

References

- Abraham R., Marsden J. E.: *Foundations of Mechanics*. Benjamin Cummings (1978).
- Abraham R., Robbins J.: *Transversal Mapping and Flows*. Benjamin, New York (1967).
- Arnold V. I.: Characteristic class entering in quantization conditions. *Funct. Anal. Appl.* **1**, 1–13 (1967).
- Arnold V. I.: *Mathematical Methods of Classical Mechanics*. Springer GTM **60** (1978).
- Arnold V. I.: *Chapitres Supplémentaires de la Théorie des Équations Différentielles Ordinaires*. Edition Mir, Moscou (1980).
- Arnold V. I., Novikov S.: *Dynamical Systems IV*. E.M.S. Springer (1990) (Russian original edition, 1985).
- Arnold V. I., Varchenko A., Goussein-Zadé S.: *Singularités des Applications Différentiables, 1.re partie, Classification des Points Critiques, des Causiques et des Fronts d'Onde*. Editions Mir, Moscou (1986).
- Benenti S.: (a) Symplectic relations in Analytical Mechanics. *Atti Accad. Sci. Torino Cl. Sc. Fis. Mat.* **117**, Suppl. I, 39–91 (1983). (b) The category of symplectic reductions. *Proceedings of the International Meeting on Geometry and Physics*. Florence, October 1982, Modugno M. Ed., Pitagora Editrice, Bologna, 11–41 (1983). (c) Linear symplectic relations, in *Symplectic Geometry*. Crumeyrolle A., Grifone J. Eds., *Research Notes in Math.* **80**, Pitman Advanced Publishing Program (1983).
- Benenti S.: *Relazioni Simplettiche, la Trasformazione di Legendre e la Teoria di Hamilton-Jacobi*. *Quaderni U.M.I.* **33**, Pitagora Editrice, Bologna (1988).
- Benenti S., Tulczyjew W. M.: (a) Relazioni lineari binarie tra spazi vettoriali di dimensione finita. *Memorie Accad. Sc. Torino* **3**, 67–113 (1979). (b) Relazioni lineari simplettiche, *Memorie Accad. Sc. Torino* **5**, 71–140 (1979).

- Benenti S., Tulczyjew W. M.: The geometrical meaning and the globalization of the Hamilton–Jacobi method. *Lecture Notes in Math.* **836**, 484–497 (1980).
- Benenti S., Tulczyjew W. M.: (a) Remarques sur les reductions symplectiques, *C.R.A.S.* **294**, 561–564 (1982). (b) Sur le théorème de Jacobi en mécanique analytique. *C.R.A.S. Paris* **294**, 677–680 (1982).
- Bott R.: Nondegenerate critical manifolds. *Ann. Mat.* **60**, 248–261 (1954).
- Buchdahl H. A.: *An Introduction to Hamiltonian Optics*. Cambridge University Press (1970).
- Callen H. B.: *Thermodynamics and an Introduction to Thermostatistics*. John Wiley (1985).
- Carathéodory C., Untersuchungen über die Grundlagen der Thermodynamik. *Math. Ann.* **67**, 335–386 (1909).
- Cardin F.: On the geometrical Cauchy problem for the Hamilton–Jacobi equation. *Nuovo Cimento* **104** (5), 525–544 (1989).
- Cardin F.: The global finite structure of generic envelope loci for Hamilton–Jacobi equations. *J. Math. Phys.* **43**, 417–430 (2002).
- Chaperon M.: On generating families. The Floer Memorial Volume, Hofer H., Taubes C.H., Weinstein A., Zehnder E. Eds., *Progress in Mathematics* **133**, 283–296, Birkhäuser (1995).
- Cordani B.: Conformal regularization of the Kepler problem. *Comm. Math. Phys.* **103**, 403–413 (1986).
- Cordani B.: The Kepler Problem. Group theoretical aspects, regularization and quantization, with application to the study of perturbations. *Progress in Mathematical Physics* **29**, Birkhäuser (2003).
- Dubois J.-G., Dufour J.-P.: La théorie des catastrophes. I. La machine à catastrophe. *Ann. Inst. H. Poincaré* **20**, 113–134 (1974).
- Dubois J.-G., Dufour J.-P.: La théorie des catastrophes. III. Caustiques de l’optique géométrique. *Ann. Inst. H. Poincaré* **24**, 243–260 (1976).
- Dubois J.-G., Dufour J.-P.: La théorie des catastrophes. V. Transformées de Legendre et thermodynamique. *Ann. Inst. H. Poincaré* **29**, 1–50 (1978).
- Fermi E.: *Thermodynamics*. Italian edition: *Termodinamica*, Boringhieri (1972).
- Frölicher A., Nijenhuis A.: Theory of vector-valued differential forms. *Nederl. Akad. Wetensch. Proc.* **59**, 338–359 (1956).
- Guillemin V., Sternberg S.: *Geometric Asymptotics*. A.M.S. *Math. Surveys* **14** (1977).
- Guillemin V., Sternberg S., *Symplectic Techniques in Physics*. Cambridge Univ. Press (1984).
- Hamilton W. R.: Theory of systems of rays. *Trans. R. Irish Acad.* **15**, 69–174 (1828).
- Hermann R.: *Geometry, Physics and Systems*. Dekker, New York (1973).
- Hörmander L.: Fourier integral operators. *Acta Math.* **127**, 79–183 (1971).
- Huang K.: *Statistical Mechanics*. Wiley and Sons (1987).

