

Epilogue

On Mathematical and Theoretical Biology

This book attempts to bridge the gap between mathematics and population biology. It is intended to show students of biology how to apply mathematics to the study of some questions of importance to population biology and to introduce modeling in the natural sciences to students of mathematics. It may also be used as a reference on mathematical methods for working biological scientists.

For the most part, we have given little description of the background of the subject, and we urge the reader to explore the history of population ecology in such sources as Kingsland (1985). We also suggest exploring Real and Brown (1991), a collection of 40 classic papers in ecology over the period 1887 to 1974.

Naturally, there are many topics in the mathematics of population biology that have been omitted from this book. An important omission involves the use of stochastic models. In any real-life situation there are random effects. If population sizes are large, these effects are often small enough to be ignored, but when population sizes are small, then their use is essential. Some references for such models are Durrett and Levin (1994), Nisbet and Gurney (1982), Pollard (1973), and Renshaw (1991).

There is also no consideration of structured models that take into account nonlinear birth, death, and infection processes. Some references in this area are Castillo-Chavez (1987), Diekmann and Metz (1986), Gurney and Nisbet (1998), Gurtin and McCamy (1974), and Hoppensteadt (1975).

There has been a great deal of work in the mathematical theory of epidemics; see for example [Anderson and May (1991), Brauer, van den Driessche, and Wu (2008), Castillo-Chavez (1989), Castillo-Chavez, Blower, van den Driessche, Kirschner, and Yakubu (2001a, 2001b), Diekmann and Heesterbeek (2000), Diekmann, Heesterbeek, and Metz (1990), Haderl (1989a, 1989b, 1992, 1993), Murray (1989), Thieme (2003), and Thieme and Castillo-Chavez (1989, 1993)] for a few of the directions of current interest.

Because both of us are currently interested in the study of disease dynamics we have emphasized epidemiological systems throughout the book. In fact, we are planning to write a book on the subject that will go well beyond the introduction in Chapters 9 and 10 of this book. Mathematical and theoretical epidemiology have experienced a great deal of growth over the last two decades due to the emergence and reemergence of diseases like tuberculosis and AIDS, and epidemic disease outbreaks such as SARS (2002–2003), recurring threats of avian influenza, and the H1N1 influenza pandemic of 2009. In our discussion of disease dynamics we have shown our biases. We have focused on the impact at the population (organizational) level and have ignored the outstanding research that is being carried out in immunology. Fortunately, many of the techniques we have illustrated in the context of population dynamics and epidemiology are also of use in population genetics,

mathematical physiology, immunology, and other areas of biology. A relatively new development in the modeling of disease transmission has been the study of networks; see for example, [Newman (2002, 2003), and Strogatz (2001)]. This subject is developing rapidly, and promises to be one of the key areas of mathematical epidemiology.

In this book little has been said about evolution, units of selection, levels of aggregation, and scales (temporal or spatial). Consideration of these factors in the study of population biology and epidemiology is critical. In the study of a disease such as influenza, multiple scales come into play. We have fast (disease dynamics), slow (host demography), and super slow (evolution of cross immunity) time scales. Influenza spreads locally (as in schools) and globally. Influenza epidemic waves move across cities, countries, and continents. Locally they are driven by age structure in contact rates and by public transportation, while globally they may be driven by train or airplane flow. We hope that the mathematical techniques and modeling approaches of this book will be useful for the interdisciplinary groups of scientists working on the types of challenges posed by influenza epidemics.

We may summarize our goals by saying that if our book facilitates communication between biologists and mathematicians, then we will feel that we have made a contribution to science.

Answers to Selected Exercises

Chapter 1

Section 1.1

- 106.
- 435.
- 30.
- (i) $x = 10e^{ct}$.
(ii) $c = 0.0347$.
(iii) 60 hours, 66.4 hours.

Section 1.2

- 75.
 - 9.758×10^7 kg, 1.547 years.
- 9b For spherical cow, $MR = kW^{3/2}$; for cubical cow, $MR = k_cW^{3/2}$ with $k = k_c\pi/6^{1/3}$.
- 10 (a) $W = (-t/12 + 12.57)^4$ (b) About 24 days.

Section 1.4

- $x = 0$ unstable, $x = K$ asymptotically stable (if $r > 0, K > 0$).
- $x = 0$ unstable, $x = K$ asymptotically stable (if $r > 0, K > 0$).
- $x = 0$ unstable, $x = K(1 + \log \frac{r}{d})$ asymptotically stable (if $r > 0, K > 0$).
- All values of $y(0)$ less than the solution of $e^{-y} = 2y$.

9. (i) If $e > \beta$, equilibrium 0 is asymptotically stable and equilibrium $1 - \frac{e}{\beta} < 0$ is unstable. If $e < \beta$, equilibrium 0 is unstable and equilibrium $1 - \frac{e}{\beta} > 0$ is asymptotically stable.
 (ii) The only equilibrium is 0, asymptotically stable if $a > 0$ and unstable if $a < 0$.
11. (i) Unstable.
 (ii) Asymptotically stable.
 (iii) Asymptotically stable.
15. For $H = 10$, there is one equilibrium ($V \approx 22$); for $H = 20$, there are two equilibria ($V \approx 16, V \approx 2$); for $H = 30$, there is only one equilibrium ($V \approx 1$). A large herd depletes vegetation discontinuously.

Section 1.5

1. Critical harvest rate 4800, equilibrium with harvest of 3000 per year is 156,900.
 3. 7066
 5. $\frac{rK}{e}$.
 7. Maximum yield is the value of $rx e^{1-\frac{x}{K}} - dx$, with x defined by $re^{1-\frac{x}{K}} [1 - \frac{x}{K}] = d$; cannot be evaluated explicitly.

Section 1.6

1. Lake would be eutrophic.
 3. Dump would move lake from current equilibrium past unstable equilibrium to eutrophic equilibrium.

Section 1.7

1. (i) Mean is $\frac{b^2 - a^2}{2}$.
 (ii) Cumulative distribution function is

$$f(t) = \begin{cases} 0, & \text{if } t \leq a, \\ \frac{t-a}{b-a} & \text{if } a < t < b, \\ 1, & \text{if } t \geq b. \end{cases}$$

3. Mean is $\frac{\alpha_1 + \alpha_2}{\alpha_1 \alpha_2}$. Probability density function is $\frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} (e^{-\alpha_1 t} - e^{-\alpha_2 t})$. If $\alpha_1 = \alpha_2 = \alpha$, probability density function is $\alpha^2 t e^{-\alpha t}$.

