

References

1. Alberti G, Serra Cassano F (1994) Non-occurrence of gap for one-dimensional autonomous functionals, Calculus of variations, homogenization and continuum mechanics. World Scientific Publishing, River Edge, NJ, pp 1–17
2. Angell TS (1976) Existence theorems for hereditary Lagrange and Mayer problems of optimal control. *SIAM J Contr Optim* 14:1–18
3. Arkin VI, Evstigneev IV (1987) Stochastic models of control and economic dynamics. Academic Press, London
4. Aseev SM, Kryazhimskiy AV (2004) The Pontryagin Maximum principle and transversality conditions for a class of optimal control problems with infinite time horizons. *SIAM J Contr Optim* 43:1094–1119
5. Aseev SM, Veliov VM (2012) Maximum principle for infinite-horizon optimal control problems with dominating discount. *Dynam Contin Discrete Impuls Syst Ser B* 19:43–63
6. Aubin JP, Ekeland I (1984) Applied nonlinear analysis. Wiley Interscience, New York
7. Aubry S, Le Daeron PY (1983) The discrete Frenkel-Kontorova model and its extensions I. *Phys D* 8:381–422
8. Ball JM, Mizel VJ (1984) Singular minimizers for regular one-dimensional problems in the calculus of variations. *Bull Amer Math Soc* 11:143–146
9. Ball JM, Mizel VJ (1985) One-dimensional variational problems whose minimizers do not satisfy the Euler-Lagrange equation. *Arch Ration Mech Anal* 90:325–388
10. Baumeister J, Leitao A, Silva GN (2007) On the value function for nonautonomous optimal control problem with infinite horizon. *Syst Contr Lett* 56:188–196
11. Benyamini Y, Lindenstrauss J (2000) Geometric nonlinear functional analysis. American Mathematical Society, Providence, RI
12. Berkovitz LD (1974) Optimal control theory. Springer, New York
13. Berkovitz LD (1974) Lower semicontinuity of integral functionals. *Trans Amer Math Soc* 192:51–57
14. Blot J, Cartigny P (2000) Optimality in infinite-horizon variational problems under sign conditions. *J Optim Theory Appl* 106:411–419
15. Blot J, Hayek N (2000) Sufficient conditions for infinite-horizon calculus of variations problems. *ESAIM Contr Optim Calc Var* 5:279–292
16. Brezis H (1973) Opérateurs maximaux monotones. North Holland, Amsterdam
17. Carlson DA, Haurie A, Leizarowitz A (1991) Infinite horizon optimal control. Springer, Berlin
18. Cartigny P, Michel P (2003) On a sufficient transversality condition for infinite horizon optimal control problems. *Automatica J IFAC* 39:1007–1010
19. Cellina A, Colombo G (1990) On a classical problem of the calculus of variations without convexity assumptions. *Ann Inst H Poincaré Anal Non linéaire* 7:97–106

