

Answers to Exercises

§2.

(α) {t}

(β) {t²/2}

(γ) {-4 sin t - t}

(δ) {2 sin t - 2 cos t + 2}

§9.

$$1. \quad \lim_{z \rightarrow 1} \frac{1}{(z+1)(z-2)} = \frac{-1}{2}, \quad \lim_{z \rightarrow 1} \left\{ \frac{(z-1)}{(z-1)^2(z+1)(z-2)} - \frac{-1 \cdot (z-1)}{2(z-1)^2} \right\}$$

$$= \lim_{z \rightarrow 1} \frac{z(z-1)}{2(z-1)(z+1)(z-2)} = \frac{-1}{4}, \quad \lim_{z \rightarrow -1} \frac{1}{(z-1)^2(z-2)} = \frac{-1}{12},$$

$$\lim_{z \rightarrow 2} \frac{1}{(z-1)^2(z+1)} = \frac{1}{3}$$

$$\text{Hence } \frac{1}{(s-1)^2(s+1)(s-2)} = \frac{1}{12} \left[\frac{-6}{(s-1)^2} + \frac{-3}{s-1} + \frac{-1}{s+1} + \frac{4}{s-2} \right].$$

$$2. \quad \frac{1}{(s-\alpha)^2 - \beta^2} = \frac{-1}{2\beta} \left(\frac{1}{(s-\alpha)+\beta} - \frac{1}{(s-\alpha)-\beta} \right)$$

$$3. \quad \frac{s-\alpha}{(s-\alpha)^2 - \beta^2} = \frac{1}{2} \left(\frac{1}{s-\alpha+\beta} + \frac{1}{s-\alpha-\beta} \right)$$

§10.

1. $e^{-t}(t^2 - 2t + 1)$

2. $y(t) = (t-1)^2 e^{-t}$

3. $y(t) = -\frac{1}{2} + \frac{1}{10} e^{2t} + \frac{2}{5} \cos t - \frac{1}{5} \sin t$

$$4. \quad y(t) = -\frac{1}{2} + e^t - \frac{11}{34} e^{4t} - \frac{3}{17} \cos t + \frac{5}{17} \sin t,$$

$$z(t) = -\frac{2}{3} e^t + \frac{22}{51} e^{4t} + \frac{4}{17} \cos t - \frac{1}{17} \sin t$$

$$5. \quad x(t) = 2 - e^t, \quad y(t) = -2 + 4e^t - te^t, \quad z(t) = -2 + 5e^t - te^t$$

$$6. \quad x(t) = \frac{28}{9} e^{3t} - e^{-t} - \frac{1}{9} - \frac{t}{3}, \quad y(t) = \frac{28}{9} e^{3t} + e^{-t} - \frac{1}{9} - \frac{t}{3}$$

$$7. \quad x(t) = 2e^{2t} - 2 \cos t - 3t \sin t$$

$$y(t) = \sin t + t \cos t - 2t \sin t$$

§11.

1. Solvable if and only if $\cos 2\pi\alpha \neq 0$; and then

$$y(t) = \frac{1}{\alpha \cos 2\pi\alpha} \sin \alpha t.$$

$$2. \quad y(t) = \frac{e^{\alpha t} - e^{-\alpha t}}{\alpha(e^{2\pi\alpha} + e^{-2\pi\alpha})}$$

§13.

$$\begin{aligned} \frac{1}{s^{1/2}(s-\alpha)} &= h^{1/2}\{e^{\alpha t}\} = \frac{1}{\sqrt{\pi}} \int_0^t \frac{1}{\sqrt{t-u}} e^{\alpha u} du \\ &= \frac{1}{\sqrt{\pi}} \int_0^t e^{\alpha(t-u)} \frac{1}{\sqrt{u}} du = \frac{e^{\alpha t}}{\sqrt{\pi}} \int_0^t e^{-\alpha u} \frac{1}{\sqrt{u}} du \\ &= \frac{2e^{\alpha t}}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-\alpha \lambda^2} d\lambda = \frac{2e^{\alpha t}}{\sqrt{\pi\alpha}} \int_0^{\sqrt{\alpha t}} e^{-x^2} dx = \frac{e^{\alpha t}}{\sqrt{\alpha}} \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\alpha t}} e^{-x^2} dx \\ &= \frac{e^{\alpha t}}{\sqrt{\alpha}} \operatorname{Erf}(\sqrt{\alpha t}) \end{aligned}$$

