

Appendix A

Trigonometric Formulas

Definitions in Terms of Exponentials

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \tag{A.1}$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \tag{A.2}$$

$$\tan z = \frac{e^{iz} - e^{-iz}}{i(e^{iz} + e^{-iz})} \tag{A.3}$$

$$e^{iz} = \cos z + i \sin z \tag{A.4}$$

$$e^{-iz} = \cos z - i \sin z \tag{A.5}$$

Angle Sum and Difference

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \tag{A.6}$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \tag{A.7}$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \tag{A.8}$$

$$\cot(\alpha \pm \beta) = \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \alpha} \tag{A.9}$$

Symmetry

$$\sin(-\alpha) = -\sin \alpha \quad (\text{A.10})$$

$$\cos(-\alpha) = \cos \alpha \quad (\text{A.11})$$

$$\tan(-\alpha) = -\tan \alpha \quad (\text{A.12})$$

Multiple Angles

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha} \quad (\text{A.13})$$

$$\cos(2\alpha) = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha = \cos^2 \alpha - \sin^2 \alpha \quad (\text{A.14})$$

$$\tan(2\alpha) = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \quad (\text{A.15})$$

$$\cot(2\alpha) = \frac{\cot^2 \alpha - 1}{2 \cot \alpha} \quad (\text{A.16})$$

$$\sin(3\alpha) = -4 \sin^3 \alpha + 3 \sin \alpha \quad (\text{A.17})$$

$$\cos(3\alpha) = 4 \cos^3 \alpha - 3 \cos \alpha \quad (\text{A.18})$$

$$\tan(3\alpha) = \frac{-\tan^3 \alpha + 3 \tan \alpha}{-3 \tan^2 \alpha + 1} \quad (\text{A.19})$$

$$\sin(4\alpha) = -8 \sin^3 \alpha \cos \alpha + 4 \sin \alpha \cos \alpha \quad (\text{A.20})$$

$$\cos(4\alpha) = 8 \cos^4 \alpha - 8 \cos^2 \alpha + 1 \quad (\text{A.21})$$

$$\tan(4\alpha) = \frac{-4 \tan^3 \alpha + 4 \tan \alpha}{\tan^4 \alpha - 6 \tan^2 \alpha + 1} \quad (\text{A.22})$$

$$\sin(5\alpha) = 16 \sin^5 \alpha - 20 \sin^3 \alpha + 5 \sin \alpha \quad (\text{A.23})$$

$$\cos(5\alpha) = 16 \cos^5 \alpha - 20 \cos^3 \alpha + 5 \cos \alpha \quad (\text{A.24})$$

$$\sin(n\alpha) = 2 \sin((n-1)\alpha) \cos \alpha - \sin((n-2)\alpha) \quad (\text{A.25})$$

$$\cos(n\alpha) = 2 \cos((n-1)\alpha) \cos \alpha - \cos((n-2)\alpha) \quad (\text{A.26})$$

$$\tan(n\alpha) = \frac{\tan((n-1)\alpha) + \tan \alpha}{1 - \tan((n-1)\alpha) \tan \alpha} \quad (\text{A.27})$$

Half Angle

$$\cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 + \cos \alpha}{2}} \quad (\text{A.28})$$

$$\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{2}} \quad (\text{A.29})$$

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} \quad (\text{A.30})$$

$$\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \quad (\text{A.31})$$

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \quad (\text{A.32})$$

Powers of Functions

$$\sin^2 \alpha = \frac{1}{2} (1 - \cos(2\alpha)) \quad (\text{A.33})$$

$$\sin \alpha \cos \alpha = \frac{1}{2} \sin(2\alpha) \quad (\text{A.34})$$

$$\cos^2 \alpha = \frac{1}{2} (1 + \cos(2\alpha)) \quad (\text{A.35})$$

$$\sin^3 \alpha = \frac{1}{4} (3 \sin(\alpha) - \sin(3\alpha)) \quad (\text{A.36})$$

$$\sin^2 \alpha \cos \alpha = \frac{1}{4} (\cos \alpha - 3 \cos(3\alpha)) \quad (\text{A.37})$$

