_i > 0$, induces $p_k' > p_k^\star$ for $k \neq i$ if $\boldsymbol{p}' \in \mathcal{S}$ [11, Section 2.2.2].

In contrast, $\mathcal{S} \cap \mathcal{T}(P^\star)$ contains also other solutions than $\boldsymbol{p}^\star$ if there is a $\boldsymbol{p}' \in \mathcal{S}$, $\boldsymbol{p}' \geq \boldsymbol{p}^\star$, that satisfies the inequality condition

$$\mathbf{e}_n^{\mathrm{T}} \tilde{\boldsymbol{A}} \boldsymbol{p}^\star < \mathbf{e}_n^{\mathrm{T}} \tilde{\boldsymbol{A}} \boldsymbol{p}' < \max_{\ell} \mathbf{e}_\ell^{\mathrm{T}} \tilde{\boldsymbol{A}} \boldsymbol{p}^\star.$$

This property depends on the structure of $\boldsymbol{I}(\cdot;\boldsymbol{\rho})$ and $\tilde{\boldsymbol{A}}$. For example, let $\boldsymbol{p} = [\boldsymbol{p}_1^{\mathrm{T}}, \boldsymbol{p}_2^{\mathrm{T}}]^{\mathrm{T}}$ and $\boldsymbol{I}(\boldsymbol{p};\boldsymbol{\rho}) = [\boldsymbol{I}_1^{\mathrm{T}}(\boldsymbol{p}_1), \boldsymbol{I}_2^{\mathrm{T}}(\boldsymbol{p}_2)]^{\mathrm{T}}$ such that $\boldsymbol{p} \in \mathcal{S}$ is equivalent to $\boldsymbol{p}_i \in \mathcal{S}_i = \{\boldsymbol{p}_i \in \mathbb{R}_+^{K_i} : \boldsymbol{p}_i \geq \boldsymbol{I}_i(\boldsymbol{p}_i)\}$, $i = 1, 2$. Furthermore, let $\tilde{\boldsymbol{A}}$ be block diagonal with blocks $\tilde{\boldsymbol{A}}_1, \tilde{\boldsymbol{A}}_2$, such that $\tilde{\boldsymbol{A}}\boldsymbol{p} = [(\tilde{\boldsymbol{A}}_1\boldsymbol{p}_1)^{\mathrm{T}}, (\tilde{\boldsymbol{A}}_2\boldsymbol{p}_2)^{\mathrm{T}}]^{\mathrm{T}}$ and assume that the following inequality holds:

$$\max_n \mathbf{e}_n^{\mathrm{T}} \tilde{\boldsymbol{A}}_2 \boldsymbol{p}_2^\star < \max_\ell \mathbf{e}_\ell^{\mathrm{T}} \tilde{\boldsymbol{A}}_1 \boldsymbol{p}_1^\star.$$

Then, any $\boldsymbol{p}' \in \mathcal{T}(P^\star)$, $\boldsymbol{p}' \neq \boldsymbol{p}^\star$, where $\boldsymbol{p}_1' = \boldsymbol{p}_1^\star$ and $\boldsymbol{p}_2' = \alpha \boldsymbol{p}_2^\star$ with

$$\alpha \in \left(1, \max_\ell \mathbf{e}_\ell^{\mathrm{T}} \tilde{\boldsymbol{A}}_1 \boldsymbol{p}_1^\star / \max_n \mathbf{e}_n^{\mathrm{T}} \tilde{\boldsymbol{A}}_2 \boldsymbol{p}_2^\star\right],$$

is also an element of $\mathcal{S}$ due to the independence of $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$ and sublinearity of $\boldsymbol{I}_2$. Hence, any $\boldsymbol{p}' \in \mathcal{S} \cap \mathcal{T}(P^\star)$ is a solution of the optimization in (3.48).

## A.6 Duality for Second Order Cone Programs

A standard (dual) form of the general SOCP reads as[14]

$$\max_{\boldsymbol{y},\boldsymbol{z}} f(\boldsymbol{y}) \quad \text{s.t.} \quad \boldsymbol{c}_i - \boldsymbol{A}_i^{\mathrm{T}} \boldsymbol{y} = \boldsymbol{z}_i, \quad \boldsymbol{z}_i \in \mathcal{L}^{q_i}, \quad i = 1, \ldots, m, \tag{A.18}$$

where $f(\boldsymbol{y})$ is a concave function in $\boldsymbol{y} \in \mathbb{R}^n$ and $\boldsymbol{z} = [\boldsymbol{z}_1^{\mathrm{T}}, \ldots, \boldsymbol{z}_{q_i}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^q$, $\boldsymbol{z}_i = [z_{i1}, \boldsymbol{z}_{i2}^{\mathrm{T}}]^{\mathrm{T}}$. An example for $f(\boldsymbol{y})$ is $-\boldsymbol{b}^{\mathrm{T}}\boldsymbol{y}$, but we focus on $-|\boldsymbol{b}^{\mathrm{T}}\boldsymbol{y}|^2$ instead. Using multipliers $\boldsymbol{x}_i \in \mathbb{R}^{q_i}$ and $\mu_i \geq 0$, $i = 1, \ldots, m$, the Lagrangian function reads as

[14] Alternatively, the equality and conic constraints of (A.18) may be combined to $\boldsymbol{c}_i - \boldsymbol{A}_i^{\mathrm{T}} \boldsymbol{y} \in \mathcal{L}^{q_i}$.

$$L(\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\mu}) = f(\boldsymbol{y}) + \sum_{i=1}^{m} \boldsymbol{x}_i^{\mathrm{T}}(\boldsymbol{c}_i - \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{y}) - \boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{z}_i + \mu_i(z_{i1} - \|\boldsymbol{z}_{i2}\|_2). \tag{A.19}$$

This function allows to recast (A.18) as the max–min optimization

$$\max_{\boldsymbol{y},\boldsymbol{z}} \inf_{\boldsymbol{x},\boldsymbol{\mu}} L(\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\mu}) \quad \text{s.t.} \quad \boldsymbol{z}_i \in \mathcal{L}^{q_i}, \quad \mu_i \geq 0, \quad i = 1, \ldots, m \tag{A.20}$$

and the Lagrangian dual problem with exchanged min- and maximization as[15]

$$\min_{\boldsymbol{x},\boldsymbol{\mu}} \sup_{\boldsymbol{y},\boldsymbol{z}} L(\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{x}, \boldsymbol{\mu}) \quad \text{s.t.} \quad \boldsymbol{z}_i \in \mathcal{L}^{q_i}, \quad \mu_i \geq 0, \quad i = 1, \ldots, m. \tag{A.22}$$

Due to convexity of (A.18), the duality gap between (A.20) and (A.22) is zero, that is, strong duality holds at the solutions of these problem formulations [103, Chapter 2]. Otherwise, the objective of (A.22) upper bounds the optimum of (A.18).

