

Bibliography

1. M. Arioli, The use of QR factorization in sparse quadratic programming and backward error issues. *SIAM J. Matrix Anal. Appl.* **21**, 825–839 (2000)
2. M. Arioli, L. Baldini, A backward error analysis of a null space algorithm in sparse quadratic programming. *SIAM J. Matrix Anal. Appl.* **23**, 425–442 (2001)
3. M. Arioli, G. Manzini, A network programming approach in solving Darcy's equations by mixed finite-element methods. *ETNA* **22**, 41–70 (2006)
4. M. Arioli, J. Maryška, M. Rozložník, M. Tůma, Dual variable methods for mixed-hybrid finite element approximation of the potential fluid flow problem in porous media. *ETNA* **22**, 17–40 (2006)
5. D.N. Arnold, R.S. Falk, R. Winther, Preconditioning in $H(\text{div})$ and applications. *Math. Comput.* **66**(219), 957–984 (1997)
6. O. Axelsson, J. Karátson, Equivalent operator preconditioning for elliptic problems. *Numer. Algorithms* **50**(3), 297–380 (2009)
7. Z.Z. Bai, Solutions of Linear Systems of Block Two-by-Two Structures. <http://lsec.cc.ac.cn/~bzz/Public/psfiles/slice22a.pdf>
8. Z.Z. Bai, G.H. Golub, M.K. Ng, Hermitian and skew-Hermitian splitting methods for non-Hermitian positive definite linear systems. *SIAM J. Matrix Anal. Appl.* **24**, 603–626 (2003)
9. P. Bastian, M. Blatt, R. Scheichl, Algebraic multigrid for discontinuous Galerkin discretizations of heterogeneous elliptic problems. *Numer. Linear Algebra Appl.* **19**(2), 367–388 (2012)
10. M. Benzi, Preconditioning techniques for large linear systems: a survey. *J. Comput. Phys.* **182**, 418–477 (2002)
11. M. Benzi, V. Simoncini, On the eigenvalues of a class of saddle point matrices. *Numer. Math.* **103**, 173–196 (2006)
12. M. Benzi, G.H. Golub, J. Liesen, Numerical solution of saddle point problems. *Acta Numerica* **14**, 1–137 (2005)
13. L. Bergamaschi, J. Gondzio, G. Zilli, Preconditioning indefinite systems in interior point methods for optimization. *Comput. Optim. Appl.* **28**(2), 149–171 (2004)
14. A. Björck, *Numerical Methods for Least Squares Problems* (SIAM, Philadelphia, 1996)
15. D. Braess, R. Sarazin, An efficient smoother for the Stokes problem. *Appl. Numer. Math.* **23**, 3–20 (1997)
16. J.H. Bramble, J.E. Pasciak, A preconditioning technique for indefinite systems resulting from mixed approximations of elliptic problems. *Math. Comput.* **50**, 1–17 (1988)
17. F. Brezzi, M. Fortin, *Mixed and Hybrid Finite Element Methods* (Springer, New York, 1991)
18. Y. Chabrilac, J.P. Crouzeix, Definiteness and semidefiniteness of quadratic forms revisited. *Linear Algebra Appl.* **63**(1), 283–292 (1984)

