

Answers to Selected Exercises

Exercises of Chap. 2

2.1

$$x^*(t) = e^t + \left(\frac{e^5}{e^5 - e^{-5}} \right) (e^{-t} - e^t), \quad u^*(t) = -2 \left(\frac{e^5}{e^5 - e^{-5}} \right) e^{-t}, \quad (0 \leq t \leq 5)$$

Optimal, because $H_{uu}^* = 1 > 0$.

2.2

$$y^*(t) = -20t^3 + 30t^2, \quad u^*(t) = -120t + 60, \quad (0 \leq t \leq 1)$$

Optimal, because $H_{uu}^* = 1 > 0$.

2.3

$$y^*(t) = \frac{10k_1^2 \left[-\frac{1}{3}t^3 + t^2 \left(\frac{1+k_2^2}{1+2k_2^2} \right) \right]}{\frac{5}{6} + \frac{1}{2} \left(\frac{1+k_2^2}{1+2k_2^2} \right)},$$
$$u^*(t) = \frac{20k_1^2}{\frac{5}{6} + \frac{1}{2} \left(\frac{1+k_2^2}{1+2k_2^2} \right)} \left[-t + \left(\frac{1+k_2^2}{1+2k_2^2} \right) \right] \quad (0 \leq t \leq 1)$$

$k_1^2 = 6.5$, $k_2^2 = 3000$ (Fig. 1).

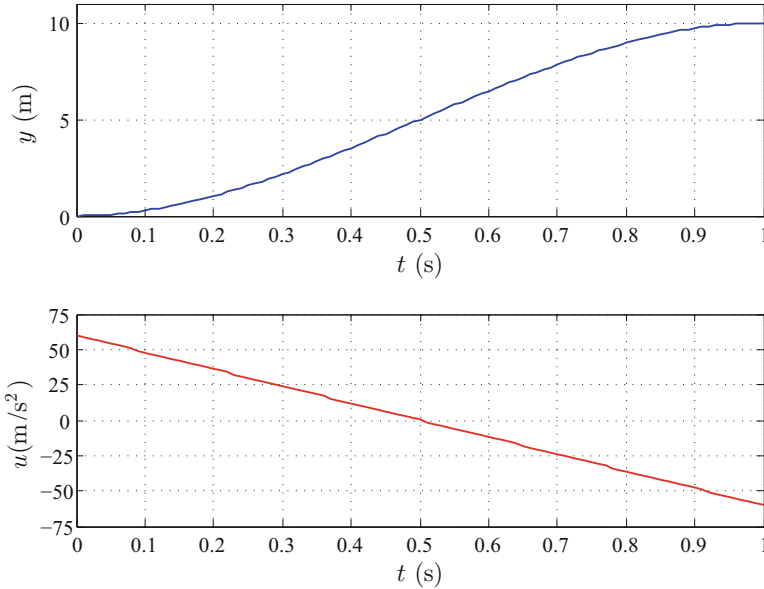


Fig. 1 The optimal trajectory and the corresponding control history for $k_1^2 = 6.5$, $k_2^2 = 3000$ (Exercise 2.3)

2.4 $t_f = 1$, $y^*(t) = -20t^3 + 30t^2$, $u^*(t) = -120t + 60$ ($0 \leq t \leq t_f$)

2.5 No, because it results in the necessary condition, $(e^{t_f} + e^{-t_f})^2 = -1$, which does not have a solution for any real value of t_f .

2.7 The optimal displacement and velocity for this case are plotted in Fig. 2: The final value of vertical acceleration input is $u(t_f) = g = 1.6$ m/s², because $\dot{x}_2(t_f) = 0$.

2.8

$$\left(H + \frac{\partial \phi}{\partial t}\right)_{t=t_f}^* = \frac{1}{2} [u^*(t_f)]^2 + \lambda_1^*(t_f) x_2^*(t_f) + \lambda_2^*(t_f) u^*(t_f) = 0$$

or

$$-\frac{1}{2} (c_1 t_f - c_2)^2 + c_1 g t_f + k^2 t_f = 0$$

implying

$$t_f = \frac{1}{c_1^2} \left[(c_1 c_2 + c_1 g + k^2) \pm \sqrt{(c_1 c_2 + c_1 g + k^2)^2 - c_1^2 c_2^2} \right]$$

