A

Some Mathematical Background

A.1 Green's Functions for the Homogeneous Grad-Shafranov Equation

The material covered in this section can be found in many advanced calculus book, however we prefer to cite the classic book by Morse and Feshbach [87].

The Δ^* operator introduced in Chapter 2 plays an important role in determining the plasma equilibrium. In particular, given the toroidal current density, the poloidal flux function can be determined by solving the partial differential Equation 2.15. Analytical solutions for this equation can be found with the aid of Green's function method. Consider the generalized partial differential equation

$$\Delta^* G(\boldsymbol{r}, \tilde{\boldsymbol{r}}) = -\mu_0 r \delta(\boldsymbol{r} - \tilde{\boldsymbol{r}}) \,, \tag{A.1}$$

where δ is the Dirac function, and the operator Δ^* is applied with respect to the $\boldsymbol{r}$ variable. If boundary conditions are not specified, the solutions of Equation A.1 are infinite, differing from each other by a function χ solving the homogenous equation $\Delta^* \chi = 0$. It can be shown by using Green's third identity that given a Green's function and assigning Dirichlet and Neumann type boundary conditions on a domain D for the poloidal flux function, the solution of Equation 2.15 can be expressed as

$$\begin{aligned} \psi(\boldsymbol{r}) = &\int_D G(\boldsymbol{r}, \tilde{\boldsymbol{r}}) J_\varphi(\tilde{\boldsymbol{r}}) \, \mathrm{d}\tilde{S} \\ &- \oint_{\partial D} \frac{1}{\mu_0 \tilde{r}} \left(\psi(\tilde{\boldsymbol{r}}) \frac{\partial}{\partial n} G(\boldsymbol{r}, \tilde{\boldsymbol{r}}) - G(\boldsymbol{r}, \tilde{\boldsymbol{r}}) \frac{\partial}{\partial n} \psi(\tilde{\boldsymbol{r}}) \right) \mathrm{d}\tilde{l} \,. \end{aligned} \tag{A.2}$$

Hence Equation A.2 allows us to evaluate the poloidal flux function inside a domain D if a Green's function ψ and $\frac{\partial \psi}{\partial n}$ on ∂D are given. Note that if a Green's function satisfies a homogeneous Neumann condition on ∂D then Dirichlet conditions on ψ are sufficient to evaluate the poloidal flux function.

On the other hand if Green's function satisfies a homogeneous Dirichlet condition then just Neumann conditions on ψ have to be assigned. It is interesting to note that of the two terms that appear on the right-hand side of Equation A.2 the former one represents the contribution given to the ψ function by the toroidal current density inside the domain D, while the latter term represents the contribution to the ψ function given by the toroidal current density external to the domain D. Allowing the D domain to cover the whole poloidal plane and imposing the following boundary conditions on Green's function

$$\lim_{\boldsymbol{r}\to\infty} G(\boldsymbol{r},\tilde{\boldsymbol{r}}) = 0 \quad \text{if } \|\tilde{\boldsymbol{r}}\| < M \in \mathbb{R}$$
$$\lim_{\boldsymbol{r}\to 0} G(\boldsymbol{r},\tilde{\boldsymbol{r}}) = 0 \quad \text{if } \|\tilde{\boldsymbol{r}}\| > M \in \mathbb{R} - \{0\}\,,$$

one obtains the so-called free space Green's function G_0. For the free space, Green's function Equation A.2 reduces to

$$\psi(\boldsymbol{r}) = \int_{\mathbb{R}^2} G_0(\boldsymbol{r},\tilde{\boldsymbol{r}}) J_\varphi(\tilde{\boldsymbol{r}})\,\mathrm{d}\tilde{S}\,. \tag{A.3}$$

The function G_0 represents the flux produced by an infinitesimal toroidal filament carrying a unitary current; the analytical expression for G_0 is

$$\begin{aligned} G_0(\boldsymbol{r},\tilde{\boldsymbol{r}}) &= \mu_0 \frac{\sqrt{r\tilde{r}}}{\pi k}\left[\left(1-\frac{k^2}{2}\right)K(k^2) - E(k^2)\right] \\ &= \mu_0 \frac{\sqrt{r\tilde{r}}}{\pi k}\left[\frac{1}{3}R_D(0,1-k^2,1) - \frac{1}{2}R_F(0,1-k^2,1)\right]\,, \end{aligned} \tag{A.4}$$

where

$$k^2 = \frac{4r\tilde{r}}{(r+\tilde{r})^2 + (z-\tilde{z})^2}\,.$$

The functions K and E are the elliptic integral of the first and second kind respectively [88]. The functions R_F and R_D have been introduced by Carlson [89] and enable one to evaluate (A.4) with better accuracy (see also [90]).