The Karush–Kuhn–Tucker (KKT) optimality conditions provide a necessary and sufficient characterization for the solutions (e.g., [102, 227]):

1. *Feasibility:* The vectors $\boldsymbol{y}$, $\boldsymbol{z}$, and $\boldsymbol{\mu}$ are primal and dual feasible, respectively, if

$$\boldsymbol{c}_i - \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{y} = \boldsymbol{z}_i, \quad \boldsymbol{z}_i \in \mathcal{L}^{q_i}, \quad \mu_i \geq 0, \quad i = 1, \ldots, m. \tag{A.23}$$

2. *Complementary Slackness:* The following products including $\boldsymbol{x}$ and $\boldsymbol{\mu}$ are zero:

$$\boldsymbol{x}_i^{\mathrm{T}}\left(\boldsymbol{c}_i - \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{y} - \boldsymbol{z}_i\right) = \boldsymbol{0}, \quad \mu_i(z_{i1} - \|\boldsymbol{z}_{i2}\|_2) = 0, \quad i = 1, \ldots, m. \tag{A.24}$$

3. *First Order Conditions:* The derivatives of the Lagrangian function are zero, i.e.,

$$\frac{\partial}{\partial \boldsymbol{y}} f(\boldsymbol{y}) = \sum_{i=1}^{m} \boldsymbol{A}_i\boldsymbol{x}_i, \quad \boldsymbol{x}_i = \mu_i[1, -\boldsymbol{z}_{i2}^{\mathrm{T}}\|\boldsymbol{z}_{i2}\|_2^{-1}]^{\mathrm{T}}, \quad i = 1, \ldots, m. \tag{A.25}$$

**Lemma A.4** *In particular, solutions* $\boldsymbol{x}_i \in \mathcal{L}^{q_i}$ *for* (A.21) *have the structure* [73, 74]

$$\boldsymbol{x}_i = \lambda_i \, \mathrm{bdiag}(1, -\mathbf{I}) \left(\boldsymbol{c}_i - \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{y}\right), \quad \lambda_i \in \mathbb{R}_+, \quad i = 1, \ldots, m, \tag{A.26}$$

*where* $\lambda_i = 0$ *if* $c_{i1} - \boldsymbol{a}_{i1}^{\mathrm{T}}\boldsymbol{y} < \|\boldsymbol{c}_{i2} - \boldsymbol{A}_{i2}^{\mathrm{T}}\boldsymbol{y}\|_2$, $\boldsymbol{c} = \left[c_{i1}, \boldsymbol{c}_{i2}^{\mathrm{T}}\right]^{\mathrm{T}}$ *and* $\boldsymbol{A}_i = [\boldsymbol{a}_{i1}, \boldsymbol{A}_{i2}]$.

[15] For the linear objective $f(\boldsymbol{y}) = -\boldsymbol{b}^{\mathrm{T}}\boldsymbol{y}$, a conic form representation of (A.22) reads as

$$\min_{\boldsymbol{x},\boldsymbol{\mu}} \sum_{i=1}^{m} \boldsymbol{c}_i^{\mathrm{T}}\boldsymbol{x}_i \quad \text{s.t.} \quad \sum_{i=1}^{m} \boldsymbol{A}_i\boldsymbol{x}_i = \boldsymbol{b}, \quad \boldsymbol{x}_i \in \mathcal{L}^{q_i}, \quad i = 1, \ldots, m. \tag{A.21}$$

*Proof* The latter condition in (A.24) shows that $\mu_i > 0$ only if $z_{i1} = \|\boldsymbol{z}_{i2}\|_2$ and $\mu_i = 0$ exactly if $z_{i1} > \|\boldsymbol{z}_{i2}\|_2$. This allows to write the structure for $\boldsymbol{x}_i$ in (A.25) as

$$\boldsymbol{x}_i = \mu_i \|\boldsymbol{z}_{i2}\|_2^{-1} [z_{i1}, -\boldsymbol{z}_{i2}^{\mathrm{T}}]^{\mathrm{T}}, \tag{A.27}$$

where $z_{i1}$ replaces $\|\boldsymbol{z}_{i2}\|_2$. Now, defining the non-negative variable $\lambda_i = \mu_i \|\boldsymbol{z}_{i2}\|_2^{-1} \in \mathbb{R}_+$ and replacing $\boldsymbol{z}_i$ according to (A.23), we obtain (A.26). □

Therewith, we aim at a dual representation when $f(\boldsymbol{y}) = -|\boldsymbol{b}^{\mathrm{T}}\boldsymbol{y}|^2$ and the parameters $\boldsymbol{A}_i$, $\boldsymbol{b}$, and $\boldsymbol{c}_i$ have similar properties as for (4.30):[16]

$$c_{i1} = 0, \qquad \boldsymbol{A}_{i2}\boldsymbol{c}_{i2} = \boldsymbol{0}, \quad i = 1, \dots, m, \tag{A.28}$$

$$\boldsymbol{A}_{i2}^{\mathrm{T}}\boldsymbol{b} = \boldsymbol{0}, \qquad \boldsymbol{A}_{i2}^{\mathrm{T}}\boldsymbol{a}_{\ell 1} = \boldsymbol{0}, \quad i = 1, \dots, m, \quad \ell = r+1, \dots, m, \tag{A.29}$$

$$\boldsymbol{a}_{j1}^{\mathrm{T}}\boldsymbol{a}_{i1} = 0, \qquad \boldsymbol{a}_{i1} \in \mathrm{range}\{\boldsymbol{A}_{i2}\}, \quad i \neq j, \quad i = 1, \dots, r, \quad j = 1, \dots, m. \tag{A.30}$$

**Theorem A.2** *Given* (A.28)–(A.30) *and Lemma A.4, the minimization problem*

$$\min_{\boldsymbol{\lambda} \geq \boldsymbol{0}} -\sum_{i=1}^{\kappa} \lambda_i \|\boldsymbol{c}_{i2}\|_2^2 \quad \text{s.t.} \quad 1 - \lambda_k \boldsymbol{a}_{k1}^{\mathrm{T}} \Big( \sum_{\ell=1}^{\kappa} \lambda_\ell \boldsymbol{A}_{\ell 2} \boldsymbol{A}_{\ell 2}^{\mathrm{T}} \Big)^{\dagger} \boldsymbol{a}_{k1} \geq 0, \quad k = 1, \dots, r,$$
$$\|\boldsymbol{b}\|_2^2 - \sum_{\ell=r+1}^{\kappa} \lambda_\ell \|\boldsymbol{a}_{\ell 1}\|_2^2 \geq 0 \tag{A.31}$$

*is a Lagrangian dual of* (A.22) *if* $\boldsymbol{a}_{\ell 1}$, $\ell = r+1, \dots, \kappa$, $\kappa \leq m$ *are collinear to* $\boldsymbol{b}$.

*Proof* Inserting (A.26) and in (A.25) for $\boldsymbol{x}_i$, at the first and second position in (A.19), respectively, the Lagrangian function becomes[17]

[16] These properties also apply for the SINR constrained power minimization of [67, 73, 74].

