

Appendix i. \mathbf{O}_L Actions: Product Rule $e_a e_{a+1} = e_{a+5}$.

Given the octonion multiplication,

$$e_a e_{a+1} = e_{a+5},$$

the resulting multiplication table is

$$\begin{bmatrix} 1 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ e_1 & -1 & e_6 & e_4 & -e_3 & e_7 & -e_2 & -e_5 \\ e_2 & -e_6 & -1 & e_7 & e_5 & -e_4 & e_1 & -e_3 \\ e_3 & -e_4 & -e_7 & -1 & e_1 & e_6 & -e_5 & e_2 \\ e_4 & e_3 & -e_5 & -e_1 & -1 & e_2 & e_7 & -e_6 \\ e_5 & -e_7 & e_4 & -e_6 & -e_2 & -1 & e_3 & e_1 \\ e_6 & e_2 & -e_1 & e_5 & -e_7 & -e_3 & -1 & e_4 \\ e_7 & e_5 & e_3 & -e_2 & e_6 & -e_1 & -e_4 & -1 \end{bmatrix}. \quad (\text{i.0})$$

\mathbf{O} is the object space of the 64-dimensional \mathbf{O}_L . In the tables listed below are the actions of the basis elements of \mathbf{O}_L on the basis of \mathbf{O} .

As an example of how to use these tables, consider the bold-faced elements in table i.0. These indicate that

$$e_{L621}[e_3] = -e_3,$$

where the sign is drawn from the slot in the relevant row and column. As another example, from table i.1 we see that

$$e_{L34}[e_5] = -e_7,$$

where e_5 is on the left side, determining the row, e_7 is on the right at the end of the same row, and e_{L34} is on the top, determining the column. A

minus sign is in the appropriate row and column, which is the sign of the action. The i.a table is associated with e_a , where $e_0 = 1$. Note that the adjoint elements along the top of table i.0 are the diagonal elements of \mathbf{O}_L , denoted I_a in 2.46. These were used to resolve the identity of \mathbf{O}_L in section 2.4.

	1	e_{L476}	e_{L517}	e_{L621}	e_{L732}	e_{L143}	e_{L254}	e_{L365}	
1	+	+	+	+	+	+	+	+	1
e_1	+	-	+	+	-	+	-	-	e_1
e_2	+	-	-	+	+	-	+	-	e_2
e_3	+	-	-	-	+	+	-	+	e_3
e_4	+	+	-	-	-	+	+	-	e_4
e_5	+	-	+	-	-	-	+	+	e_5
e_6	+	+	-	+	-	-	-	+	e_6
e_7	+	+	+	-	+	-	-	-	e_7

(i.1)

	e_{L1}	e_{L26}	e_{L34}	e_{L57}	e_{L546}	e_{L235}	e_{L427}	e_{L367}	
1	+	+	+	+	+	+	+	+	e_1
e_1	-	-	-	-	+	+	+	+	1
e_2	+	+	-	-	+	-	-	+	e_6
e_3	+	-	+	-	+	-	+	-	e_4
e_4	-	+	-	+	+	-	+	-	e_3
e_5	+	-	-	+	-	-	+	+	e_7
e_6	-	-	+	+	+	-	-	+	e_2
e_7	-	+	+	-	-	-	+	+	e_5

(i.2)

	e_{L2}	e_{L37}	e_{L45}	e_{L61}	e_{L657}	e_{L346}	e_{L531}	e_{L471}	
1	+	+	+	+	+	+	+	+	e_2
e_1	-	+	+	-	-	-	+	+	e_6
e_2	-	-	-	-	+	+	+	+	1
e_3	+	+	-	-	+	-	-	+	e_7
e_4	+	-	+	-	+	-	+	-	e_5
e_5	-	+	-	+	+	-	+	-	e_4
e_6	+	-	-	+	-	-	+	+	e_1
e_7	-	-	+	+	+	-	-	+	e_3

