

# Index

- abuse of notation 20
- addition
  - of linear transformations 67
  - of matrices 108
  - of position vectors 2
  - of vectors 13
- adjoint of  $T$  248
- affine subspace 162
- algebraic multiplicity 206
- angle between  $\mathbf{A}$  and  $\mathbf{B}$  216, 224
- associated homogeneous system 165
- associative law 13
  - general 18
- axiomatic method 13
- axioms of a vector space 13–14
- basic rules
  - of matrix operations 112
  - of vector algebra 2–3
- basis
  - ordered 47, 93
  - orthonormal 231
  - standard 82, 101, 126
  - usual 126
- Bessel's inequality 242
- Cartesian  $k$ -space
  - (real) 16
  - complex 19
- characteristic function of  $s$  35
- characteristic polynomial of  $T$  199
- cofactor 196
- cofactor matrix 196
- column of a matrix 62, 100, 101, 107
- column space of  $\mathbf{A}$  160
- column vectors of  $\mathbf{A}$  160
- commutative law of vector addition 13
  - generalized 18
- component
  - of  $\mathbf{A}$  along  $\mathbf{A}_i$  227
  - of a matrix 100
  - of a position vector 5
  - of a vector 16
    - relative to the ordered basis 47, 227
- coordinate transformation 13
- coordinates
  - of a point 4
  - of a vector (relative to the ordered basis) 47
- Cramer's rule 214
- criteria for Eigenvalues 199
- cyclic transformation 153
- cyclic vector for  $T$  153
- determinant 174, 193
  - properties of 195
- diagonal entries 114
- diagonalizable 186
- diagonalize or diagonalizing  $T$  186, 202
- differentiation operator 71, 155
- dimension of a vector space 46
- displacement 1
- distance 223
- dot product 217
- echelon form 166

## Index

- eigenspace of  $T$  190
- eigenvalue 183
  - of  $T$  185
- eigenvector 183
  - of  $T$  associated to  $e$  185
- endomorphisms 183
- equation of the line through  $P, Q$  5
  - of the plane 9
- Euclidean space 16
  
- finite dimensional vector space 40
- force 1
- Fourier coefficients 227
- Fundamental theorem of algebra 208
  
- general solution 167
- geometric multiplicity 206
- Gram–Schmidt orthonormalization process 230
  
- homogeneous equation, linear 28
- homogeneous quadratic function 265
  
- identity matrix 113
  - operator 78
  - transformation 78
- index of nilpotence 115
- infinite dimensional vector space 45
- initial point 1
- injective 85
- inner product 217, 221, 244
  - trace 220
- inner product space
  - (real) 217
  - complex 244
- intersection (of sets) 19
- involution 118
- isometric 235
  - embedding 234
  - isomorphism 235
- isometry 235
- isomorphic 73
- isomorphism 73
  - natural 82
  
- Jordan canonical form 191
  
- $k$ -tuple 16
  
- Law of cosines 242
- length function 222
- length of  $A$  221, 245
- linear
  - algebra 27
  - combination 21
  - dependence 33
  - extension 82
  - homogeneous equation 28
  - independence 33
  - relation 33
  - span 21, 22
  - subspace 20
  - transformation 27
- linear equations
  - coefficients of 157
  - homogeneous system of 162
  - solution of 157
  - system of 157, 160
- linear transformation
  - composition of 67, 102
  - definition of 63
  - homogeneity of 75
  - image of a 69
  - kernel of a 69
  - rank of a 183
  - scalar multiplication of 68
  - self-adjoint 250
  - symmetric 252
- linear transformations
  - sum of 67
  - vector space of 68
- linearly dependent 33
  - on a set of vectors 36
  - set of vectors 33
- linearly independent 33
  - set of vectors 33
  
- Matrix**
  - augmented 168
  - coefficient 159
  - cofactor 196
  - definition of 100, 108
  - diagonal 114
  - echelon 166
  - equality of 108
  - idempotent 115
  - inverse 117
  - invertible 117
  - involutory 118
  - $i$ th row of 100, 107
  - $j$ th column of 101, 107
  - lower triangular 115
  - minor 193
  - $m \times n$ - 107
  - multiplication of 103
  - nilpotent 115
  - nonsingular 117
  - scalar 113

- skew symmetric 119, 123
- square (of size  $n$ ) 111
- strictly triangular 115
- sum of 108
- symmetric 119, 123, 246
- the identity 113
- transpose of a 123, 195
- triangular 115
- zero 115
- Matrix
  - equation of a linear system 159
  - of constants of a linear system 159
  - of ST 102
  - product 109, 110
- model 15
- multiplication by scalar numbers 13, 14, 108
- multiplicity
  - algebraic 206
  - geometric 206
- nilpotent 72
  - matrix 115
  - of index  $k$  151
  - transformation 151
- normalize 231
- number 13
- orthogonal
  - complement 237
  - projection 149
  - set of vectors 225
- orthonormal 225
  - basis 231
  - set 231
- parallel (translation) of  $\mathcal{U}$  162
- Parseval's inequality 242
- pivot operation 174
- point
  - initial 1
  - terminal 1
- principal axes 265
- principal axis theorem 261, 265
- product
  - dot 217
  - inner 217
  - scalar 217
  - scalar on  $\mathcal{P}_k(\mathbb{R})$  219
  - standard scalar on  $\mathbb{R}^n$  218
  - standard scalar on  $\mathcal{V}$  218
  - trace scalar 220
- projection 15, 124, 146
  - definition of 148
  - orthogonal 149
  - self-adjoint 149
  - skew 149
- properties of
  - determinant 195
  - inner product 217
  - trace 244
- quadratic form 246, 264, 265, 271
- quadratic function 264
  - homogeneous 265
- rank of the linear transformation 183
- real vector space of polynomials 27
- real vector space of degrees less than or equal to  $n$  22
- reduced echelon form 166
- reduction to the echelon form 166
- reflection 124
- representing a linear transformation by matrix 125, 146
- restriction of  $T$  to  $\mathcal{W}$  273
- rotation 64, 243
- row of a matrix 62, 100, 107
- scalar 13, 14
- scalar multiplication 13, 14, 108
  - of position vectors 2, 3
- scalar product 217
  - on  $\mathcal{P}_k(\mathbb{R})$  219
  - standard scalar product on  $\mathbb{R}^n$  218
  - standard scalar product on  $\mathcal{V}$  218
  - trace 220
- self-adjoint 250
  - projection 149
- shift 84
- Schwarz inequality 222
- simplex method 173, 174
- singular endomorphism 191
- size of a matrix 107
- solution space 162, 165, 260
- space 13
  - column 160
  - inner product 216, 221
  - linear 13
  - vector 13
- spectral theorem 254
- standard basis 82, 101, 106
- subspace 20
  - affine 162
  - linear 20
- sum
  - of linear spaces 23
  - of matrices 108