20. Cellina A, Zagatti S (1995) An existence result in a problem of the vectorial case of calculus of variations. *SIAM J Contr Optim* 33:960–970
21. Cesari L (1983) *Optimization - theory and applications*. Springer, New York
22. Cinquni S (1936) Sopra l'esistenza della soluzione nei problemi di calcolo delle variazioni di ordine n. *Ann Scuola Norm Sup Pisa* 5:169–190
23. Clarke FH (1983) *Optimization and nonsmooth analysis*. Wiley Interscience, New York
24. Clarke FH, Loewen PD (1989) An intermediate existence theory in the calculus of variations. *Ann Scuola Norm Sup Pisa* 16:487–526
25. Clarke FH, Vinter RB (1985) Regularity properties of solutions to the basic problem in the calculus of variations. *Trans Amer Math Soc* 289:73–98
26. Clarke FH, Vinter RB (1986) Regularity of solutions to variational problems with polynomial Lagrangians. *Bull Polish Acad Sci* 34:73–81
27. Coleman BD, Marcus M, Mizel VJ (1992) On the thermodynamics of periodic phases. *Arch Ration Mech Anal* 117:321–347
28. Crasta G, Malusa A (1996) Existence results for noncoercive variational problems. *SIAM J Contr Optim* 34:2064–2076
29. De Blasi FS, Myjak J (1998) On a generalized best approximation problem. *J Approx Theory* 94:54–72
30. Deville R, Godefroy R, Zizler V (1993) *Smoothness and renorming in Banach spaces*. Longman Scientific and Technical, Harlow
31. Deville R, Revalski J (2000) Porosity of ill-posed problems. *Proc Amer Math Soc* 128:1117–1124
32. Diestel J, Uhl JJ (1977) *Vector measures*. American Mathematical Society, Providence, RI
33. Doob JL (1994) *Measure theory*. Springer, New York
34. Evtstigneev IV, Flam SD (1998) Rapid growth paths in multivalued dynamical systems generated by homogeneous convex stochastic operators. *Set-Valued Anal* 6:61–81
35. Ferriero A (2005) The approximation of higher-order integrals of the calculus of variations and the Lavrentiev phenomenon. *SIAM J Contr Optim* 44:99–110
36. Gaitsgory V, Rossomakhine S, Thatcher N (2012) Approximate solution of the HJB inequality related to the infinite horizon optimal control problem with discounting. *Dynam Contin Discrete Impuls Syst Ser B* 19:65–92
37. Gale D (1967) On optimal development in a multi-sector economy. *Rev Econ Stud* 34:1–18
38. Guo X, Hernandez-Lerma O (2005) Zero-sum continuous-time Markov games with unbounded transition and discounted payoff rates. *Bernoulli* 11:1009–1029
39. Ioffe AD (1972) An existence theorem for problems of the calculus of variations. *Sov Math Dokl* 13:919–923
40. Ioffe AD (1976) An existence theorem for a general Bolza problem. *SIAM J Contr Optim* 14:458–466
41. Ioffe AD, Tikhomirov VM (1979) *Theory of extremal problems*. North-Holland, New York
42. Ioffe AD, Zaslavski AJ (2000) Variational principles and well-posedness in optimization and calculus of variations. *SIAM J Contr Optim* 38:566–581
43. Jasso-Fuentes H, Hernandez-Lerma O (2008) Characterizations of overtaking optimality for controlled diffusion processes. *Appl Math Optim* 57:349–369
44. Kelley JL (1955) *General topology*. Van Nostrand, Princeton, NJ
45. Lavrentiev M (1926) Sur quelques problemes du calcul des variations. *Ann Math Pura Appl* 4:107–124
46. Leizarowitz A (1985) Infinite horizon autonomous systems with unbounded cost. *Appl Math Opt* 13:19–43
47. Leizarowitz A (1986) Tracking nonperiodic trajectories with the overtaking criterion. *Appl Math Opt* 14:155–171
48. Leizarowitz A, Mizel VJ (1989) One dimensional infinite horizon variational problems arising in continuum mechanics. *Arch Ration Mech Anal* 106:161–194
49. Loewen PD (1987) On the Lavrentiev phenomenon. *Canad Math Bull* 30:102–108