Chapter 2

Section 2.1

1. $x_n = 2^{1-n}$.
3. $x_{2n} = \frac{1}{2^n}, x_{2n+1} = -\frac{1}{2^n}$.
5. $x_{2n} = r^n, x_{2n+1} = -r^n$.
9. 0, 1, 1, 2, 3, 5, 8, 13.

Section 2.2

1. [Figure 10.9](#).

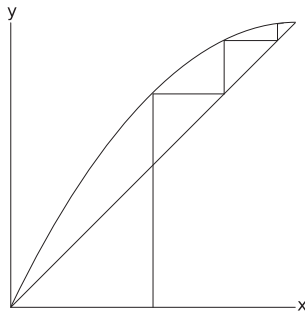


Fig. 10.9

3. [Figure 10.10](#).

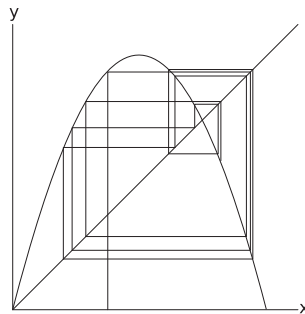


Fig. 10.10

5. x_n approaches 1000 if $R = 2$.

Section 2.3

- If $r^2 - 4A < 0$, the only equilibrium is 0 (asymptotically stable). If $r^2 - 4A \geq 0$, equilibria are 0, $\frac{r + \sqrt{r^2 - 4A}}{2}$ (asymptotically stable) and $\frac{r - \sqrt{r^2 - 4A}}{2}$ (unstable).
- $x = 0$ (stable if $r < 1$), $x = \frac{1}{\alpha} \left(r^{\frac{1}{\beta}} - 1 \right)$, asymptotically stable if $\left(r^{\frac{1}{\beta}} - 1 \right) (\beta - 2) < 2$.
- $x = 0$ (unstable), $x = 1$ (asymptotically stable).
- (b) $p < 1 - \frac{1}{e} = 0.632$.
- (i) $x = 0$, $x = 2$ asymptotically stable, $x = 1$ unstable.
(ii) $\sqrt{\frac{9}{8}} - 1 = 0.06 < a < 1$.
- $x = \frac{1}{2}$ is asymptotically stable if $0 \leq \alpha < \frac{1}{2\pi}$. Periodic orbit for $\alpha = \frac{1}{4}$ is $\left\{ \frac{1}{4}, \frac{3}{4}, \frac{1}{4}, \dots \right\}$.
- $x = 0$ (asymptotically stable if $a < 1$). $x = \log a$ (asymptotically stable if $1 < a < e^2$). Period doubling bifurcation appears when $a = e^2$. Population goes extinct if $a < 1$.

Section 2.4

- $r = e$.
- Fixed points 0, $\frac{5}{2}$, $\frac{7}{12}$, 1 all unstable. Cycle of period 2 with initial value $\frac{1}{4}$.
- $r = e$.

Section 2.5

- $x = 0$ is asymptotically stable for $0 < \alpha < 1$, $x = \frac{\log \alpha}{\beta}$ is asymptotically stable for $1 < \alpha < e^2$.
- $x = 0$ is asymptotically stable for $1 < \alpha < 1$, $x = \frac{\log \alpha}{\beta}$ is asymptotically stable for $\alpha > 1$.

Section 2.6

- $x_{n+1} = 6x_n$; (2,3), (6,6), (12,18).
- (-1, 1, 1, -1, -1, 1, 1, ...).
- (0,0) and $\left(\frac{a}{a-1} \frac{\log a}{bc}, \frac{1}{b} \log a \right)$ [if $a > 1$], both unstable.

Section 2.7

- Equilibria are solutions of $y = \alpha B(y) - D(y)$, with x given by $B(y)$. Asymptotic stability conditions are $|D'(y)| < 1 - \alpha B'(y)$, $-\alpha B'(y) < 1$.

Section 2.8

- (i) $A = 104.85, L = 179.32, P = 143.46$.
 (ii) $A = 2.75, L = 13.06, P = 10.45$.

Chapter 3

Section 3.2

6. $x = \frac{K \pm \sqrt{K^2 - \frac{4KH}{r}}}{2}$.

Section 3.3

- $x = 0$ asymptotically stable for all T if $r < dA$. $x = \frac{r - Ad}{d}$, asymptotically stable for all T if $r > dA$.
- (i) $x = 0$ is unstable for all T , $x = 3$ is asymptotically stable if $\sec z < -2$, where $z = -T \tan z$.
 (ii) $x = 0$ is unstable for all T , $x = 3 - \log(1 + p)$ is asymptotically stable if $\log(1 + p) - 2 > \sec z$, where $z = -(1 + p)T \tan z$.

Section 3.4

- $x = 0$ asymptotically stable for all T if $r < \pi Ad$, $x = \frac{r}{\pi d} - A$ asymptotically stable for all T if $r > \pi Ad$ [for both parts (a) and (b)].
- $x = 0$ asymptotically stable for all T if $r < \pi d$. $x = \log \frac{r}{\pi d}$, existing only if $r > \pi d$, asymptotically stable for all T .

Section 3.5

1. Positive equilibrium x_∞ is asymptotically stable for all T if it exists, which happens if $H < \max x(e^{3-x} - 1) = 6.45$.

Chapter 4

Section 4.3

1. $u' = u - v, v' = u + v$ at $(1, 1)$.
3. $u' = v, v' = 2u + v$ at $(1, -1)$ and $u' = v, v' = -2u + v$ at $(-1, 1)$.
5. No equilibrium.
7. $u' = \lambda u, v' = \mu v$ at $(0, 0)$, $u' = -ax_\infty u - by_\infty v, v' = -cx_\infty u - dy_\infty v$ at $x_\infty = \frac{d\lambda - b\mu}{ad - bc}, y_\infty = \frac{a\mu - c\lambda}{ad - bc}$.
9. $(0, 0)$ with community matrix $\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, asymptotically stable. $(1, 1)$ with community matrix $\begin{bmatrix} -1 & 1 \\ 5 & -1 \end{bmatrix}$, unstable. $(4, 4)$ with community matrix $\begin{bmatrix} -1 & 1 \\ 5 & -1 \end{bmatrix}$, asymptotically stable.
11. (i) $S' = -\frac{\beta SI}{N} + \gamma(N - I - S)$ $I' = \frac{\beta SI}{N} - \gamma I$ [N constant].
 (ii) Disease-free equilibrium $S = N, I = 0$ is asymptotically stable if $\beta < \nu$.
 Endemic equilibrium with $\beta S = \nu N$ is asymptotically stable if it exists ($\beta > \nu$).
13. (i) Λ -people/time, β, μ , and γ -1/time.
 (ii) $N' = \Lambda - \mu N$.
 (iii) $N(t) = \frac{\Lambda}{\mu}(1 - e^{-\mu t}) + N(0)e^{-\mu t}$.
 (v) $R_0 = \frac{\beta}{\mu + \gamma}$, equilibrium $S = K, I = 0$ is asymptotically stable if $R_0 < 1$ and equilibrium $S = \frac{\mu + \gamma}{\beta}$ is asymptotically stable if $R_0 > 1$.
15. (a) $\begin{bmatrix} -\mu & -\beta \\ 0 & \beta - (\mu + \gamma) \\ 0 & \gamma & -\mu \end{bmatrix}$, with eigenvalues $-\mu, -\mu, \beta - (\mu + \gamma)$.
 (c) $R_0 = \frac{\beta}{\mu + \gamma}$, same as for (4.9).
 (d) Disease-free equilibrium is asymptotically stable if $R_0 < 1$.