19. D. Drzisga, L. John, U. Rude, B. Wohlmuth, W. Zulehner, On the analysis of block smoothers for saddle point problems. *SIAM J. Matrix Anal. Appl.* **39**(2), 932–960 (2018)
20. C. Durazzi, V. Ruggiero, Indefinitely preconditioned conjugate gradient method for large sparse equality and inequality constrained quadratic problems. *Numer. Linear Algebra Appl.* **10**(8), 673–688 (2002)
21. H.C. Elman, Multigrid and Krylov subspace methods for the discrete Stokes equations. *Int. J. Numer. Meth. Fluids* **22**, 755–770 (1996)
22. H.C. Elman, G.H. Golub, Inexact and preconditioned Uzawa algorithms for saddle point problems. *SIAM J. Numer. Anal.* **31**(6), 1645–1661 (1994)
23. H.C. Elman, D.J. Silvester, A.J. Wathen, Iterative methods for problems in computational fluid dynamics, in *Iterative Methods in Scientific Computing*, ed. by R.H. Chan, C.T. Chan, G.H. Golub (Springer, Singapore, 1997), pp. 271–327
24. H. Elman, D.J. Silvester, A.J. Wathen, Block preconditioners for the discrete incompressible Navier-Stokes equations. *Int. J. Numer. Meth. Fluids* **40**, 333–344 (2002)
25. H. Elman, D.J. Silvester, A.J. Wathen, *Finite Elements and Fast Iterative Solvers with Applications in Incompressible Fluid Dynamics* (Oxford University Press, Oxford, 2005)
26. R. Estrin, C. Greif, On nonsingular saddle-point systems with a maximally rank deficient leading block. *SIAM J. Matrix Anal. Appl.* **36**(2), 367–384 (2015)
27. R. Estrin, C. Greif, Towards an optimal condition number of certain augmented Lagrangian-type saddle-point matrices. *Numer. Linear Algebra Appl.* **23**(4), 693–705 (2016)
28. V. Faber, T.A. Manteuffel, S.V. Parter, On the theory of equivalent operators and application to the numerical solution of uniformly elliptic partial differential equations. *Adv. Appl. Math.* **11**(2), 109–163 (1990)
29. B. Fischer, A. Ramage, D.J. Silvester, A.J. Wathen, Minimum residual methods for augmented systems. *BIT* **38**, 527–543 (1998)
30. R.W. Freund, N.M. Nachtigal, QMR: a quasi-minimal residual method for non-Hermitian linear systems. *Numer. Math.* **60**, 315–339 (1991)
31. R.W. Freund, N.M. Nachtigal, Software for simplified Lanczos and QMR algorithms. *Appl. Numer. Math.* **19**, 319–341 (1995)
32. R. Freund, G.H. Golub, N.M. Nachtigal, Iterative solution of linear systems. *Acta Numerica* **1**, 1–44 (1992)
33. J.F. Gerbeau, C. Farhat, CME358: The Finite Element Method for Fluid Mechanics. Stanford University, Spring 2009. <http://www.stanford.edu/class/cme358/>
34. T. Gergelits, Z. Strakoš, Composite convergence bounds based on Chebyshev polynomials and finite precision conjugate gradient computations. *Numer. Algorithms* **65**(4), 759–782 (2014)
35. G. Golub, C. Greif, On solving block-structured indefinite linear systems. *SIAM J. Sci. Comput.* **24**(6), 2076–2092 (2003)
36. N.I.M. Gould, V. Simoncini, Spectral analysis of saddle point matrices with indefinite leading blocks. *SIAM J. Matrix Anal. Appl.* **31**(3), 1152–1171 (2009)
37. A. Greenbaum, *Iterative Methods for Solving Linear Systems* (SIAM, Philadelphia, 1997)
38. A. Greenbaum, V. Pták, Z. Strakoš, Any convergence curve is possible for GMRES. *SIAM J. Matrix Anal. Appl.* **17**, 465–470 (1996)
39. M.R. Hestenes, E. Stiefel, Methods of conjugate gradients for solving linear systems. *J. Res. Nat. Bur. Stand.* **49**, 409–436 (1952)
40. N.J. Higham, *Accuracy and Stability of Numerical Algorithms*, 2nd edn. (SIAM, Philadelphia, 2002)
41. R. Hiptmair, Operator preconditioning. *Comput. Math. Appl.* **52**(5), 699–706 (2006)
42. R.A. Horn, C.R. Johnson, *Matrix Analysis*, 2nd edn. (Cambridge University Press, New York, 2012)
43. J. Hrnčíř, I. Pultarová, Z. Strakoš, Decomposition into subspaces and operator preconditioning I: abstract framework. Preprint CNMM/2017/06, Prague (2017)
44. I.C.F. Ipsen, A note on preconditioning non-symmetric matrices. *SIAM J. Sci. Comput.* **23**(3), 1050–1051 (2001)