[17] Note that (A.33) corresponds to the Quadratically constrained program (QCP)

$$\max_{\boldsymbol{y}} f(\boldsymbol{y}) \quad \text{s.t.} \quad (c_{i1} - \boldsymbol{a}_{i1}^{\mathrm{T}}\boldsymbol{y})^2 \geq \|\boldsymbol{c}_{i2} - \boldsymbol{A}_{i2}^{\mathrm{T}}\boldsymbol{y}\|_2^2, \quad i = 1, \dots, m, \tag{A.32}$$

whose necessary KKT optimality conditions are sufficient for satisfying (A.23)–(A.25) by construction of (A.33) with (A.27) if the KKT points additionally fulfill $c_{i1} - \boldsymbol{a}_{i1}^{\mathrm{T}}\boldsymbol{y} \geq 0$, $i = 1, \dots, m$ [73, Appendix A]. While KKT points can exist for (A.32), that violate this necessary requirement, these points are obviously infeasible for (A.18) [see (A.23)]. Hence, solving (A.32) generally provides a valid solution for (A.18) only if there is a mapping for KKT points $\boldsymbol{y}$ with $c_{i1} - \boldsymbol{a}_{i1}^{\mathrm{T}}\boldsymbol{y} < 0$ into KKT points $\boldsymbol{y}'$ with $c_{i1} - \boldsymbol{a}_{i1}^{\mathrm{T}}\boldsymbol{y}' > 0$.

$$L(\boldsymbol{y},\boldsymbol{\lambda}) = f(\boldsymbol{y}) + \sum_{i=1}^{m} \lambda_i \big((c_{i1} - \boldsymbol{a}_{i1}^{\mathrm{T}}\boldsymbol{y})^2 - \|\boldsymbol{c}_{i2} - \boldsymbol{A}_{i2}^{\mathrm{T}}\boldsymbol{y}\|_2^2\big). \tag{A.33}$$

With property (A.28) and $f(\boldsymbol{y}) = -|\boldsymbol{b}^{\mathrm{T}}\boldsymbol{y}|^2$, this Lagrangian function becomes

$$L(\boldsymbol{y},\boldsymbol{\lambda}) = -\sum_{i=1}^{m} \lambda_i \|\boldsymbol{c}_{i2}\|_2^2 + \boldsymbol{y}^{\mathrm{T}}\Big(-\boldsymbol{b}\boldsymbol{b}^{\mathrm{T}} + \sum_{i=1}^{m} \lambda_i \big(\boldsymbol{a}_{i1}\boldsymbol{a}_{i1}^{\mathrm{T}} - \boldsymbol{A}_{i2}\boldsymbol{A}_{i2}^{\mathrm{T}}\big)\Big)\boldsymbol{y}. \tag{A.34}$$

Here, the dual objective $g(\boldsymbol{\lambda}) = \sup_{\boldsymbol{y}} L(\boldsymbol{y},\boldsymbol{\lambda})$ in (A.22) is unbounded above unless

$$\boldsymbol{b}\boldsymbol{b}^{\mathrm{T}} - \sum_{i=1}^{m} \lambda_i \big(\boldsymbol{a}_{i1}\boldsymbol{a}_{i1}^{\mathrm{T}} - \boldsymbol{A}_{i2}\boldsymbol{A}_{i2}^{\mathrm{T}}\big) \succeq \boldsymbol{0}, \tag{A.35}$$

where $g(\boldsymbol{\lambda}) = -\sum_{i=1}^{m} \lambda_i \|\boldsymbol{c}_{i2}\|_2^2$ and $\boldsymbol{y} = \boldsymbol{0}$ if the linear matrix inequality is strict and $\boldsymbol{y}$ lies in the nullspace of the singular matrix. Due to orthogonality (A.29) and positive semidefiniteness, this condition holds if and only if

$$\boldsymbol{b}\boldsymbol{b}^{\mathrm{T}} - \sum_{\ell=r+1}^{m} \lambda_\ell \boldsymbol{a}_{\ell 1}\boldsymbol{a}_{\ell 1}^{\mathrm{T}} \succeq \boldsymbol{0}, \tag{A.36}$$

$$\sum_{i=1}^{r} \lambda_i \boldsymbol{a}_{i1}\boldsymbol{a}_{i1}^{\mathrm{T}} - \sum_{i=1}^{m} \lambda_i \boldsymbol{A}_{i2}\boldsymbol{A}_{i2}^{\mathrm{T}} \succeq \boldsymbol{0}. \tag{A.37}$$

Due to (A.36), $\lambda_\ell > 0$ only holds for collinear $\boldsymbol{a}_{\ell 1}$ and $\boldsymbol{b}$ and $\lambda_\ell = 0$ otherwise. Let $\ell = r+1, \ldots, \kappa$ be the indices of these vectors. Then, (A.36) becomes the second constraint in (A.31) and $g(\boldsymbol{y})$ reduces to the first $\kappa - r + 1$ summands.

Furthermore, because of mutual independence (orthogonality) between the vectors $\boldsymbol{a}_{ij}$, $j = 1, \ldots, r$, (A.37) holds if and only if

$$\sum_{j=1}^{m} \lambda_j \boldsymbol{A}_{j2}\boldsymbol{A}_{j2}^{\mathrm{T}} - \lambda_k \boldsymbol{a}_{k1}\boldsymbol{a}_{k1}^{\mathrm{T}} \succeq 0, \quad k = 1, \ldots, r.$$

Applying Schur's complement relation [293], these positive semidefiniteness conditions transform to the second constraints in (A.31). The range-space property in (A.30), which can be relaxed to $\boldsymbol{a}_{i1} \in \operatorname{range}\Big\{\sum_{j=1}^{m} \lambda_j \boldsymbol{A}_{j2}\Big\}$, ensures sufficiency of the reformulated constraints in (A.31) for (A.37). Otherwise, the multiplier $\lambda_i = 0$ if $\boldsymbol{a}_{i1} \notin \operatorname{range}\Big\{\sum_{j=1}^{m} \lambda_j \boldsymbol{A}_{j2}\Big\}$. □