§16.

$$1. \quad \frac{1}{s^2} e^{-\alpha/s} = \sum_{n=0}^{\infty} (-1)^n \frac{\alpha^n h^{n+2}}{n!} = \left\{ \sum_{n=0}^{\infty} (-1)^n \frac{\alpha^n t^{n+1}}{n!(n+1)!} \right\}$$

$$= \left\{ (\sqrt{t/\alpha}) \sum_{n=0}^{\infty} (-1)^n \frac{(2\sqrt{\alpha t})^{2n+1}}{2^{2n+1} n!(n+1)!} \right\} = \left\{ (\sqrt{t/\alpha}) J_1(2\sqrt{\alpha t}) \right\}$$

$$2. \quad \frac{1}{\sqrt{s}} e^{\alpha/s} = \sum_{n=0}^{\infty} \frac{\alpha^n h^{n+1/2}}{n!} = \left\{ \sum_{n=0}^{\infty} \frac{\alpha^n t^{n-1/2}}{n! \Gamma(n+1/2)} \right\}$$

$$= \left\{ \sum_{n=0}^{\infty} \frac{\alpha^n t^{n-1/2}}{n(n-1) \cdots 1(n-1/2)(n-3/2) \cdots (1/2) \Gamma(1/2)} \right\}$$

$$= \left\{ \sum_{n=0}^{\infty} \frac{2^{2n} \alpha^n t^{n-1/2}}{(2n)(2n-1)\cdots 1} \right\} = \left\{ \frac{1}{\sqrt{\pi t}} \cosh (2\sqrt{\alpha t}) \right\}$$

3. See (16.13).

§20.

(α) Since $e^{-\beta/s^m} = e^{-\beta h^m} = I + \sum_{k=1}^{\infty} (-\beta)^k \frac{h^{mk}}{k!}$,

$$\begin{aligned} D e^{-\beta/s^m} &= D I + \sum_{k=1}^{\infty} (-\beta)^k \frac{1}{k!} (D h^{mk}) \\ &= 0 + \sum_{k=1}^{\infty} (-\beta)^k \frac{1}{k!} (-mk h^{mk+1}) \\ &= \sum_{k=1}^{\infty} (-\beta)^{k-1} \frac{h^{m(k-1)}}{(k-1)!} m\beta h^{m+1} \\ &= e^{-\beta/s^m} \frac{m\beta}{s^{m+1}} \end{aligned}$$

(β) $\frac{Dy}{y} = \frac{(-2+2b+2)s + c}{s^2 + 1/4} = \frac{2bs + c}{s^2 + 1/4}$.

$$= \frac{b + ci}{s + i/2} + \frac{b - ci}{s - i/2},$$

so that

$$y = C(s + i/2)^{b+ci} (s - i/2)^{b-ci} \quad (i = \sqrt{-1}).$$

§29.

1. $y_0(\lambda) = \frac{-6\lambda I}{s^3} + \frac{-\lambda^3 I}{s} = \{-3t^2\lambda - \lambda^3\}$

2. $y_0(\lambda) = \frac{-24I}{s^5} + \frac{-12\lambda^2 I}{s^3} + \frac{-\lambda^4 I}{s} = \{-t^4 - 6t^2\lambda^2 - \lambda^4\}$

§36.

(α) $\frac{1}{s^2+a^2} = \left\{ \frac{1}{a} \sin at \right\} < \frac{1}{a}$

(β) $\frac{1}{\sqrt{s}} e^{-\lambda\sqrt{s}} = \left\{ \frac{1}{\sqrt{t}} \exp(-\lambda^2/4t) \right\}$.

The function on the right-hand side takes its maximum value

$$\frac{1}{\lambda} \frac{2}{\sqrt{\pi e}} \quad \text{at} \quad t = \frac{\lambda^2}{2}.$$