Section 4.4

1. $(1, 1)$ unstable.
3. $(-1, -1)$ unstable, $(1, 1)$ unstable.
5. No equilibrium.
7. Orbits depend on values of the parameters; solution is given in Section 5.1.
9. Orbits depend on the values of the parameters; solution is given in Section 5.2.

- 11. (b) $y \approx \frac{\alpha N}{\gamma - \alpha} (e^{-\alpha t} - e^{-\gamma t})$.
 (c) $y \approx \frac{N}{1000}$ for $t = 6.915$.
- 17. Competitive system; all solutions approach $(\frac{2}{3}, \frac{2}{3})$.
- 19. $(0, 0)$ has matrix $\begin{bmatrix} -0 & -1 \\ -1 & -0 \end{bmatrix}$, (saddle point). $(1, 1)$ has matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (unstable node).

Section 4.5

- 1. There are three equilibria $(0, 0)$, $(20, 0)$, and $(10, 30)$. All of them are unstable. There is a stable limit cycle around $(10, 30)$. Two species coexist with oscillation.
- 3. There are three equilibria $(0, 0)$, $(20, 0)$, and $(2, 32.4)$. All of them are unstable. There is a stable limit cycle around $(2, 42.4)$. Two species coexist with oscillation.
- 5. There are two equilibria $(0, 0)$, $(20, 0)$; $(20, 0)$ is asymptotically stable. Thus the predator species goes to extinction.
- 7. The equilibrium $(60, 20)$ is asymptotically stable and all trajectories approach it. Two species coexist.
- 9. The equilibrium $(0, 45)$ is asymptotically stable and all trajectories approach it. The x -species goes to extinction and the y -species wins the competition.
- 11. The equilibrium $(0, 16)$ is asymptotically stable and all trajectories approach it. The x -species goes to extinction and the y -species wins the competition.
- 13. If $n < 8$, steady state is an asymptotically stable, a spiral point if $n < \sqrt{48}$ and a node if $\sqrt{48} < n < 8$. If $n > 8$, steady state is a saddle point.

Chapter 5

Section 5.1

- 1. The equilibrium $(60, 20)$ is asymptotically stable and all trajectories approach it. Two species coexist.
- 3. The equilibrium $(0, 45)$ is asymptotically stable and all trajectories approach it. The x -species goes to extinction and the y -species wins the competition.

5. The equilibrium $(0, 16)$ is asymptotically stable and all trajectories approach it. The x -species goes to extinction and the y -species wins the competition.

Section 5.2

1. There are three equilibria $(0, 0)$, $(20, 0)$, and $(10, 30)$. All of them are unstable. There is a stable limit cycle around $(10, 30)$. Two species coexist with oscillation.
3. There are three equilibria $(0, 0)$, $(20, 0)$, and $(2, 32.4)$. All of them are unstable. There is a stable limit cycle around $(2, 42.4)$. Two species coexist with oscillation.
5. There are two equilibria $(0, 0)$, $(20, 0)$. $(20, 0)$ is asymptotically stable. Thus the predator species goes to extinction.

Section 5.3

1. [Figure 10.11](#).

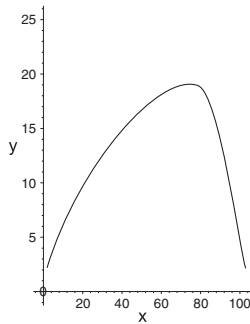


Fig. 10.11

Section 5.5

1. There are two equilibria $(0, 0)$ and $(120, 70)$. The first is unstable and the second is asymptotically stable.
3. There are three equilibria $(0, 0)$, $(0, 10)$, and $(4 + \sqrt{34}, 5 + \sqrt{34})$. The first two are unstable and the third is asymptotically stable.
5. There are three equilibria $(0, 0)$, $(0, M)$, and $(K, 0)$. They are all unstable. If $ab \geq 1$, orbits are unbounded. If $ab < 1$, a stable equilibrium $(x_\infty, y_\infty) = \left(\frac{K+aM}{1-ab}, \frac{Mb+K}{1-ab} \right)$ appears and all trajectories approach this equilibrium.

Chapter 6

Section 6.1

1. When $E = 0$, the equilibrium $(x_\infty, y_\infty) = (60, 20)$ is asymptotically stable and thus the two species coexist. When $E > 0$, the stable equilibria are on the line $x + y = 80$, and when $E = 120$, it coalesces to $(0, 80)$. Therefore, harvesting decreases the x -species size and increases the y -species, and eventually moves coexistence to x -extinction.
3. Without harvesting, the x -species goes to extinction and the y -species wins the competition. Harvesting the x -species will speed up extinction of the x -species.
5. Four equilibria $O_1 = (40 + \sqrt{40^2 - H}, 0)$, $O_2 = (40 - \sqrt{40^2 - H}, 0)$, $O_3 = \left(\frac{60 + \sqrt{60^2 - 6H}}{2}, 40 - \frac{60 + \sqrt{60^2 - 6H}}{6} \right)$ and $O_4 = \left(\frac{60 - \sqrt{60^2 - 6H}}{2}, 40 - \frac{60 - \sqrt{60^2 - 6H}}{6} \right)$. O_1 and O_2 are always unstable; O_4 is unstable and O_3 is stable. With harvesting of the x -species, as H increases O_3 and O_4 move along the line $x + 3y = 120$ until they coalesce when $H = 60$, resulting in coexistence at $(30, 30)$.
7. There is no equilibrium with positive population sizes. The x -species becomes extinct in finite time.