(i.3)

	e_{L3}	e_{L41}	e_{L56}	e_{L72}	e_{L761}	e_{L457}	e_{L642}	e_{L512}	
1	+	+	+	+	+	+	+	+	e_3
e_1	-	-	+	+	+	-	-	+	e_4
e_2	-	+	+	-	-	-	+	+	e_7
e_3	-	-	-	-	+	+	+	+	1
e_4	+	+	-	-	+	-	-	+	e_1
e_5	+	-	+	-	+	-	+	-	e_6
e_6	-	+	-	+	+	-	+	-	e_5
e_7	+	-	-	+	-	-	+	+	e_2

(i.4)

	e_{L4}	e_{L52}	e_{L67}	e_{L13}	e_{L172}	e_{L561}	e_{L753}	e_{L623}	
1	+	+	+	+	+	+	+	+	e_4
e_1	+	-	-	+	-	-	+	+	e_3
e_2	-	-	+	+	+	-	-	+	e_5
e_3	-	+	+	-	-	-	+	+	e_1
e_4	-	-	-	-	+	+	+	+	1
e_5	+	+	-	-	+	-	-	+	e_2
e_6	+	-	+	-	+	-	+	-	e_7
e_7	-	+	-	+	+	-	+	-	e_6

(i.5)

	e_{L5}	e_{L63}	e_{L71}	e_{L24}	e_{L213}	e_{L672}	e_{L164}	e_{L734}	
1	+	+	+	+	+	+	+	+	e_5
e_1	-	+	-	+	+	-	+	-	e_7
e_2	+	-	-	+	-	-	+	+	e_4
e_3	-	-	+	+	+	-	-	+	e_6
e_4	-	+	+	-	-	-	+	+	e_2
e_5	-	-	-	-	+	+	+	+	1
e_6	+	+	-	-	+	-	-	+	e_3
e_7	+	-	+	-	+	-	+	-	e_1

(i.6)

	e_{L6}	e_{L74}	e_{L12}	e_{L35}	e_{L324}	e_{L713}	e_{L275}	e_{L145}	
1	+	+	+	+	+	+	+	+	e_6
e_1	+	-	+	-	+	-	+	-	e_2
e_2	-	+	-	+	+	-	+	-	e_1
e_3	+	-	-	+	-	-	+	+	e_5
e_4	-	-	+	+	+	-	-	+	e_7
e_5	-	+	+	-	-	-	+	+	e_3
e_6	-	-	-	-	+	+	+	+	1
e_7	+	+	-	-	+	-	-	+	e_4

(i.7)

	e_{L7}	e_{L15}	e_{L23}	e_{L46}	e_{L435}	e_{L124}	e_{L316}	e_{L256}	
1	+	+	+	+	+	+	+	+	e_7
e_1	+	+	-	-	+	-	-	+	e_5
e_2	+	-	+	-	+	-	+	-	e_3
e_3	-	+	-	+	+	-	+	-	e_2
e_4	+	-	-	+	-	-	+	+	e_6
e_5	-	-	+	+	+	-	-	+	e_1
e_6	-	+	+	-	-	-	+	+	e_4
e_7	-	-	-	-	+	+	+	+	1

(i.8)

Appendix ii. O_R Actions: Product Rule $e_a e_{a+1} = e_{a+5}$.

For the sake of completeness, the actions of the basis of O_R on O are also given.

	1	e_{R476}	e_{R517}	e_{R621}	e_{R732}	e_{R143}	e_{R254}	e_{R365}	
1	+	+	+	+	+	+	+	+	1
e_1	+	-	+	+	-	+	-	-	e_1
e_2	+	-	-	+	+	-	+	-	e_2
e_3	+	-	-	-	+	+	-	+	e_3
e_4	+	+	-	-	-	+	+	-	e_4
e_5	+	-	+	-	-	-	+	+	e_5
e_6	+	+	-	+	-	-	-	+	e_6
e_7	+	+	+	-	+	-	-	-	e_7

(ii.0)

	e_{R1}	e_{R26}	e_{R34}	e_{R57}	e_{R564}	e_{R253}	e_{R472}	e_{R376}	
1	+	+	+	+	+	+	+	+	e_1
e_1	-	-	-	-	+	+	+	+	1
e_2	-	-	+	+	-	+	+	-	e_6
e_3	-	+	-	+	-	+	-	+	e_4
e_4	+	-	+	-	-	+	-	+	e_3
e_5	-	+	+	-	+	+	-	-	e_7
e_6	+	+	-	-	-	+	+	-	e_2
e_7	+	-	-	+	+	+	-	-	e_5