### A.6.1 Application to Uplink–Downlink Duality for MSE Based QoS Optimization

With the extended precoder $\tilde{\boldsymbol{t}} = [p, \mathrm{Re}(\boldsymbol{t}^{\mathrm{T}}), \mathrm{Im}(\boldsymbol{t}^{\mathrm{T}})]^{\mathrm{T}}$, the real valued equivalent standard SOC constraint formulation of (4.30) reads as [cf. (A.18)][18]

$$\max_{\tilde{\boldsymbol{t}},z} -(\mathbf{e}_1^{\mathrm{T}}\tilde{\boldsymbol{t}})^2 \quad \text{s.t.} \quad -\tilde{\boldsymbol{A}}_\ell^{\mathrm{T}}\tilde{\boldsymbol{t}} = z_{\ell+K}, \quad z_{\ell+K} \in \mathcal{L}^{NK}, \quad \ell = 1,\ldots,L,$$
$$\boldsymbol{c}_k - \tilde{\boldsymbol{R}}_k^{\mathrm{T}}\tilde{\boldsymbol{t}} = z_k, \qquad z_k \in \mathcal{L}^{NK+1}, \quad k = 1,\ldots,K, \tag{A.38}$$

where we introduced $\boldsymbol{c}_k = [c_{1k}, \boldsymbol{c}_{2k}^{\mathrm{T}}]^{\mathrm{T}}$ and $\tilde{\boldsymbol{R}}_k = [\tilde{\boldsymbol{r}}_{1k}, \tilde{\boldsymbol{R}}_{2,k}]$ as

$$\boldsymbol{c}_k = \begin{bmatrix} \mathbf{0} \\ \tilde{\sigma}_k \end{bmatrix}, \quad \tilde{\boldsymbol{R}}_k^{\mathrm{T}} = \begin{bmatrix} 0 & \mathbf{e}_k^{\mathrm{T}} \otimes (1-\varepsilon_k)^{-1/2}\,\mathrm{Re}(\bar{\boldsymbol{h}}_k^{\mathrm{H}}) & \mathbf{e}_k^{\mathrm{T}} \otimes (1-\varepsilon_k)^{-1/2}\,\mathrm{Im}(\bar{\boldsymbol{h}}_k^{\mathrm{T}}) \\ \mathbf{0} & \mathrm{Re}(\hat{\boldsymbol{R}}_k^{1/2}) & -\mathrm{Im}(\hat{\boldsymbol{R}}_k^{1/2}) \\ \mathbf{0} & \mathrm{Im}(\hat{\boldsymbol{R}}_k^{1/2}) & \mathrm{Im}(\hat{\boldsymbol{R}}_k^{1/2}) \\ 0 & \mathbf{0}^{\mathrm{T}} & \mathbf{0}^{\mathrm{T}} \end{bmatrix}$$

for the MSE constraints and, for the power constraints, $\tilde{\boldsymbol{A}}_\ell = [\tilde{\boldsymbol{a}}_{1\ell}, \tilde{\boldsymbol{A}}_{2\ell}]$ reads as

$$\tilde{\boldsymbol{A}}_\ell^{\mathrm{T}} = \begin{bmatrix} \sqrt{P_\ell} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathrm{Re}(\boldsymbol{A}_\ell^{1/2}) & -\mathrm{Im}(\boldsymbol{A}_\ell^{1/2}) \\ \mathbf{0} & \mathrm{Im}(\boldsymbol{A}_\ell^{1/2}) & \mathrm{Re}(\boldsymbol{A}_\ell^{1/2}) \end{bmatrix}, \quad \ell = 1,\ldots,L.$$

Since $\boldsymbol{c}_{2k}$, $\tilde{\boldsymbol{r}}_{1k}$, and $\tilde{\boldsymbol{a}}_{1\ell}$ meet the orthogonality requirements [cf. (A.28)–(A.30)]

$$\begin{aligned} &&\tilde{\boldsymbol{R}}_{2k}\boldsymbol{c}_{2k} = \mathbf{0}, \quad & k = 1,\ldots,K, \\ \tilde{\boldsymbol{R}}_{2k}\mathbf{e}_1 = \mathbf{0}, \quad &&\tilde{\boldsymbol{A}}_{2m}\mathbf{e}_1 = \mathbf{0}, \quad & k = 1,\ldots,K, \quad m = 1,\ldots,L, \\ \tilde{\boldsymbol{R}}_{2k}\boldsymbol{a}_{1\ell} = \mathbf{0}, \quad &&\tilde{\boldsymbol{A}}_{2m}\boldsymbol{a}_{1\ell} = \mathbf{0}, \quad & k = 1,\ldots,K, \quad m,\ell = 1,\ldots,L, \\ \tilde{\boldsymbol{r}}_{2k}^{\mathrm{T}}\tilde{\boldsymbol{a}}_{1\ell} = \mathbf{0}, \quad &&\tilde{\boldsymbol{r}}_{1k}^{\mathrm{T}}\tilde{\boldsymbol{r}}_{1j} = \mathbf{0}, \quad & k \neq j, \quad k,j = 1,\ldots,K, \quad \ell = 1,\ldots,L, \end{aligned}$$

we can apply Theorem A.2 to write the dual optimization of (A.38) as

$$\min_{\boldsymbol{\mu}\geq\mathbf{0},\boldsymbol{\lambda}\geq\mathbf{0}} -\sum_{i=1}^{K}\lambda_i\|\boldsymbol{c}_{i2}\|_2^2 \quad \text{s.t.} \quad 1-\sum_{\ell=1}^{L}\mu_\ell\|\tilde{\boldsymbol{a}}_{\ell 1}\|_2^2 \geq 0$$
$$1-\lambda_k\tilde{\boldsymbol{r}}_{k1}^{\mathrm{T}}\Big(\sum_{i=1}^{K}\lambda_i\tilde{\boldsymbol{R}}_{i2}\tilde{\boldsymbol{R}}_{i2}^{\mathrm{T}} + \sum_{\ell=1}^{L}\mu_\ell\tilde{\boldsymbol{A}}_{\ell 2}\tilde{\boldsymbol{A}}_{\ell 2}^{\mathrm{T}}\Big)^{\dagger}\tilde{\boldsymbol{r}}_{k1} \geq 0, \quad k = 1,\ldots,K. \tag{A.39}$$

[18]This is actually the dual form of a standard SOCP [102, Section 5.2.1] as shown in (A.18).

Inserting the substitutes for $\boldsymbol{c}_k$, $\tilde{\boldsymbol{R}}_k$, and $\tilde{\boldsymbol{A}}_\ell$, the equivalent complex form of (A.39) reads as[19]

$$\begin{aligned}
&\min_{\boldsymbol{\mu}\geq\mathbf{0},\boldsymbol{\lambda}\geq\mathbf{0}} -\sum_{i=1}^{K}\lambda_i\tilde{\sigma}_i^2 \quad \text{s.t.} \quad 1-\sum_{\ell=1}^{L}\mu_\ell P_\ell \geq 0 \\
&1-\frac{\lambda_k}{1-\varepsilon_k}\bar{\boldsymbol{h}}_k^{\mathrm{H}}\Big(\sum_{i=1}^{K}\lambda_i\boldsymbol{R}_k+\sum_{\ell=1}^{L}\mu_\ell\boldsymbol{A}_{k,\ell}\Big)^{\dagger}\bar{\boldsymbol{h}}_k \geq 0, \quad k=1,\ldots,K.
\end{aligned} \tag{A.40}$$

This formulation is equivalent to the dual formulation of the per-user MSE constrained QoS optimization in Corollary 4.1.