45. P. Jiránek, M. Rozložník, Maximum attainable accuracy of inexact saddle point solvers. *SIAM J. Matrix Anal. Appl.* **29**(4), 1297–1321 (2008)
46. P. Jiránek, M. Rozložník, Limiting accuracy of segregated solution methods for nonsymmetric saddle point problems. *J. Comput. Appl. Math.* **215**, 28–37 (2008)
47. E.F. Kaasschieter, A.J.M. Huijben, Mixed hybrid finite elements and streamline computation for the potential flow problem. *Numer. Method Partial Differ. Equ.* **8**(3), 221–266 (1992)
48. A. Klawonn, Block-triangular preconditioners for saddle point problems with a penalty term. *SIAM J. Sci. Comput.* **19**(1), 172–184 (1998)
49. A. Klawonn, An optimal preconditioner for a class of saddle point problems with a penalty term. *SIAM J. Sci. Comput.* **19**(2), 540–552 (1998)
50. M. Kroupa, J. Mužák, J. Trojáček, Remediation of former uranium in-situ leaching area at Stráž pod Ralskem – Hamr na Jezeře, Czech Republic. The Uranium Mining and Remediation Exchange Group (UMREG) and other uranium production cycle technical meetings – Selected papers 2012–2015. IAEA-TECDOC, Vienna (in preparation)
51. J. Liesen, Z. Strakoš, Convergence of GMRES for tridiagonal toeplitz matrices. *SIAM J. Matrix Anal. Appl.* **26**, 233–251 (2004)
52. J. Liesen, Z. Strakoš, GMRES convergence analysis for a convection-diffusion model problem. *SIAM J. Sci. Comput.* **26**, 1989–2009 (2005)
53. J. Liesen, B. Parlett, On nonsymmetric saddle point matrices that allow conjugate gradient iterations. *Numer. Math.* **108**, 605–624 (2008)
54. J. Liesen, Z. Strakoš, *Krylov Subspace Methods, Principles and Analysis* (Oxford University Press, Oxford, 2013)
55. L. Lukšan, J. Vlček, Indefinitely preconditioned inexact Newton method for large sparse equality constrained non-linear programming problems. *Numer. Linear Algebra Appl.* **5**(3), 1099–1506 (1999)
56. J. Málek, Z. Strakoš, *Preconditioning and the Conjugate Gradient Method in the Context of Solving PDEs*. SIAM Spotlight Series (SIAM, Philadelphia, 2015)
57. J. Maryška, M. Rozložník, M. Tůma, Mixed hybrid finite element approximation of the potential fluid flow problem. *J. Comput. Appl. Math.* **63**, 383–392 (1995)
58. J. Maryška, M. Rozložník, M. Tůma, The potential fluid flow problem and the convergence rate of the minimal residual method. *Numer. Linear Algebra Appl.* **3**, 525–542 (1996)
59. J. Maryška, M. Rozložník, M. Tůma, Schur complement systems in the mixed-hybrid finite element approximation of the potential fluid flow problem. *SIAM J. Sci. Comput.* **22**, 704–723 (2000)
60. J. Maryška, M. Rozložník, M. Tůma, Schur complement reduction in the mixed-hybrid approximation of Darcy’s law: rounding error analysis. *J. Comput. Appl. Math.* **117**, 159–173 (2000)
61. M.F. Murphy, G.H. Golub, A.J. Wathen, A note on preconditioning for indefinite linear systems. *SIAM J. Sci. Comput.* **21**, 1969–1972 (2000)
62. J. Nocedal, S.J. Wright, *Numerical Optimization*, 2nd edn. (Springer, New York, 2006)
63. D. Orban, M. Arioli, *Iterative Solution of Symmetric Quasi-Definite Linear Systems*. SIAM Spotlight Series (SIAM, Philadelphia, 2017)
64. C.C. Paige, M.A. Saunders, Solution of sparse indefinite systems of linear equations. *SIAM J. Numer. Anal.* **12**, 617–629 (1975)
65. I. Perugia, V. Simoncini, Block-diagonal and indefinite symmetric preconditioners for mixed finite element formulations. *Numer. Linear Algebra Appl.* **7**(7–8), 585–616 (2000)
66. J. Pestana, A.J. Wathen, The antitriangular factorization of saddle point matrices. *SIAM J. Matrix Anal. Appl.* **35**(2), 339–353 (2014)
67. J. Pestana, A.J. Wathen, Natural preconditioning and iterative methods for saddle point systems. *SIAM Rev.* **57**, 71–91 (2015)
68. C. Powell, D.J. Silvester, Optimal preconditioning for Raviart–Thomas mixed formulation of second-order elliptic problems. *SIAM J. Matrix Anal. Appl.* **25**(3), 718–738 (2003)
69. A. Quarteroni, A. Viali, *Numerical Approximation of Partial Differential Equations* (Springer, Berlin/Heidelberg, 1994)