(ii.1)

	<i>eR2</i>	<i>eR37</i>	<i>eR45</i>	<i>eR61</i>	<i>eR675</i>	<i>eR364</i>	<i>eR513</i>	<i>eR417</i>	
1	+	+	+	+	+	+	+	+	<i>e2</i>
<i>e1</i>	+	-	-	+	+	+	-	-	<i>e6</i>
<i>e2</i>	-	-	-	-	+	+	+	+	1
<i>e3</i>	-	-	+	+	-	+	+	-	<i>e7</i>
<i>e4</i>	-	+	-	+	-	+	-	+	<i>e5</i>
<i>e5</i>	+	-	+	-	-	+	-	+	<i>e4</i>
<i>e6</i>	-	+	+	-	+	+	-	-	<i>e1</i>
<i>e7</i>	+	+	-	-	-	+	+	-	<i>e3</i>

(ii.2)

	<i>eR3</i>	<i>eR41</i>	<i>eR56</i>	<i>eR72</i>	<i>eR716</i>	<i>eR475</i>	<i>eR624</i>	<i>eR521</i>	
1	+	+	+	+	+	+	+	+	<i>e3</i>
<i>e1</i>	+	+	-	-	-	+	+	-	<i>e4</i>
<i>e2</i>	+	-	-	+	+	+	-	-	<i>e7</i>
<i>e3</i>	-	-	-	-	+	+	+	+	1
<i>e4</i>	-	-	+	+	-	+	+	-	<i>e1</i>
<i>e5</i>	-	+	-	+	-	+	-	+	<i>e6</i>
<i>e6</i>	+	-	+	-	-	+	-	+	<i>e5</i>
<i>e7</i>	-	+	+	-	+	+	-	-	<i>e2</i>

(ii.3)

	<i>eR4</i>	<i>eR52</i>	<i>eR67</i>	<i>eR13</i>	<i>eR127</i>	<i>eR516</i>	<i>eR735</i>	<i>eR632</i>	
1	+	+	+	+	+	+	+	+	<i>e4</i>
<i>e1</i>	-	+	+	-	+	+	-	-	<i>e3</i>
<i>e2</i>	+	+	-	-	-	+	+	-	<i>e5</i>
<i>e3</i>	+	-	-	+	+	+	-	-	<i>e1</i>
<i>e4</i>	-	-	-	-	+	+	+	+	1
<i>e5</i>	-	-	+	+	-	+	+	-	<i>e2</i>
<i>e6</i>	-	+	-	+	-	+	-	+	<i>e7</i>
<i>e7</i>	+	-	+	-	-	+	-	+	<i>e6</i>

(ii.4)

	e_{R5}	e_{R63}	e_{R71}	e_{R24}	e_{R231}	e_{R627}	e_{R146}	e_{R743}	
1	+	+	+	+	+	+	+	+	e_5
e_1	+	-	+	-	-	+	-	+	e_5
e_2	-	+	+	-	+	+	-	-	e_7
e_3	+	+	-	-	-	+	+	-	e_4
e_4	+	-	-	+	+	+	-	-	e_6
e_5	-	-	-	-	+	+	+	+	1
e_6	-	-	+	+	-	+	+	-	e_3
e_7	-	+	-	+	-	+	-	+	e_1

(ii.5)

	e_{R6}	e_{R74}	e_{R12}	e_{R35}	e_{R342}	e_{R731}	e_{R257}	e_{R154}	
1	+	+	+	+	+	+	+	+	e_6
e_1	-	+	-	+	-	+	-	+	e_2
e_2	+	-	+	-	-	+	-	+	e_1
e_3	-	+	+	-	+	+	-	-	e_5
e_4	+	+	-	-	-	+	+	-	e_7
e_5	+	-	-	+	+	+	-	-	e_3
e_6	-	-	-	-	+	+	+	+	1
e_7	-	-	+	+	-	+	+	-	e_4