Formulas and Tables

I. Special Functions

$$\Gamma(\lambda) = \int_0^{\infty} t^{\lambda-1} e^{-t} dt \quad (\text{Re } \lambda > 0) \quad (\S 12)$$

$$\Gamma(\lambda+1) = \lambda\Gamma(\lambda), \quad \Gamma(n) = (n-1)! \quad (n = 1, 2, \dots)$$

$$B(\lambda, \mu) = \frac{\Gamma(\lambda)\Gamma(\mu)}{\Gamma(\lambda+\mu)} \int_0^1 t^{\lambda-1}(1-t)^{\mu-1} dt \quad (\text{Re } \lambda > 0, \text{Re } \mu > 0) \quad (\S 12)$$

$$\Gamma(1/2) = \sqrt{\pi} = 2 \int_0^{\infty} e^{-v^2} dv$$

$$\text{Erf } t = \frac{2}{\sqrt{\pi}} \int_0^t e^{-v^2} dv \quad (\S 13)$$

$$\text{Cerf } t = \frac{2}{\sqrt{\pi}} \int_t^{\infty} e^{-v^2} dv = 1 - \text{Erf } t$$

$$J_n(t) = \sum_{k=0}^{\infty} (-1)^k \frac{t^{n+2k}}{2^{n+2k} k! (k+n)!} \quad (n = 0, 1, 2, \dots) \quad (\S 15, \S 20)$$

$$J_n(it) = i^n \sum_{k=0}^{\infty} \frac{t^{n+2k}}{2^{n+2k} k! (k+n)!} \quad (n = 0, 1, 2, \dots)$$

$$L_n^\alpha(t) = \sum_{k=0}^{\infty} \binom{n+\alpha}{n-k} \frac{(-t)^k}{k!} \quad (\S 20)$$

$$(1+z)^\gamma = \exp(\gamma \log(1+z)) \quad \text{for complex numbers } \gamma \text{ and } z$$

$$= \sum_{k=0}^{\infty} \binom{\gamma}{k} z^k \quad (\text{convergent for } |z| < 1) \quad (\S 14)$$

II. Formulas of the Operational Calculus

$$C = C[0, \infty) \tag{§1}$$

$$C_H = \left\{ \frac{f}{h^k}; f \in C \text{ and } k = 1, 2, \dots \right\} \tag{§3}$$

$$C/C: \text{ convolution quotients } \frac{f}{g} \quad (f, g \in C \text{ with } g \neq 0) \tag{§18}$$

$$\{a(t)\} + \{b(t)\} = \{a(t) + b(t)\},$$

$$\{a(t)\}\{b(t)\} = \left\{ \int_0^t a(t-u)b(u)du \right\},$$

$$\alpha\{f(t)\} = \{\alpha f(t)\}$$

$$h\{f(t)\} = \left\{ \int_0^t f(u)du \right\}$$

$$s\{f(t)\} = \{f'(t)\} + [f(0)], [f(0)] = s\{f(0)\} \tag{§5}$$

$$s^n f = f^{(n)} + s^{n-1}[f(0)] + \dots + s[f^{(n-2)}(0)] + [f^{(n-1)}(0)]$$

$$T^\alpha\{f(t)\} = \{e^{\alpha t}f(t)\}, \quad T^\alpha R(s) = R(s-\alpha)$$

$$T^\alpha \frac{b}{a} = \frac{T^\alpha b}{T^\alpha a} \tag{§20, §37}$$

$$Df = \{-tf(t)\}, \quad D(fg) = (Df)g + f(Dg)$$

$$D \frac{b}{a} = \frac{(Db)a - b(Da)}{a^2} \tag{§19}$$

$$\int_0^\infty e^{-\lambda s} f(\lambda) d\lambda = \{f(t)\} \tag{§26}$$

$$\frac{\partial}{\partial \lambda} e^{-\lambda s} = -s e^{-\lambda s}, \quad e^0 = I \tag{§23}$$

$$\frac{\partial}{\partial \lambda} e^{-\lambda s^{1/2}} = -s^{1/2} e^{-\lambda s^{1/2}}, \quad e^0 = I \tag{§26}$$

III. Tables of Hyperfunctions $\subseteq C/C$

$$\frac{I}{s} = h = \{1\}, \quad I = \frac{f}{f} \quad (f(t) \not\equiv 0) \tag{§5}$$

$$\frac{I}{s^n} = h^n = \left\{ \frac{t^{n-1}}{(n-1)!} \right\} \quad (n = 1, 2, \dots) \tag{§6}$$

$$\frac{I}{s^\lambda} = h^\lambda = \left\{ \frac{t^{\lambda-1}}{\Gamma(\lambda)} \right\} \quad (\text{Re } \lambda > 0) \tag{§13}$$

$$h^\gamma = \frac{h^{\gamma+n}}{h^n} = \frac{\Gamma(\gamma+n)^{-1} t^{\gamma+n-1}}{\Gamma(n)^{-1} t^{n-1}} \quad \text{for complex numbers } \gamma$$