### A.6.2 Reconstruction of the Primal Variables

The duality gap between the optima of the dual and the primal problem is zero. Both problems have conic convex formulations and the solution $\boldsymbol{\lambda}^\star$ for (A.31) satisfies the KKT conditions (A.23)–(A.25). Given $\boldsymbol{\lambda}^\star$, reconstructing the primal solution $\boldsymbol{y}$ is by the zero-derivative condition (A.25). We separate $\boldsymbol{y}$ into the sum of two vectors

$$\boldsymbol{y}=\boldsymbol{\Pi_b}\boldsymbol{y}+(\mathbf{I}-\boldsymbol{\Pi_b})\boldsymbol{y}=\boldsymbol{y}_1+\boldsymbol{y}_2, \tag{A.41}$$

where $\boldsymbol{y}_1=\boldsymbol{\Pi_b}\boldsymbol{y}$, $\boldsymbol{\Pi_b}=\|\boldsymbol{b}\|_2^{-1}\boldsymbol{b}\boldsymbol{b}^{\mathrm{T}}$ is the projection into the span of $\boldsymbol{b}$, and $\boldsymbol{y}_2=(\mathbf{I}-\boldsymbol{\Pi_b})\boldsymbol{y}$ is the projection into the orthogonal complement space.

Inserting (A.28)–(A.30), (A.25), and the zero duality gap provides the solution for $\boldsymbol{y}_1$ and the structure of $\boldsymbol{y}_2$, which read as

$$\boldsymbol{y}_1=\frac{\sqrt{\sum_{i=1}^{m}\lambda_i\|\boldsymbol{c}_{i2}\|_2^2}}{\|\boldsymbol{b}\|_2}\boldsymbol{b}, \tag{A.42}$$

$$\boldsymbol{y}_2=\sum_{j=1}^{r}\beta_j\boldsymbol{y}_{j2}=\sum_{j=1}^{r}\boldsymbol{a}_{j1}^{\mathrm{T}}\boldsymbol{y}_2\lambda_j\Big(\sum_{i=1}^{m}\lambda_i\boldsymbol{A}_{i2}\boldsymbol{A}_{i2}^{\mathrm{T}}\Big)^{\dagger}\boldsymbol{a}_{j1}. \tag{A.43}$$

That is, $\boldsymbol{y}_2$ is a linear combination of the vectors $\boldsymbol{y}_{j2}=\lambda_j(\sum_{i=1}^{m}\lambda_i\boldsymbol{A}_{i2}\boldsymbol{A}_{i2}^{\mathrm{T}})^{\dagger}\boldsymbol{a}_{j1}$, with weights $\beta_j$, $j=1,\ldots,r$. Finding the weights $\beta_j$, $j=1,\ldots,r$ is in turn the solution of the conic inequality conditions from primal feasibility (A.23), that is,

$$-\boldsymbol{a}_{i1}^{\mathrm{T}}\bar{\boldsymbol{Y}}_2\boldsymbol{\beta}\geq\sqrt{\|\boldsymbol{c}_{i2}\|_2^2+\|\boldsymbol{A}_{i2}^{\mathrm{T}}\bar{\boldsymbol{Y}}_2\boldsymbol{\beta}\|_2^2}, \quad i=1,\ldots,r, \tag{A.44}$$

[19] For the complex reconstruction, we used the equivalent complex expressions for real valued representations and matrix vector multiplications in [325, Lemma 1], for example.

where $\bar{\boldsymbol{Y}}_2 = [\boldsymbol{y}_{12}, \ldots, \boldsymbol{y}_{r2}]$, $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_r]^{\mathrm{T}}$, and we inserted the orthogonality of (A.28). Since existence of such a $\boldsymbol{\beta}$ is ensured, its computation is a feasibility problem over a set of conic convex constraints.

Computing $\boldsymbol{\beta}$ simplifies for the considered applications, where we can exploit the block structures of $\boldsymbol{a}_{i1}$, $i = 1, \ldots, r$ and $\boldsymbol{A}_{i2}$, $i = 1, \ldots, m$, e.g.,

$$\boldsymbol{a}_{i1} = \mathbf{e}_i \otimes \tilde{\boldsymbol{a}}_{i1}, \quad i = 1, \ldots, r, \qquad \boldsymbol{A}_{i2} = \mathbf{I}_m \otimes \tilde{\boldsymbol{A}}_{i2}, \quad i = 1, \ldots, m, \tag{A.45}$$

with auxiliary vectors $\tilde{\boldsymbol{a}}_{i1} \in \mathbb{R}^{n/m}$ and $\tilde{\boldsymbol{A}}_{i2} \in \mathbb{R}^{n/m \times n/m}$ and the canonical unit vector $\mathbf{e}_i \in \mathbb{R}^m$. The structure particularly leads to

$$\boldsymbol{a}_{i1}^{\mathrm{T}} \boldsymbol{y}_{i2} > 0, \quad \boldsymbol{a}_{i1}^{\mathrm{T}} \boldsymbol{y}_{j2} = 0, \quad j \neq i, \quad i, j = 1, \ldots, r \tag{A.46}$$

$$\bar{\boldsymbol{Y}}_2^{\mathrm{T}} \boldsymbol{A}_{i2} \boldsymbol{A}_{i2}^{\mathrm{T}} \bar{\boldsymbol{Y}}_2 = \operatorname{diag}\left(\|\boldsymbol{A}_{i2}^{\mathrm{T}} \boldsymbol{y}_{12}\|_2^2, \ldots, \|\boldsymbol{A}_{i2}^{\mathrm{T}} \boldsymbol{y}_{r2}\|_2^2\right), \quad i = 1, \ldots, r. \tag{A.47}$$

The conic form equality condition in (A.44), therefore, becomes equivalent to

$$(\boldsymbol{a}_{i1}^{\mathrm{T}} \boldsymbol{y}_{i2})^2 \beta_i^2 = \|\boldsymbol{c}_{i2}\|_2^2 + \sum_{j=1}^{r} \|\boldsymbol{A}_{i2}^{\mathrm{T}} \boldsymbol{y}_{j2}\|_2^2 \beta_j^2, \quad -\boldsymbol{a}_{i1}^{\mathrm{T}} \boldsymbol{y}_{i2} \beta_i > 0, \quad i = 1, \ldots, r. \tag{A.48}$$

Hence, the search for $\boldsymbol{\beta}$ is the solution to a linear equation system[20]

$$\boldsymbol{\Psi} \boldsymbol{\beta}^2 = \boldsymbol{\zeta}, \tag{A.49}$$

where we substituted $\boldsymbol{\beta}^2 = [\beta_1^2, \ldots, \beta_r^2]^{\mathrm{T}}$, $\boldsymbol{\zeta} = [\|\boldsymbol{c}_{12}\|_2^2, \ldots, \|\boldsymbol{c}_{r2}\|_2^2]^{\mathrm{T}}$, and

$$[\boldsymbol{\Psi}]_{i,j} = -\|\boldsymbol{A}_{i2}^{\mathrm{T}} \boldsymbol{y}_{j2}\|_2^2, \quad i \neq j,$$
$$[\boldsymbol{\Psi}]_{i,i} = (\boldsymbol{a}_{i1} \boldsymbol{y}_{i2})^2 - \|\boldsymbol{A}_{i2}^{\mathrm{T}} \boldsymbol{y}_{i2}\|_2^2, \quad i, j = 1, \ldots, r.$$

## A.7 Properties of the Dual Uplink MSE Optimizations

**Lemma A.5** *The outer maximization of the dual max–min QoS optimization with weighted sum MSE constraints* (4.34) *is a convex optimization problem.*

*Proof* The constraint $\sum_{\ell=1}^{L} P_\ell \mu_\ell \leq 1$ is affine in $\boldsymbol{\mu}$ and the objective $\sum_{j=1}^{M} \lambda_j^\star \hat{\sigma}_j^2$ is linear in the limit point $\boldsymbol{\lambda}^\star$ of the sequence $\{\boldsymbol{\lambda}^{(n)}\}_n$ with update rule (4.56). Hence, the outer maximization of (4.34) is a convex problem if the limit point

[20] This is similar to the available MSE dualities with a single sum power constraint [161].