$$\sin \alpha \cos^2 \alpha = \frac{1}{4} (\sin \alpha + \sin(3\alpha)) \quad (\text{A.38})$$

$$\cos^3 \alpha = \frac{1}{4} (\cos(3\alpha) + 3 \cos \alpha) \quad (\text{A.39})$$

$$\sin^4 \alpha = \frac{1}{8} (3 - 4 \cos(2\alpha) + \cos(4\alpha)) \quad (\text{A.40})$$

$$\sin^3 \alpha \cos \alpha = \frac{1}{8} (2 \sin(2\alpha) - \sin(4\alpha)) \quad (\text{A.41})$$

$$\sin^2 \alpha \cos^2 \alpha = \frac{1}{8} (1 - \cos(4\alpha)) \quad (\text{A.42})$$

$$\sin \alpha \cos^3 \alpha = \frac{1}{8} (2 \sin(2\alpha) + \sin(4\alpha)) \quad (\text{A.43})$$

$$\cos^4 \alpha = \frac{1}{8} (3 + 4 \cos(2\alpha) + \cos(4\alpha)) \quad (\text{A.44})$$

$$\sin^5 \alpha = \frac{1}{16} (10 \sin \alpha - 5 \sin(3\alpha) + \sin(5\alpha)) \quad (\text{A.45})$$

$$\sin^4 \alpha \cos \alpha = \frac{1}{16} (2 \cos \alpha - 3 \cos(3\alpha) + \cos(5\alpha)) \quad (\text{A.46})$$

$$\sin^3 \alpha \cos^2 \alpha = \frac{1}{16} (2 \sin \alpha + \sin(3\alpha) - \sin(5\alpha)) \quad (\text{A.47})$$

$$\sin^2 \alpha \cos^3 \alpha = \frac{1}{16} (2 \cos \alpha - 3 \cos(3\alpha) - 5 \cos(5\alpha)) \quad (\text{A.48})$$

$$\sin \alpha \cos^4 \alpha = \frac{1}{16} (2 \sin \alpha + 3 \sin(3\alpha) + \sin(5\alpha)) \quad (\text{A.49})$$

$$\cos^5 \alpha = \frac{1}{16} (10 \cos \alpha + 5 \cos(3\alpha) + \cos(5\alpha)) \quad (\text{A.50})$$

$$\tan^2 \alpha = \frac{1 - \cos(2\alpha)}{1 + \cos(2\alpha)} \quad (\text{A.51})$$

Products of sin and cos

$$\cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta) \quad (\text{A.52})$$

$$\sin \alpha \sin \beta = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta) \quad (\text{A.53})$$

$$\sin \alpha \cos \beta = \frac{1}{2} \sin(\alpha - \beta) + \frac{1}{2} \sin(\alpha + \beta) \quad (\text{A.54})$$

$$\cos \alpha \sin \beta = \frac{1}{2} \sin(\alpha + \beta) - \frac{1}{2} \sin(\alpha - \beta) \quad (\text{A.55})$$

$$\sin(\alpha + \beta) \sin(\alpha - \beta) = \cos^2 \beta - \cos^2 \alpha = \sin^2 \alpha - \sin^2 \beta \quad (\text{A.56})$$

$$\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \beta + \sin^2 \alpha \quad (\text{A.57})$$

Sum of Functions

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{\alpha \pm \beta}{2} \cos \frac{\alpha \pm \beta}{2} \quad (\text{A.58})$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} \quad (\text{A.59})$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2} \quad (\text{A.60})$$

$$\tan \alpha \pm \tan \beta = \frac{\sin(\alpha \pm \beta)}{\cos \alpha \cos \beta} \quad (\text{A.61})$$

$$\cot \alpha \pm \cot \beta = \frac{\sin(\beta \pm \alpha)}{\sin \alpha \sin \beta} \quad (\text{A.62})$$

$$\frac{\sin \alpha + \sin \beta}{\sin \alpha - \sin \beta} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}} \quad (\text{A.63})$$

$$\frac{\sin \alpha + \sin \beta}{\cos \alpha - \cos \beta} = \cot \frac{-\alpha + \beta}{2} \quad (\text{A.64})$$

$$\frac{\sin \alpha + \sin \beta}{\cos \alpha + \cos \beta} = \tan \frac{\alpha + \beta}{2} \quad (\text{A.65})$$

$$\frac{\sin \alpha - \sin \beta}{\cos \alpha + \cos \beta} = \tan \frac{\alpha - \beta}{2} \quad (\text{A.66})$$