- Janeczko S.: (a) Geometric approach to coexistence of phases and singularities of Lagrangian submanifolds. Geometrical methods in physics, Proceedings, J.E. Purkině University, Brno (1983). (b) Geometrical approach to phase transitions and singularities of Lagrangian submanifolds. *Demonstratio Mathematica* **16**, 487–502 (1983). (c) Geometric approach to equilibrium thermodynamics, thermostatics of phase transitions. Notes of a lecture delivered at the Istituto di Fisica Matematica J. L. Lagrange, University of Torino (1983).
- Kalnins E. G.: Separation of Variables for Riemannian Spaces of Constant Curvature. Pitman Monographs and Surveys in Pure and Applied Mathematics **28**. Longman Scientific & Technical, John Wiley & Sons, Inc., New York (1986).
- Kalnins E. G., Miller W. Jr., Pogosyan G. S.: Superintegrability on the two-dimensional hyperboloid. *J. Math. Phys.*, **38** (10), 5416–5433 (1997).
- Kalnins E. G., Miller W. Jr., Hakobyan Ye. M., Pogosyan G. S.: Superintegrability on the two-dimensional hyperboloid. II. *J. Math. Phys.* **40** (5), 2291–2306 (1999).
- Kijowski J., Tulczyjew W. M.: A Symplectic Framework for Fields Theories. *Lecture Notes in Physics* **107** (1979).
- Koslov V. V.: *Dynamical Systems X – General Theory of Vortices*. Springer (2003).
- Lang S.: *Differential Manifolds*. Addison Wesley, Reading (1972).
- Lawruk B., Sniatycki J., Tulczyjew W. M.: Special symplectic spaces. *J. Diff. Equations* **17**, 477–497 (1975).
- Leray J.: The meaning of Maslov’s asymptotic method: the need of Plank’s constant in mathematics. *Bull. Am. Math. Society* **5** (1), 15–27 (1981).
- Levi-Civita T., Amaldi U.: *Lezioni di Meccanica Razionale*, Vol. II, parte II. Zanichelli (1989).
- Libermann P., Marle C.-M.: *Symplectic Geometry and Analytical Mechanics*. Reidel (1987).
- Lichnerowicz A., *C.R.A.S. Paris* **280**, 523–527 (1975).
- Lunenburg R. K.: *Mathematical Theory of Optics*. University of California Press, Berkeley and Los Angeles (1964).
- Marmo G., Morandi G., Mukunda N.: A Geometrical Approach to the Hamilton–Jacobi Form of Dynamics and Its Generalizations. *Rivista del Nuovo Cimento* **13** (1990).
- Maslov V. P.: *Théorie des Perturbations et Méthodes Asymptotiques*. Gauthier-Villiar, Paris (1971).
- Mishchenko A. S., Shatalov V. E., Sternin B. Y.: *Lagrangian Manifolds and the Maslov Operator*. Springer (1990) (Russian original edition, 1978).
- Moser J.: Regularization of Kepler’s problem and averaging method on a manifold. *Comm. Pure Appl. Math.* **23**, 609–636 (1970).
- Mruęała R.: Lie, Jacobi, Poisson and quasi-Poisson structures in thermodynamics. *Tensor* **56**, 37–45 (1995).
- Petersen P.: *Riemannian Geometry*. Springer (1998).