(ii.6)

	e_{R7}	e_{R15}	e_{R23}	e_{R46}	e_{R453}	e_{R142}	e_{R361}	e_{R265}	
1	+	+	+	+	+	+	+	+	e_7
e_1	-	-	+	+	-	+	+	-	e_5
e_2	-	+	-	+	-	+	-	+	e_3
e_3	+	-	+	-	-	+	-	+	e_2
e_4	-	+	+	-	+	+	-	-	e_6
e_5	+	+	-	-	-	+	+	-	e_1
e_6	+	-	-	+	+	+	-	-	e_4
e_7	-	-	-	-	+	+	+	+	1

(ii.7)

Appendix iii. O_L Actions: Product Rule $e_a e_{a+1} = e_{a+3}$.

Given the octonion multiplication,

$$e_a e_{a+1} = e_{a+3},$$

the resulting multiplication table is

$$\begin{bmatrix} 1 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ e_1 & -1 & e_4 & e_7 & -e_2 & e_6 & -e_5 & -e_3 \\ e_2 & -e_4 & -1 & e_5 & e_1 & -e_3 & e_7 & -e_6 \\ e_3 & -e_7 & -e_5 & -1 & e_6 & e_2 & -e_4 & e_1 \\ e_4 & e_2 & -e_1 & -e_6 & -1 & e_7 & e_3 & -e_5 \\ e_5 & -e_6 & e_3 & -e_2 & -e_7 & -1 & e_1 & e_4 \\ e_6 & e_5 & -e_7 & e_4 & -e_3 & -e_1 & -1 & e_2 \\ e_7 & e_3 & e_6 & -e_1 & e_5 & -e_4 & -e_2 & -1 \end{bmatrix}. \quad (\text{iii.0})$$

The resulting adjoint actions are

	1	e_{L325}	e_{L436}	e_{L547}	e_{L651}	e_{L762}	e_{L173}	e_{L214}	
1	+	+	+	+	+	+	+	+	1
e_1	+	-	-	-	+	-	+	+	e_1
e_2	+	+	-	-	-	+	-	+	e_2
e_3	+	+	+	-	-	-	+	-	e_3
e_4	+	-	+	+	-	-	-	+	e_4
e_5	+	+	-	+	+	-	-	-	e_5
e_6	+	-	+	-	+	+	-	-	e_6
e_7	+	-	-	+	-	+	+	-	e_7

(iii.1)

	e_{L1}	e_{L37}	e_{L56}	e_{L24}	e_{L453}	e_{L764}	e_{L572}	e_{L632}	
1	+	+	+	+	+	+	+	+	e_1
e_1	-	-	-	-	+	+	+	+	1
e_2	+	-	-	+	+	+	-	-	e_4
e_3	+	+	-	-	-	+	+	-	e_7
e_4	-	+	+	-	+	+	-	-	e_2
e_5	+	-	+	-	-	+	-	+	e_6
e_6	-	+	-	+	-	+	-	+	e_5
e_7	-	-	+	+	-	+	+	-	e_3

(iii.2)

	e_{L2}	e_{L41}	e_{L67}	e_{L35}	e_{L564}	e_{L175}	e_{L613}	e_{L743}	
1	+	+	+	+	+	+	+	+	e_2
e_1	-	-	+	+	-	+	+	-	e_4
e_2	-	-	-	-	+	+	+	+	1
e_3	+	-	-	+	+	+	-	-	e_5
e_4	+	+	-	-	-	+	+	-	e_1
e_5	-	+	+	-	+	+	-	-	e_3
e_6	+	-	+	-	-	+	-	+	e_7
e_7	-	+	-	+	-	+	-	+	e_6

(iii.3)

	e_{L3}	e_{L52}	e_{L71}	e_{L46}	e_{L675}	e_{L216}	e_{L724}	e_{L154}	
1	+	+	+	+	+	+	+	+	e_3
e_1	-	+	-	+	-	+	-	+	e_7
e_2	-	-	+	+	-	+	+	-	e_5
e_3	-	-	-	-	+	+	+	+	1
e_4	+	-	-	+	+	+	-	-	e_6
e_5	+	+	-	-	-	+	+	-	e_2
e_6	-	+	+	-	+	+	-	-	e_4
e_7	+	-	+	-	-	+	-	+	e_1