Trigonometric Relations

$$\sin^2 \alpha - \sin^2 \beta = \sin(\alpha + \beta) \sin(\alpha - \beta) \quad (\text{A.67})$$

$$\cos^2 \alpha - \cos^2 \beta = -\sin(\alpha + \beta) \sin(\alpha - \beta) \quad (\text{A.68})$$

Appendix B

Unit Conversions

General Conversion Formulas

$$\begin{aligned} \text{N}^a \text{ m}^b \text{ s}^c &\approx 4.448^a \times 0.3048^b \times \text{lb}^a \text{ ft}^b \text{ s}^c \\ &\approx 4.448^a \times 0.0254^b \times \text{lb}^a \text{ in}^b \text{ s}^c \\ \text{lb}^a \text{ ft}^b \text{ s}^c &\approx 0.2248^a \times 3.2808^b \times \text{N}^a \text{ m}^b \text{ s}^c \\ \text{lb}^a \text{ in}^b \text{ s}^c &\approx 0.2248^a \times 39.37^b \times \text{N}^a \text{ m}^b \text{ s}^c \end{aligned}$$

Conversion Factors

Acceleration

$$1 \text{ ft/s}^2 \approx 0.3048 \text{ m/s}^2 \quad 1 \text{ m/s}^2 \approx 3.2808 \text{ ft/s}^2$$

Angle

$$1 \text{ deg} \approx 0.01745 \text{ rad} \quad 1 \text{ rad} \approx 57.307 \text{ deg}$$

Area

$$\begin{array}{ll}
 1 \text{ in}^2 \approx 6.4516 \text{ cm}^2 & 1 \text{ cm}^2 \approx 0.155 \text{ in}^2 \\
 1 \text{ ft}^2 \approx 0.09290304 \text{ m}^2 & 1 \text{ m}^2 \approx 10.764 \text{ ft}^2 \\
 1 \text{ acre} \approx 4046.86 \text{ m}^2 & 1 \text{ m}^2 \approx 2.471 \times 10^{-4} \text{ acre} \\
 1 \text{ acre} \approx 0.4047 \text{ ha} & 1 \text{ ha} \approx 2.471 \text{ acre}
 \end{array}$$

Damping

$$\begin{array}{ll}
 1 \text{ N s/m} \approx 6.85218 \times 10^{-2} \text{ lb s/ft} & 1 \text{ lb s/ft} \approx 14.594 \text{ N s/m} \\
 1 \text{ N s/m} \approx 5.71015 \times 10^{-3} \text{ lb s/in} & 1 \text{ lb s/in} \approx 175.13 \text{ N s/m}
 \end{array}$$

Energy and Heat

$$\begin{array}{ll}
 1 \text{ Btu} \approx 1055.056 \text{ J} & 1 \text{ J} \approx 9.4782 \times 10^{-4} \text{ Btu} \\
 1 \text{ cal} \approx 4.1868 \text{ J} & 1 \text{ J} \approx 0.23885 \text{ cal} \\
 1 \text{ kW h} \approx 3600 \text{ kJ} & 1 \text{ MJ} \approx 0.27778 \text{ kW h} \\
 1 \text{ ft lbf} \approx 1.355818 \text{ J} & 1 \text{ J} \approx 0.737562 \text{ ft lbf}
 \end{array}$$

Force

$$1 \text{ lb} \approx 4.448222 \text{ N} \quad 1 \text{ N} \approx 0.22481 \text{ lb}$$

Fuel Consumption

$$\begin{array}{ll}
 1 \text{ l/100 km} \approx 235.214583 \text{ mi/gal} & 1 \text{ mi/gal} \approx 235.214583 \text{ l/100 km} \\
 1 \text{ l/100 km} = 100 \text{ km/l} & 1 \text{ km/l} = 100 \text{ l/100 km} \\
 1 \text{ mi/gal} \approx 0.425144 \text{ km/l} & 1 \text{ km/l} \approx 2.352146 \text{ mi/gal}
 \end{array}$$