Section 6.2

1. $H_c = H^* = \frac{5}{6}$.
3. $H_c = 5.787$, $H_1^* = 3.7037$, $H_2^* = 5.5556$. H must be in the interval $(0, H_1^*)$ or in the interval (H_2^*, H_c) .
5. $H_c = 8.8889$. Trace of community matrix is positive, meaning that equilibrium is unstable. There exists a stable limit cycle.

Section 6.3

1. If we repeat the process 5 times, the estimated average instantaneous harvest is 34.5526. If we repeat the process 10 times, the estimated average instantaneous harvest is 34.2373. If we repeat the process 15 times, the estimated average instantaneous harvest is 34.1638. If we repeat the process 100 times, the estimated average instantaneous harvest is 34.0324. If we repeat the process 500 times, the estimated average instantaneous harvest is 34.0318.

Section 6.4

1. $Y(E) = \frac{aE}{E+b}$, always less than a and approaches a as $E \rightarrow \infty$. However, maximum fish population size is $\frac{a}{b}$. Thus the yield cannot exceed $\frac{a}{b}$.
3. E_{MSY} is the unique solution of the equation $\log\left(\frac{r}{d+E}\right) - \frac{E}{d+E} = 0$ for E in the interval $(0, r-d)$. The maximum sustainable yield $Y(E_{MSY}) = \frac{E_{MSY}^2}{d+E_{MSY}}$.

Section 6.5

1. $F(y) = ry \log \frac{K}{y}$, $h = Ey$, $C(y) = \frac{c}{y}$; y^* is the unique solution to the equation $rp \log \frac{K}{y} - (\delta + r)(p - \frac{c}{y}) = 0$. If $\frac{K}{e} < \frac{c}{p}$, $y^* < \frac{K}{e} = y_{MSY}$, implying overfishing.

Chapter 7

Section 7.1

1. $\lambda_0 = 0.9491$, $v_0 = [0.8021, 0.4534, 0.1979]$,
stable age distribution $[0.5131, 0.3604, 0.1226]$.
3. $\lambda_0 = 1.6826$, $v_0 = [0.9552, 0.2838, 0.0843]$,
stable age distribution $[0.7281, 0.2145, 0.0637]$.
5. Leslie matrix is

$$A = \begin{bmatrix} 1 & 3 \\ 2/3 & 0 \end{bmatrix},$$

$\lambda_0 = 2$, $v_0 = [0.9487, 0.3162]$, the stable age distribution is $[0.75, 0.25]$. The total population $P_n \approx c2^n$ when n is large.

Section 7.3

- 1(d). $B(t) = \left(\beta \int_0^\infty \phi(s) ds \right) e^{(\beta - \mu)t}$.
4. $P(t) = \varphi_0(a) \frac{\pi(a+t)}{\pi(a)} + \int_0^t P(t-a) \pi(a) da$.

Chapter 8

Section 8.2

4. $p_1' = cp_1p_2 - (d+e)p_1$, $p_2' = \lambda + (d-\lambda)p_1 - (e+\lambda)p_2 - cp_1p_2$.
Equilibria are $(0, \lambda/(e+\lambda))$, $(\lambda/(\lambda+e) - \lambda(d+e)/c, 0)$.
- (ii) Threshold quantity is

$$\frac{c\lambda}{(e+\lambda)(d+e)}.$$

Section 8.4

2. Unique equilibrium

$$y_1 = \frac{\mu + d + 2\sigma_1}{(\mu + d)(\mu + \sigma_1) + \mu\sigma_2} \Lambda, \quad y_2 = \frac{\mu + 2\sigma_1}{(\mu + d)(\mu + \sigma_1) + \mu\sigma_2} \Lambda,$$

asymptotically stable.

Section 8.6

- $u_n(x, t) = B_n e^{-D(\frac{2(n-1)\pi}{2L})^2 t} \sin \frac{2(n-1)\pi}{2L} x$
 $u(x, t) = \sum_{n=1}^{\infty} B_n e^{-D(\frac{2(n-1)\pi}{2L})^2 t} \sin \frac{2(n-1)\pi}{2L} x$
- $u_n(x, t) = (A_n \cos \lambda_n x + B_n \sin \lambda_n x) e^{-D(\lambda_n)^2 t}$
 where λ_n are roots of $\beta \tan \lambda L = -\lambda$,
 $u(x, t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos \lambda_n x + B_n \sin \lambda_n x) e^{-D(\lambda_n)^2 t}$,
 $u(x, 0) = f(x) = A_0 + \sum_{n=1}^{\infty} (A_n \cos \lambda_n x + B_n \sin \lambda_n x)$.
- From Problem 1, $u(x, t) = \sum_{n=1}^{\infty} B_n e^{-D(\frac{2(n-1)\pi}{2L})^2 t} \sin \frac{2(n-1)\pi}{2L} x$, $u(x, 0) = x = \sum_{n=1}^{\infty} B_n \sin \frac{2(n-1)\pi}{2L} x$, $B_n = \int_0^L x \sin \frac{2(n-1)\pi}{2L} x dx = \frac{8L}{2(n-1)\pi}$.
- $u_n(x, t) = B_n e^{-D(\frac{2(n-1)\pi}{2L})^2 t} \sin \frac{2(n-1)\pi}{2L} x$, $u(x, t) = \sum_{n=1}^{\infty} B_n e^{-D(\frac{2(n-1)\pi}{2L})^2 t} \sin \frac{2(n-1)\pi}{2L} x$,
 $u(x, 0) = f(x) = \sum_{n=1}^{\infty} B_n \sin \frac{2(n-1)\pi}{2L} x$, $B_n = \int_0^L f(x) \sin \frac{2(n-1)\pi}{2L} x dx$
- Let $u(x, t) = v(x) + w(x, t)$ where $v(x) = c_1 + c_2 x$ with $v(0) = c_1 = u_1$ and $v'(0) = c_2 = 0$. Therefore, $v(x) = u_1$ and $u(x, 0) = v(0) + w(x, 0) = f(x)$, leading to $w(x, 0) = f(x) - u_1$. Now, $w(x, t)$ is a solution to Problem 1; $w(x, t) = \sum_{n=1}^{\infty} B_n e^{-D(\frac{2(n-1)\pi}{2L})^2 t} \sin \frac{2(n-1)\pi}{2L} x$, $B_n = \int_0^L (f(x) - u_1) \sin \frac{2(n-1)\pi}{2L} x dx$.
- $X(x) = A \cos \lambda x + B \sin \lambda x$ with $X(0) = 0$ and $X'(L) = -\alpha X(L)$
 $X_n(x) = B_n \sin \lambda_n x$ where λ_n are roots of $\alpha \tan \lambda L = -\lambda$. Therefore, $u(x, t) = \sum_{n=1}^{\infty} B_n \sin \lambda_n x e^{-D(\lambda_n)^2 t}$.
- $u(x, 0) = u_0 + u_0 \cos \frac{\pi x}{L}$.