(iii.4)

	e_{L4}	e_{L63}	e_{L12}	e_{L57}	e_{L716}	e_{L327}	e_{L135}	e_{L265}	
1	+	+	+	+	+	+	+	+	e_4
e_1	+	-	+	-	-	+	-	+	e_2
e_2	-	+	-	+	-	+	-	+	e_1
e_3	-	-	+	+	-	+	+	-	e_6
e_4	-	-	-	-	+	+	+	+	1
e_5	+	-	-	+	+	+	-	-	e_7
e_6	+	+	-	-	-	+	+	-	e_3
e_7	-	+	+	-	+	+	-	-	e_5

(iii.5)

	e_{L5}	e_{L74}	e_{L23}	e_{L61}	e_{L127}	e_{L431}	e_{L246}	e_{L376}	
1	+	+	+	+	+	+	+	+	e_5
e_1	-	+	+	-	+	+	-	-	e_6
e_2	+	-	+	-	-	+	-	+	e_3
e_3	-	+	-	+	-	+	-	+	e_2
e_4	-	-	+	+	-	+	+	-	e_7
e_5	-	-	-	-	+	+	+	+	1
e_6	+	-	-	+	+	+	-	-	e_1
e_7	+	+	-	-	-	+	+	-	e_4

(iii.6)

	e_{L6}	e_{L15}	e_{L34}	e_{L72}	e_{L231}	e_{L542}	e_{L357}	e_{L417}	
1	+	+	+	+	+	+	+	+	e_6
e_1	+	+	-	-	-	+	+	-	e_5
e_2	-	+	+	-	+	+	-	-	e_7
e_3	+	-	+	-	-	+	-	+	e_4
e_4	-	+	-	+	-	+	-	+	e_3
e_5	-	-	+	+	-	+	+	-	e_1
e_6	-	-	-	-	+	+	+	+	1
e_7	+	-	-	+	+	+	-	-	e_2

(iii.7)

	<i>e_{L7}</i>	<i>e_{L26}</i>	<i>e_{L45}</i>	<i>e_{L13}</i>	<i>e_{L342}</i>	<i>e_{L653}</i>	<i>e_{L461}</i>	<i>e_{L521}</i>	
1	+	+	+	+	+	+	+	+	<i>e₇</i>
<i>e₁</i>	+	-	-	+	+	+	-	-	<i>e₃</i>
<i>e₂</i>	+	+	-	-	-	+	+	-	<i>e₆</i>
<i>e₃</i>	-	+	+	-	+	+	-	-	<i>e₁</i>
<i>e₄</i>	+	-	+	-	-	+	-	+	<i>e₅</i>
<i>e₅</i>	-	+	-	+	-	+	-	+	<i>e₄</i>
<i>e₆</i>	-	-	+	+	-	+	+	-	<i>e₂</i>
<i>e₇</i>	-	-	-	-	+	+	+	+	1

(iii.8)

Appendix iv. O_R Actions: Product Rule $e_a e_{a+1} = e_{a+3}$.

Finally

	1	e_{R325}	e_{R436}	e_{R547}	e_{R651}	e_{R762}	e_{R173}	e_{R214}	
1	+	+	+	+	+	+	+	+	1
e_1	+	-	-	-	+	-	+	+	e_1
e_2	+	+	-	-	-	+	-	+	e_2
e_3	+	+	+	-	-	-	+	-	e_3
e_4	+	-	+	+	-	-	-	+	e_4
e_5	+	+	-	+	+	-	-	-	e_5
e_6	+	-	+	-	+	+	-	-	e_6
e_7	+	-	-	+	-	+	+	-	e_7

(iv.0)

	e_{R1}	e_{R37}	e_{R56}	e_{R24}	e_{R453}	e_{R764}	e_{R572}	e_{R632}	
1	+	+	+	+	+	+	+	+	e_1
e_1	-	-	-	-	+	+	+	+	1
e_2	-	+	+	-	-	-	+	+	e_4
e_3	-	-	+	+	+	-	-	+	e_7
e_4	+	-	-	+	-	-	+	+	e_2
e_5	-	+	-	+	+	-	+	-	e_6
e_6	+	-	+	-	+	-	+	-	e_5
e_7	+	+	-	-	+	-	-	+	e_3