(integer $n \geq 1$ such that $\operatorname{Re}(\gamma+n) > 1$) (§19)

$$Dh^\gamma = -\gamma h^{\gamma+1} \quad (\S 19)$$

$$D(I-\alpha h)^\gamma = \gamma(I-\alpha h)^{\gamma-1} \alpha h^2, \quad \text{where}$$

$$(I-\alpha h)^\gamma = \sum_{k=0}^{\infty} \binom{\gamma}{k} (-\alpha)^k h^k \quad (\S 19)$$

$$D(s-\alpha I)^\gamma = D \frac{(I-\alpha h)^\gamma}{h^\gamma} = \gamma(s-\alpha I)^{\gamma-1} \quad (\S 19)$$

$$\frac{I}{\sqrt{s}} = h^{1/2} = \left\{ \frac{1}{\sqrt{\pi t}} \right\} \quad (\S 13)$$

$$\frac{I}{s-\alpha} = \{e^{\alpha t}\} \quad (\S 6)$$

$$\frac{I}{(s-\alpha)^\lambda} = \left\{ \frac{t^{\lambda-1}}{\Gamma(\lambda)} e^{\alpha t} \right\} \quad (\operatorname{Re} \lambda > 0) \quad (\S 13)$$

$$\frac{I}{\sqrt{s+\alpha}} = \left\{ \frac{1}{\sqrt{\pi t}} e^{-\alpha t} \right\} \quad (\S 13)$$

$$\frac{I}{s\sqrt{s+\alpha}} = \left\{ \frac{1}{\sqrt{\alpha}} \operatorname{Erf} \sqrt{\alpha t} \right\} \quad (\alpha > 0) \quad (\S 13)$$

$$s^\gamma = h^{-\gamma}, \quad \text{in particular } s^0 = h^0 = I \quad \text{and } Ds = I \quad (\S 19)$$

$$(s-\alpha I)^\gamma (s-\alpha I)^\delta \quad \text{for complex numbers } \gamma \quad \text{and } \delta$$

$$= (s-\alpha I)^{\gamma+\delta} \quad (\S 19)$$

$$h^\gamma h^\delta = h^{\gamma+\delta} \quad (\S 19)$$

$$\frac{I}{(s-\alpha)^2 + \beta^2} = \left\{ \frac{1}{\beta} e^{\alpha t} \sin \beta t \right\}$$

$$\frac{I}{[(s-\alpha)^2 + \beta^2]^2} = \left\{ \frac{e^{\alpha t}}{2\beta^2} \left[\frac{1}{\beta} \sin \beta t - t \cos \beta t \right] \right\}$$

$$\frac{s-\alpha}{(s-\alpha)^2 + \beta^2} = \{e^{\alpha t} \cos \beta t\} \quad (\S 9)$$

$$\frac{I}{(s-\alpha)^2 - \beta^2} = \left\{ \frac{1}{\beta} e^{\alpha t} \sinh \beta t \right\}$$

$$\frac{s-\alpha}{(s-\alpha)^2 - \beta^2} = \{e^{\alpha t} \cosh \beta t\}$$

$$\frac{1}{(s^2 + \alpha^2)^{1/2}} = \{J_0(\alpha t)\}$$

$$\frac{1}{(s^2 - \alpha^2)^{1/2}} = \{J_0(i\alpha t)\}$$

$$\frac{(s^2 + \alpha^2)^{1/2} - s}{(s^2 + \alpha^2)^{1/2}} = \{\alpha J_1(\alpha t)\} \tag{§16}$$

$$((s^2 + \alpha^2)^{1/2} - s)^n = \left\{ \frac{n\alpha^n}{t} J_n(\alpha t) \right\} \quad (n = 1, 2, \dots)$$

$$\frac{((s^2 + \alpha^2)^{1/2} - s)^n}{(s^2 + \alpha^2)^{1/2}} = \{\alpha^n J_n(\alpha t)\} \quad (n = 0, 1, 2, \dots)$$

$$\frac{1}{s} e^{-\lambda/s} = \{J_0(2\sqrt{\lambda t})\}$$

$$\frac{1}{s^2} e^{-\lambda/s} = \{\sqrt{t/\lambda} J_1(2\sqrt{\lambda t})\} \tag{§16}$$

$$\frac{1}{\sqrt{s}} e^{-\lambda/s} = \left\{ \frac{1}{\sqrt{\pi t}} \cos 2\sqrt{\lambda t} \right\}$$

