Appendix
Loaded Transformer Input Impedance

Figure A.1a shows a possible representation of the transformer model, where a series resistance is added to the model in [Long00]. Figure A.1b shows the transformer model when loaded with a complex impedance.

The input impedance can be written as following:

$$
\begin{aligned}
Z_{in} &= (Z_{L_1} - X_M) + X_M//(Z_{L_2} - X_M + Z_L) \\
&= Z_{L_1} - X_M + \frac{X_M(Z_{L_2} - X_M + Z_L)}{Z_{L_2} + Z_L} \\
&= Z_{L_1} - \frac{X_M^2}{Z_{L_2} + Z_L}
\end{aligned} \tag{A.1}
$$

X_M in A.1 is jωM, where M is the mutual inductance, and can be written as:

$$M = k \times \sqrt{L_1 L_2} \tag{A.2}$$

Equation A.1 can then be written as:

$$Z_{in} = r_1 + j\omega L_1 + \frac{k^2\omega^2 L_1 L_2}{r_2 + j\omega L_2 + \dfrac{R_L}{1 + j\omega C_L R_L}} \tag{A.3}$$

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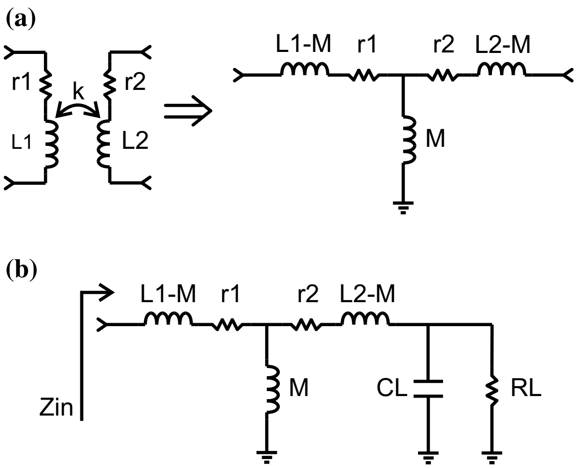

Fig. A.1 **a** Equivalent transformer model and **b** when loaded with a complex impedance

A.1 Real Part of the Input Impedance

The real part of Eq. A.3 can be written as following:

$$Real(Z_{in}) = r_1 + \frac{k^2\omega^2 L_1 L_2[r_2 + R_L(1-\omega^2 L_2 C_L) + \omega^2 R_L C_L(L_2 + r_2 R_L C_L)]}{[r_2 + R_L(1-\omega^2 L_2 C_L)]^2 + \omega^2(L_2 + r_2 R_L C_L)^2} \tag{A.4}$$

It can be shown that the real part of the input impedance can be written as:

$$Real(Z_{in}) = r_1 + \frac{k^2\omega^2 L_1 L_2\{R_L + r_2[1 + (\omega R_L C_L)^2]\}}{r_2\{2R_L + r_2[1 + (\omega R_L C_L)^2]^2\} + R_L^2(1-\omega^2 L_2 C_L)^2 + (\omega L_2)^2} \tag{A.5}$$

A.2 Imaginary Part of the Input Impedance

The imaginary part of Eq. A.3 can be written as following:

$$\begin{aligned} Imag(Z_{in}) &= \omega L_1 \\ &+ \frac{k^2\omega^2 L_1 L_2\{j\omega R_L C_L(r_2 + R_L - \omega^2 R_L C_L L_2) - j\omega(L_2 + r_2 R_L C_L)\}}{r_2\{2R_L + r_2[1 + (\omega R_L C_L)^2]^2\} + R_L^2(1-\omega^2 L_2 C_L)^2 + (\omega L_2)^2} \end{aligned} \tag{A.6}$$

It can be shown that the imaginary part of the input impedance can be written as:

$$Imag(Z_{in}) = \omega L_1 (1 + \frac{k^2\omega^2 L_2 [R_L^2 C_L (1 - \omega^2 L_2 C_L) - L_2]}{r_2\{2R_L + r_2[1 + (\omega R_L C_L)^2]\} + R_L^2 (1 - \omega^2 L_2 C_L)^2 + (\omega L_2)^2}) \quad (A.7)$$

A.3 Simplified Input Impedance Equations

If we assume an infinite quality factor, Eqs. A.5 and A.7 can be rewritten as following:

$$Real(Z_{in}) = \frac{k^2\omega^2 L_1 L_2 R_L}{R_L^2 (1 - \omega^2 L_2 C_L)^2 + (\omega L_2)^2} \quad (A.8)$$

$$Imag(Z_{in}) = \omega L_1 (1 + \frac{k^2\omega^2 L_2 [R_L^2 C_L (1 - \omega^2 L_2 C_L) - L_2]}{R_L^2 (1 - \omega^2 L_2 C_L)^2 + (\omega L_2)^2}) \quad (A.9)$$

If the load is only capacitive (i.e., R_L is approaching infinity), the equations become:

$$Real(Z_{in}) = \frac{k^2\omega^2 L_1 L_2}{R_L (1 - \omega^2 L_2 C_L)^2} = 0 \quad (A.10)$$

$$Imag(Z_{in}) = \omega L_1 (1 + \frac{k^2\omega^2 L_2 C_L}{1 - \omega^2 L_2 C_L}) \quad (A.11)$$

If the load is only resistive, Eqs. A.8 and A.9 become:

$$Real(Z_{in}) = \frac{k^2\omega^2 L_1 L_2 R_L}{R_L^2 + (\omega L_2)^2} \quad (A.12)$$

$$Imag(Z_{in}) = \omega L_1 (1 - \frac{k^2 (\omega L_2)^2}{R_L^2 + (\omega L_2)^2}) \quad (A.13)$$

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