

# References

1. N. Aronszajn: Theory of reproducing kernels. *Transactions of the AMS* **68**, 1950, 337–404.
2. R. Askey: Radial characteristic functions. TSR # 1262, Univ. Wisconsin, 1973.
3. S. Banach, S. Mazur: Zur Theorie der linearen Dimension. *Studia Mathematica* **4**, 1933, 100–112.
4. M. Beckmann, A. Iske: Error estimates and convergence rates for filtered back projection. *Mathematics of Computation*, published electronically on April 30, 2018, <https://doi.org/10.1090/mcom/3343>.
5. M. Beckmann, A. Iske: Approximation of bivariate functions from fractional Sobolev spaces by filtered back projection. HBAM 2017-05, U. Hamburg, 2017.
6. A. Beer: Bestimmung der Absorption des rothen Lichts in farbigen Flüssigkeiten. *Annalen der Physik und Chemie* **86**, 1852, 78–88.
7. Å. Björck: *Numerical Methods for Least Squares Problems*. SIAM, 1996.
8. S. Bochner: *Vorlesungen über Fouriersche Integrale*. Akademische Verlagsgesellschaft, Leipzig, 1932.
9. D. Braess: *Nonlinear Approximation Theory*. Springer, Berlin, 1986.
10. M.D. Buhmann: *Radial Basis Functions*. Cambridge University Press, Cambridge, UK, 2003.
11. E.W. Cheney: *Introduction to Approximation Theory*. Second edition, McGraw Hill, New York, NY, U.S.A., 1982.
12. W. Cheney, W. Light: *A Course in Approximation Theory*. Graduate Studies in Mathematics, vol. 101, AMS, Providence, RI, U.S.A., 2000.
13. O. Christensen: *An Introduction to Frames and Riesz Bases*. Second expanded edition, Birkhäuser, 2016.
14. C.K. Chui: *Wavelets: A Mathematical Tool for Signal Analysis*. Monographs on Mathematical Modeling and Computation. SIAM, 1997.
15. C.W. Clenshaw: A note on the summation of Chebyshev series. *Mathematics of Computation* **9**(51), 1955, 118–120.
16. J.W. Cooley, J.W. Tukey. An algorithm for the machine calculation of complex Fourier series. *Mathematics of Computation* **19**, 1965, 297–301.
17. P.C. Curtis Jr.: N-parameter families and best approximation. *Pacific Journal of Mathematics* **9**, 1959, 1013–1027.
18. I. Daubechies: *Ten Lectures on Wavelets*. SIAM, Philadelphia, 1992.
19. P.J. Davis: *Interpolation and Approximation*. 2nd edition, Dover, NY, 1975.
20. C. de Boor: *A Practical Guide to Splines*. Revised edition, Applied Mathematical Sciences, vol. 27, Springer, New York, 2001.
21. R.A. DeVore: Nonlinear approximation. *Acta Numerica*, 1998, 51–150.