A.2 Solutions of the Homogeneous Grad–Shafranov Equation

The homogeneous Grad–Shafranov equation

$$\Delta^* \psi = 0 \quad \Longleftrightarrow \quad r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\psi\right) + \frac{\partial^2}{\partial z^2}\psi = 0 \tag{A.5}$$

plays an important role in the plasma shape identification algorithms discussed in Chapter 3. In particular, its solutions in the not simply connected set Ω_v are used to obtain an approximation of the poloidal flux function fitting the available magnetic measurements.

A.2.1 Green's Functions

The Green's functions have been defined in Section A.1; from Equation A.1 it is evident that a Green's function $G(\boldsymbol{r}, \tilde{\boldsymbol{r}})$ satisfies the homogeneous Grad–Shafranov equation everywhere except at the point $\tilde{\boldsymbol{r}}$ where the impulsive source is located. A Green's function is, thus, a solution of Equation (A.5) in Ω_v if $\tilde{\boldsymbol{r}} \notin \Omega_v$.

A.2.2 Toroidal Harmonics

A toroidal coordinate system (η, ζ, φ) is defined with respect to the cylindrical coordinate system (r, z, φ) introduced in Section 2.2 by the relations [87, 91]

$$r = \frac{r_0 \sinh(\zeta)}{\cosh\zeta - \cos\eta} \tag{A.6}$$

$$z = \frac{r_0 \sin\eta}{\cosh\zeta - \cos\eta} + z_0 \tag{A.7}$$

where $\boldsymbol{r}_0 = (r_0, z_0)$ is the position on the poloidal plane of the singular point $(\zeta = +\infty)$ of the toroidal coordinate system.

In a toroidal coordinate system Equation A.5 is written as

$$\frac{\cosh\zeta - \cos\eta}{r_0}\left[\sinh\zeta \frac{\partial}{\partial\zeta}\left(\frac{\cosh\zeta - \cos\eta}{r_0}\frac{\partial}{\partial\zeta}\psi\right) + \frac{\partial}{\partial\eta}\left(\frac{\cosh\zeta - \cos\eta}{r_0}\frac{\partial}{\partial\eta}\psi\right)\right] = 0\,. \tag{A.8}$$

Analytical solutions to the partial differential Equation A.8 can be found using the variable separation method. Let

$$\psi(\eta, \zeta) = \frac{r_0 \sinh\zeta}{\sqrt{\cosh\zeta - \cos\eta}} h(\zeta) g(\eta)\,; \tag{A.9}$$

by substituting Equation A.9 in A.8 one obtains the two *separated* equations

$$\frac{1}{\sinh\zeta}\frac{\partial}{\partial\zeta}\left(\sinh\zeta \frac{\partial}{\partial\zeta} h\right) - \frac{1}{\sinh^2\zeta} h - \left(\lambda^2 - \frac{1}{4}\right) h = 0 \tag{A.10a}$$

$$\frac{\partial^2}{\partial\eta^2} g + \lambda^2 g = 0\,, \tag{A.10b}$$

where λ is the separation constant. Due to the fact that the coordinate η represents an angle, the function g must be periodic at 2π. From Equation A.10b it follows that λ^2 must be an integer. Moreover, with the change of variable $s = \cosh\zeta$ it is possible to show that Equation A.10a becomes the Legendre differential equation [88, 87]. From these considerations we obtain a pair of linearly independent solutions for each of the Equations A.10

$$h_1(\zeta) = P^1_{n-\frac{1}{2}}(\cosh\zeta) \tag{A.11a}$$

$$h_2(\zeta) = Q^1_{n-\frac{1}{2}}(\cosh\zeta) \tag{A.11b}$$

$$g_1(\eta) = \cos(n\eta) \tag{A.11c}$$

$$g_2(\eta) = \sin(n\eta)\,, \tag{A.11d}$$

where $n = \lambda^2$ and P^α_β, and Q^α_β are the grade α and order β *associate Legendre function* of first and second kind, respectively [88, 92]. With the aid of the functions defined in Equations A.11 the following set of solutions to (A.5) are obtained