Length

$$\begin{array}{ll}
 1 \text{ in} \approx 25.4 \text{ mm} & 1 \text{ cm} \approx 0.3937 \text{ in} \\
 1 \text{ ft} \approx 30.48 \text{ cm} & 1 \text{ m} \approx 3.28084 \text{ ft} \\
 1 \text{ mi} \approx 1.609347 \text{ km} & 1 \text{ km} \approx 0.62137 \text{ mi}
 \end{array}$$

Mass

$$\begin{array}{ll}
 1 \text{ lb} \approx 0.45359 \text{ kg} & 1 \text{ kg} \approx 2.204623 \text{ lb} \\
 1 \text{ slug} \approx 14.5939 \text{ kg} & 1 \text{ kg} \approx 0.068522 \text{ slug} \\
 1 \text{ slug} \approx 32.174 \text{ lb} & 1 \text{ lb} \approx 0.03.1081 \text{ slug}
 \end{array}$$

Moment and Torque

$$\begin{array}{ll}
 1 \text{ lb ft} \approx 1.35582 \text{ N m} & 1 \text{ N m} \approx 0.73746 \text{ lb ft} \\
 1 \text{ lb in} \approx 8.85075 \text{ N m} & 1 \text{ N m} \approx 0.11298 \text{ lb in}
 \end{array}$$

Mass Moment

$$1 \text{ lb ft}^2 \approx 0.04214 \text{ kg m}^2 \quad 1 \text{ kg m}^2 \approx 23.73 \text{ lb ft}^2$$

Power

$$\begin{array}{ll}
 1 \text{ Btu/h} \approx 0.2930711 \text{ W} & 1 \text{ W} \approx 3.4121 \text{ Btu/h} \\
 1 \text{ hp} \approx 745.6999 \text{ W} & 1 \text{ kW} \approx 1.341 \text{ hp} \\
 1 \text{ hp} \approx 550 \text{ lb ft/s} & 1 \text{ lb ft/s} \approx 1.8182 \times 10^{-3} \text{ hp} \\
 1 \text{ lb ft/h} \approx 3.76616 \times 10^{-4} \text{ W} & 1 \text{ W} \approx 2655.2 \text{ lb ft/h} \\
 1 \text{ lb ft/min} \approx 2.2597 \times 10^{-2} \text{ W} & 1 \text{ W} \approx 44.254 \text{ lb ft/min}
 \end{array}$$

Pressure and Stress

$$\begin{aligned}
 1 \text{ lb/in}^2 &\approx 6894.757 \text{ Pa} & 1 \text{ MPa} &\approx 145.04 \text{ lb/in}^2 \\
 1 \text{ lb/ft}^2 &\approx 47.88 \text{ Pa} & 1 \text{ Pa} &\approx 2.0886 \times 10^{-2} \text{ lb/ft}^2 \\
 1 \text{ Pa} &\approx 0.00001 \text{ atm} & 1 \text{ atm} &\approx 101,325 \text{ Pa}
 \end{aligned}$$

Stiffness

$$\begin{aligned}
 1 \text{ N/m} &\approx 6.85218 \times 10^{-2} \text{ lb/ft} & 1 \text{ lb/ft} &\approx 14.594 \text{ N/m} \\
 1 \text{ N/m} &\approx 5.71015 \times 10^{-3} \text{ lb/in} & 1 \text{ lb/in} &\approx 175.13 \text{ N/m}
 \end{aligned}$$

Temperature

$$\begin{aligned}
 ^\circ\text{C} &= (^\circ\text{F} - 32)/1.8 \\
 ^\circ\text{F} &= 1.8^\circ\text{C} + 32
 \end{aligned}$$

Velocity

$$\begin{aligned}
 1 \text{ mi/h} &\approx 1.60934 \text{ km/h} & 1 \text{ km/h} &\approx 0.62137 \text{ mi/h} \\
 1 \text{ mi/h} &\approx 0.44704 \text{ m/s} & 1 \text{ m/s} &\approx 2.2369 \text{ mi/h} \\
 1 \text{ ft/s} &\approx 0.3048 \text{ m/s} & 1 \text{ m/s} &\approx 3.2808 \text{ ft/s} \\
 1 \text{ ft/min} &\approx 5.08 \times 10^{-3} \text{ m/s} & 1 \text{ m/s} &\approx 196.85 \text{ ft/min}
 \end{aligned}$$