$$\frac{1}{\sqrt{s}} e^{\lambda s} = \left\{ \frac{1}{\sqrt{\pi t}} \cosh(2\sqrt{\lambda t}) \right\}$$

$$e^{-\lambda s} = s^2 \{h_1(\lambda, t)\} = s^2 \left\{ \int_0^t H_\lambda(u) du \right\} = s^2 \left\{ \begin{array}{l} 0, \quad 0 \leq t < \lambda \\ t - \lambda, \quad 0 \leq \lambda < t \end{array} \right\} \tag{§22}$$

$$\exp(-\lambda\sqrt{s}) = \left\{ \frac{\lambda}{2\sqrt{\pi t^3}} \exp(-\lambda^2/4t) \right\}$$

$$\frac{1}{\sqrt{s}} \exp(-\lambda\sqrt{s}) = \left\{ \frac{1}{\sqrt{\pi t}} \exp(-\lambda^2/4t) \right\} \tag{§27}$$

$$\frac{1}{s} \exp(-\lambda\sqrt{s}) = \{\text{Cerf}(\lambda/2\sqrt{t})\}$$

$$\exp(\lambda[s - (s^2 + \alpha^2)^{1/2}]) = I - \left\{ \frac{\lambda}{\sqrt{t^2 + 2\lambda t}} \alpha J_1(\alpha \sqrt{t^2 + 2\lambda t}) \right\}$$

$$\frac{\exp(\lambda[s - (s^2 + \alpha^2)^{1/2}])}{(s^2 + \alpha^2)^{1/2}} = \left\{ J_0(\alpha \sqrt{t^2 + 2\lambda t}) \right\} \tag{§28}$$

$$\exp(\lambda[s - (s^2 - \alpha^2)^{1/2}]) = I - \left\{ \frac{\lambda}{\sqrt{t^2 + 2\lambda t}} i\alpha J_1(i\alpha \sqrt{t^2 + 2\lambda t}) \right\}$$

$$\frac{\exp(\lambda[s - (s^2 - \alpha^2)^{1/2}])}{(s^2 - \alpha^2)^{1/2}} = \left\{ J_0(i\alpha \sqrt{t^2 + 2\lambda t}) \right\}$$

$$\exp(\lambda[s-(s^2+2\alpha s)^{1/2}]) = e^{-\alpha\lambda} - \frac{\lambda}{\sqrt{t^2+2\lambda t}} e^{-\alpha(\lambda+t)} i\alpha J_1(i\alpha\sqrt{t^2+2\lambda t})$$

$$\frac{\exp(\lambda[s-(s^2+2\alpha s)^{1/2}])}{(s^2+2\alpha s)^{1/2}} = e^{-\alpha(\lambda+t)} J_0(i\alpha\sqrt{t^2+2\lambda t}) \quad (\S 37)$$

$$\exp(-\lambda(s^2+\alpha^2)^{1/2}) = e^{-\lambda s} - \left\{ \begin{array}{l} 0, \quad 0 \leq t < \lambda \\ \frac{\lambda}{\sqrt{t^2-\lambda^2}} \alpha J_1(\alpha\sqrt{t^2-\lambda^2}), \quad 0 \leq \lambda < t \end{array} \right\}$$

$$\frac{\exp(-\lambda(s^2+\alpha^2)^{1/2})}{(s^2+\alpha^2)^{1/2}} = \left\{ \begin{array}{l} 0, \quad 0 \leq t < \lambda \\ J_0(\alpha\sqrt{t^2-\lambda^2}), \quad 0 \leq \lambda < t \end{array} \right\} \quad (\S 28)$$

$$\exp(-\lambda(s^2-\alpha^2)^{1/2}) = e^{-\lambda s} - \left\{ \begin{array}{l} 0, \quad 0 \leq t < \lambda \\ \frac{\lambda}{\sqrt{t^2-\lambda^2}} i\alpha J_1(i\alpha\sqrt{t^2-\lambda^2}), \quad 0 \leq \lambda < t \end{array} \right\}$$

$$\frac{\exp(-\lambda(s^2-\alpha^2)^{1/2})}{(s^2-\alpha^2)^{1/2}} = \left\{ \begin{array}{l} 0, \quad 0 \leq t < \lambda \\ J_0(i\alpha\sqrt{t^2-\lambda^2}), \quad 0 \leq \lambda < t \end{array} \right\}$$