Section 8.7

$$3. u(x, t) = \int_0^{\infty} u_0 \frac{1}{\sqrt{4D\pi t}} e^{-\frac{|x-y|^2}{4Dt}} dy.$$

Section 8.8

$$1. u(x, t) = \int_{-\infty}^{\infty} f(y) \frac{1}{\sqrt{4D\pi t}} e^{-\frac{|x-y|^2}{4Dt}} dy + \int_0^{\infty} g(s) \frac{1}{\sqrt{4D\pi(t-s)}} e^{-\frac{|x|^2}{4D(t-s)}} ds.$$

Chapter 9

Section 9.2

1. 1.4416.
3. 2.827%.
5. 17.56%.
7. 58.33%.
10. 0.9125 foxes/km².
11. 23.1%.
13. No.

Section 9.3

3. Epidemic probability is 0.583.

Section 9.6

1. $S'(t) = -\beta S(t)I(t)$, $I'(t) = I'_0(t) + \beta S(t)I(t) - \beta S(t - \sigma)I(t - \sigma)$.

Section 9.9

1. (i) Model

$$\begin{aligned} \dot{S} &= -\beta(I + \delta A)S - \nu S, \\ \dot{S}_v &= -\varepsilon\beta(I + \delta A)S_v + \nu S - \phi S_v, \\ \dot{P} &= \phi S_v, \\ \dot{L} &= \beta(I + \delta A)S + \varepsilon\beta(I + \delta A)S_v - \kappa L, \\ \dot{I} &= p\kappa L - \gamma I, \\ \dot{A} &= (1 - p)\kappa L - \eta A, \\ \dot{R} &= \eta A + \gamma I, \end{aligned}$$

where S_v is the vaccinated class, P is the protected class, ν is the vaccination rate, and ϕ is the rate of advancement to protection from vaccination. Yes, the population is constant. No, there are no endemic equilibria. This can be seen since S tends to zero, causing S_v to tend toward zero, causing L to tend to zero, which ultimately causes I to tend to zero as t goes to infinity.

(ii) As the vaccination rate is increased, the number of infected individuals will decrease. Yes, vaccination can prevent an outbreak.

Section 9.10

3. Model

$$\begin{aligned}\dot{S} &= -\beta(I + \delta T)S, \\ \dot{E} &= \beta(I + \delta T)S - \kappa E, \\ \dot{I} &= \kappa E - (\alpha + \gamma)I, \\ \dot{T} &= \gamma I - \eta T, \\ \dot{R} &= \eta T + \alpha I.\end{aligned}$$

Then

$$\mathcal{R}_0 = \frac{\beta N}{\alpha + \gamma}.$$

Chapter 10

Section 10.1

3. (i) $C = C_0 e^{-\alpha t}$. (ii) $S_0 e^{\frac{\beta C_0}{\alpha}(e^{-\alpha t} - 1)}$. (iii) $\lim_{t \rightarrow \infty} S(t) = S_0 e^{-\frac{\beta}{\alpha} C_0}$.
5. About 4 1/2 hours.

Section 10.3

1. (i) Check $\frac{dN}{dt} = 0$. (ii) $\frac{1}{\mu}$ average life span; $\frac{1}{\gamma}$ average infectious period; $\frac{1}{\xi}$ average quarantine period. Their units are the same, namely time. (iii) $\nu = \frac{\mu}{\sigma}$, $\theta = \frac{\gamma}{\sigma}$, $\zeta = \frac{\xi}{\sigma}$. (iv) $\mathcal{R}_0 = \frac{\sigma}{\mu + \gamma}$, $\mathcal{R}_0 \leq 1$, $(0, 0, 0)$ is asymptotically stable. $\mathcal{R}_0 > 1$, $(0, 0, 0)$ is unstable.

Section 10.4

3. There is a disease-free equilibrium $S = 0$, $V = N$, $I = 0$. To find endemic equilibria, substitute

$$S + \frac{\alpha I}{\varphi + \beta I}, \quad \sigma \beta V = \frac{\alpha \varphi}{\varphi + \beta I},$$

into

$$(\alpha + \theta)I + \theta S + \theta V = \theta N$$

to obtain a quadratic equation for I that may have either one positive root or two positive roots.

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Index

R_0 , *see* basic reproductive number

absorption, 24, 31
action potential, 119, 120, 219
activator, 328
adjoint variable, 255, 262, 263
age at infection, 421, 422
age distribution
 stable, 270–273, 278, 279, 287
 persistent, 278
age group, 289
age of infection, 351, 384
age progression rate, 289
age structure, 288
age-structured population, 79
agriculture, 31, 37, 432
AIDS, 345–347, 377, 411
Allee effect, 22, 55
allele, 332
analytic expressions, 17
Antarctic fin whale, 74
antibiotic, xiv
Antonine plagues, 347
aperiodic, *see* periodic
Assyria, 346
asymptotic stability, 20, *see* stability
asymptotic velocity, 322
attack rate, 354
autocatalytic reaction, 332
autonomous, 18
Aztecs, 347, 424

backward bifurcation, 441
bacteria, xviii, 346, 350
bacteria agents, 411
balance equation, 302, 330
bang-bang, *see* control

 control, 238
basic reproduction number, 353, 368, 412, 418
basic reproductive number, xv, 5
Bayes's theorem, 40
Bendixson theorem, 152
Bernardelli population wave, 272
Bernouilli, 349
Beverton-Holt model, 69
bifurcation
 backward, 441
 period-doubling, 88
 Hopf, 431
bifurcation curve, 44
biomass, 3
bionomic equilibrium, 244
birth, 432
birth control, xiii
birth cycle, 68
birth modulus, 274
birth rate, 8, 134, 289
 per capita, xiv, 4
births, 349, 411, 413
bistable, 45
Black Death, *see* bubonic plague
blood cells, 116
blood sample, 356
blood stream, xv
blood–brain barrier, 144
boundary condition, 303
branching process, 351, 362, 373
bubonic plague, xiii, 347, 356, 432, 434
Burgers equation, 339

cable equation, 333, 334
cannibalism, 80, 81
canonical form, 156
capacitance, 334