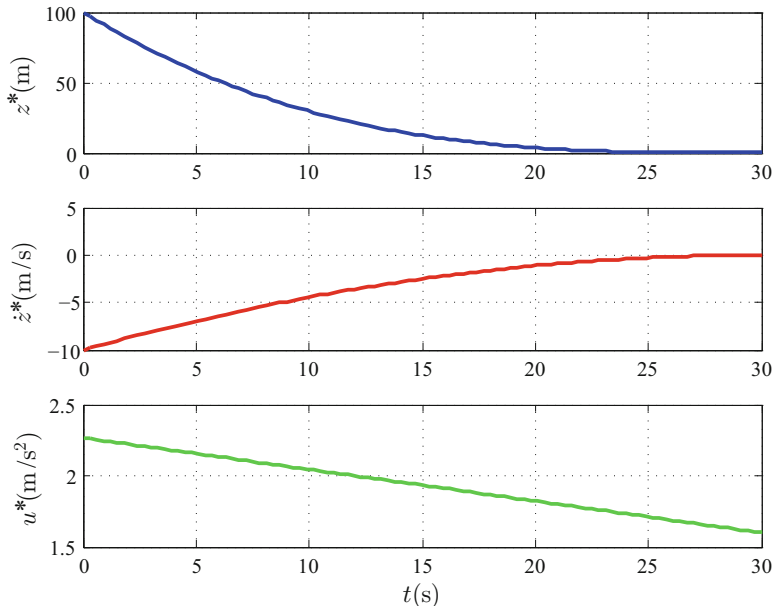


Fig. 2 The optimal trajectory for lunar landing (Exercise 2.7)

A real and positive solution for t_f is derived for $k = 1/100$, for which $t_f = 57.0248$ s, $c_1 = 0.0249$, and $c_2 = -0.714$. The optimal displacement and velocity for this case are plotted in Fig. 3.

2.15

$$\hat{u}(t) = \begin{cases} -1, & \hat{\lambda}(t) > 0 \\ 1, & \hat{\lambda}(t) < 0 \end{cases}$$

$H = 1 + \lambda u = 0$, $\hat{x}(0) = 1$, $\hat{u} = -1$, ($\hat{\lambda} = 1$), $\hat{x}(t) = 1 - t$, ($0 \leq t \leq t_f$), $\hat{x}(t_f) = 0$, $t_f = \hat{t} = 1$.

2.18

(a) No.

(b) For $a = 1.5g = 2.4 \text{ m/s}^2$,

$$t_f = \frac{2}{5g} \left\{ \dot{z}(0) + \sqrt{\dot{z}^2(0) + 5[6gz(0) + \dot{z}^2(0)]} \right\} = 15.8712 \text{ s}$$

$$\hat{t} = \frac{\dot{z}(0) + \frac{1}{2}gt_f}{3g} = 0.5619 \text{ s}$$

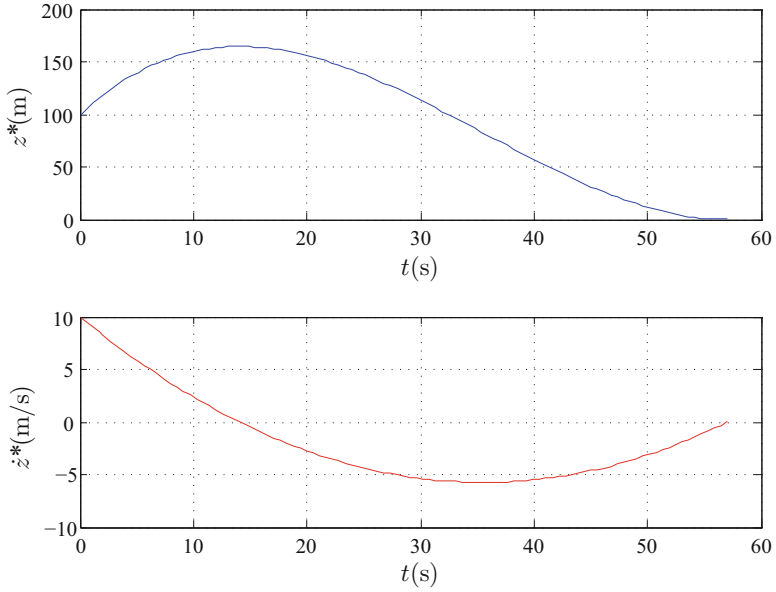


Fig. 3 The optimal trajectory for lunar landing (Exercise 2.8)