70. T. Rees, J. Scott, The null-space method and its relationship with matrix factorizations for sparse saddle point systems. *Numer. Linear Algebra Appl.* **25**(1), 1–17 (2018)
71. M. Rozložník, V. Simoncini, Krylov subspace methods for saddle point problems with indefinite preconditioning. *SIAM J. Matrix Anal. Appl.* **24**(2), 368–391 (2002)
72. M. Rozložník, F. Okulicka-Dłużewska, A. Smoktunowicz, Cholesky-like factorization of symmetric indefinite matrices and orthogonalization with respect to bilinear forms. *SIAM J. Matrix Anal. Appl.* **36**(2), 727–751 (2015)
73. T. Rusten, R. Winther, A preconditioned iterative method for saddlepoint problems. *SIAM J. Matrix Anal. Appl.* **13**(3), 887–904 (1992)
74. Y. Saad, *Iterative Methods for Sparse Linear Systems* (PWS Pub. Co., Boston, 1996) (2nd edition: SIAM, Philadelphia, 2003)
75. Y. Saad, M.H. Schultz, GMRES: a generalized minimal residual algorithm for solving nonsymmetric linear systems. *SIAM J. Sci. Stat. Comput.* **7**, 856–869 (1986)
76. D.J. Silvester, A.J. Wathen, Fast iterative solution of stabilized Stokes systems, part II: using block preconditioners. *SIAM J. Numer. Anal.* **31**, 1352–1367 (1994)
77. J. Stoer, Solution of large linear systems of conjugate gradient type methods, in *Mathematical Programming* (Springer, Berlin, 1983), pp. 540–565
78. J. Stoer, R. Freund, On the solution of large indefinite systems of linear equations by conjugate gradients algorithm, in *Computing Methods in Applied Sciences and Engineering V*, ed. by R. Glowinski, J.L. Lions (INRIA, North Holland, 1982), pp. 35–53
79. M. Stoll, A. Wathen, Combination preconditioning and the Bramble-Pasciak+ preconditioner. *SIAM J. Matrix Anal. Appl.* **30**(2), 582–608 (2008)
80. R.J. Vanderbei, Symmetric quasi-definite matrices. *SIAM J. Optim.* **5**(1), 100–113 (1995)
81. A.J. Wathen, Preconditioning. *Acta Numerica* **24**, 329–376 (2015)
82. A.J. Wathen, B. Fischer, D.J. Silvester, The convergence rate of the minimal residual method for the Stokes problem. *Numer. Math.* **71**, 121–134 (1995)
83. A.J. Wathen, B. Fischer, D.J. Silvester, The convergence of iterative solution methods for symmetric and indefinite linear systems, in *Numerical Analysis*, ed. by D. Griffiths, D. Higham, A.G. Watson (Longman Scientific, Harlow, 1998), pp. 230–240
84. W. Zulehner, A class of smoothers for saddle point problems. *Computing* **65**, 227–246 (2000)
85. W. Zulehner, Analysis of iterative methods for saddle point problems: a unified approach. *Math. Comput.* **71**, 479–505 (2002)
86. W. Zulehner, Nonstandard norms and robust estimates for saddle point problems. *SIAM J. Matrix Anal. Appl.* **32**(2), 536–560 (2011)

