Answers to Exercises

§2.

(α) $\{t\}$

(β) $\{t^2/2\}$

(γ) $\{-4 \sin t - t\}$

(δ) $\{2 \sin t - 2 \cos t + 2\}$

§9.

1. $\lim_{z\to 1} \frac{1}{(z+1)(z-2)} = \frac{-1}{2}, \quad \lim_{z\to 1} \left\{\frac{(z-1)}{(z-1)^2(z+1)(z-2)} - \frac{-1\cdot(z-1)}{2(z-1)^2}\right\}$

$$= \lim_{z\to 1} \frac{z(z-1)}{2(z-1)(z+1)(z-2)} = \frac{-1}{4}, \quad \lim_{z\to -1} \frac{1}{(z-1)^2(z-2)} = \frac{-1}{12},$$

$$\lim_{z\to 2} \frac{1}{(z-1)^2(z+1)} = \frac{1}{3}$$

Hence $\frac{I}{(s-1)^2(s+1)(s-2)} = \frac{1}{12}\left[\frac{-6}{(s-1)^2} + \frac{-3}{s-1} + \frac{-1}{s+1} + \frac{4}{s-2}\right]$.

2. $\frac{I}{(s-\alpha)^2-\beta^2} = \frac{-1}{2\beta}\left(\frac{I}{(s-\alpha)+\beta} - \frac{I}{(s-\alpha)-\beta}\right)$

3. $\frac{s-\alpha}{(s-\alpha)^2-\beta^2} = \frac{1}{2}\left(\frac{I}{s-\alpha+\beta} + \frac{I}{s-\alpha-\beta}\right)$

§10.

1. $e^{-t}(t^2 - 2t + 1)$

2. $y(t) = (t-1)^2e^{-t}$

3. $y(t) = -\frac{1}{2} + \frac{1}{10}e^{2t} + \frac{2}{5}\cos t - \frac{1}{5}\sin t$

4. $y(t) = -\frac{1}{2} + e^{t} - \frac{11}{34} e^{4t} - \frac{3}{17} \cos t + \frac{5}{17} \sin t,$

$z(t) = -\frac{2}{3} e^{t} + \frac{22}{51} e^{4t} + \frac{4}{17} \cos t - \frac{1}{17} \sin t$

5. $x(t) = 2 - e^{t}, \quad y(t) = -2 + 4e^{t} - te^{t}, \quad z(t) = -2 + 5e^{t} - te^{t}$

6. $x(t) = \frac{28}{9} e^{3t} - e^{-t} - \frac{1}{9} - \frac{t}{3}, \quad y(t) = \frac{28}{9} e^{3t} + e^{-t} - \frac{1}{9} - \frac{t}{3}$

7. $x(t) = 2e^{2t} - 2 \cos t - 3t \sin t$

$y(t) = \sin t + t \cos t - 2t \sin t$

§11.

1. Solvable if and only if $\cos 2\pi\alpha \neq 0$; and then

$$y(t) = \frac{1}{\alpha \cos 2\pi\alpha} \sin \alpha t.$$

2. $y(t) = \dfrac{e^{\alpha t} - e^{-\alpha t}}{\alpha(e^{2\pi\alpha} + e^{-2\pi\alpha})}$

§13.

$$\frac{1}{s^{1/2}(s-\alpha)} = h^{1/2}\{e^{\alpha t}\} = \frac{1}{\sqrt{\pi}} \int_0^t \frac{1}{\sqrt{t-u}} e^{\alpha u}\, du$$

$$= \frac{1}{\sqrt{\pi}} \int_0^t e^{\alpha(t-u)} \frac{1}{\sqrt{u}}\, du = \frac{e^{\alpha t}}{\sqrt{\pi}} \int_0^t e^{-\alpha u} \frac{1}{\sqrt{u}}\, du$$

$$= \frac{2e^{\alpha t}}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-\alpha\lambda^2} d\lambda = \frac{2e^{\alpha t}}{\sqrt{\pi\alpha}} \int_0^{\sqrt{\alpha t}} e^{-x^2} dx = \frac{e^{\alpha t}}{\sqrt{\alpha}} \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\alpha t}} e^{-x^2} dx$$

$$= \frac{e^{\alpha t}}{\sqrt{\alpha}} \operatorname{Erf}(\sqrt{\alpha t})$$

§16.