Volume

$$\begin{aligned}
 1 \text{ in}^3 &\approx 16.39 \text{ cm}^3 & 1 \text{ cm}^3 &\approx 0.0061013 \text{ in}^3 \\
 1 \text{ ft}^3 &\approx 0.02831685 \text{ m}^3 & 1 \text{ m}^3 &\approx 35.315 \text{ ft}^3 \\
 1 \text{ gal} &\approx 3.7851 & 1 \text{ l} &\approx 0.2642 \text{ gal} \\
 1 \text{ gal} &\approx 3785.41 \text{ cm}^3 & 1 \text{ l} &\approx 1000 \text{ cm}^3
 \end{aligned}$$

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Index

A

Ackerman

- geometry, 328
- mechanism, 328
- steering, 178, 325, 328

Ackerman condition, 178, 325

Activation functions, 24

Aerodynamic force, 122

Aligning moment, 37, 38

Angle

- body-wheel sideslip, 27
- camber, 26
- sideslip, 26
- steering, 326
- tire contact, 20
- tireprint, 20
- tire sideslip, 27
- wheel-body sideslip, 27

Arc length, 314

Arctan function, 25

Attitude angle, 80

B

Bernoulli, Jacques, 325

B-frame, 2, 11

Bicycle model, 131, 143, 163, 172, 239

- body force components, 131
- coefficient matrix, 240
- control variables, 147, 240, 241
- coordinate frame, 78, 79
- curvature response, 163
- equations of motion, 239, 241

force system coefficients, 238

global sideslip angle, 136

input vector, 147, 241

kinematic steering, 138

neutral steer, 170

Newton–Euler equations, 144, 174

oversteer, 170

roll damping, 237, 238

roll stiffness, 237, 238

sideslip coefficient, 238

sideslip response, 164

stability factor, 170, 171

torque coefficient, 237

understeer, 170

vehicle velocity vector, 136

Body

frame, 1, 11

Body-wheel sideslip, 27

C

Camber

angle, 26, 43, 46

force, 43

moment, 46

stiffness, 44, 49

torque, 45

trail, 45

trust, 43

Cesàro, Ernesto, 325

C-frame, 2, 5, 7, 9, 11, 130

Circumferential slip, 18

- Clothoid, 297
 - arc length, 297
 - curvature, 298
 - figure 8, 315, 319
 - history, 325
 - radius, 298
 - road, 297
 - scaling parameter, 297
 - sharpness, 300
 - tangent angle, 298
- Combined force, 49, 63, 65, 68
 - approximate elliptic model, 68
 - camber effect, 68
 - diamond mode, 69
 - elliptic model, 50
 - experimental data, 65
 - velocity dependency, 63
- Combined slip, 50
- Contact patch, 2
- Coordinate frame
 - body, 80
 - global, 80
 - tire, 12
 - vehicle, 12, 78, 80, 215
 - wheel, 12, 127
 - wheel-body, 12, 127
- Cornering stiffness, 31
- Cornu, Alfred, 325
- Cornu spiral, 325
- Crouse angle, 80
- Cubic
 - function, 36
- Curvature response, 163

- D**
- Differential geometry
 - space curve, 314
- Directional
 - cosine, 323
- Dynamics
 - lateral force, 25
 - tire, 1, 13, 15, 17, 25, 31, 40–44, 48–50, 68, 69

- E**
- Effective tire radius, 19
- Ellipse condition, 58, 59
- Euler, Leonard, 325