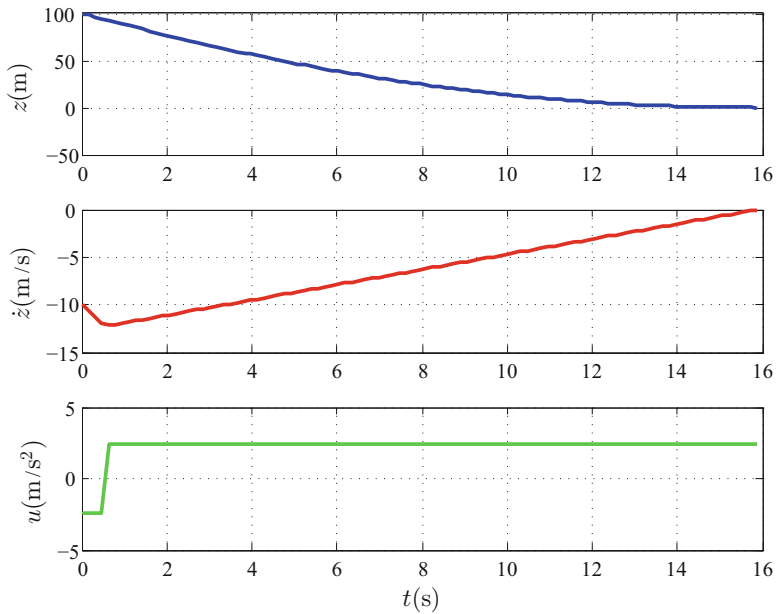


Fig. 4 Time-optimal trajectory for lunar landing (Exercise 2.18)

$$u(t) = \begin{cases} -1.5g & (0 \leq t \leq \hat{t}) \\ 1.5g & (\hat{t} < t \leq t_f) \end{cases}$$

$$z(t) = \begin{cases} z(0) + \dot{z}(0)t - \frac{5}{4}gt^2 & (0 \leq t \leq \hat{t}) \\ z(0) + \dot{z}(0)t + \frac{1}{4}gt^2 + \frac{3}{2}g\hat{t}^2 - 3g\hat{t}t & (\hat{t} < t \leq t_f) \end{cases}$$

The time-optimal displacement and velocity for this case are plotted in Fig. 4.

Exercises of Chap. 3

3.3

- (a) $v = 0.1995$ km/s, $\phi = 53.643^\circ$.
 (b) $v = 0.1289$ km/s, $\phi = -31.357^\circ$.
 (c) 1018.3 s.
 (d) C is 25.4246° behind the radial position of A .
 (e) $z = 0.0024$, $x = 0.6588$, $a = 606.591$ km, $e = 0.3820$.
 (f) Zero.

3.4

- (a) $v = 0.4164$ km/s, $\phi = 0$.
 (b) $v = 0.3875$ km/s, $\phi = 0$.

3.5 $a = 3875.483$ km, $e = 0.950225$, $v'_1 = 3.32175$ km/s, $\phi'_1 = 58.221^\circ$, $v'_2 = 1.855$ km/s, $\phi'_2 = -28.344^\circ$. No, because the orbit is Earth intersecting.

3.6 $a = 8003.0968$ km, $e = 0.1754826$, $i = 63.6658^\circ$, $\Omega = 98.1301^\circ$, $\omega = 59.3285^\circ$, $\tau = -373.756$ s.

3.7

$$\Delta \mathbf{v}_1 = -0.193607\mathbf{I} - 0.41983\mathbf{J} + 3.32604\mathbf{K} \text{ km/s}$$

$$\Delta \mathbf{v}_2 = 7.28047\mathbf{I} + 0.16851\mathbf{J} + 6.2595\mathbf{K} \text{ km/s}$$

3.8 5.5939 km/s.

3.9

$$\lim_{r_t \rightarrow \infty} (\Delta v_1 + \Delta v_2 + \Delta v_3) = 14.586 \text{ km/s}$$

For $r_t = 100$ A.U., $\Delta v_1 + \Delta v_2 + \Delta v_3 = 15.04$ km/s.