- capital asset, 261, 263
- carriers, 414
- carrying capacity, xiv, xv, 9, 413
- catastrophe, 27, 224
- catastrophic outbreak, 203
- Cauchy initial value problem, 336, 340
- census, 7, 10, 81, 448
- center, 140, 167, 188
- Chagas, 346
- chaos, 63
- chaotic behavior, 61, 65
- characteristic equation, 140
- characteristic equation, 72, 96, 336, 379, 419
- chemostat, 183
- chicken pox, 346, 422
- cholera, 345
- closed season, 240
- cluster, 346, 367
- coalescence, 33, 201, 225
- cobwebbing method, 53
- coexistence, xviii, 130, 218, 224
- cohort, 273, 352
- collapse, 55
- colonization, 294
- commensalism, 193
- communicable disease, 345, 413, 442
- community matrix, 129, 206, 207, 232
- compartment
 - infected, 395
 - uninfected, 394
- compartmental model, 349, 442
- compensation, 21
- competing disease strains, 445
- competition
 - contest, 65
 - intraspecies, 65
 - scramble, 65, 98
 - unqualified, 168
- competitive exclusion, xviii, 165, 170
- complex dynamics, xvii
- compounding, 247
- computer algebra system, *see* Maple, *see* Mathematica
- configuration model, 370
- connecting orbit, 235, 325
- connection matrix, 297
- constraint, 229
- contact, 367, 368, 412
- contact intervention, 368
- contact rate, 416
- contest competition, 98
- control
 - bang-bang, 238, 249
 - optimal, 238
- control group, 348
- control reproduction number, 379, 392
- control, optimal, 238
- convolution, 276
- Cortez, *see* Aztecs
- critical depensation, 21
- critical harvest rate, 28, 237
- critical transmissibility, 368
- culture, 126
 - batch, 126
 - continuous, 126
- cumulative distribution, 38
- current shadow price, 263
- cusp, 45
- data, 419, 431, 434
- death modulus, 274
- death rate, *see* mortality rate
 - per capita, 4
- debilitating, 345, 433
- degree, 362, 367
- degree distribution, 362, 367
- delay, xvii, 19
- delay logistic equation, 94
- delta function, 317
- demand curve, 244
- demographic, 346, 394, 400
- demographic equation, 446
- demographic equilibrium, 87, 446
- demographic process, xv
- dendritic branch, 333, 340
- density dependence, 289
- Density-dependent, 413, 416
- depensation model, 21
- destabilization, 101, 323
- determinant, 140
- deterministic model, *see* model
- diarrhea, 345
- difference equation
 - differential, 105
 - linear, 72
 - linear homogeneous, 49
 - nonlinear, 51
- differential equation
 - autonomous, 21
 - linear homogeneous, 19
 - nonautonomous, 16
- differential infectivity, 377
- differential–difference equation, 349
- differential–difference equations, 118
- diffusion, 24, 91, 293, 442
- diffusion coefficient, 303, 331
- diffusion equation, 301
- diffusive instability, 327

- diffusivity, 303, 331
- dimensionless, xv, 36, 112
- direction field, 20
- discount rate, 247
- discrete, viii
- discrete generations, 112
- disease
 - fatal, 425, 432
- disease evolution, 346
- disease mortality, 432
- disease outbreak, 367
- disease transmission, 301, 348, 405
- dispersal, 448
- distributed
 - delay, xvii, 91
- divergence, 330
- divorce, 160
- doubling time, 6
- drug use, 346
- Dulac criterion, 172

- Ebola virus, 345
- economic models, 78, 244
- economic optimization, 37
- edge, 368
- effort, 29
- eigenvalue, 140, 295, 306
 - complex, 272
 - dominant, 273
 - double, 140
- eigenvector, 156
- elimination, 349, 434
- emigration, 50
- endangered species, xv
- endemic, 133
- energy reserve, 202
- English boarding school, 360
- environmental activists, 37
- environmental landscape, xiii
- epidemic, v, xv, 346
- epidemiology, viii
- equilibrium, 19
 - age distribution, 279, 283, 285, 290
 - bionomic, 244
 - endemic, 133
 - eutrophic, 32
 - isolated, 136
 - monotone, 57
 - oligotrophia, 32
 - oscillatory, 57
 - stable, 150
- equilibrium analysis, 56, 413
- eradication, 420
- escapement, 68

- eutrophic equilibrium, 32
- eutrophication, 31
- excess
 - degree, 363
- excitable cell, 333
- exponential distribution, 40, 369
- exponential growth, xvi
- exposed period, 373
- extinction, xv
- extinction rate, 294
- Eyam, 356

- family
 - one-parameter, 5
- fecundity, 268
- Feigenbaum constant, 64
- fertility, 68, 272
- Fibonacci sequence, 52
- Ficks law, 302, 330
- final size relation, 354, 374, 392
- finite time horizon, 248
- fishery, 11, 21, 33, 68, 242, 244, *see* harvesting
 - collapse, 33
 - open access, 242
- FitzHugh–Nagumo system, 219
- fleas, 357
- flour beetles, 80, 289
- flow, 350, 404, 411
- fluid convection, 209
- flux, 302, 330
- focus, 140
- foliage, 43, 202
- foraging rate, 333
- forcing
 - periodic, 84
- forest, 199
- forestry, 31
- Fourier integral, 314, 315
- Fourier series, 307, 314
 - full, 313
- Fourier sine integral, 318, 320
- Fourier sine series, 307, 310, 311
- fox rabies, 360
- functional equation, 286
- functional response
 - predator, 43, 175, 176
- functional response predator, 236
- fundamental solution, 317, 338, 339

- Garki project, 349
- gender, xviii, 160, 346
- gene, 60, 279
- generating function, 367
- generations