$\lambda_j^\star = \lim_{n\to\infty} \lambda_j^{(n)}$ is concave in $\boldsymbol{\mu}$. The proof of this statement is by induction. Given a starting point $\boldsymbol{\lambda}'$, $\lambda_j^{(1)} = I_j(\boldsymbol{\lambda}', \boldsymbol{\mu})$ is the concatenation of the affine function

$$\boldsymbol{X}(\boldsymbol{\lambda}', \boldsymbol{\mu}) = \sum_{i=1}^{M} \lambda_i' \hat{\boldsymbol{R}}_i + \sum_{\ell=1}^{L} \mu_\ell \boldsymbol{A}_\ell \tag{A.50}$$

with the concave function[21]

$$f_j(\boldsymbol{X}) = \frac{m_j - \varepsilon_j}{\hat{\boldsymbol{h}}^{\mathrm{H}} \boldsymbol{G}_j \boldsymbol{X}^\dagger \boldsymbol{G}_j \hat{\boldsymbol{h}}} \tag{A.51}$$

and, hence, concave increasing in $\boldsymbol{\mu}$. By the same argument, $\lambda_j^{(n+1)} = I_j(\boldsymbol{\lambda}^{(n)}, \boldsymbol{\mu})$ is jointly concave increasing in $\boldsymbol{\lambda}^{(n)}$ and $\boldsymbol{\mu}$. Furthermore, since the concatenation of two concave increasing functions is again concave and increasing, also $\lambda_j^{(n+1)} = I_j(I_j(\boldsymbol{\lambda}^{(n-1)}, \boldsymbol{\mu}), \boldsymbol{\mu})$ is concave in $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}^{(n-1)}$. Applying this, each sequence element is concave in $\boldsymbol{\mu}$, such that also the convergence point $\boldsymbol{\lambda}^\star$ shares this property. □

**Lemma A.6** *Let* $\boldsymbol{\lambda}^\star$ *be a solution to either of the inner minimizations in* (4.91) *and* (4.89) *with given* $\boldsymbol{\mu}$. *Then,* $\mathrm{WAMSE}_j(\boldsymbol{\lambda}^\star, \boldsymbol{\mu}) = \varepsilon\varepsilon_j$ *for all indices* $j \in \mathcal{J}$ *within the active set*

$$\mathcal{J} = \{j \in \{1, \ldots, K\} : \varepsilon\varepsilon_j \leq m_j\}.$$

*Proof* Assume the contrary for the proof, i.e., a solution $\boldsymbol{\lambda}'$ exists with $\sum_{j=1}^{M} \lambda_j' \hat{\sigma}_j^2 = p$ and $\varepsilon_i^{-1} \mathrm{WAMSE}_i(\boldsymbol{\lambda}', \boldsymbol{\mu}) < \varepsilon$ for at least on $i \in \mathcal{J}$, where $\varepsilon = \max_j \varepsilon_j^{-1} \mathrm{WAMSE}_j(\boldsymbol{\lambda}', \boldsymbol{\mu})$. Constructing $\boldsymbol{\lambda} \leq \boldsymbol{\lambda}'$ as $\lambda_i = \lambda_i' - \epsilon$ with sufficiently small $\epsilon > 0$ and $\lambda_k = \lambda_k'$ for $k \neq i$, $\boldsymbol{\lambda}$ is in the $\epsilon$-neighborhood of $\boldsymbol{\lambda}'$. However, $\mathrm{WAMSE}_j(\boldsymbol{\lambda}, \boldsymbol{\mu}) < \mathrm{WAMSE}_j(\boldsymbol{\lambda}', \boldsymbol{\mu}) = \varepsilon\varepsilon_j$ for $j \in \mathcal{J}$, $j \neq i$ and $\varepsilon\varepsilon_i > \mathrm{WAMSE}_i(\boldsymbol{\lambda}, \boldsymbol{\mu}) > \mathrm{WAMSE}_i(\boldsymbol{\lambda}', \boldsymbol{\mu})$, while the power is reduced to $\sum_{j=1}^{M} \lambda_j \hat{\sigma}_j^2 = p - \hat{\sigma}_j^2 \epsilon < p$ by $\boldsymbol{\lambda}$. This contradicts the initial optimality assumption of the power allocation $\boldsymbol{\lambda}'$. □

**Lemma A.7** *The downlink optima* $\rho^{(\mathrm{dl})}(p)$ *and* $p^{(\mathrm{dl})}(\boldsymbol{\varepsilon}(\rho))$ *for* (4.109) *and* (4.19), *respectively, are inverse functions if the QoS targets are* $\varepsilon_k(\rho)$, $k = 1, \ldots, K$. *Then, also the uplink optima* $\rho^{(\mathrm{ul})}(p)$ *and* $p^{(\mathrm{ul})}(\boldsymbol{\varepsilon}(\rho))$ *of* (4.110) *and* (4.34), *respectively, are inverse to each other.*

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[21] The function $g : \mathcal{H}_+^N \to \mathbb{R}_+$, with $g(\boldsymbol{X}) = (\boldsymbol{a}^{\mathrm{H}} \boldsymbol{X}^{-1} \boldsymbol{a})^{-1}$ is concave if $\frac{\mathrm{d}^2}{\mathrm{d}\alpha^2} g(\boldsymbol{A} + \alpha \boldsymbol{B}) \leq 0$. Writing

$$\frac{\mathrm{d}^2}{\mathrm{d}\alpha^2} g(\boldsymbol{A} + \alpha \boldsymbol{B}) = \frac{2}{(\boldsymbol{a}^{\mathrm{H}} \boldsymbol{X}^{-1} \boldsymbol{a})^3} \left( |(\boldsymbol{a}^{\mathrm{H}} \boldsymbol{X}^{-1/2})(\boldsymbol{X}^{-1/2} \boldsymbol{B} \boldsymbol{X}^{-1} \boldsymbol{a})|^2 - \|\boldsymbol{X}^{-1/2} \boldsymbol{B} \boldsymbol{X}^{-1} \boldsymbol{a}\|_2^2 \|\boldsymbol{X}^{-1/2} \boldsymbol{a}\|_2^2 \right) \Big|_{\boldsymbol{X} = \boldsymbol{A} + \alpha \boldsymbol{B}},$$

the inequality is by the Cauchy–Schwarz inequality. Therefore, also (A.51) is concave in $\boldsymbol{\mu}$.