50. Lykina V, Pickenhain S, Wagner M (2008) Different interpretations of the improper integral objective in an infinite horizon control problem. *J Math Anal Appl* 340:498–510
51. Makarov VL, Rubinov AM (1977) *Mathematical theory of economic dynamics and equilibria*. Springer, New York
52. Malinowska AB, Martins N, Torres DFM (2011) Transversality conditions for infinite horizon variational problems on time scales. *Optim Lett* 5:41–53
53. Mania B (1934) Sopra un esempio di Lavrentieff. *Boll Un Mat Ital* 13:146–153
54. Marcus M, Zaslavski AJ (1999) On a class of second order variational problems with constraints. *Isr J Math* 111:1–28
55. Marcus M, Zaslavski AJ (1999) The structure of extremals of a class of second order variational problems. *Ann Inst H Poincaré Anal Non linéaire* 16:593–629
56. Marcus M, Zaslavski AJ (2002) The structure and limiting behavior of locally optimal minimizers. *Ann Inst H Poincaré Anal Non linéaire* 19:343–370
57. Maz'ja VG (1985) *Sobolev spaces*. Springer, Berlin
58. McKenzie LW (1976) Turnpike theory. *Econometrica* 44:841–866
59. McShane EJ (1934) Existence theorem for the ordinary problem of the calculus of variations. *Ann Scuola Norm Pisa* 3:181–211
60. Mizel VJ (2000) *New developments concerning the Lavrentiev phenomenon, Calculus of variations and differential equations*. CRC Press, Boca Raton, FL, pp 185–191
61. Mordukhovich BS (1988) *Approximation methods in optimization and control*. Nauka, Moscow
62. Mordukhovich BS (1990) Minimax design for a class of distributed parameter systems. *Automat Remote Contr* 50:1333–1340
63. Mordukhovich BS (1999) Existence theorems in nonconvex optimal control, *Calculus of variations and optimal control*. CRC Press, Boca Raton, FL, pp 175–197
64. Mordukhovich BS, Shvartsman I (2004). *Optimization and feedback control of constrained parabolic systems under uncertain perturbations, Optimal control, stabilization and nonsmooth analysis*. Lecture Notes Control Information Sciences. Springer, New York, pp 121–132
65. Morrey CH (1967) *Multiple integrals in the calculus of variations*. Springer, Berlin
66. Moser J (1986) Minimal solutions of variational problems on a torus. *Ann Inst H Poincaré Anal Non linéaire* 3:229–272
67. Nitecki Z (1971) *Differentiable dynamics. An introduction to the orbit structure of diffeomorphisms*. MIT Press, Cambridge
68. Pickenhain S, Lykina V, Wagner M (2008) On the lower semicontinuity of functionals involving Lebesgue or improper Riemann integrals in infinite horizon optimal control problems. *Contr Cybern* 37:451–468
69. Reich S, Zaslavski AJ (1999) Convergence of generic infinite products of nonexpansive and uniformly continuous operators. *Nonlinear Anal* 36:1049–1065
70. Reich S, Zaslavski AJ (2000) Asymptotic behavior of dynamical systems with a convex Lyapunov function. *J Nonlinear Convex Anal* 1:107–113
71. Reich S, Zaslavski AJ (2001) The set of divergent descent methods in a Banach space is σ -porous. *SIAM J Optim* 11:1003–1018
72. Reich S, Zaslavski AJ (2001) Well-posedness and porosity in best approximation problems. *Topol Meth Nonlinear Anal* 18:395–408
73. Reich S, Zaslavski AJ (2002) Generic existence of fixed points for set-valued mappings. *Set-Valued Anal* 10:287–296
74. Rockafellar RT (1971) Existence and duality theorems for convex problems of Bolza. *Trans Amer Math Soc* 159:1–40
75. Rockafellar RT (1975) Existence theorems for general control problems of Bolza and Lagrange. *Adv Math* 15:312–333
76. Rubinov AM (1984) *Economic dynamics*. *J Sov Math* 26:1975–2012
77. Samuelson PA (1965) A catenary turnpike theorem involving consumption and the golden rule. *Amer Econ Rev* 55:486–496