- nonoverlapping, xvii
 - overlapping, xvii
- genesis model, 279
- genetic, 346
- genetic disease, 346
- global behavior, 172
- globally asymptotically stable, 150
- gonorrhea, 13, 41, 346, 400, 411
- granulocytes, 116
- graph, 297
 - bi-directional, 297
- graphic solution, vii, 53, 58
- grazing, 91, 94
- Great Plague, 347, 356
- Green's theorem, 253
- growth curve, 22
- growth rate, xiv
 - delayed, 93
 - intrinsic, 11
 - per capita, 8
- haddock, 69
- half-life, 6
- halibut fishery, 11
- Hamiltonian, 255
- Han empire, 347
- harvesting
 - constant effort, 27, 28
 - constant rate, 27
 - instantaneous, 240
 - proportional, 28
- hazard of infection, 370
- health care, xiv, 405, 432
- heat equation, 340, 341
- heat kernel, 336, 338
- helminth agents, 411
- herbivore, 130, 221
- herd immunity, 359, 420
- heterogeneity, xviii, 372, 442
- heterosexual, 346, 404, 442
- HIV/AIDS, 346, 377
- home patch, 299, 300
- homoclinic, 233
- homosexual, 13, 41
- Hopf
 - bifurcation, 431
- host, 3, 466
- Hudson's Bay Company, 179
- hunting, 22, 240, 361
- hyperbola, 141, 224
- hysteresis, 32
- immature members, 70, 109
- immigration, 46, 50, 189, 349, 394
- immune response, 356
- immunity
 - herd, 359, 420
 - temporary, 349, 350, 427, 428, 445
- immunization, 350, 359, 420
- Incas, 347
- infected, 346, 367
- infected edge, 367
- infection
 - secondary, 42
- infectiousness, 348
- infective, 13
- infective period distribution, 441
- infectivity variable, 374
- inflection point, 18
- influenza, 345, 376, 437
- inhibitor, 328
- initial age distribution, 275
- initial boundary value problem, 303
- initial condition, 5
- initial data, 190
- initial value problem, 5
- inoculation, 349, 422
- instability, xviii
- integral equation, 276, 284, 349
- integro-differential equation, 91
- interaction, 368
 - facultative, 192
 - multispecies, xviii
 - obligatory, 192
 - terms, 130
- interepidemic period, 422
- internal patch dynamics, 298, 299
- interpolation, 448
- intra-specific competition, 289
- intracellular current, 333
- intracranial tumor, 144
- intraspecific competition, 289
- intrinsic growth rate, *see* growth rate
- invading species, 212
- invariant set, 148
- Invasive pneumococcal disease, 457
- ion channel, 218
- island, 296
- isocline, 170
 - predator, 176
 - prey, 176
- isolation, 350, 426
- iterate, 60, 63
- Jordan curve theorem, 149
- Jury criterion, 75, 78
- juveniles, 289, 458

- Kermack-McKendrick model, 350
 Kolmogorov forward equation, 341
 Kolmogorov model, 190
 Kolmogorov theorem, 191
- lags, *see* delay
 lake
 - eutrophic, xvii
 - eutrophication, xvii
 - irreversible, 35
 - oligotrophies, 31
 - reversible, 34
- Laplace equation, 331
 Laplace transform, 106, 277, 287
 Laplacian, 331
 larva, 80, 112
 least squares, 7
 Leishmaniasis, 453
 Leslie matrix, 268
 Leslie model, 190
 life expectancy, 421
 life span
 - average, 5, 345, 423
 - budworm, 43
 - generation, xv
 - tree, 43
- limit set, 148
 limiting system, 134
 limnological mechanisms, 31
 limpet, 145
 linearization, 19, 56
 logistic (differential) equation, xvi
 logistic difference equation, 58
 Lorenz equations, 208
 Lotka–Volterra equations, 123
 Lotka–Sharpe equation, 279
 lynx, Canadian, 179
- M-matrix, 397
 macroparasite, 492
 major epidemic, 362, 368
 malaria, xv, 345, 442
 Malthus, xiii, *see also* population model, 5
 management model, 380
 Maple, vii, 36
 marginal value, 263
 marriage function, 89
 mass action incidence, 351, 388
 Mathematica, vii, 36, *see* Maple, 92
 mating function, *see also* marriage function, 89
 Matlab, vii
 matrix, 417, 418
 maximum sustainable yield (MSY), 243
 McKendrick equation, 274
- mean field approximation, 332
 mean transmissibility, 367
 measles, 345, 347
 measurement, 43
 membrane battery, 333
 metapopulation, 293, 297
 metered model, 68
 microorganisms, 4, 345
 migration rate, *see* emigration
 minimum velocity, 325
 minor outbreak, 365, 368
 mixing
 - homogeneous, xviii, 14, 361, 442
- mobility, 299, 301
 model, vii, *see also* population model
 - Rosenzweig–MacArthur, 175
 - Beverton–Holt, 69
 - compartmental, 349, 442
 - depensation, 21
 - deterministic, 358
 - differential infectivity, 377
 - economic, 78, 244
 - genesis, 279
 - Kermack–McKendrick, 350, 354
 - Kolmogorov, 190
 - Leslie, 190
 - Leslie matrix, 268
 - metered, 68
 - multispecies, 126
 - neuron, 218
 - Nicholson’s blowflies, 112
 - Nicholson–Bailey, 75
 - Ricker, 69
 - stochastic, 465
- model fitting, 348
 modeling assumptions, xv, *see* specific model
 modulus, birth/death, 274
 monotone
 - convergence, 100
 - decrease, 20, 182, 270, 355
- mortality, 345, 418
 mortality function, *see* modulus, death
 - modulus, 71
- mortality rate, xvi, 401
 mortality–harvesting curve, 109
 mosquitoes, 349, 401, 404, 442
 motion of particles, 302
 multispecies model, 126
 mutated strain, 400
 mutual inhibition, 156
 mutualism, xviii, 192
- natural death, 349, 419, 427
 neighbor, 364, 371

- network, 367, 368
- neural network, 60, 333
- neuron, 333
- neuron model, 218
- newborn, 68, 86, 422, 426, 435
 - vaccination, 135
- next generation matrix, 394, 396
- Nicholson's blowflies model, 112
- Nicholson-Bailey model, 75
- node, 139, 325, 370, 372
- nonlinear, 323
- North Atlantic plaice, 69
- nullcline, 176
- numerical simulation, 112, 205, 237, 380
- nutrients, 10, 126

- objective function, 238, 460
- observation, xiii
- occupancy model, 294
- occupied edge, 367, 368
- Ohms law, 333, 334
- optimal, *see* control
 - equilibrium population, 248
- optimal control, 238
- optimization, 251, 438
- orbit, 123
- Ordinary Least Squares, 434, 436
- Ornstein - Uhlenbeck process, 341
- oscillation, 65, 80, 173, 174, 180, 185, 205, 245, 272
- outbreak, 202, 346, 351, 465
- overfishing, 246
- overharvesting, 111
- overpopulation, xiii

