A popular method to extract cyclostationary features is to implement the spectral correlation function (SCF) [65]. In cyclostationary feature detectors, the incoming signal is modeled as a cyclostationary random process with multiple periodicities, as compared to the traditional wide sense stationary model for energy detection where the cyclostationary information is discarded. Then, the autocorrelation function for a zero-mean process $x(t)$ is defined as

$$R_X(t + \frac{\tau}{2}, t - \frac{\tau}{2}) = E[x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2})]$$

which, in the ‘$t$’ domain, exhibits the multiple periodicities of the incoming signal. The Fourier transform of the autocorrelation over ‘$t$’ as expressed in

$$R_\alpha^X(\tau) = \lim_{Z \to \infty} \frac{1}{Z} \int_{-\frac{Z}{2}}^{\frac{Z}{2}} R_X(t + \frac{\tau}{2}, t - \frac{\tau}{2}) e^{-j2\pi \alpha t} dt$$

is called the cyclic autocorrelation in the cycle frequency ($\alpha$) domain, and is expected to capture these periodicities. For a cycloergodic process, the definition simplifies to

$$R_\alpha^X(\tau) = \lim_{Z \to \infty} \frac{1}{Z} \int_{-\frac{Z}{2}}^{\frac{Z}{2}} x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) e^{-j2\pi \alpha t} dt$$

Then, using the cyclic Wiener-Khinchin relation on $R_\alpha^X(\tau)$, the spectral correlation function (SCF) can then be defined as

$$S_\alpha^X(f) = \int_{-\infty}^{\infty} R_\alpha^X(\tau) e^{-j2\pi \alpha \tau} d\tau$$

Here, we note that the cyclic autocorrelation function, $R_\alpha^X(\tau)$, and the SCF, $S_\alpha^X(f)$, reduce to the conventional definitions of autocorrelation function $R_X(\tau)$, and the power spectral density, $S_X(f)$, for the case when $\alpha = 0$.

Again, an alternative, and more convenient equivalent definition of the SCF can be expressed in terms of the short time Fourier transform (STFT) of the received signal $x(t)$. The STFT of a signal is just the Fourier transform at the particular instance ‘$t$’, defined as

$$X_T(t, f) = \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(u) e^{-j2\pi fu} du$$
The SCF can then be defined in terms of the STFT as

\[
S^\alpha_X(f) = \lim_{T \to \infty} \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \int_{\frac{\alpha}{2}}^{\frac{\alpha}{2}} X_T(t, f + \frac{\alpha}{2}) X^*_T(t, f - \frac{\alpha}{2}) \, dt
\]

thus better justifying its nomenclature. As is evident, this second definition lends itself to an easier execution in hardware and has been used in hardware implementations of cyclostationary feature detectors [113]. For discrete time implementation, the discrete and finite time equivalent definition of the SCF is derived as

\[
\hat{S}^\alpha_X(f) = \frac{1}{N} \frac{1}{T} \sum_{n=0}^{N} X_{T_{DFT}}(n, f + \frac{\alpha}{2}) X_{T_{DFT}}^*(n, f - \frac{\alpha}{2})
\]

where \(X_{T_{DFT}}(n, f)\) represents the \(N\) point DFT around sample \(n\).

The superiority of cyclostationary feature detectors over energy detection can be visualized in Fig. A.1, where the power spectral density (PSD) and SCF of a BPSK modulated signal are simulated in MATLAB®. The PSD, as would be calculated by an energy detector is shown in Fig. A.1. The energy detection technique is based on comparing the signal energy with a threshold that is dependent on an estimation of the noise power. As evident in the Fig. A.1, any estimation error due to interference or changing noise variance could easily change the absolute energy detected and cause errors, especially in the case of weak signals. Also, the PSD cannot discriminate between signal energy and noise/interferer energy. However, when the SCF is calculated as in Fig. A.1, the additive white Gaussian noise appears only along the main diagonal, which represents the PSD as estimated by energy detectors. Since the cross-correlation of the noise tends to zero for long observation times, the BPSK signal is clearly visible along the \(\alpha\) axis (second diagonal). Moreover, the energy along the \(\alpha\) axis is independent of the noise variance.
Appendix A: SCF

Fig. A.1 (a) PSD of a BPSK signal with $-60$ dB SNR and two interferers, and (b) SCF of the same signal, where the diagonal represents the PSD along the $f$ axis as shown in (a).
References


References


