

Answers to Problems

Chapter 2

2.9. Yes, it can. Consider, for example, probability space $\Omega = [0, 1]$ with Borel sigma-algebra and Lebesgue measure, and let $\xi(\omega) = \frac{1}{\omega}\mathbb{I}\{\omega < 1/2\}$ and $\eta(\omega) = \xi(1 - \omega)$. Then $\min(\xi, \eta) = 0$.

2.14. Exponential distribution, for example.

2.18. (i) $\alpha > 2$.

2.19. $n \geq 3$. Yes, the product is long-tailed.

2.20. $\beta < n$.

2.21. (ii) The same result holds for any distribution with negative mean.

2.26. (i) *Solution.* Denote by τ the first return time to the state 1, that is, $\tau = \min(n \geq 1 : X_n = 1)$ given $X_0 = 1$. Then $\tau = n$ if and only if $X_1 = n$, so that $\mathbb{P}\{\tau = n\} = F\{n - 1\}$ and $\mathbb{E}\tau = \sum_{n \geq 1} nF\{n - 1\}$. Therefore, the Markov chain is positive recurrent if and only if F has finite mean.

Let $\{\pi_i\}_{i \geq 1}$ be the stationary distribution. For any $i \geq 1$,

$$\begin{aligned} \pi_i &= \pi_{i+1} + \pi_1 F\{i - 1\} \\ &= \pi_{i+2} + \pi_1 F\{i\} + \pi_1 F\{i - 1\} \\ &= \dots \\ &= \pi_1 F[i - 1, \infty). \end{aligned}$$

Then

$$\begin{aligned} \sum_{i \geq 1} \pi_i &= \pi_1 \sum_{i \geq 1} \sum_{j \geq i-1} F\{j\} \\ &= \pi_1 \sum_{j \geq 0} (j + 1)F\{j\}, \end{aligned}$$

which implies that the invariant distribution is given by the residual distribution F_r .

Chapter 3

3.8. Let the ξ 's have exponential distribution with parameter λ .

- (i) $n_1 e^{-\lambda x^{1/\beta_1}}$ where $\beta_1 := \max \alpha_i$ and n_1 is the number of α_i that are equal to β_1 ;
- (ii) $n_2 \lambda x^{1/\beta_2}$ where $\beta_2 := \min \alpha_i$ and n_2 is the number of α_i that are equal to β_2 ;
- (iii) $n_1 e^{-\lambda x^{1/\beta_1}}$ if $\beta_2 > 1$ and $n_2 \lambda x^{1/\beta_2}$ if $\beta_2 < 0$.

3.9. $\alpha < 1 - \beta$.

3.10. For all positive values.

3.12. $\mathbb{P}\{S_n > x\} \leq n\mathbb{P}\{\xi_1 > x - 1\}$.

3.13. $\lambda t \bar{F}(x)$ where λ is the intensity and F is the jump distribution.

3.15. Proportional to $\bar{F}(x)$ with the following coefficients:

- (i) $\mathbb{P}\{X_0 = 1\}(2p_{11} + p_{12}(1+c)) + \mathbb{P}\{X_0 = 2\}(p_{22}2c + p_{21}(1+c))$;
- (ii) $\mathbb{P}\{X_0 = 1\} \left[1 + p_{11} + \sum_{j=0}^{\infty} p_{12} p_{21} p_{22}^j ((j+1)c + 1) \right] + \mathbb{P}\{X_0 = 2\} \left[c + p_{22}c + \sum_{j=0}^{\infty} p_{21} p_{11}^j p_{12}(j+1+c) \right]$;
- (iii) $\mathbb{P}\{X_0 = 1\} \left[1 + kp_{11} + k \sum_{j=0}^{\infty} p_{12} p_{22}^j p_{21} ((j+1)c + 1) \right] + \mathbb{P}\{X_0 = 2\} \left[c + kp_{22}c + k \sum_{j=0}^{\infty} p_{21} p_{11}^j p_{12}(j+1+c) \right]$.

3.16. (i) $H(x) = x$; (ii) $H(x) = x/\log x$; (iii) $H(x) = x^{1-\beta}$; (iv) $H(x) = \log x$.

3.19. (i) $\frac{1}{x \log x}$ and $\frac{2}{x \log x}$; (ii) $\frac{e^{-\sqrt{x}}}{\sqrt{x} \log x}$ and $\frac{2e^{-\sqrt{x}}}{\sqrt{x} \log x}$; (iii) $\frac{c_1}{\log x}$ and $\frac{c_2}{\log x}$ where $c_2 < 2c_1$.

Solution. The tail distribution function of ξ_i is equal to

$$\bar{F}(x) = \int_0^1 e^{-x^y} dy = \frac{1}{\log x} \int_1^x \frac{e^{-u}}{u} du \sim \frac{c_1}{\log x} \quad \text{as } x \rightarrow \infty,$$

where

$$c_1 := \int_1^{\infty} \frac{e^{-u}}{u} du.$$

Further,

$$\begin{aligned} \mathbb{P}\{\xi_1 + \xi_2 > x\} &= \mathbb{P}\{\xi_1 > x\} + \mathbb{P}\{\xi_2 > x\} - \mathbb{P}\{\xi_1 > x, \xi_2 > x\} \\ &\quad + \mathbb{P}\{\xi_1 \leq x, \xi_2 \leq x, \xi_1 + \xi_2 > x\}. \end{aligned}$$

By the conditional independence,

$$\begin{aligned} \mathbb{P}\{\xi_1 > x, \xi_2 > x\} &= \mathbb{E}\mathbb{P}\{\xi_1 > x, \xi_2 > x \mid \eta\} \\ &= \int_0^1 e^{-2x^t} dt \sim \frac{1}{\log x} \int_2^{\infty} \frac{e^{-u}}{u} du \quad \text{as } x \rightarrow \infty. \end{aligned}$$