- Pham Mau Quan: Géométrie du problème de Kepler: orbites et variétés des orbites. C.R.A.S. Paris **291**, 299–301 (1980).
- Pham Mau Quan: (a) Régularisation riemannienne des singularités d'équations différentielles. C.R.A.S. Paris **296**, 241–244 (1983). (b) Riemannian regularization of singularities. Application to the Kepler problem. Proceedings of the IUTAM-ISIMM Symposium on Modern developments in Analytical Mechanics, Torino, June 7–11, 1982. Atti dell'Accademia delle Scienze di Torino **117**, 341–348 (1983).
- Poincaré H.: Thermodynamique. Cours de la Faculté des Sciences de Paris. G. Carré, Paris (1892).
- Poston T., Stewart I.: Catastrophe Theory and its Applications. Pitman (1978).
- Sniatycki J., Tulczyjew W. M.: Generating forms of Lagrangian submanifolds. Indiana Univ. Math. J. **22** (3) (1972).
- Souriau J. M.: Structures des Systèmes Dynamiques. Dunod, Paris (1970).
- Souriau J. M.: Géométrie globale du problème à deux corps. Proceedings of the IUTAM-ISIMM Symposium on Modern developments in Analytical Mechanics, Torino, June 7–11, 1982. Atti dell'Accademia delle Scienze di Torino **117**, 369–418 (1983).
- Straumann N., Jetzer P., Kaplan J.: Topics on Gravitational Lensing, Napoli Series on Physics and Astrophysics, Vol. 1, Bibliopolis, Naples (1998).
- Synge J.L.: Relativity: the General Theory. North-Holland (1960).
- Synge J. L.: Geometrical Optics. An Introduction to Hamilton's Method. Cambridge University Press (1962).
- Théret D.: A complete proof of Viterbo's uniqueness theorem on generating functions. Topology and Its Applications **96**, 249–266 (1999).
- Tulczyjew W. M.: Hamiltonian systems, Lagrangian systems and the Legendre transformation, Istituto Nazionale di Alta Matematica, Symposia Mathematica **14**, 247–258 (1974).
- Tulczyjew W. M.: Relations symplectiques et les équations d'Hamilton-Jacobi relativistes, C.R.A.S. Paris **281**, 545–547 (1975).
- Tulczyjew W. M.: (a) The Legendre transformation. Ann. Inst. H. Poincaré **27**, 101–114 (1977). (b) A symplectic formulation of relativistic particle dynamics, Acta Phys. Polonica **B 8**, 431–447 (1977).
- Tulczyjew W. M.: Geometric Formulation of Physical Theories, Statics and Dynamics of Mechanical Systems. Bibliopolis, Napoli (1989).
- Tulczyjew W. M., Urbanski P.: A slow and careful Legendre transformation for singular Lagrangians, Acta Phys. Polonica **B 30**, 2909–2978 (1999).
- Vaisman I.: Lectures on the Geometry of Poisson Manifolds. Progress in Mathematics **118**, Birkäuser (1994).
- Vinogradov A. M., Kuperschmidt B. A.: The structures of Hamiltonian mechanics. Russian Math. Surveys **32**, 177–243 (1977).
- Viterbo C.: Symplectic topology as the geometry of generating functions. Math. Ann. **292**, 685–710 (1992).

- Weinstein A.: Symplectic manifolds and their Lagrangian submanifolds, *Advances in Math.* **6**, 329–346 (1971).
- Weinstein A.: Lagrangian submanifolds and Hamiltonian systems. *Ann. of Math.* **98**, 377–410 (1973).
- Weinstein A.: Lectures on symplectic manifolds. *C.B.M.S. Conf. Series in Math., A.M.S.* **29** (1977).
- Weinstein A.: Symplectic geometry. *Bull. Amer. Math. Soc.* **5** (1), 1–13 (1981).
- Weinstein A.: Poisson geometry, *Diff. Geometry and its Applications* **9**, 213–238 (1998).
- Whittaker E. T.: *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies*. Cambridge University Press (1927).
- Wolf J. A.: *Spaces of Constant Curvature*. Publish or Perish (1984).

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