(iv.1)

	e_{R2}	e_{R41}	e_{R67}	e_{R35}	e_{R564}	e_{R175}	e_{R613}	e_{R743}	
1	+	+	+	+	+	+	+	+	e_2
e_1	+	+	-	-	+	-	-	+	e_4
e_2	-	-	-	-	+	+	+	+	1
e_3	-	+	+	-	-	-	+	+	e_5
e_4	-	-	+	+	+	-	-	+	e_1
e_5	+	-	-	+	-	-	+	+	e_3
e_6	-	+	-	+	+	-	+	-	e_7
e_7	+	-	+	-	+	-	+	-	e_6

(iv.2)

	e_{R3}	e_{R52}	e_{R71}	e_{R46}	e_{R675}	e_{R216}	e_{R724}	e_{R154}	
1	+	+	+	+	+	+	+	+	e_3
e_1	+	-	+	-	+	-	+	-	e_7
e_2	+	+	-	-	+	-	-	+	e_5
e_3	-	-	-	-	+	+	+	+	1
e_4	-	+	+	-	-	-	+	+	e_6
e_5	-	-	+	+	+	-	-	+	e_2
e_6	+	-	-	+	-	-	+	+	e_4
e_7	-	+	-	+	+	-	+	-	e_1

(iv.3)

	e_{R4}	e_{R63}	e_{R12}	e_{R57}	e_{R716}	e_{R327}	e_{R135}	e_{R265}	
1	+	+	+	+	+	+	+	+	e_4
e_1	-	+	-	+	+	-	+	-	e_2
e_2	+	-	+	-	+	-	+	-	e_1
e_3	+	+	-	-	+	-	-	+	e_6
e_4	-	-	-	-	+	+	+	+	1
e_5	-	+	+	-	-	-	+	+	e_7
e_6	-	-	+	+	+	-	-	+	e_3
e_7	+	-	-	+	-	-	+	+	e_5

(iv.4)

	e_{R5}	e_{R74}	e_{R23}	e_{R61}	e_{R127}	e_{R431}	e_{R246}	e_{R376}	
1	+	+	+	+	+	+	+	+	e_5
e_1	+	-	-	+	-	-	+	+	e_6
e_2	-	+	-	+	+	-	+	-	e_3
e_3	+	-	+	-	+	-	+	-	e_2
e_4	+	+	-	-	+	-	-	+	e_7
e_5	-	-	-	-	+	+	+	+	1
e_6	-	+	+	-	-	-	+	+	e_1
e_7	-	-	+	+	+	-	-	+	e_4

(iv.5)

	e_{R6}	e_{R15}	e_{R34}	e_{R72}	e_{R231}	e_{R542}	e_{R357}	e_{R417}	
1	+	+	+	+	+	+	+	+	e_6
e_1	-	-	+	+	+	-	-	+	e_5
e_2	+	-	-	+	-	-	+	+	e_7
e_3	-	+	-	+	+	-	+	-	e_4
e_4	+	-	+	-	+	-	+	-	e_3
e_5	+	+	-	-	+	-	-	+	e_1
e_6	-	-	-	-	+	+	+	+	1
e_7	-	+	+	-	-	-	+	+	e_2

(iv.6)

	e_{R7}	e_{R26}	e_{R45}	e_{R13}	e_{R342}	e_{R653}	e_{R461}	e_{R521}	
1	+	+	+	+	+	+	+	+	e_7
e_1	-	+	+	-	-	-	+	+	e_3
e_2	-	-	+	+	+	-	-	+	e_6
e_3	+	-	-	+	-	-	+	+	e_1
e_4	-	+	-	+	+	-	+	-	e_5
e_5	+	-	+	-	+	-	+	-	e_4
e_6	+	+	-	-	+	-	-	+	e_2
e_7	-	-	-	-	+	+	+	+	1

(iv.7)

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