Glossary

$\mathscr{B}(E)$ The set of measurable functions on E.

$\mathscr{B}_b(E)$ The set of bounded measurable functions on E.

$\mathscr{B}_b^+(E)$ The set of nonnegative bounded measurable functions on E.

$C(E)$ The set of continuous functions on E.

$C_b(E)$ The set of bounded continuous functions on E.

$C_b^+(E)$ The set of bounded nonnegative continuous functions on E.

$C_0(E)$ The set of continuous functions with compact support on E.

$C^p(E)$ The set of functions on E with continuous derivatives up to order p.

$C_b^p(E)$ The set of functions on E with bounded continuous derivatives up to order p.

$C_0^p(E)$ The set of functions on E with compact support and continuous derivatives up to order p.

$\mathscr{C}(E) = C([-r_0, 0]; E)$ For a metric space E.

D The Malliavin gradient operator with respect to the underlying Brownian motion.

$\mathbb{E}$ The expectation with respect to the underlying probability measure $\mathbb{P}$.

$\mathbb{E}_{\mathbb{Q}}$ The expectation with respect to the (changed) probability measure $\mathbb{Q}$.

$\mathbb{H}$ Hilbert space.

$\mathbb{H}^1$ The Cameron–Martin space over $\mathbb{H}$.

$\mathbb{H}_0^{1,p}(D)$ The Sobolev space on an open domain $D \subset \mathbb{R}^d$, which is the closure of $C_0^\infty(D)$ under the norm $\|f\|_{1,p} := \|f\|_{L^p(\mathbf{m})} + \|\nabla f\|_{L^p(\mathbf{m})}$, where $\mathbf{m}$ is the Lebesgue measure on D.

$\mathscr{L}(\mathbb{H})$ The set of densely defined linear operators on $\mathbb{H}$.

$\mathscr{L}_b(\mathbb{H})$ The set of bounded linear operators on $\mathbb{H}$.

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$\mathscr{L}_{HS}(\mathbb{H})$ The set of Hilbert–Schmidt operators on $\mathbb{H}$.

$\mathscr{L}_S(\mathbb{H})$ The set of densely defined closed linear operators on $\mathbb{H}$.

X_t The segment process associated to an SDDE with time delay r_0, i.e., $X_t(s) := X(t+s)$, $s \in [-r_0, 0]$.

$|\cdot|$ The norm in the underlying Hilbert space $\mathbb{H}$ or the Euclidean space $\mathbb{R}^d$.

$\|\cdot\|$ The operator norm for linear operators.

$\|\cdot\|_{HS}$ The Hilbert–Schmidt norm of linear operators.

$\|\cdot\|_\sigma$ The intrinsic norm induced by a linear operator σ, i.e., $\|x\|_\sigma = \inf\{|y| : \sigma y = x\}$ and $\inf\emptyset = \infty$ by convention.

$\|\cdot\|_\infty$ The uniform norm, i.e., $\|f\|_\infty := \sup|f|$ for a function f.

$\langle\cdot,\cdot\rangle$ The inner product in the underlying Hilbert space $\mathbb{H}$.

$\langle\cdot,\cdot\rangle_2$ The inner product in $L^2(\mathbf{m})$ for a reference measure $\mathbf{m}$.

${}_{\mathbb{V}^*}\langle\cdot,\cdot\rangle_{\mathbb{V}}$ The dualization between a Banach space $\mathbb{V}$ and its duality $\mathbb{V}^*$ with respect to a Hilbert space into which $\mathbb{V}$ is continuously and densely embedded.

∇ The gradient operator with respect to the underlying space variable.

$a \vee b$ $\max\{a,b\}$.

$a \wedge b$ $\min\{a,b\}$.

PDE Partial differential equation.

SDE Stochastic differential equation.

SDDE Stochastic delay differential equation.

SDPDE Stochastic delay partial differential equation.

SPDE Stochastic partial differential equation.

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