

NOTATIONS

\mathbf{R}	the real numbers
\mathbf{R}^+	the nonnegative real numbers
\mathbf{N}	the natural numbers
\mathbf{R}^n	n -dimensional Euclidean space
E, X	Banach spaces
H	Hilbert space
$E = X \oplus Y$	direct sum decomposition
$\ \cdot\ , \ \cdot\ _X$	norm in the Banach space X or its dual X^*
$\langle \cdot, \cdot \rangle$	duality pairing between the space X and X^*
(\cdot, \cdot)	scalar product in the Hilbert space H
$L^p(\Omega)$	the Lebesgue space of measurable functions, whose p -th power is integrable on Ω
$L^\infty(\Omega)$	the space of measurable functions, which are bounded almost everywhere
$C(X, \mathbf{R})$	the space of continuous functionals from X to \mathbf{R}
$C^k(X, \mathbf{R})$	the space of k -times continuously differentiable functionals
$x_j \rightarrow x$	the strong convergence of sequence $(x_j)_j$ to x in X
$x_j \rightharpoonup x$	the weak convergence of sequence

$$K = \{x \in E : f'(x) = 0\}, \quad K_c = \{x \in K : f(x) = c\}$$

$$f^c = \{x \in E : f(x) \leq c\}, \quad f_c = \{x \in E : f(x) \geq c\}$$

$$f_a^b = \{x \in E : a \leq f(x) \leq b\}$$

$$B_\rho = \{x \in E : \|x\| \leq \rho\}, \quad S_\rho = \{x \in E : \|x\| = \rho\}$$

$$B_\rho(x_0) = \{x \in E : \|x - x_0\| \leq \rho\}, \quad S_\rho(x_0) = \{x \in E : \|x - x_0\| = \rho\}$$

$$d(x, F) = \inf \{\|x - y\|, y \in F\}, \quad F_\delta = \{x \in E : d(x, F) < \delta\}$$
$$\text{supp}\varphi = \overline{\{x : \varphi(x) \neq 0\}}$$

$ \Omega $	the measure of the set Ω
$f \circ g$	the composition of the mappings f and g
$W_p^k(\Omega)$	the Sobolev space of order k
$H^1(\Omega)$	the Sobolev space of order 1
$C_0^\infty(\Omega)$	the space of C^∞ -functions with compact support in Ω
$H_0^1(\Omega)$	the closure of $C_0^\infty(\Omega)$ in $H^1(\Omega)$
$H_{2\pi}^1(\Omega)$	the Sobolev space of 2π -periodic absolutely continuous functions on $[0, 2\pi]$
$\ \cdot\ _{k,p}$	the norm in $W_p^k(\Omega)$
$\ \cdot\ _p$	the norm in $L^p(\Omega)$
$LL(x, \mathbf{R})$	the space of locally Lipschitz functions from X to \mathbf{R}
$C_{loc}^k(\mathbf{R}^n)$	the space of locally k -times continuous functions on \mathbf{R}^n
$L_{loc}^p(\mathbf{R}^n)$	the space of locally p -integrable functions
$H_{loc}^k(\mathbf{R}^n)$	the space of locally k -times weakly differentiable functions on \mathbf{R}^n

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