$$\exp(-\lambda(s^2+2\alpha s)^{1/2}) = e^{-\alpha\lambda} e^{-\lambda s} - \left\{ \begin{array}{l} 0, \quad 0 \leq t < \lambda \\ \frac{\lambda}{\sqrt{t^2-\lambda^2}} e^{-\alpha t} i\alpha J_1(i\alpha\sqrt{t^2-\lambda^2}), \quad 0 \leq \lambda < t \end{array} \right\}$$

$$\frac{\exp(-\lambda(s^2+2\alpha s)^{1/2})}{(s^2+2\alpha s)^{1/2}} = \left\{ \begin{array}{l} 0, \quad 0 \leq t < \lambda \\ e^{-\alpha t} J_0(i\alpha\sqrt{t^2-\lambda^2}), \quad 0 \leq \lambda < t \end{array} \right\}$$

$$\exp(\lambda[s-((s-\alpha)^2+\beta^2)^{1/2}]) = e^{\alpha\lambda} - \left\{ \frac{\lambda}{\sqrt{t^2+2\lambda t}} e^{\alpha(\lambda+t)} \beta J_1(\beta\sqrt{t^2+2\lambda t}) \right\} \quad (\S 37)$$

$$\frac{\exp(\lambda[s-((s-\alpha)^2+\beta^2)^{1/2}])}{((s-\alpha)^2+\beta^2)^{1/2}} = \left\{ e^{\alpha(\lambda+t)} J_0(\beta\sqrt{t^2+2\lambda t}) \right\}$$

$$\exp(-\lambda[(s-\alpha)^2+\beta^2]^{1/2}) = e^{\alpha\lambda} e^{-\lambda s} - \left\{ \begin{array}{l} 0, \quad 0 \leq t < \lambda \\ \frac{\lambda}{\sqrt{t^2-\lambda^2}} e^{\alpha t} \beta J_1(\beta\sqrt{t^2-\lambda^2}), \quad 0 \leq \lambda < t \end{array} \right\}$$

$$\frac{\exp(-\lambda[(s-\alpha)^2+\beta^2]^{1/2})}{((s-\alpha)^2+\beta^2)^{1/2}} = \left\{ \begin{array}{l} 0, \quad 0 \leq t < \lambda \\ e^{\alpha t} J_0(\beta\sqrt{t^2-\lambda^2}), \quad 0 \leq \lambda < t \end{array} \right\}$$

(There are also those obtained by substituting $\sqrt{-1} \beta$ for β in the last two formulas above.)

$$e^{-\lambda\sqrt{s}} \leq \left[\frac{3}{\lambda^2} (\sqrt{6/\pi e^3}) \right] h \quad (\lambda > 0) \tag{536}$$

$$\frac{1}{\sqrt{s}} e^{-\lambda\sqrt{s}} \leq \left[(\sqrt{2/\pi e}) \frac{1}{\lambda} \right] h \quad (\lambda > 0)$$

References

1. Berg, E. J.: Heaviside's Operational Calculus as Applied to Engineering and Physics, McGraw-Hill Book Company (1936).
2. Doetsch, Gustav: Einführung in Theorie und Anwendung der Laplace-Transformation, Birkhäuser Verlag (1958).
3. Erdélyi, Arthur: Operational Calculus and Generalized Functions, Holt, Reinhart, and Winston (1962).
4. Heaviside, Oliver: Electromagnetic Theory, I-III, London (1893-1899).
5. Mikusiński, Jan: Operational Calculus, Pergamon Press (1959).
6. Mikusiński, Jan: The Bochner Integral, Academic Press (1978).
7. Krabbe, Gregers: Operational Calculus, Springer-Verlag (1970).
8. Okamoto, Shuichi: A simplified derivation of Mikusiński's operational calculus, Proc. Jap Acad. 50, Ser. A, No. 1 (1979), 1-5.
9. Yosida, Kôzaku-Okamoto, Shûichi: A note on Mikusiński's operational calculus, Proc. Jap. Acad. 56, Ser. A, No. 1 (1980), 1-3.
10. Yosida, Kôzaku: The algebraic derivative and Laplace's differential equation, Proc. Jap. Acad. 59, Ser. A, No. 1 (1983), 1-4.
11. Yosida, Kôzaku-Matsuura, Shigetake: A note on Mikusiński's proof of the Titchmarsh convolution theorem, to be published in the Contemporary Mathematics Series of the Amer. Math. Soc.

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