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# List of Symbols

- $(A, \boldsymbol{\varphi})$ , 75
- $(V, \mathcal{O})$ , 129
- $(\widehat{A}, \widehat{\boldsymbol{\varphi}})$ , 223
- $(\widehat{A}, \boldsymbol{\varphi})$ , 215
- $-\boldsymbol{\gamma}$ , 38
- $\langle \cdot, \cdot \rangle$ , 3
- $B(\mathbf{a}, r)$ , 9
- $C^1$ , 11
- $C^\infty$ , 11
- $C^p$ , 11
- $D_{\mathbf{y}}f(\mathbf{a})$ , 10
- $G_g$ , 79
- $I_\varepsilon$ , 303
- $Jf(\mathbf{a})$ , 10
- $K_\varepsilon$ , 303
- $L(\boldsymbol{\gamma})$ , 23
- $L(\boldsymbol{\gamma}, P)$ , 23
- $N_{x_0}$ , 101
- $T_{x_0}M$ , 97
- $d\boldsymbol{\omega}$ , 155
- $df(\mathbf{a})$ , 10
- $dx_1 \wedge \cdots \wedge dx_j \wedge \cdots \wedge dx_n$ , 149
- $dx_{\mathbb{I}}$ , 148
- $\partial^{\text{top}}(A)$ , 219
- $\Lambda^k(\mathbb{R}^n)$ , 147
- $N(\mathbf{x})$ , 195
- $T^*\boldsymbol{\omega}$ , 161
- $\boldsymbol{\alpha} \cup \boldsymbol{\beta}$ , 38
- $\boldsymbol{\alpha} \sim \boldsymbol{\beta}$ , 35
- $\boldsymbol{\omega}$ , 151
- $\boldsymbol{\omega} \wedge \boldsymbol{\varphi}$ , 153
- $f'(\mathbf{a})$ , 10
- $\frac{\partial(T_{i_1}, \dots, T_{i_k})}{\partial(u_1, \dots, u_k)}(\mathbf{u})$ , 167
- $\frac{\partial f}{\partial x_i}(\mathbf{a})$ , 10
- $\int_A f = \int_A f(\mathbf{x})d\mathbf{x}$ , 55
- $\int_M \mathbf{F} \cdot d\mathbf{s} = \int_M \langle \mathbf{F}, \mathbf{T} \rangle d\mathbf{s}$ , 204
- $\int_M \mathbf{F} \cdot \mathbf{N} dV_{n-1}$ , 199
- $\int_M \boldsymbol{\omega}$ , 197
- $\int_M \langle \mathbf{F}, \mathbf{N} \rangle dS = \int_M \mathbf{F} \cdot d\mathbf{S}$ , 199
- $\int_{(\Delta, \boldsymbol{\varphi})} \boldsymbol{\omega} = \int_{\boldsymbol{\varphi}(\Delta)} \boldsymbol{\omega}$ , 187
- $\int_{\boldsymbol{\gamma}} \mathbf{F}$ , 29
- $\int_{\boldsymbol{\gamma}} \boldsymbol{\omega}$ , 33
- $\langle \mathbf{a}, \mathbf{b} \times \mathbf{c} \rangle$ , 7
- $\nabla f(\mathbf{a})$ , 11
- $\|P\|$ , 24
- $\|\cdot\|$ , 3
- $\bar{A}$ , 9
- $\partial M$ , 218
- $\partial \mathbb{S}$ , 221
- $\theta_x$ , 134
- $\{v_j\}_{j=1}^n \sim \{w_j\}_{j=1}^n$ , 129
- $\mathbf{e}_1$ , 148
- $k\text{-area}(P)$ , 110
- $\nabla \times \mathbf{F}$ , 14
- $\mathbf{a} \times \mathbf{b}$ , 6
- $\mathbb{H}^k$ , 209
- $\mathbb{S}$ , 220
- $\text{area}(K, \boldsymbol{\varphi})$ , 116
- $\text{area}(M)$ , 118
- Curl  $\mathbf{F}$ , 14
- Div  $\mathbf{F}$ , 14
- int(A), 9

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