$$\psi^{\rm c}_{{\rm int},n}(\eta,\zeta) = \frac{r_0\sinh\zeta}{\sqrt{\cosh\zeta-\cos\eta}} P^1_{n-\frac{1}{2}}(\cosh\zeta)\cos(n\eta) \tag{A.12a}$$

$$\psi^{\rm s}_{{\rm int},n}(\eta,\zeta) = \frac{r_0\sinh\zeta}{\sqrt{\cosh\zeta-\cos\eta}} P^1_{n-\frac{1}{2}}(\cosh\zeta)\sin(n\eta) \tag{A.12b}$$

$$\psi^{\rm c}_{{\rm ext},n}(\eta,\zeta) = \frac{r_0\sinh\zeta}{\sqrt{\cosh\zeta-\cos\eta}} Q^1_{n-\frac{1}{2}}(\cosh\zeta)\cos(n\eta) \tag{A.12c}$$

$$\psi^{\rm s}_{{\rm ext},n}(\eta,\zeta) = \frac{r_0\sinh\zeta}{\sqrt{\cosh\zeta-\cos\eta}} Q^1_{n-\frac{1}{2}}(\cosh\zeta)\sin(n\eta)\,. \tag{A.12d}$$

It is possible to give a physical meaning to the functions defined in Equations A.12, which are called toroidal harmonics. In particular, the functions $\psi^{\rm c}_{{\rm int},n}$ ($\psi^{\rm s}_{{\rm int},n}$) give the poloidal flux produced by a toroidal current multipole of order $2n$; the multipole is located at the point $\boldsymbol{r}_0$ and is arranged in such a way that it produces a symmetric (antisymmetric) flux with respect to the $z = z_0$ line. The functions $\psi^{\rm c}_{{\rm ext},n}$ ($\psi^{\rm s}_{{\rm ext},n}$) give the flux produced by a current multipole located at the point $(0, z_0)$ of the poloidal plane. It is therefore clear that the functions (A.12) solve the homogeneous Grad–Shafranov equation everywhere in the poloidal plane except at the points (r_0, z_0) and $(0, z_0)$. The level line of some such functions are shown in Figure A.1 and A.2. Assuming that the singular points (r_0, z_0) and $(0, z_0)$ do not belong to Ω_v it is possible to show that the toroidal harmonics constitute a complete set of solution of Equation A.5 so that each solution of this equation can be written as

$$\psi = \sum_{n=0}^{\infty} \left[a^{\rm c}_n\psi^{\rm c}_{{\rm int},n} + a^{\rm s}_n\psi^{\rm s}_{{\rm int},n} + b^{\rm c}_n\psi^{\rm c}_{{\rm ext},n} + b^{\rm s}_n\psi^{\rm s}_{{\rm ext},n}\right]. \tag{A.13}$$

Note that if the point (r_0, z_0) is located in the plasma region Ω_p the first two terms in the summation (A.13) represent the flux produced by the current distribution in the plasma, while the last two terms represent the flux produced by the current distribution outside the Ω_v region.