It is left to prove that

$$\mathbb{P}\{\xi_1 \leq x, \xi_2 \leq x, \xi_1 + \xi_2 > x\} = o(\bar{F}(x)) \quad \text{as } x \rightarrow \infty. \quad (5.60)$$

- (ii) $n_2 \lambda x^{1/\beta_2} / |\beta_2|$ where $\beta_2 := \min \alpha_i$ and n_2 is the number of α_i that are equal to β_2 ;
 (iii) $n_1 \lambda x^{1/\beta_1 - 1} e^{-\lambda x^{1/\beta_1}}$ if $\beta_2 > 1$ and $n_2 \lambda x^{1/\beta_2} / |\beta_2|$ if $\beta_2 < 0$.

$$4.12. \sum_n \frac{n \mathbb{P}\{\tau=n\}}{\pi(n+x^2)}.$$

$$4.14. \lambda t f(x).$$

4.19.

- (i) $p_{i,i+1} = p - c/i + o(1/i)$ and $p_{i,i-1} = p + c/i + o(1/i)$ as $i \rightarrow \infty$ where $p < 1/2$ and $c > p/2$;
 (ii) $p_{i,i+1} = p - c/i^\beta + o(1/i^\beta)$ and $p_{i,i-1} = p + c/i^\beta + o(1/i^\beta)$ as $i \rightarrow \infty$ where $p < 1/2$, $c > 0$ and $0 < \beta < 1$.

4.20. *Solution.* It follows from the solution to Problem 2.26 that the invariant distribution for the Markov chain coincides with the residual distribution F_r . Then the result follows from Theorem 4.32 with $T = 1$.

4.21. All the distributions F_i are tail equivalent to some distribution from \mathcal{S}^* .

$$4.22. \frac{(1-p)^2}{(2\pi)^{d/2} \sqrt{\det B}}.$$

Chapter 5

5.3. Let the ξ 's have exponential distribution with parameter λ .

$$(i) n e^{-\lambda x^{1/\alpha}}; \quad (ii) n \lambda x^{1/\alpha}.$$

$$5.4. \frac{\alpha}{\varepsilon(1-\alpha)} x^{1-1/\alpha}.$$

5.5. It is proportional to the tail of the jump distribution with coefficient λT where λ is the intensity of the jumps.

5.6. It is proportional to the tail \bar{F}_I with coefficient λ/a where λ is the intensity of the jumps.

5.8. $\bar{F}_I(x)/a$. If the left tail of the distribution of η is much heavier than the right tail, then the asymptotic tail behaviour of M will be determined by the left tail of η .

5.9. $\mathbb{P}\{M > x\} \sim c \bar{F}_I(x/c)/a$ where $c := c_0 + \dots + c_k$.

5.12. *Solution.* We observe that

$$\mathbb{P}\{X_n > x \mid X_0 = 1\} = \sum_{k=1}^n \mathbb{P}\{X_{n-k} = 1, X_{n-k+1} \geq 2, \dots, X_{n-1} \geq 2, X_n > x \mid X_0 = 1\}.$$

Then, by the Markov property,

$$\begin{aligned} \mathbb{P}\{X_n > x \mid X_0 = 1\} &= \sum_{k=1}^n \mathbb{P}\{X_{n-k} = 1 \mid X_0 = 1\} \\ &\quad \times \mathbb{P}\{X_k > x, X_{k-1} \geq 2, \dots, X_1 \geq 2 \mid X_0 = 1\} \\ &= \sum_{k=1}^n \mathbb{P}\{X_{n-k} = 1 \mid X_0 = 1\} F[x+k, \infty). \end{aligned}$$

By the ergodic theorem for Markov chains,

$$\mathbb{P}\{X_n = 1 \mid X_0 = 1\} \rightarrow 1/\mathbb{E}\{\tau \mid X_0 = 1\} \quad \text{as } n \rightarrow \infty,$$

where $\tau = \min\{n \geq 1 : X_n = 1\}$. Taking also into account that the distribution F is long-tailed, we deduce that, as $n, x \rightarrow \infty$,

$$\begin{aligned} \mathbb{P}\{X_n > x \mid X_0 = 1\} &\sim \frac{1}{\mathbb{E}\{\tau \mid X_0 = 1\}} \sum_{k=1}^n F[x+k, \infty) \\ &\sim \frac{1}{\mathbb{E}\{\tau \mid X_0 = 1\}} \int_x^{x+n} \bar{F}(y) dy. \end{aligned}$$

Since $\mathbb{E}\{\tau \mid X_0 = 1\} = \mathbb{E}\{X_1 \mid X_0 = 1\}$, the result follows.

5.13. The answers differ for even and odd valued of n . If n is even number, $n = 2k$, then

$$\mathbb{P}\left\{\max_{0 \leq i \leq n} S_i > x\right\} \sim \frac{(c^{(1)} + c^{(2)})}{|a^{(1)} + a^{(2)}|} \int_0^{k|a^{(1)}+a^{(2)}|} \bar{F}(x+y) dy$$

as $x \rightarrow \infty$ uniformly in $k \geq 1$. If n is odd, $n = 2k + 1$, then

$$\mathbb{P}\left\{\max_{0 \leq i \leq n} S_i > x\right\} \sim \frac{(c^{(1)} + c^{(2)})}{|a^{(1)} + a^{(2)}|} \int_0^{k|a^{(1)}+a^{(2)}|} \bar{F}(x+y) dy + c^{(2)} \bar{F}(x).$$

5.14. $\frac{c}{a} \int_0^{n|a|} \bar{F}(x+y) dy$ as $n, x \rightarrow \infty$.

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