Further Reading

In this book we have not discussed many applications of functional analysis. This is not because of a lack of such applications, but, conversely, because there are so many that their inclusion would have made this text far too long. Nevertheless, applications to other areas can provide a stimulus for the study of further developments of functional analysis. Mathematical areas in which functional analysis plays a major role include ordinary and partial differential equations, integral equations, complex analysis and numerical analysis. There are also many uses of functional analysis in more applied sciences. Most notable perhaps is quantum theory in physics, where functional analysis provides the very foundation of the subject.

Often, functional analysis provides a general framework and language which allows other subjects to be developed succinctly and effectively. In particular, many applications involve a linear structure of some kind and so lead to vector spaces and linear transformations on these spaces. When these vector spaces are finite-dimensional, standard linear algebra often plays a crucial role, whereas when the spaces are infinite-dimensional functional analysis is likely to be called upon. However, although we have only mentioned the linear theory, there is more to functional analysis than this. In fact, there is an extensive theory of non-linear functional analysis, which has many applications to inherently non-linear fields such as fluid dynamics and elasticity. Indeed, non-linear functional analysis, together with its applications, is a major topic of current research. Although we have not been able to touch on this, much of this theory depends crucially on a sound knowledge of the linear functional analysis that has been developed in this book.

There are a large number of functional analysis books available, many at a very advanced level. For the reader who wishes to explore some of these areas further we now mention some books which could provide a suitable starting point. References [8], [11], [12] and [16] discuss similar material to that in this book, and are written at about the same level (some of these assume different prerequisites to those of this book, for example, a knowledge of topology or Zorn's lemma). In addition, [8] contains several applications of functional analysis, in particular to ordinary and partial differential equations, and to numerical analysis. It also studies several topics in non-linear functional analysis.

For a more advanced treatment of general functional analysis, a reasonably wide-ranging text for which knowledge of the topics in this book would be a prerequisite is [14]. An alternative is [15]. Rather more specialized and advanced textbooks which discuss particular aspects of the theory are as follows:

- for further topics in the theory of Banach spaces see [6];
- for the theory of algebras of operators defined on Hilbert spaces see [9];
- for integral equations see [10];
- for partial differential equations see [13];
- for non-linear functional analysis see [17].

References

[1] T. S. Blyth and E. F. Robertson, *Basic Linear Algebra*, Springer, SUMS, 1998.

[2] J. C. Burkill, *A First Course in Mathematical Analysis*, CUP, Cambridge, 1962.

[3] J. C. Burkill and H. Burkill, *A Second Course in Mathematical Analysis*, CUP, Cambridge, 1970.

[4] M. Capiński and E. Kopp, *Measure, Integral and Probability*, Springer, SUMS, 1999.

[5] C. W. Curtis, *Linear Algebra, an Introductory Approach*, Springer, New York, 1984.

[6] J. Diestel, *Sequences and Series in Banach Spaces*, Springer, New York, 1984.

[7] P. R. Halmos, *Naive Set Theory*, Springer, New York, 1974.

[8] V. Hutson and J. S. Pym, *Applications of Functional Analysis and Operator Theory*, Academic Press, New York, 1980.

[9] R. V. Kadison and J. R. Ringrose, *Fundamentals of the Theory of Operator Algebras, Vol. I*, Academic Press, New York, 1983.

[10] R. Kress, *Linear Integral Equations*, Springer, New York, 1989.

[11] E. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley, New York, 1978.

[12] J. D. Pryce, *Basic Methods of Linear Functional Analysis*, Hutchinson, London, 1973.

[13] M. Renardy and R. C. Rogers, *An Introduction to Partial Differential Equations*, Springer, New York, 1993.

[14] W. Rudin, *Functional Analysis*, McGraw-Hill, New York, 1973.

[15] A. E. Taylor and D. C. Lay, *Introduction to Functional Analysis*, Wiley, New York, 1980.

[16] N. Young, *An Introduction to Hilbert Space*, CUP, Cambridge, 1988.

[17] E. Zeidler, *Nonlinear Functional Analysis and its Applications, Vol. I*, Springer, New York, 1986.

Notation Index

Index