*Proof* Since $\mathrm{AMSE}_k(\boldsymbol{t}) = \varepsilon_k(\rho)$ holds for all $k = 1, \ldots, K$ at the optimum of both optimizations and the entries of $\boldsymbol{\varepsilon}(\rho)$ are strict monotonically decreasing in $\rho$, $p^{(\mathrm{dl})}(\boldsymbol{\varepsilon}(\rho))$ strictly increases with $\rho$ and vice versa $\rho^{(\mathrm{dl})}(p)$ strictly increases with $p$. This shows the inversion in the downlink. Since $\mathrm{AMSE}_k^{(\mathrm{ul})} = \varepsilon_k(\rho)$, $k = 1, \ldots, K$ also holds at the optimum of the uplink balancing and QoS optimization (cf. Proof of Lemma 4.2), the inversion property is also valid for these optimizations. □

With this lemma, the same argumentation as for the proof of Theorem 4.2 also provides strong duality between (4.109) and (4.110).

## A.8 Some Distribution and Quantile Functions

The distribution function (CDF) $F_z : \mathbb{R} \to [0, 1]$ of a random $z \in \mathbb{R}$ reads as

$$F_z(x) = \Pr(z \leq x) \tag{A.52}$$

and its quantile $q \in \mathbb{R}$ at $\epsilon \in [0, 1]$ is $q = \mathrm{q}_z(\epsilon)$, where $\mathrm{q}_z : [0, 1] \to \mathbb{R}$ with

$$\mathrm{q}_z(\epsilon) = \inf\left\{x \in \mathbb{R} : \; F_z(x) \geq \epsilon\right\}. \tag{A.53}$$

If the CDF $F_z$ is continuously increasing in its domain and $\epsilon \in (0, 1)$, the Quantile function (QF) simply becomes the inverse of the CDF, i.e.,

$$\mathrm{q}_z(\epsilon) = F_z^{-1}(\epsilon). \tag{A.54}$$

The CDF of a standard normal distributed random variable $z \sim \mathcal{N}(0, 1)$ shall be denoted by $\Phi : \mathbb{R} \to [0, 1]$, which is given by [203, Equation 26.2.2]

$$\Phi(x) = (2\pi)^{-1/2} \int_{-\infty}^{x} \mathrm{e}^{-t^2/2} \, \mathrm{d}t. \tag{A.55}$$

The related distribution of a log-normal random $z \in \mathbb{R}_+$, $\ln(z) \sim \mathcal{N}(\mu, \sigma^2)$ is

$$F_z(x) = \Phi\Big(\frac{\ln(x) - \mu}{\sigma}\Big). \tag{A.56}$$

If in turn $z_{\mathrm{dB}} = 10 \log_{10}(z)$ dB is log-normal distributed, the CDF reads as

$$F_z(x) = \Phi\Big(\frac{\ln(10 \log_{10}(x)) - \mu}{\sigma}\Big). \tag{A.57}$$

Since these CDFs are continuous, the corresponding quantile functions are completely defined by $\Phi^{-1} : (0, 1) \to \mathbb{R}$. The $\epsilon$-quantile for the latter example is

$$F_z^{-1}(\epsilon) = 10^{10^{-1} F_{z_{\text{dB}}}^{-1}(\epsilon)}, \tag{A.58}$$

$$F_{z_{\text{dB}}}^{-1}(\epsilon) = \exp\big(\mu + \sigma \Phi^{-1}(\epsilon)\big). \tag{A.59}$$

The CDF of a (central) chi-square random variable $z \sim \mathcal{X}_k^2$, i.e., $z = \sum_{i=1}^k w_i^2 \in \mathbb{R}_+$ with i.i.d. normal distributed $w_i \sim \mathcal{N}(0, 1)$ and degree $k$, is denoted by $F_{\chi^2}(\cdot; k) : \mathbb{R}_+ \to [0, 1]$ and reads as (cf. [203, Equation 26.4.1])

$$F_{\chi^2}(x; k) = \frac{1}{\Gamma(k/2)} \int_0^{x/2} t^{k/2-1} \mathrm{e}^{-t} \, \mathrm{d}t. \tag{A.60}$$

For $k = 2$, i.e., $z \sim \mathcal{X}_2^2$ this distribution function simplifies to

$$F_{\chi^2}(x; 2) = 1 - \mathrm{e}^{-x/2}. \tag{A.61}$$

If $z$ is exponentially distributed, i.e., $2z \sim \mathcal{X}_2^2$ for $z = |y|^2$ and $y \sim \mathcal{N}_{\mathbb{C}}(0, 1)$, then

$$F_z(x) = F_{\chi^2}(2x; 2) = 1 - \mathrm{e}^{-x}. \tag{A.62}$$

The corresponding $\epsilon$-quantile is $F_z^{-1}(\epsilon) = -\ln(1 - \epsilon)$.

The series expansion for the CDF $F_{\chi^2(\lambda)}(\cdot; k) : \mathbb{R}_+ \to [0, 1]$ of a non-central chi-square-distributed random variable $z \sim \mathcal{X}_k^2(\lambda)$, i.e., $z = \sum_{i=1}^k (\nu_i - w_i)^2$ with $\nu_i \in \mathbb{R}_+$, non-centrality parameter $\lambda = \sum_{i=1}^k \nu_i^2$, and degree $k$, reads as [203, Equation 26.4.25]

$$F_{\chi^2(\lambda)}(x; k) = \mathrm{e}^{-(\lambda/2)} \sum_{j=0}^{\infty} \frac{(\lambda/2)^j}{j!} F_{\chi^2}(x; k + 2j). \tag{A.63}$$

This expression leads to an alternative series expansion for (5.28), which reads as

$$F_{\zeta_k^{-1}}(x) = \mathrm{e}^{-\sigma_{\xi_k}^{-2}} \sum_{j=0}^{\infty} \frac{\sigma_{\xi_k}^{-2j}}{j!} F_{\chi^2}(2\sigma_{\xi_k}^{-2} x; 2 + 2j). \tag{A.64}$$

The integral representation (5.28) in turn follows with an alternative representation of the non-central chi-square-distribution $F_{\chi^2(\lambda)}(x; k)$ from Nuttall [278]:

$$F_{\chi^2(\lambda)}(x; k) = 1 - Q_{k/2}(\sqrt{\lambda}, \sqrt{x}), \tag{A.65}$$

where $Q_M(a, b)$ denotes the Marcum-Q function

$$Q_M(a, b) = \frac{1}{a^{M-1}} \int_b^{\infty} s^M \mathrm{e}^{-\frac{x^2+a^2}{2}} I_{M-1}(as) \, \mathrm{d}s \tag{A.66}$$

and $I_N(x)$ is the modified Bessel function of the first kind, i.e., [203, Equation 9.6.10]

$$I_N(x) = \left(\frac{x}{2}\right)^N \sum_{n=0}^{\infty} \frac{(x/2)^{2n}}{n!\Gamma(N+n+1)}. \tag{A.67}$$