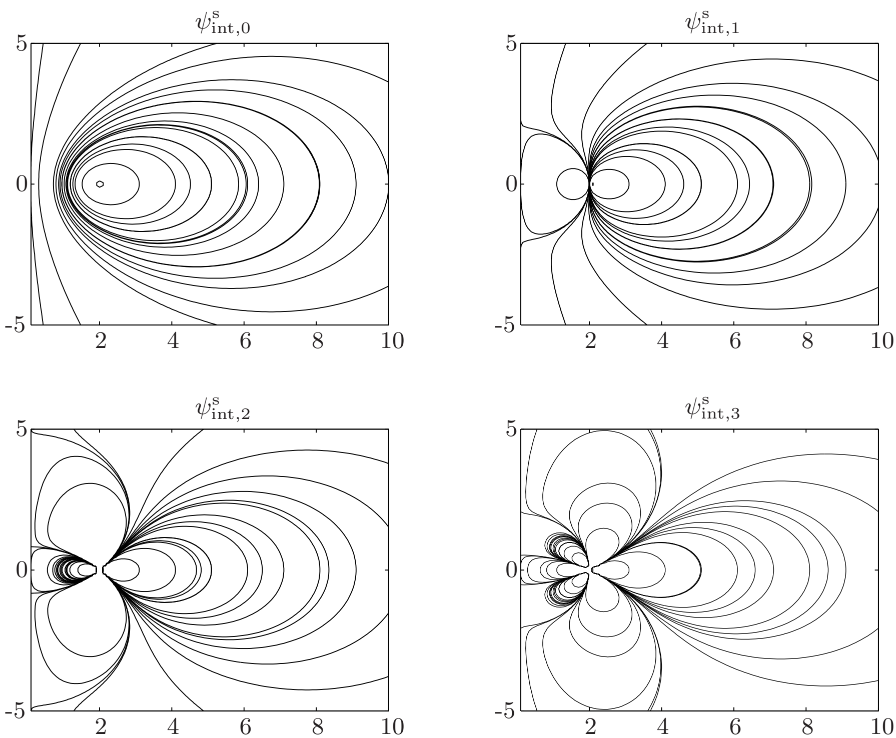

Figure A.1. Level lines of some toroidal harmonics of internal type, with $r_0 = 2\,, z_0 = 0$

A.3 Ill-posedness and Plasma Shape Identification Problem

From a mathematical point of view, estimation of the plasma boundary is the problem of evaluating the function ψ, satisfying Equation 3.14, with certain boundary conditions given on the measurement contour Γ_m (Section 3.4). These boundary conditions can be of Dirichlet type (when flux sensors are used)

$$\psi(\boldsymbol{r}_m) = f_m\,, \quad \boldsymbol{r}_m \in \Gamma_m\,, \tag{A.14}$$

or they can be of Neumann type (when tangential magnetic field sensors are used)

$$-\frac{\partial}{\partial z}\,\psi(\boldsymbol{r}_m)\cos(\theta_m) + \frac{\partial}{\partial r}\,\psi(\boldsymbol{r}_m)\sin(\theta_m) = r_m f_m\,, \tag{A.15}$$

where the angle θ_m individuates, with respect to the radial axis, the direction of the tangent in $\boldsymbol{r}_m$. From Equation 2.8, it is simple to recognize that the right-hand side of Equation A.15 is the normal (with respect to Γ_m) derivative of the ψ function, so that this equation can also be written as

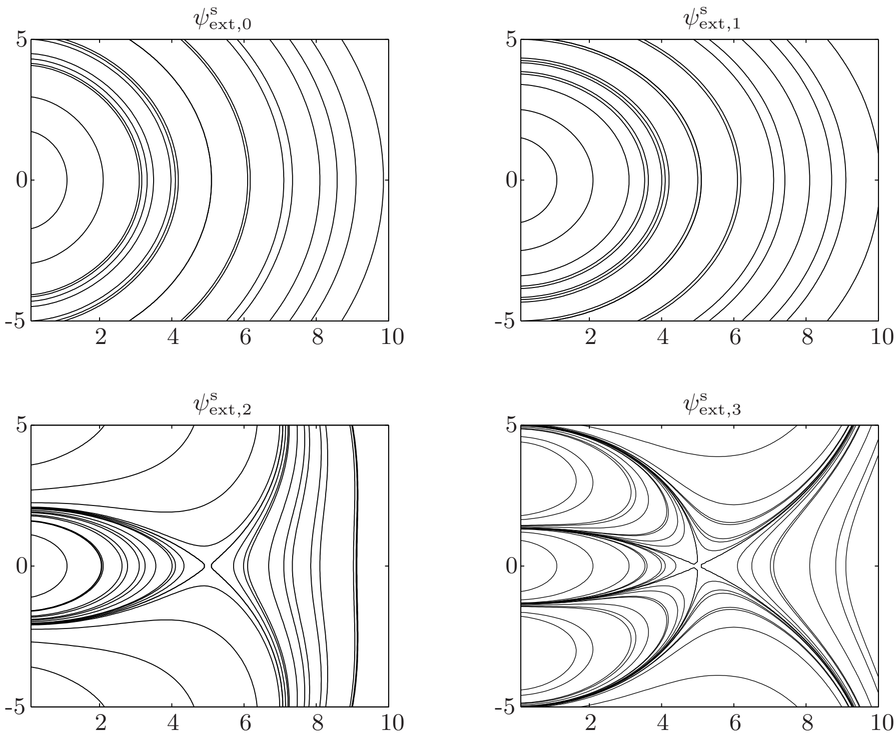

Figure A.2. The level lines of some toroidal harmonics of external type, with $r_0 = 2\,, z_0 = 0$