22. B. Diederichs, A. Iske: Improved estimates for condition numbers of radial basis function interpolation matrices. *J. Approximation Theory*, published electronically on October 16, 2017, <https://doi.org/10.1016/j.jat.2017.10.004>.
23. G. Faber: Über die interpolatorische Darstellung stetiger Funktionen. *Jahresbericht der Deutschen Mathematiker-Vereinigung* **23**, 1914, 192–210.
24. G.E. Fasshauer: *Meshfree Approximation Methods with Matlab*. World Scientific, Singapore, 2007.
25. G.E. Fasshauer, M. McCourt: *Kernel-based Approximation Methods using Matlab*. World Scientific, Singapore, 2015.
26. G.B. Folland: *Fourier Analysis and its Applications*. Brooks/Cole, Pacific Grove, CA, U.S.A., 1992.
27. B. Fornberg, N. Flyer: *A Primer on Radial Basis Functions with Applications to the Geosciences*. SIAM, Philadelphia, 2015.
28. W. Gander, M.J. Gander, F. Kwok: *Scientific Computing – An Introduction using Maple and MATLAB*. Texts in CSE, volume 11, Springer, 2014.
29. C. Gasquet, P. Witomski: *Fourier Analysis and Applications*. Springer Science+Business Media, New York, 1999.
30. M. v. Golitschek: Penalized least squares approximation problems. *Jaen Journal on Approximation Theory* **1**(1), 2009, 83–96.
31. J. Gomes, L. Velho: *From Fourier Analysis to Wavelets*. Springer, 2015.
32. A. Haar: Zur Theorie der orthogonalen Funktionensysteme. *Mathematische Annalen* **69**, 1910, 331–371.
33. M. Haase: *Functional Analysis: An Elementary Introduction*. American Mathematical Society, Providence, RI, U.S.A., 2014.
34. P.C. Hansen, J.G. Nagy, D.P. O’Leary: *Deblurring Images: Matrices, Spectra, and Filtering*. Fundamentals of Algorithms. SIAM, Philadelphia, 2006.
35. E. Hewitt, K.A. Ross: *Abstract Harmonic Analysis I*. Springer, Berlin, 1963.
36. K. Höllig, J. Hörner: *Approximation and Modeling with B-Splines*. SIAM, Philadelphia, 2013.
37. A. Iske: *Charakterisierung bedingt positiv definiten Funktionen für multivariate Interpolationsmethoden mit radialen Basisfunktionen*. Dissertation, Universität Göttingen, 1994.
38. A. Iske: *Multiresolution Methods in Scattered Data Modelling*. Lecture Notes in Computational Science and Engineering, vol. 37, Springer, Berlin, 2004.
39. J.L.W.V. Jensen: Sur les fonctions convexes et les inégalités entre les valeurs moyennes. *Acta Mathematica* **30**, 1906, 175–193.
40. P. Jordan, J. von Neumann: On inner products in linear, metric spaces. *Annals of Mathematics* **36**(3), 1935, 719–723.
41. C.L. Lawson, R.J. Hanson: *Solving Least Squares Problems*. Prentice-Hall, Englewood Cliffs, NJ, U.S.A., 1974.
42. P.D. Lax: *Functional Analysis*. Wiley-Interscience, New York, U.S.A., 2002.
43. G.G. Lorentz, M. v. Golitschek, Y. Makovoz: *Constructive Approximation*. Grundlehren der mathematischen Wissenschaften, Band 304, Springer, 2011.
44. W.R. Madych: Summability and approximate reconstruction from Radon transform data. In: *Integral Geometry and Tomography*, E. Grinberg and T. Quinto (eds.), AMS, Providence, RI, U.S.A., 1990, 189–219.
45. W.R. Madych, S.A. Nelson: *Multivariate Interpolation: A Variational Theory*. Technical Report, Iowa State University, 1983.
46. W.R. Madych, S.A. Nelson: Multivariate interpolation and conditionally positive definite functions. *Approx. Theory Appl.* **4**, 1988, 77–89.