78. Sarychev AV (1997) First-and second order integral functionals of the calculus of variations which exhibit the Lavrentiev phenomenon. *J Dynam Contr Syst* 3:565–588
79. Sarychev AV, Torres DFM (2000) Lipschitzian regularity of minimizers for optimal control problems with control-affine dynamics. *Appl Math Optim* 41:237–254
80. Sychev MA, Mizel VJ (1998) A condition on the value function both necessary and sufficient for full regularity of minimizers of one-dimensional variational problems. *Trans Amer Math Soc* 350:119–133
81. Tonelli L (1921) *Fondamenti di calcolo delle variazioni*. Zanichelli, Bologna
82. Zajicek L (1987) Porosity and σ -porosity. *Real Anal Exchange* 13:314–350
83. Zaslavski AJ (1987) Ground states in Frenkel-Kontorova model. *Math USSR Izvestiya* 29:323–354
84. Zaslavski AJ (1995) Optimal programs on infinite horizon 1. *SIAM J Contr Optim* 33:1643–1660
85. Zaslavski AJ (1995) Optimal programs on infinite horizon 2. *SIAM J Contr Optim* 33:1661–1686
86. Zaslavski AJ (1996) Generic existence of solutions of optimal control problems without convexity assumptions, preprint
87. Zaslavski AJ (2000) Generic well-posedness of optimal control problems without convexity assumptions. *SIAM J Contr Optim* 39:250–280
88. Zaslavski AJ (2001) Existence of solutions of optimal control problems without convexity assumptions. *Nonlinear Anal* 43:339–361
89. Zaslavski AJ (2001) Well-posedness and porosity in optimal control without convexity assumptions. *Calc Var* 13:265–293
90. Zaslavski AJ (2001) Generic well-posedness for a class of optimal control problems. *J Nonlinear Convex Anal* 2:249–263
91. Zaslavski AJ (2002) Generic existence of solutions of variational problems without convexity assumptions. *Comm Appl Anal* 6:457–477
92. Zaslavski AJ (2003) Generic well-posedness of variational problems without convexity assumptions. *J Math Anal Appl* 279:22–42
93. Zaslavski AJ (2003) Well-posedness and porosity in the calculus of variations without convexity assumptions. *Nonlinear Anal* 53:1–22
94. Zaslavski AJ (2004) Generic well-posedness of nonconvex optimal control problems. *Nonlinear Anal* 59:1091–1124
95. Zaslavski AJ (2004) Existence and uniform boundedness of approximate solutions of variational problems without convexity assumptions. *Dynam Syst Appl* 13:161–178
96. Zaslavski AJ (2004) The turnpike property for approximate solutions of variational problems without convexity. *Nonlinear Anal* 58:547–569
97. Zaslavski AJ (2005) Nonoccurrence of the Lavrentiev phenomenon for nonconvex variational problems. *Ann Inst H Poincaré Anal Non linéaire* 22:579–596
98. Zaslavski AJ (2006) Nonoccurrence of gap for infinite dimensional control problems with nonconvex integrands. *Optimization* 55:171–186
99. Zaslavski AJ (2006) *Turnpike properties in the calculus of variations and optimal control*. Springer, New York
100. Zaslavski AJ (2006) Nonoccurrence of the Lavrentiev phenomenon for many optimal control problems. *SIAM J Contr Optim* 45:1116–1146
101. Zaslavski AJ (2007) Nonoccurrence of the Lavrentiev phenomenon for many nonconvex constrained variational problems. *Calc Var* 28:351–381
102. Zaslavski AJ (2007) Turnpike results for a discrete-time optimal control systems arising in economic dynamics. *Nonlinear Anal* 67:2024–2049
103. Zaslavski AJ (2008) Generic nonoccurrence of the Lavrentiev phenomenon for a class of optimal control problems. *J Dynam Contr Syst* 14:95–119
104. Zaslavski AJ (2008) Nonoccurrence of the Lavrentiev phenomenon for many infinite dimensional linear control problems with nonconvex integrands. *Dynam Syst Appl* 17:407–434

105. Zaslavski AJ (2009) Two turnpike results for a discrete-time optimal control systems. *Nonlinear Anal* 71:902–909
106. Zaslavski AJ (2010) *Optimization on metric and normed spaces*. Springer, New York
107. Zaslavski AJ (2012) A generic turnpike result for a class of discrete-time optimal control systems. *Dynam Contin Discrete Impuls Syst Ser B* 19:225–265
108. Zeimer WP (1989) *Weakly differentiable functions*. Springer, New York
109. Zolezzi T (1995) Well-posedness criteria in optimization with application to the calculus of variations. *Nonlinear Anal* 25:437–453

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