$$\frac{\partial}{\partial n}\,\psi(\boldsymbol{r}_m) = r_m f_m\,. \tag{A.16}$$

Following the arguments in Chapter 6 of [87], and considering that (3.14) is an elliptic equation, it results that if Γ_m is an open contour, neither Dirichlet nor Neumann conditions are sufficient to univocally determine the function ψ. On the other hand the knowledge of both the values of ψ and of its normal derivative $\partial\psi/\partial n$ (giving rise to a Cauchy problem for Equation 3.14) enables one to determine ψ; unfortunately in this case the solution will not depend continuously on the boundary data (in other words the solution is unstable). But even if Γ_m is a closed contour, the plasma shape identification problem remains a difficult mathematical problem; indeed for an elliptic equation, with boundary conditions given on a closed contour, Dirichlet or Neumann conditions are sufficient to unambiguously determine the solution, if the domain where the equation must be satisfied is simply connected. This is not the case for the problem under consideration, where the domain of validity of Equation 3.14 is a doughnut shaped area surrounding the plasma (Figure 3.5). Taking also into account that in a realistic situation the conditions of Equations A.14 and A.16 are not given at all points of Γ_m but only at a finite number of points (where the magnetic sensors are located), it is possible to conclude that the problem of identification of the plasma shape from the magnetic measurements is, as defined by Hadamard [93], an ill-posed problem. This is not surprising since this is often the case in inverse problems like the one under consideration. The book [55] is a good reference for this kind of problem; concerning the plasma shape identification problem, a viable solution is a procedure based on representing the flux function through a truncated Fourier expansion as detailed in Section 3.4.

B

Units Used in Plasma Physics

In this chapter we summarize the dimensions of the most common quantities involved in the electromagnetic and plasma physics fields (Table B.2). We refer to the International System (SI) nomenclature, which is based on the units reported in Table B.1.

Table B.1. The International System (SI) base units

Physical Quantities	Symbol for Quantities	Name of Unit	Symbol for Unit
Length	l	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Current	I, i	ampere	A
Temperature	T	kelvin	K
Amount of substance		mole	mol
Luminous intensity		candela	cd

Table B.2. Units of some physical quantities encountered in plasma physics. Dimensions are expressed in SI base units where l = length in metres, m = mass in kg, and t = time in seconds.

Physical Quantity	Symbol	Dimensions	SI units	Comment
Capacitance	C	$\frac{t^2q^2}{ml^2}$	farad (F)	
Charge	q	q	coulomb (C)	
Charge density	ρ	$\frac{q}{l^3}$	C/m^3	
Conductance		$\frac{tq^2}{ml^2}$	siemens (S)	
Conductivity	σ	$\frac{tq^2}{ml^3}$	S/m	
Current	I, i	$\frac{q}{t}$	ampere (A)	
Current density	$\boldsymbol{J}$, $\boldsymbol{j}$	$\frac{q}{l^2t}$	A/m^2	
Displacement	$\boldsymbol{D}$	$\frac{q}{l^2}$	C/m^2	$\boldsymbol{D} = \epsilon\boldsymbol{E}$
Electric field	$\boldsymbol{E}$	$\frac{ml}{t^2q}$	V/m	
Energy	U, W	$\frac{ml^2}{t^2}$	joule (J)	electronvolt (eV) is also used; $1\text{eV} \simeq 1.60 \times 10^{-19}\text{J}$
Inductance	L	$\frac{ml^2}{q^2}$	henry (H)	
Magnetic intensity	H	$\frac{q}{lt}$	A/m	$\boldsymbol{B} = \mu\boldsymbol{H}$
Magnetic flux	ϕ, ψ	$\frac{ml^2}{tq}$	weber (Wb)	$\psi = \int_S \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S}$
Magnetic induction	$\boldsymbol{B}$	$\frac{m}{tq}$	tesla (T)	gauss (G) is also used; $1\text{G} = 10^{-4}\text{T}$
Permeability	μ	$\frac{ml}{q^2}$	H/m	In plasma $\mu \simeq \mu_0 = 4\pi \times 10^{-7}\text{Hm}^{-1}$
Permittivity	ϵ	$\frac{t^2q^2}{ml^3}$	F/m	In plasma $\epsilon \simeq \epsilon_0 = 8.8542 \times 10^{-12}\text{Fm}^{-1}$
Potential	V	$\frac{ml^2}{t^2q}$	volt (V)	
Power	P	$\frac{ml^2}{t^3}$	watt (W)	
Resistance	R	$\frac{ml^2}{tq^2}$	ohm (Ω)	
Resistivity	η, ρ	$\frac{ml^3}{tq^2}$	Ωm	
Magnetic vector potential	$\boldsymbol{A}$	$\frac{ml}{tq}$	Wb/m	$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$

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