47. W.R. Madych, S.A. Nelson: Multivariate interpolation and conditionally positive definite functions II. *Mathematics of Computation* **54**, 1990, 211–230.
48. J. Mairhuber: On Haar’s theorem concerning Chebyshev problems having unique solutions. *Proc. Am. Math. Soc.* **7**, 1956, 609–615.
49. S. Mallat: *A Wavelet Tour of Signal Processing*. Academic Press, 1998.
50. G. Meinardus: *Approximation of Functions: Theory and Numerical Methods*. Springer, Berlin, 1967.
51. V. Michel: *Lectures on Constructive Approximation*. Birkhäuser, NY, 2013.
52. P. Munshi: Error analysis of tomographic filters I: theory. *NDT & E Int.* **25**, 1992, 191–194.
53. P. Munshi, R.K.S. Rathore, K.S. Ram, M.S. Kalra: Error estimates for tomographic inversion. *Inverse Problems* **7**, 1991, 399–408.
54. P. Munshi, R.K.S. Rathore, K.S. Ram, M.S. Kalra: Error analysis of tomographic filters II: results. *NDT & E Int.* **26**, 1993, 235–240.
55. J.J. O’Connor, E.F. Robertson: *MacTutor History of Mathematics archive*. <http://www-history.mcs.st-andrews.ac.uk>.
56. M.J.D. Powell: *Approximation Theory and Methods*. Cambridge University Press, Cambridge, UK, 1981.
57. A. Quarteroni, R. Sacco, F. Saleri: *Numerical Mathematics*. Springer, New York, 2000.
58. M. Reed, B. Simon: *Fourier Analysis, Self-Adjointness*. In: *Methods of Modern Mathematical Physics II*, Academic Press, New York, 1975.
59. E.Y. Remez: Sur le calcul effectif des polynômes d’approximation des Tschebyscheff. *Compt. Rend. Acad. Sc.* **199**, 1934, 337.
60. E.Y. Remez: Sur un procédé convergent d’approximations successives pour déterminer les polynômes d’approximation. *Compt. Rend. Acad. Sc.* **198**, 1934, 2063.
61. R. Schaback: Creating surfaces from scattered data using radial basis functions. In: *Mathematical Methods for Curves and Surfaces*, M. Dæhlen, T. Lyche, and L.L. Schumaker (eds.), Vanderbilt University Press, Nashville, 1995, 477–496.
62. R. Schaback, H. Wendland: *Special Cases of Compactly Supported Radial Basis Functions*. Technical Report, Universität Göttingen, 1993.
63. R. Schaback, H. Wendland: *Numerische Mathematik*. Springer, Berlin, 2005.
64. L.L. Schumaker: *Spline Functions: Basic Theory*. Third Edition, Cambridge University Press, Cambridge, UK, 2007.
65. L.L. Schumaker: *Spline Functions: Computational Methods*. SIAM, 2015.
66. L.A. Shepp, B.F. Logan: The Fourier reconstruction of a head section. *IEEE Trans. Nucl. Sci.* **21**, 1974, 21–43.
67. G. Szegő: *Orthogonal Polynomials*. AMS, Providence, RI, U.S.A., 1939.
68. L.N. Trefethen: *Approximation Theory and Approximation Practice*. SIAM, Philadelphia, 2013.
69. D.F. Walnut: *An Introduction to Wavelet Analysis*. Birkhäuser Basel, 2004.
70. G.A. Watson: *Approximation Theory and Numerical Methods*. John Wiley & Sons, Chichester, 1980.
71. H. Wendland: Piecewise polynomial, positive definite and compactly supported radial functions of minimal degree. *Advances in Comp. Math.* **4**, 1995, 389–396.
72. H. Wendland: *Scattered Data Approximation*. Cambridge University Press, Cambridge, UK, 2005.
73. Wikipedia. The free encyclopedia. <https://en.wikipedia.org/wiki/>
74. Z. Wu: Multivariate compactly supported positive definite radial functions. *Advances in Comp. Math.* **4**, 1995, 283–292.

# Subject Index

- Algorithm
  - Clenshaw, 125, 126, 136
  - divided differences, 33
  - filtered back projection, 344
  - Gram-Schmidt, 120
  - Neville-Aitken, 27
  - pyramid, 270
  - Remez, 167, 173
- alternation
  - condition, 142, 167
  - matrix, 164
  - set, 165
  - theorem, 165
- autocorrelation, 247, 259, 282
  
- back projection, 325
- Banach space, 2
- band-limited function, 255
- bandwidth, 255, 329
- Bernstein
  - operator, 188
  - polynomial, 187
- Bessel inequality, 109
- best approximation, 61
  - direct characterization, 87
  - dual characterization, 86
  - strongly unique, 92
  
- Chebyshev
  - approximation, 139
  - knots, 43
  - norm, 139
  - partial sum, 125, 231
  - polynomials, 43, 123
- Cholesky decomposition, 300
- complete
  - orthogonal system, 196
  - orthonormal system, 196
  
- completeness criterion, 197
- computerized tomography, 317
- condition number, 305
- connected, 159
- convergence rate, 206
- convex
  - function, 73
  - functional, 74
  - hull, 148
  - set, 69
- convolution, 246, 259, 334
  - kernel, 334
  - theorem
    - Fourier transform, 259
    - Radon transform, 346
  
- dense subset, 186
- Dirac
  - approximation theorem, 249
  - evaluation functional, 283
  - sequence, 248
- Dirichlet kernel, 209
- discrete Fourier transform, 53
- divided difference, 29, 168
- dual
  - functional, 84
  - space, 84
  
- Euclidean space, 3
- extremal points, 140
  
- fill distance, 295
- filtered back projection, 328, 344
- formula
  - Euler, 49
  - Hermite-Genocchi, 36
  - Leibniz, 37
  - Rodrigues, 127