- F**
- Force
 - aerodynamic, 122
 - bicycle roll vehicle, 227
 - gravitation, 124
 - longitudinal, 16
 - shear, 49
 - tangential, 49
 - wind, 123
- Force system
 - planar, 125
 - unsaturated, 139
- Formula
 - Leibniz, 300
- Four-wheel model
 - Newton–Euler equations, 260
- Four wheel steering, 330
- Four-wheel vehicle, 140
 - dynamics, 140
 - Newton–Euler equations, 260
- Frame
 - body, 1, 11
 - tire, 1, 4, 5, 9, 11
 - wheel, 1, 5, 11
 - wheel-body, 1, 5, 9, 11
- Fresnel Integrals, 297
- Friction ellipse, 58
- Front-wheel-steering, 325
- Function
 - activation, 24
 - arctan, 25
 - cubic, 36
 - Heaviside, 15, 24
 - logistic, 24
 - nonlinear-saturation, 35
 - proportional-saturation, 16, 34
 - saturation, 15, 24, 25, 49
 - Sigmoid, 24
 - TV-shaped, 65

- G**
- Global sideslip angle, 27, 136, 137
- Gravitation force, 124
- Grip, 39

- H**
- Heading angle, 80
- Heaviside function, 15, 24

L

- Lateral
 - tire force, 25
- Lateral force
 - cubic model, 36
- Lateral stiffness, 49
- Leibniz formula, 300
- Logistic function, 24
- Longitudinal
 - tire force, 14, 16
- Longitudinal force, 15
 - velocity-dependent, 23
- Longitudinal friction, 16
- Longitudinal slip, 15–18
- Longitudinal slip ratio
 - rate, 23

M

- Mechanism
 - steering, 328
 - trapezoidal steering, 328

N

- Neutral steer, 170–172
- Nonlinear-saturation
 - function, 35

O

- Oversteer, 170–172

P

- Pitch moment, 79
- Planar
 - vehicle dynamics, 115, 119, 122, 125, 139
- Planar dynamics, 143, 172
 - body force components, 131
 - control variables, 147
 - coordinate frame, 78, 79
 - curvature response, 163
 - global sideslip angle, 136
 - input vector, 147
 - kinematic steering, 138
 - neutral steer, 170
 - Newton–Euler equations, 115, 144, 174
 - oversteer, 170
 - sideslip response, 164
 - stability factor, 170, 171
 - steady-state turning, 163
 - understeer, 170
 - vehicle velocity vector, 136
 - wheel number, 84

- Pneumatic trail, 37, 38
- Proportional-saturation
 - function, 34

R

- Road
 - bank angle, 94, 96
 - banked, 96, 97
 - clothoid, 297, 298, 315, 319
 - curvature, 298
 - design, 297, 302, 303, 305, 308–310, 314, 315, 319, 323, 324
 - history, 325
 - radius, 298
 - sharpness, 300
 - spatial, 323, 324
 - spiral, 297
 - tangent angle, 298
- Road design, 297
- Road dynamics, 297
- Roll angle, 79, 216
- Roll dynamics, 215
 - coefficient matrix, 240
 - control variables, 240, 241
 - equations of motion, 239, 241
 - force system coefficients, 238
 - input vector, 241
 - lateral force, 230
 - Newton–Euler equations, 215, 219, 260
 - roll-steering angle, 231
 - roll damping, 237, 238
 - roll stiffness, 237, 238
 - sideslip angle, 231
 - sideslip coefficient, 238
 - tire slip coefficient, 230
 - torque coefficient, 237
 - vehicle slip coefficient, 234
 - wheel force system, 221
- Roll moment, 79
- Roll thrust, 233, 290
- Roll-steer, 261
- Roll-steer angle, 233, 290
- Rotation
 - radius of, 326

S

- Saturation function, 15, 16, 24, 25, 49
- Sideslip angle, 26, 27, 80
 - tire, 27
 - wheel-body, 27
- Sideslip coefficient, 28
- Sideslip response, 164

- Sideslip stiffness, 31
- Sigmoid function, 24
- Slip moment, 38
- Slip ratio, 15, 18
- Space
 - curve, 314
- Spatial
 - road, 323, 324
- Spiral
 - clothoid, 310
 - Cornu, 325
 - Fresnel, 325
- Stability factor, 170, 171
- Stall, 93
- Steady state
 - center of rotation, 170
 - curvature response, 163
 - sideslip response, 164
 - stability factor, 170
- Steering, 178, 297, 325, 326, 331
 - Ackerman condition, 178, 325
 - autodriver, 339
 - bicycle model, 326, 328
 - counter steer, 336
 - four wheel, 330–339
 - front wheel, 178, 325
 - inner-outer relationship, 326, 329
 - inner steer angle, 178, 325, 326, 331
 - inner wheel, 178, 325, 326, 331
 - kinematic, 297
 - kinematic condition, 178, 325
 - mechanism, 328
 - outer steer angle, 325, 326, 331
 - outer wheel, 178, 325, 326, 331
 - radius of curvature, 338
 - same steer, 336
 - sign convection, 336
 - sign convention, 331
 - static steering, 326
 - static steering condition, 326
 - steer angle, 326
 - trapezoidal mechanism, 328, 329
 - turning center, 325, 330, 336–338
 - turning radius, 326, 328, 335, 336
- Stiffness
 - camber, 49
 - lateral, 49
- Symbols, xi