- Paramecium, 185, 186
 - aurelia, 185, 188
 - caudatum, 185
- parameter estimation, 348, 434, 460
- parasite, xv, 75, 199, 453
 - helminth agents, 411
- partial differential equation, 288, 293, 301, 302
 - linear, 306
- partition, 288
- partner, 14, 160
- passenger pigeon, 22
- patches, xviii, 293, 294, 323, 447
- pathogen, 4, 419
- pattern in space, 324
- per capita, xiii, *see* asobirth, death, growth rates, xiii
 - birth rate, xiv, 4
 - death rate, 4
 - growth rate, 8
- period-doubling, 63, 100, 238
- period-doubling bifurcation, 88
- periodic orbits, 125, 155, 240
- periodic solution, 61, 148, 150, 221, 232
- periodic wave train, 332
- perpetual outbreak, 203
- Perron-Frobenius theorem, 271
- persistence, 212
- persistent
 - age distribution, 278
- perturbation, 3, 20, 142, 173, 195, 212
- pest, xv
- petri dish, 6
- phase plane, 123
- phase portrait, 125
- phosphorus, 31
- pigeon, *see* passenger
- plague, 346, 411, *see also* bubonic plague
- Poincaré-Bendixson theorem, 149, 172, 178
- pollinator, 191
- pollutants, 91
- Pontryagin maximum principle, 238
- population
 - decay, xv
 - density, xvi, 3, 61, 418
 - dynamics, xiii, 465
 - explosion, xv
 - model
 - exponential, xiv, *see also* Malthus
 - logistic, xiv
- population control, 431
- positive feedback, 328
- positive semiorbit, 148
- predation, 8, 43, 199, 333
- predator functional response, 176
- predator numerical response, 175
- predator-prey, 123, 229, 294, 329, 433
- predator-induced coexistence, 214
- predator-induced exclusion, 216
- predator-prey, xviii, 173
- prediction, 5
- probability, 38, 299, 367, 368
- probability density, 39
- probability distribution, 38
- protozoa, *see* Paramecium
- public health, 347, 348, 404, 415, 435
- pulse vaccination, 443

- qualitative
 - analysis, 17, 92, 424
 - behavior, xvii, 8, 135
 - dynamics, 16, 134
 - prediction, 5, 65, 193
 - quality of life, xiv

- quantitative prediction, 348
- quarantine, 348, 354, 377, 415, 426
- random contact, 351
- random motion, 303, 330
- rat, 357
- rate of transfer, 349, 427
- reaction–diffusion equation, 321, 329
- recolonization, 294
- recovery, 350, 395, 404, 411, 413, 445
- recruitment, 16, *see also* birth cycle
 - cycle, 67
 - delayed, 70, 98
 - function, 65
- recursive calculation, 52
- recycling, 31
 - forest, 205
 - phosphorus, 32
 - rate, 31
- refuge, 45, 189
- reinfection, 119, 346, 350, 411
- removed, 350, 411, 415
- renewal condition, 274
- renewal equation, 276
 - nonlinear, 284
- reproduction matrix, 71
- reproductive
 - curve, 68
 - life, xv
 - number, *see* basic reproductive
- resource limitations, xiii, 9, *see also*
 - competition, harvesting, 229
- resources, 289
- respiratory infection, 345, 457
- response delay, 91
- Riccati equation, 335
- Ricker model, 69
- robust, 17, 51
- Rosenzweig–MacArthur model, 175
- Routh–Hurwitz
 - conditions, 208
- Routh–Hurwitz conditions, 106, 419
- rubella, 345, 422
- rumor, 346, 414
- runoff, 31
- Saccharomyces exiguus*, 188
- saddle point, 139, 141
- salmon, xvii, 69
- sandhill crane, 29
- SARS, 345, 435
- schistosomiasis, 345
- second order, 52
 - difference equation, 76
 - partial differential equations, 304
- secondary infection, 348, 412
- sedimentation, 31
- selection, 406, 466
- self-limiting, 125, 177
- self-regulating, 130
- semiorbit, 148
 - bounded, 148
 - positive, 148
- Sennacherib, 346
- separation of variables, 8, 68, 124, 304, 413
- separatrix, 149
- SEQIJR model, 378
- sequence, 49
- serological study, 356
- sexually transmitted disease, 389, 411
- shadow price, 263
- shark, 123
- Sharpe, 279
- sigmoid function, 31, 452
- singular control, 257, 258
- SIR model, 351, 407, 411, 419
- SIS model, 411, 425, 445, 447
- SITR model, 375
- sleeping sickness, 345
- smallpox, 347, 420, 434
- smoking, 158
- snowshoe hare, 179
- social interaction, 407
- Spaniards, 347
- spatial location, 293
- spiral point, 141
- spiral wave, 332
- spruce budworm, xvii, 25, 199, 204, 206
- stability, 17
 - absolute, 100
 - asymptotic, 23, 58
 - exponential, 114
 - neutral, 143
 - oscillatory, 114
- stable
 - age distribution, 270–273, 278, 279, 287
- stable age distribution, *see* age distribution
- stable pattern, 324, 332
- standard incidence, 388
- standardized solution, 337
- stasis, xv
- stochastic, 3, 341, 358, 426
- stock recruit, 69
- stress, 202
- superposition principle, 336
- surveillance data, 460
- survival, 168
 - equilibrium, 65, 81

- probability, 80
- survivor species, 213, 217
- survivorship, 69
- susceptibility, 348, 367, 431
- sustainable economic rent, 244
- switch point, 238
- switching function, 257

- Taylor approximation, viii, *see* Taylor's theorem
- Taylor's theorem, 19, 76, 95, 129, 141
- terminal condition, 251
- threshold, xv, 296, 348, 418
- tide pool, 145
- time scale of
 - births and deaths, 352
 - disease, 352
 - observation, xv
- tipping point, xv
- trajectories, 123
- trajectory, 156
- transformation property, 339
- transient, 311
- transition, 26, 195, 369, 441
- transmissibility, 367, 368
- transmission, 367, 368
- transmission intervention, 368
- transmission rate, 14, 440
- transportation network, 293
- travel between patches, 295, 296, 299
- treatment model, 354, 375
- tube, 302
 - infinite, 303
 - semi-infinite, 303
- tuberculosis, 345
- tumor, *see* intracranial
- typhus, 345

- uncertainty, 35, 438

- unchangeable, 159
- underreporting, 360, 453
- uniform persistence, 212
- uninhabitable patch, 296
- uniqueness, 12
 - of solutions, 18
- unstable, 21, 58, 433

- vaccination, xiv, 134, 348, 415
 - pulse, 443
- variable maturation, 107
- vector, 346, 404, 442, 455
- vector field, 150
- vector transmission, 404, 442
- vector-matrix notation, 23, 71, 74, 136, 207
- vegetation, 26, 130, 221
- Verhulst, xvi
 - difference equation, 51
- vertex, 362–365, 367, 368
- vertical transmission, 442
- viral agent, 345
- virus, 420, 458
- Voltorra, *see also* Lotka-Volterra
- Von Foerster equation, 274
- vortex, 139

- wave traveling, 325
- West Nile virus, 346, 404
- WinPP, vii, viii, 92, 93
- worker, 159
 - inactive, 159
 - master, 159
 - positive, 159
 - reluctant, 159

- XPP, vii, viii, 92

- Yield, 29
- yield effort curve, 29