## Fourier

- coefficient, 48, 112
- convolution theorem, 247
- inversion formula, 250, 251, 255, 258
- matrix, 53
- operator, 118, 240, 250, 258
- partial sum, 112
- series, 118
- slice theorem, 327
- spectrum, 239, 241
- transform, 240, 258, 327

frame, 202

frequency spectrum, 239

## functional

- bounded, 84
- continuous, 64
- convex, 74
- dual, 84
- linear, 84

Gâteaux derivative, 87

## Gauss

- filter, 333
- function, 245, 259, 281
- normal equation, 11

Hölder inequality, 77, 79

## Haar

- space, 158
- system, 158
- wavelet, 261

## Hermite

- function, 138, 252
- Genocchi formula, 36
- polynomials, 130

Hilbert space, 69

indicator function, 261

## inequality

- Bessel, 109
- Hölder, 77, 79
- Jensen, 73
- Minkowski, 78, 79
- Young, 77

Jackson theorems, 217

Jensen inequality, 73

Kolmogorov criterion, 92

## Lagrange

- basis, 278
- polynomial, 21
- representation, 21, 278

## Lebesgue

- constant, 211, 305
- integrable, 79

Legendre polynomial, 127

Leibniz formula, 37

## Lemma

- Aitken, 26
- Riemann-Lebesgue, 242

## Lipschitz

- constant, 222
- continuity, 222

low-pass filter, 329

## matrix

- alternation, 164
- design, 10
- Gram, 106, 286
- Toeplitz, 57
- unitriangular, 299
- Vandermonde, 20, 276

## minimal

- distance, 61
- sequence, 69

Minkowski inequality, 78, 79

modulus of continuity, 224

multiresolution analysis, 266

## Newton

- Cotes quadrature, 235
- polynomial, 28, 168

## operator

- analysis, 199
- Bernstein, 188
- difference, 29
- projection, 108
- synthesis, 199

## orthogonal

- basis, 106
- complement, 108, 266
- projection, 104, 108, 265
- system, 196

## orthonormal

- basis, 107, 267
- system, 196, 293

- parallel beam geometry, 338
- parallelogram identity, 66
- Parseval identity, 109, 195, 234, 255
- periodic function, 47
- polarization identity, 66
- polynomial
  - Bernstein, 187
  - Chebyshev, 43, 123
  - Hermite, 130
  - Lagrange, 21
  - Legendre, 127
  - Newton, 28
- positive definite function, 277
- projection
  - operator, 108
  - orthogonal, 108
- pseudoinverse, 18
- pyramid algorithm, 270
- radially symmetric, 279
- Radon transform, 321
- refinement equations, 264
- regularization method, 14
- Remez
  - algorithm, 167, 173
  - exchange, 172
- reproducing kernel, 287
- Riemann-Lebesgue lemma, 242
- Riesz
  - basis, 198, 302
  - constant, 198, 302
  - stability, 302
- scale space, 264
- scaling function, 263
- Schwartz space, 251, 260
- sequence
  - Cauchy, 69
  - Dirac, 248
  - Korovkin, 187
- sinc function, 40, 243
- sinogram, 324
- Sobolev space, 335
- space
  - Banach, 2
  - Haar, 158
  - Hilbert, 69
  - Schwartz, 251
  - Sobolev, 335
- spline filter, 346
- strictly convex
  - function, 73
  - norm, 74
  - set, 69
- support, 242
- Theorem
  - alternation, 165
  - Banach-Mazur, 86
  - Banach-Steinhaus, 214
  - Bochner, 280
  - Carathéodory, 150
  - Charshiladse-Losinski, 215
  - de La Vallée Poussin, 236
  - Dini-Lipschitz, 228, 231
  - Faber, 217
  - Freud, 93
  - Jackson, 219, 223–225, 228
  - Jordan-von Neumann, 66
  - Korovkin, 189
  - Kuzmin, 235
  - Madych-Nelson, 288
  - Mairhuber-Curtis, 160
  - Paley-Wiener, 256
  - Plancherel, 255, 260
  - Pythagoras, 108, 290
  - Shannon, 256
  - Weierstrass, 191, 192
- three-term recursion, 121
- Toeplitz matrix, 57
- topological
  - closure, 186
  - dual, 84
- translation-invariant, 264, 313
- trigonometric polynomials, 47, 48
- uniform boundedness principle, 214
- unitriangular matrix, 299
- Vandermonde matrix, 20, 159, 276
- wavelet, 260
  - analysis, 269
  - coefficient, 269
  - Haar, 261
  - space, 267
  - synthesis, 270
  - transform, 271
- window function, 329
- Young inequality, 77