- T**
- T-frame, 2, 4, 5, 9, 11
- Talbot, Arthur, 325
- Talbot curve, 325
- Tangential slip, 18
- Taylor series, 20
- Tire
 - aligning moment, 14, 38, 47
 - bank moment, 14
 - bore torque, 14
 - camber angle, 26, 46, 47, 49
 - camber arm, 46
 - camber effect, 68
 - camber force, 43, 46
 - camber moment, 46
 - camber stiffness, 44, 49
 - camber torque, 45
 - camber trail, 45
 - camber trust, 43
 - circumferential slip, 18
 - combined force, 49, 50, 58, 68, 69
 - combined slip, 58
 - contact angle, 20
 - coordinate frame, 26
 - cornering force, 39
 - cornering stiffness, 31
 - drag force, 39
 - dynamics, 1, 31, 43, 49, 50, 68, 69
 - effective radius, 19, 20
 - ellipse condition, 58, 59
 - equivalent radius, 20
 - equivalent speed, 17
 - force system, 13, 40–42, 44, 48, 49
 - forward force, 13
 - forward velocity, 19
 - frame, 1, 4, 5, 9, 11
 - friction, 22
 - friction coefficient, 16, 32
 - friction ellipse, 58
 - geometric radius, 19, 20
 - grip, 39
 - impossible force zones, 76
 - lateral drop factor, 51, 54, 59, 70, 74
 - lateral force, 13, 31, 33, 35, 36, 39, 40, 42–44, 46, 50, 51
 - laterally deflected, 32
 - lateral stiffness, 32, 49
 - limit slip curve, 55
 - limit slip line, 75
 - load, 20
 - longitudinal drop factor, 50, 54, 59, 69, 74, 75
 - longitudinal force, 13–15, 50, 68, 69
 - longitudinal friction, 22
 - longitudinally deflected, 33
 - longitudinal slip, 15, 16, 49
 - maximum force, 76
 - maximum velocity, 22