## Index

- of position vectors 2, 5
- of vectors 13
- superdiagonal (of a matrix) 127
- surjective 106
- symmetric 119, 123, 246, 252
- system of linear equations 157, 160
  
- terminal point 1
- trace 220
- transformation
  - cyclic 153
  - identity 78
  - injective 85
  - nilpotent 72, 151
  - zero 153
- transpose 123, 195
- triangle inequality 223
- triangular matrix 115
  
- union (of sets) 19
- unit vector 232, 241
  
- vector
  - bound 1
  - column (of a matrix) 160
  - equality of 1
  - free 1
  - position 2
  - same 1
  - unit 232, 241
  - zero 13
    - addition 13, 14
    - algebra 2
    - from  $\mathbf{P}$  to  $\mathbf{Q}$  1
    - in the plane and space 1-10
    - quantities 1
  - vector equation
    - on the line  $L$  4
    - of the plane  $\pi$  8
  - vector space 13
    - complex 19, 244
    - real 19
  - velocity 1
  
- weighted shift 117
  
- zero
  - matrix 115
  - transformation 68
  - vector 13

# List of Notations

$a, b, c, \dots$	as numbers	15
$A, B, \dots, P, Q, \dots, O, \dots$	as points	1
	as vectors	15
$O$	the origin	1
$\vec{P}, \vec{E}_i$		1, 10
$\overline{PQ}$		1
$\in$	“is an element of”	19
$\subset$	“is contained” or “is a subset of”	19
$(a, b)$	open interval	28
$[a, b]$	closed interval	28
$S \cap T$	set theoretic intersection	19
$S \cup T$	set theoretic union	19
$\Rightarrow$	“implies”	17
$ S $		46
$ A $	length of vector $A$	
$\langle , \rangle$	“inner product”	217
$d(A, B)$	distance	223
$A \cdot B$	dot/scalar product	217
$A^{-1}$	inverse matrix of $A$	117
$A^t$	transpose $d$ matrix of $A$	123
$A^{\text{cof}}$	cofactor matrix of $A$	197
$A^*$	adjoint of $A$	248
$A + \mathcal{U}$	parallel (translation) of $\mathcal{U}$	162
$A_{ij}$	cofactor	196
$a_{ij}$		62
$(a_{ij})$		100
$\mathbb{C}$	the set of all complex numbers	19, 27
$\mathbb{C}^k$	the complex Cartesian space	19
$\mathcal{E}(a, b)$		28
$\det C$	the determinant of the matrix $C$	174
$D$	differentiation operator	71

## List of Notations

$\Delta(t)$	characteristic polynomial	199
$\dim$	dimension of	46
$\mathbf{E}_i$		82
$E_{rs}$		108
$\mathcal{F}(S)$	the set of all real valued functions defined on $S$	30
$\mathcal{F}(S, T)$		31
$\mathcal{F}_c(S)$		31
$g _T$		77
$I$	the identity transformation	78
$I$	the identity matrix	114
$Im T$	The image of $T$	69
$\ker T$	the kernel of $T$	69
$\mathcal{L}(\ )$	the linear span of	21, 22
$l_{k(x)}$		97
$L$	integration operator	71
$M_{k(x)}$	multiplication operator by $k(x)$	97
$\mathcal{M}_{mn}$	The set of all $m \times n$ matrices	108
$\mathcal{P}(\mathbb{R})$	the set of all polynomials	27
$\mathcal{P}_n(\mathbb{R})$	the set of all polynomials of degree $< n$	26
$\mathbb{R}$	the real numbers	16
$\mathbb{R}^k$	the real Cartesian space	16
$\mathbb{R}_T$		78
$\gamma$	the solution space	162
$S$	the shift operator	84
$T_A$		247
$tr(A)$	trace of $A$	220
$T(U)$		66
$T_\varphi$		83
$T _y$		273
$\mathcal{Y}_e$		190
$\mathcal{Y}_{xy}$	the $x$ - $y$ plane	69
$\mathcal{W}^\perp$	the orthogonal complement of	237
$\mathcal{G}_s$	the characteristic function of $s$	35
$\emptyset$	the empty set	91
$\mathcal{V}, \mathcal{W}, \mathcal{U}, \mathcal{L}, \mathcal{T}$	vector spaces	20
$T, S, \dots$	Linear transformations	63