Exercises of Chap. 4

4.1

$$a_r^*(t) = -\lambda_r^*(t),$$

$$\begin{pmatrix} \lambda_r^*(t) \\ \lambda_{\dot{r}}^*(t) \\ \lambda_{\theta}^*(t) \end{pmatrix} = \begin{pmatrix} c_1 \cos nt + nc_2 \sin nt + 2c_3 \sin nt \\ -\frac{c_1}{n} \sin nt + c_2 \cos nt + 2\frac{c_3}{n}(\cos nt - 1) \\ c_3 \end{pmatrix} \quad (0 \leq t \leq t_f)$$

where c_1, c_2, c_3 are to be solved from the terminal boundary conditions, $\delta r(t_f) = \delta \dot{r}(t_f) = \delta \theta(t_f) = 0$

4.2

$$\lambda_{\theta}^*(t_f) = 0 = c_3$$

$$\left(H + \frac{\partial \phi}{\partial t} \right)_{t=t_f} = 0 = -\frac{1}{2}[\lambda_{\dot{r}}^*(t_f)]^2$$

which implies $\lambda_{\dot{r}}^*(t_f) = 0$. This along with $\delta r(t_f) = \delta \dot{r}(t_f) = 0$ yields

$$c_1 = \frac{2n^2[n\delta r(0) \cos nt_f + \delta \dot{r}(0) \sin nt_f] \cos nt_f}{nt_f - \sin nt_f \cos nt_f}$$

$$c_2 = \frac{2n[n\delta r(0) \cos nt_f + \delta \dot{r}(0) \sin nt_f] \sin nt_f}{nt_f - \sin nt_f \cos nt_f}$$

and t_f satisfies the following equation:

$$n^2 t_f \delta r(0) \sin nt_f + \delta \dot{r}(0) (\sin^3 nt_f + \cos^2 nt_f \sin nt_f - nt_f \cos nt_f) = 0$$

4.4

$$a_{\theta}^*(t) = -\lambda_{\theta}^*(t)$$

$$\begin{pmatrix} \lambda_r^*(t) \\ \lambda_{\theta}^*(t) \\ \lambda_{\dot{r}}^*(t) \\ \lambda_{\dot{\theta}}^*(t) \end{pmatrix} = \begin{pmatrix} c_1(4 - 3 \cos nt) + 6c_2(nt - \sin nt) + 3nc_3 \sin nt - 6nc_4(1 - \cos nt) \\ c_2 \\ -c_1 \frac{\sin nt}{n} - 2c_2 \frac{(1 - \cos nt)}{n} + c_3 \cos nt + 2c_4 \sin nt \\ 2c_1 \frac{(1 - \cos nt)}{n} + c_2 \frac{(3nt - 4 \sin nt)}{n} - 2c_3 \sin nt + c_4(4 \cos nt - 3) \end{pmatrix} \quad (0 \leq t \leq t_f)$$

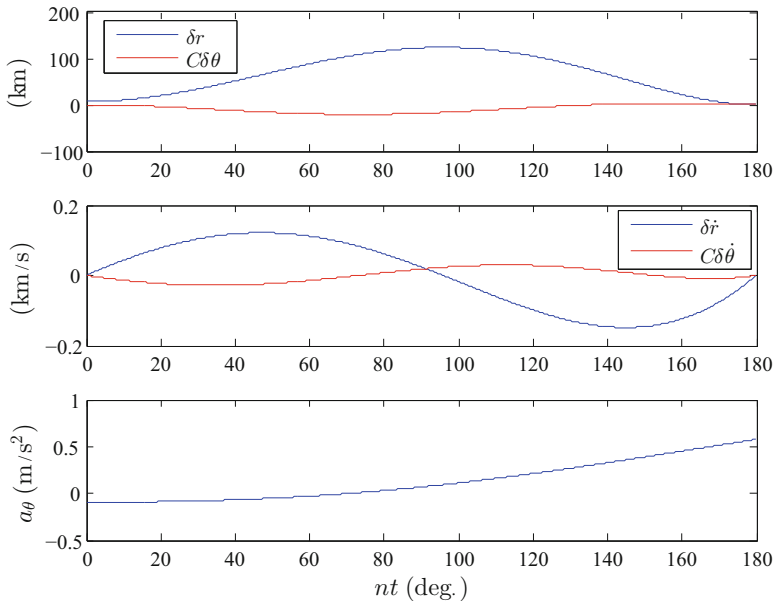


Fig. 5 The optimal trajectory for coplanar rendezvous (Exercise 4.4)