1. $\frac{I}{s^2} e^{-\alpha/s} = \sum_{n=0}^{\infty} (-1)^n \frac{\alpha^n h^{n+2}}{n!} = \left\{ \sum_{n=0}^{\infty} (-1)^n \frac{\alpha^n t^{n+1}}{n!(n+1)!} \right\}$

$$= \left\{ (\sqrt{t/\alpha}) \sum_{n=0}^{\infty} (-1)^n \frac{(2\sqrt{\alpha t})^{2n+1}}{2^{2n+1} n!(n+1)!} \right\} = \left\{ (\sqrt{t/\alpha})\ J_1(2\sqrt{\alpha t}) \right\}$$

2. $\frac{I}{\sqrt{s}} e^{\alpha/s} = \sum_{n=0}^{\infty} \frac{\alpha^n h^{n+1/2}}{n!} = \left\{ \sum_{n=0}^{\infty} \frac{\alpha^n t^{n-1/2}}{n!\Gamma(n+1/2)} \right\}$

$$= \left\{ \sum_{n=0}^{\infty} \frac{\alpha^n t^{n-1/2}}{n(n-1)\cdots 1(n-1/2)(n-3/2)\cdots(1/2)\Gamma(1/2)} \right\}$$

$$= \left\{ \sum_{n=0}^{\infty} \frac{2^{2n}\alpha^n t^{n-1/2}}{(2n)(2n-1)\cdots 1} \right\} = \left\{ \frac{1}{\sqrt{\pi t}} \cosh\ (2\sqrt{\alpha t}) \right\}$$

3. See (16.13).

§20.

(α) Since $e^{-\beta/s^m} = e^{-\beta h^m} = I + \sum_{k=1}^{\infty} (-\beta)^k \frac{h^{mk}}{k!}$,

$$De^{-\beta/s^m} = DI + \sum_{k=1}^{\infty} (-\beta)^k \frac{1}{k!} (Dh^{mk})$$

$$= 0 + \sum_{k=1}^{\infty} (-\beta)^k \frac{1}{k!} (-mk\ h^{mk+1})$$

$$= \sum_{k=1}^{\infty} (-\beta)^{k-1} \frac{h^{m(k-1)}}{(k-1)!} m\beta\ h^{m+1}$$

$$= e^{-\beta/s^m} \frac{m\beta}{s^{m+1}}$$

(β) $\frac{Dy}{y} = \frac{(-2+2b+2)s + c}{s^2 + 1/4} = \frac{2bs + c}{s^2 + 1/4}$.

$$= \frac{b + ci}{s + i/2} + \frac{b - ci}{s - i/2},$$

so that

$$y = C(s + i/2)^{b+ci}(s - i/2)^{b-ci} \qquad (i = \sqrt{-1}).$$

§29.

1. $y_0(\lambda) = \frac{-6\lambda I}{s^3} + \frac{-\lambda^3 I}{s} = \{-3t^2\lambda - \lambda^3\}$

2. $y_0(\lambda) = \frac{-24I}{s^5} + \frac{-12\lambda^2 I}{s^3} + \frac{-\lambda^4 I}{s} = \{-t^4 - 6t^2\lambda^2 - \lambda^4\}$

§36.

(α) $\frac{1}{s^2+a^2} = \{\frac{1}{a} \sin at\} < \frac{1}{a}$

(β) $\frac{1}{\sqrt{s}} e^{-\lambda\sqrt{s}} = \left\{\frac{1}{\sqrt{t}} \exp(-\lambda^2/4t)\right\}.$

The function on the right-hand side takes its maximum value

$$\frac{1}{\lambda} \frac{2}{\sqrt{\pi e}} \quad \text{at} \quad t = \frac{\lambda^2}{2}.$$