Inserting the latter two expressions with $M = 1$, $N = 0$, and $k = 2$ into (A.65), the function $F_{\chi^2(\lambda)}(x; 2)$ becomes

$$F_{\chi^2(\lambda)}(x; 2) = 1 - \int_{\sqrt{x}}^{\infty} s\, \mathrm{e}^{-\frac{s^2+\lambda}{2}} \sum_{n=0}^{\infty} \left(\frac{(\sqrt{\lambda}s/2)^n}{n!}\right)^2 \mathrm{d}s. \tag{A.68}$$

The CDF expression (5.28) in Sect. 5.2 is then obtained with $\lambda = 2\sigma_{\xi_k}^{-2}$. As for the computation of explicit values for the CDF, computing values for the quantile function of non-central chi-squared distributed random variables requires numerical evaluations, e.g., those from [203, Section 26.4] and [279, Section 4].

Moreover, the following lemma is useful to evaluate the quantile function for $z^{-1}$.

**Lemma A.8** *Let the CDFs* $F_z, F_{z^{-1}} : \mathbb{R}_+ \to [0, 1]$ *of the random variables* $z, z^{-1} \in \mathbb{R}_{++}$ *be continuously increasing in their domain and* $\epsilon \in (0, 1)$. *Then, the relation* $F_z(x) = 1 - F_{z^{-1}}(x^{-1})$ *holds and* $q = F_z^{-1}(1-\epsilon) = 1/F_{z^{-1}}^{-1}(\epsilon)$ *defines the* $1-\epsilon$*-quantile of* $F_z$.

*Proof* Let the random variable $z \in \mathbb{R}_{++}$ induce a strict monotonically increasing CDF $F_z(x)$ and $\Pr(z > x) > 0$ for $z > 0$. Then, the CDF induced by $z$ reads as

$$F_z(x) = \Pr(x^{-1} \le z^{-1}) = 1 - \Pr(x^{-1} > z^{-1}) = 1 - F_{z^{-1}}(x^{-1}),$$

where the last equality is with continuity of the CDF $F_{z^{-1}} : \mathbb{R}_+ \to [0, 1]$.[22] Therewith, $F_z(x) = 1 - \epsilon$ is equivalent to $F_{z^{-1}}(x^{-1}) = \epsilon$ for $x > 0$ and the inverse CDFs satisfy

$$x = F_z^{-1}(1-\epsilon) = (F_{z^{-1}}^{-1}(\epsilon))^{-1},$$

which proves the statement. □

Now, let $z = g(x, y)$ be a monotonic function $g : \mathbb{R}_+^2 \to \mathbb{R}$ of two independent continuous random variables $x, y \in \mathbb{R}_+$ with PDFs $f_x(x)$ and $f_y(y)$. The function $g$ shall be monotonically increasing and invertible in $y$, that is, there is a continuous function $h : \mathbb{R}^2 \to \mathbb{R}_+$ such that $y \le h(x, t)$ and $g(x, y) \le t$ are equivalent. Examples are a quotient $g(x, y) = x^{-1}y$ and a linear function $g(x, y) = y - ax$,

[22] The last equality has to be replaced by a smaller or equal sign if the CDF is non-continuous.

with $h(x,t) = xt$ and $h(x,t) = t + ax$, respectively. The CDF of such a random variable $z$ is generally unknown, but can be expressed via the integral representation

$$F_z(t) = \int_{D(X)} f_x(s)\,\Pr(y \le h(x,t)|x = s)\,\mathrm{d}s, \tag{A.69}$$

where $D(X)$ denotes the domain of $f_x$. Example $h(x,z) = xz$ is for a log-dB-normal random $y \in [1,\infty)$, where $\ln(y_{\mathrm{dB}}) \sim \mathcal{N}(\mu_{y_{\mathrm{dB}}}, \sigma^2_{y_{\mathrm{dB}}})$ and the PDF is

$$f_y(y) = \big(y\ln(y)\sqrt{(2\pi)}\sigma_{y_{\mathrm{dB}}}\big)^{-1}\exp\Big(-\frac{(\ln(10\log_{10}(y)) - \mu_{y_{\mathrm{dB}}})^2}{2\sigma^2_{y_{\mathrm{dB}}}}\Big), \tag{A.70}$$

and a non-central chi-square random variable $x \sim \mathcal{X}_2^2(\lambda)$, which PDF reads as

$$f_x(x) = \frac{1}{2}\mathrm{e}^{-\frac{x+\lambda}{2}}\sum_{j=0}^{\infty}\frac{(\lambda x/4)^j}{(j!)^2}. \tag{A.71}$$

The CDF of the random variable $z = x^{-1}y \in \mathbb{R}_+$ is thus

$$F_z(t) = \int_0^{\infty}\frac{1}{2}\mathrm{e}^{-\frac{s+\lambda}{2}}\Big(\sum_{j=0}^{\infty}\frac{(\lambda s/4)^j}{(j!)^2}\Big)\Phi\Big(\frac{\ln(10\log_{10}(st)) - \mu_{y_{\mathrm{dB}}}}{\sigma_{y_{\mathrm{dB}}}}\Big)\,\mathrm{d}s. \tag{A.72}$$

For $z = y - ax \in \mathbb{R}$, $x$ is non-central chi-square distributed of degree 2, i.e., $x \sim \mathcal{X}_2^2(\lambda)$, while $y$ is central chi-square distributed $y \sim \mathcal{X}_{2r}^2$. In this case, the CDF of $z$ reformulates to

$$F_z(t) = \int_0^{\infty}\frac{1}{2}\mathrm{e}^{-\frac{s+\lambda}{2}}\Big(\sum_{j=0}^{\infty}\frac{(\lambda s/4)^j}{(j!)^2}\Big)F_{\chi^2}(t + as; 2r)\,\mathrm{d}s. \tag{A.73}$$

*Remark A.1* The PDF of a non-central chi-squared distributed variable $x \sim \mathcal{X}_k^2(\lambda)$, with non-centrality parameter $\lambda$, reads as

$$f_x(x) = \frac{1}{2}\mathrm{e}^{-\frac{x+\lambda}{2}}\Big(\frac{x}{\lambda}\Big)^{\frac{k}{4}-\frac{1}{2}} I_{k/2-1}\big(\sqrt{\lambda x}\big) \tag{A.74}$$

with the modified Bessel function $I_N$ from (A.67). For $k = 2$, this PDF simplifies to (A.71). Furthermore, if $\lambda = 0$ and thus $x \sim \mathcal{X}_k^2$, its PDF is

$$f_x(x) = \frac{1}{2^{k/2}\Gamma(k/2)}x^{k/2-1}\mathrm{e}^{-x/2}, \tag{A.75}$$

which becomes $f_x(x) = \frac{1}{2}\mathrm{e}^{-x/2}$ for $k = 2$.

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