where c_1, c_2, c_3, c_4 are to be solved from the following terminal boundary conditions:

$$\begin{aligned} \frac{2\pi}{n^3}c_1 + \frac{2}{n^2}\left(\frac{3\pi^2}{2n} - \frac{14}{n}\right)c_2 - \frac{8}{n^2}c_3 - \frac{2\pi}{n^2}c_4 &= 0.07C \\ -\frac{2}{n^3}\left(14 - \frac{3\pi^2}{2}\right)c_1 + \frac{\pi}{n^3}\left(16 - \frac{3\pi^2}{2}\right)c_2 - \frac{2\pi}{n^2}c_3 + 3\left(16 - \frac{3\pi^2}{2}\right)c_4 &= 0.06\pi C \\ -\frac{8}{n}c_1 + \frac{2\pi}{n}c_2 - 2\pi c_3 + 12c_4 &= 0 \\ \frac{2\pi}{n^2}c_1 + \frac{3}{n^2}\left(16 - \frac{3\pi^2}{2}\right)c_2 - \frac{12}{n}c_3 - \frac{\pi}{n}c_4 &= 0.12Cn \end{aligned}$$

The optimal trajectory and control history are plotted in Fig. 5.

4.5

$$a_n^*(t) = -\lambda_z^*(t), \quad \left\{ \begin{matrix} \lambda_z^*(t) \\ \lambda_{\dot{z}}^*(t) \end{matrix} \right\} = \left\{ \begin{matrix} c_1 \cos nt + c_2 n \sin nt \\ -c_1 \frac{\sin nt}{n} + c_2 \cos nt \end{matrix} \right\} \quad (0 \leq t \leq t_f)$$

where c_1, c_2 are solved from the terminal boundary conditions to be the following:

$$c_1 = 0.002 \frac{n^3}{\pi} C, \quad c_2 = 0$$

which give the following extremal trajectory:

$$\delta z^*(t) = 0.001C \left[\cos nt + \frac{n}{\pi} \left(\frac{\sin nt}{n} - t \cos nt \right) \right] \quad (0 \leq t \leq t_f)$$

$$\delta \dot{z}^*(t) = 0.001nC \left(\frac{n}{\pi} t \sin nt - \sin nt \right) \quad (0 \leq t \leq t_f)$$

and control history, $u^*(t) = 0.002n^2C \frac{\sin nt}{\pi}$, $(0 \leq t \leq t_f)$.

4.7

$$\tan \beta^* = \frac{v_y^*(t_f) + g(t_f - t)}{v_x^*(t_f)}$$

Exercises of Chap. 5

5.2

$$\hat{a}_n(t) = \begin{cases} -1, & \lambda_n > 0 \\ 1, & \lambda_n < 0 \end{cases}$$

with $\lambda_n = \text{const}$. With $\alpha_0 > 0$, $a_n = 1$, $\alpha(t) = Ct/h$, and $t_f = \alpha_0 \sqrt{\mu/C}$.

5.3

$$\hat{\beta}(t) = \begin{cases} \cos^{-1}(b\pi/2), & \lambda_2 > 0 \\ -\cos^{-1}(b\pi/2), & \lambda_2 < 0 \end{cases}$$

where

$$\ddot{\lambda}_2 = -c_2 \frac{c}{r^2} \sin \beta - \left(3h^2/r^4 - 2\mu/r^3 + 2c \cos \beta/r^3 \right) \lambda_2$$

where c_2 is constant and $r(t), \beta(t)$ are on the optimal trajectory.

5.5

$$\hat{a}_n(t) = \begin{cases} Cn^2, & \lambda_2 < 0 \\ -Cn^2, & \lambda_2 > 0 \end{cases}$$

where

$$\lambda_2 = -\frac{c_2}{n} \sin nt + c_2 \cos nt$$

The response until the first switching time, $t_s = \tan^{-1}(nc_2/c_1)$, with $c_1 > 0$, $c_2 > 0$, is the following:

$$\delta z(t) = C(0.01 \sin nt + \cos nt - 1), \quad (0 \leq t \leq t_s)$$

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