# Name Index

- Aitken, A.C. (1895-1967), 26
- Banach, S. (1892-1945), 86, 214
- Beer, A. (1825-1863), 318
- Bernstein, S.N. (1880-1968), 187
- Bessel, F.W. (1784-1846), 109
- Bochner, S. (1899-1982), 280
- Carathéodory, C. (1873-1950), 150
- Cauchy, A.-L. (1789-1857), 69, 105
- Chebyshev, P.L. (1821-1894), 139
- Cholesky, A.-L. (1875-1918), 299
- Cooley, J.W. (1926-2016), 56
- Cotes, R. (1682-1716), 235
- Courant, R. (1888-1972), 303
- Cramer, G. (1704-1752), 166
- Curtis, P.C. Jr. (1928-2016), 160
- de L'Hôpital, M. (1661-1704), 211
- de La Vallée Poussin (1866-1962), 236
- Dini, U. (1845-1918), 228
- Dirac, P.A.M. (1902-1984), 248, 283
- Dirichlet, P.G.L. (1805-1859), 209
- Euler, L. (1707-1783), 49
- Faber, G. (1877-1966), 217
- Fischer, E.S. (1875-1954), 303
- Fourier, J.B.J. (1768-1830), 48
- Fréchet, M.R. (1878-1973), 287
- Freud, G. (1922-1979), 93
- Fubini, G. (1879-1943), 243
- Gâteaux, R. (1889-1914), 87
- Gauß, C.F. (1777-1855), 11
- Genocchi, A. (1817-1889), 34
- Gram, J.P. (1850-1916), 106
- Hölder, O. (1859-1937), 77
- Haar, A. (1885-1933), 158, 260
- Hahn, H. (1879-1934), 86
- Hermite, C. (1822-1901), 34, 130
- Hesse, L.O. (1811-1874), 11
- Hilbert, D. (1862-1943), 69
- Horner, W.G. (1786-1837), 182
- Jackson, D. (1888-1946), 218
- Jensen, J.L. (1859-1925), 73
- Jordan, P. (1902-1980), 66
- Kolmogoroff, A.N. (1903-1987), 91
- Korovkin, P.P. (1913-1985), 187
- Kotelnikov, V. (1908-2005), 257
- Kuzmin, R.O. (1891-1949), 235
- Lagrange, J.-L. (1736-1813), 21
- Lambert, J.H. (1728-1777), 318
- Laplace, P.-S. (1749-1827), 165
- Lebesgue, H.L. (1875-1941), 79, 211
- Legendre, A.-M. (1752-1833), 127
- Leibniz, G.W. (1646-1716), 37
- Lipschitz, R. (1832-1903), 222
- Machiavelli, N.B. (1469-1527), 56
- Mairhuber, J.C. (1922-2007), 160
- Mazur, S. (1905-1981), 86
- Minkowski, H. (1864-1909), 78
- Neumann, J. von (1903-1957), 66
- Neville, E.H. (1889-1961), 27
- Newton, I. (1643-1727), 28
- Nyquist, H. (1889-1976), 257
- Paley, R. (1907-1933), 256
- Parseval, M.-A. (1755-1836), 109
- Plancherel, M. (1885-1967), 254
- Pythagoras (around 570-510 BC), 108

- Radon, J. (1887-1956), [320](#)  
Rayleigh, J.W.S. (1842-1919), [303](#)  
Remez, E.Y. (1896-1975), [167](#)  
Riemann, B. (1826-1866), [242](#)  
Riesz, F. (1880-1956), [198](#), [287](#)  
Rodrigues, B.O. (1795-1851), [127](#)  
Rolle, M. (1652-1719), [162](#)
- Schmidt, E. (1876-1959), [119](#)  
Schwartz, L. (1915-2002), [251](#)  
Schwarz, H.A. (1843-1921), [105](#)  
Shannon, C.E. (1916-2001), [255](#)  
Sobolev, S.L. (1908-1989), [335](#)  
Steinhaus, H. (1887-1972), [214](#)
- Szegő, G. (1895-1985), [123](#)
- Taylor, B. (1685-1731), [98](#)  
Tikhonov, A.N. (1906-1993), [15](#)  
Toeplitz, O. (1881-1940), [57](#)  
Tukey, J.W. (1915-2000), [56](#)
- Vandermonde, A.-T. (1735-1796), [20](#)
- Weierstraß, K. (1815-1897), [186](#)  
Whittaker, E.T. (1873-1956), [257](#)  
Wiener, N. (1894-1964), [256](#)
- Young, W.H. (1863-1942), [77](#)