- neutral position, 1
 - non-radiale, 49
 - normal force, 14
 - overturning moment, 14
 - pitch moment, 14
 - plane, 26
 - pneumatic trail, 38
 - possible tire force zone, 57
 - radiale, 49
 - rest position, 1
 - rolling resistance torque, 14
 - roll moment, 14
 - saturation force, 43
 - saturation values, 50, 51, 69, 70
 - self aligning moment, 14
 - shear force, 49
 - side force, 39
 - sideslip angle, 26, 27, 31, 46
 - sliding, 64
 - sliding line, 33
 - slip coefficient, 16
 - slip factors, 50, 51
 - slip models, 22, 23
 - slip moment, 38
 - slip ratio, 15, 17, 18, 20–23, 65
 - stiffness, 32
 - stress distributions, 38
 - tangential slip, 18
 - tangetial force, 49
 - tilting torque, 14
 - tireprint, 2, 41
 - tireprint angle, 20
 - tireprint model, 48
 - traction force, 13
 - vertical force, 14
 - wheel load, 14
 - yaw moment, 14
 - Tireprint, 2, 26, 41, 42, 48
 - angle, 20
 - Track, 326
 - Transformation
 - homogeneous, 6, 7, 29
 - tire to vehicle frame, 9
 - tire to wheel-body frame, 5–7, 29
 - tire to wheel frame, 4, 5
 - wheel-body to vehicle frame, 11
 - wheel to tire frame, 2, 5
 - wheel to wheel-body frame, 7
 - Trapezoidal steering, 328, 329
 - Turning center, 330, 336–338
 - TV-shaped function, 65
 - Two-wheel vehicle, 131, 138, 143, 163, 172, 239
 - body force components, 131
 - coefficient matrix, 240
 - control variables, 147, 240, 241
 - coordinate frame, 78, 79
 - curvature response, 163
 - equations of motion, 239, 241
 - force system coefficients, 238
 - global sideslip angle, 136
 - input vector, 147, 241
 - kinematic steering, 138
 - neutral steer, 170
 - Newton–Euler equations, 144, 174
 - oversteer, 170
 - roll damping, 237, 238
 - roll stiffness, 237, 238
 - sideslip coefficient, 238
 - sideslip response, 164
 - stability factor, 170, 171
 - torque coefficient, 237
 - understeer, 170
 - vehicle velocity vector, 136
- U**
- Understeer, 170–172
 - Unit system, xi
- V**
- Vehicle
 - accelerating, 88, 90–92
 - banked road, 94, 96
 - longitudinal dynamics, 88, 90–92, 94, 96
 - maximum acceleration, 90, 92
 - stall, 93
 - Vehicle dynamics, 115, 172
 - aerodynamic effect, 147
 - aerodynamic force, 122
 - aligning moment, 80
 - attitude angle, 80
 - bank moment, 79
 - bicycle model, 131, 135, 143, 163, 172
 - bicycle roll model, 227
 - body force components, 131
 - body force system, 125, 139
 - coefficient matrix, 147, 240
 - comments, 122
 - control variables, 147, 240, 241
 - crouse angle, 80
 - curvature response, 163
 - equations of motion, 136, 238, 239, 241
 - force system, 79
 - force system coefficients, 238
 - forward force, 79
 - four-wheel planar, 172

Vehicle dynamics (*cont.*)

four-wheel roll, 255
 global equations, 119
 heading angle, 80
 input vector, 147, 241
 Lagrange method, 121
 lateral force, 27, 79, 142, 230
 lateral moment, 79
 longitudinal force, 79
 longitudinally weight transfer, 88
 neutral, 171, 172
 neutral steer, 170
 Newton–Euler equations, 115, 144, 174, 215, 260
 normal force, 79
 oversteer, 170–172
 overturning moment, 79
 path of motion, 88
 pitch angle, 79, 216
 pitch moment, 79
 pitch rate, 79, 216
 planar, 78, 115, 119, 125, 139
 planar four-wheel, 172
 rigid vehicle, 78, 215
 roll angle, 79, 216
 roll damping, 237, 238
 roll dynamics, 215, 216
 roll moment, 79
 roll rate, 79, 216
 roll-steering angle, 231
 roll stiffness, 237, 238
 roll vehicle model, 227
 sideslip angle, 80, 231
 sideslip coefficient, 28, 238
 sideslip response, 164
 six DOF, 219
 stability factor, 170, 171
 steady-state turning, 163
 steer angle, 134, 231
 tilting torque, 79
 tire force system, 125, 139
 tire lateral force, 26
 tire slip coefficient, 230
 torque coefficient, 237

traction force, 79
 two-wheel model, 131, 135, 143, 163, 172
 understeer, 170–172
 vehicle load, 79
 vehicle slip coefficient, 234
 vehicle velocity vector, 136
 vertical force, 79
 weight transfer, 88
 wheel force system, 221
 wheel frame, 127
 wheel number, 84
 yaw angle, 79, 216
 yaw moment, 80
 yaw rate, 79, 216

Vehicle kinematics

velocity distribution, 87

W

Weight transfer, 88, 91–94, 97, 98, 102, 103
 banked road, 94, 97
 inclined road, 91
 lateral acceleration, 93
 linearized, 102
 local frame, 103
 longitudinal and lateral acceleration, 98
 longitudinally, 88
 maximum acceleration, 92
 W-frame, 2, 4, 5, 7, 11, 129, 130

Wheel

 extreme velocity, 29
 forward velocity, 19
 frame, 1, 5, 11
 neutral position, 1
 rest position, 1

Wheel-body

 frame, 1, 5, 9, 11

Wheelbase, 326

Wheel number, 84

Wind force, 123

Y

Yaw moment, 80