Index

- Aasen's method, 42, 69
Arrow–Hurwicz method, 51, 52
Asymptotic convergence factor, 60, 103, 115, 121, 125, 128
Augmented Lagrangian method, 24
Augmented system, 11
Average contraction factors, 116
- Babuška–Brezzi condition, 14
Back-substitution, 33, 48, 87
Back-substitution formula, 88, 91
Bi-CG method, 57, 61, 84
Block diagonal preconditioner, 69, 70
Block triangular preconditioner, 69, 73
- CG method, 57–59, 63, 81, 83, 94, 120
CGNE method, 84
CGS method, 84
Cholesky factorization, 12, 43, 45, 81
Cholesky-like factorization, 45
Coercive bilinear form, 13
Constraint preconditioner, 70, 75, 76, 94, 128
Continuity equation, 6, 108
Convergence delay, 79
Coupled method, 33, 45, 49
- Darcy's law, 108
Diagonal scaling, 98, 115, 124, 127
DIAMO, s. e., 103, 104, 107
Direct method, 33, 41, 45
Discretization, 13
Discretization parameter, 112, 113, 119, 121, 125
- Finite element method, 13
Fixed-point iteration, 50
FOM method, 57
Fundamental cycle null-space basis, 123
- Generalized saddle-point matrix, 30
Generalized saddle-point problem, 20
GMRES method, 57, 61, 65, 73, 84
Groundwater flow modeling, 103, 107
- Hermitian/skew-Hermitian matrix splitting, 56
H-symmetric Bi-CG method, 65
H-symmetric matrix, 53, 55, 65
H-symmetric QMR method, 65
- Inclusion set, 26, 100, 113, 120, 124, 127, 128
Interior-point methods, 17
Iterative method, 33, 35, 38, 49
Iterative refinement, 50
- Krylov subspace method, 49, 57, 75, 77, 84
- Lagrange multipliers, 3, 11, 17, 19, 111
Lagrangian, 3, 5, 7, 17, 19
LDL^T factorization, 42, 69
Least squares problem, 9, 11, 38
LU factorization, 34, 47, 85
- Maximum attainable accuracy, 79, 81, 83, 85, 87, 101

- Minimum norm solution, 11
- MINRES method, 57, 59, 60, 64, 72, 103, 115, 120, 121, 124, 128
- Mixed-hybrid finite element method, 108
- Model problem, 112, 119, 121, 128
- Multigrid method, 49, 67

- Newton method, 18, 19
- Nonlinear programming, 18
- Normal equations, 10–12, 47
- Null-space, 9
 - basis, 26, 36, 46, 48
 - method, 37, 39, 46, 48, 54, 77, 82, 87, 90, 96, 122
 - projected matrix, 23, 37, 48, 98, 122
 - projected system, 24, 37, 47, 83, 97, 122

- Partial differential equation, 12
- Peak/plateau behavior, 59, 62
- Petrov–Galerkin condition, 57
- Porous media, 103, 107
- Potential fluid flow problem, 103, 107
- Preconditioned Krylov subspace method, 62, 75, 76, 94
- Preconditioning, 62, 69

- QMR method, 57, 61, 84
- QR factorization, 26, 46, 83, 85

- Quadratic programming, 3, 16
- Quasi-definite matrix, 43

- Range-space, 9
- Raviart–Thomas discretization, 111
- Real-world application, 103
- Regularized saddle-point problem, 30
- Relative residual norm, 60, 64, 66, 120, 124

- Saddle-point matrix, 2, 23, 26, 34, 42, 112
- Saddle-point problem, 4, 12, 18, 23, 24, 111
- Schur complement matrix, 23, 34, 43, 45, 72, 73, 117
- Schur complement method, 35, 36, 45, 53, 76, 81, 87, 94, 117
- Schur complement systems, 23, 34, 81, 95, 117
- Second-order elliptic equation, 5, 104
- Segregated method, 33
- Stationary iterative method, 49, 50, 69

- True residual, 81, 85, 88, 91

- Uniformly regular mesh refinement, 112
- Updated residual, 81, 85, 88, 91
- Uranium mining, 103, 104
- Uzawa’s method, 50, 51