Formulas and Tables

I. Special Functions

$$\Gamma(\lambda) = \int_0^\infty t^{\lambda-1}e^{-t}dt \quad (\text{Re } \lambda > 0) \qquad (\S 12)$$

$$\Gamma(\lambda+1) = \lambda\Gamma(\lambda), \quad \Gamma(n) = (n-1)! \quad (n = 1,2,\cdots)$$

$$B(\lambda,\mu) = \frac{\Gamma(\lambda)\Gamma(\mu)}{\Gamma(\lambda+\mu)} \int_0^1 t^{\lambda-1}(1-t)^{\mu-1}dt \quad (\text{Re } \lambda > 0, \text{ Re } \mu > 0) \qquad (\S 12)$$

$$\Gamma(1/2) = \sqrt{\pi} = 2\int_0^\infty e^{-v^2}dv$$

$$\text{Erf } t = \frac{2}{\sqrt{\pi}} \int_0^t e^{-v^2}dv \qquad (\S 13)$$

$$\text{Cerf } t = \frac{2}{\sqrt{\pi}} \int_t^\infty e^{-v^2}dv = 1 - \text{Erf } t$$

$$J_n(t) = \sum_{k=0}^\infty (-1)^k \frac{t^{n+2k}}{2^{n+2k}k!(k+n)!} \quad (n = 0,1,2,\cdots) \qquad (\S 15, \S 20)$$

$$J_n(it) = i^n \sum_{k=0}^\infty \frac{t^{n+2k}}{2^{n+2k}k!(k+n)!} \quad (n = 0,1,2,\cdots)$$

$$L_n^\alpha(t) = \sum_{k=0}^\infty \binom{n+\alpha}{n-k} \frac{(-t)^k}{k!} \qquad (\S 20)$$

$$(1+z)^\gamma = \exp(\gamma \log(1+z)) \quad \text{for complex numbers } \gamma \text{ and } z$$

$$= \sum_{k=0}^\infty \binom{\gamma}{k} z^k \quad (\text{convergent for } |z| < 1) \qquad (\S 14)$$

II. Formulas of the Operational Calculus

$\mathcal{C} = C[0,\infty)$ (§1)

$\mathcal{C}_H = \{\frac{f}{h^k};\ f \in \mathcal{C} \text{ and } k = 1,2,\ldots\}$ (§3)

$\mathcal{C}/\mathcal{C}$: convolution quotients $\frac{f}{g}$ $(f,g \in \mathcal{C}$ with $g \neq 0)$ (§18)

$\{a(t)\} + \{b(t)\} = \{a(t) + b(t)\},$

$\{a(t)\}\{b(t)\} = \left\{\int_0^t a(t-u)b(u)du\right\},$

$\alpha\{f(t)\} = \{\alpha f(t)\}$

$h\{f(t)\} = \left\{\int_0^t f(u)du\right\}$

$s\{f(t)\} = \{f'(t)\} + [f(0)],\ [f(0)] = s\{f(0)\}$ (§5)

$s^n f = f^{(n)} + s^{n-1}[f(0)] + \cdots + s[f^{(n-2)}(0)] + [f^{(n-1)}(0)]$

$T^\alpha\{f(t)\} = \{e^{\alpha t}f(t)\},\quad T^\alpha R(s) = R(s-\alpha)$

$T^\alpha \frac{b}{a} = \frac{T^\alpha b}{T^\alpha a}$ (§20, §37)

$Df = \{-tf(t)\},\quad D(fg) = (Df)g + f(Dg)$

$D\frac{b}{a} = \frac{(Db)a - b(Da)}{a^2}$ (§19)

$\int_0^\infty e^{-\lambda s} f(\lambda)d\lambda = \{f(t)\}$ (§26)

$\frac{\partial}{\partial\lambda} e^{-\lambda s} = -se^{-\lambda s},\quad e^o = I$ (§23)

$\frac{\partial}{\partial\lambda} e^{-\lambda s^{1/2}} = -s^{1/2}e^{-\lambda s^{1/2}},\quad e^o = I$ (§26)

III. Tables of Hyperfunctions $\subseteqq \mathcal{C}/\mathcal{C}$

$\frac{I}{s} = h = \{1\},\quad I = \frac{f}{f}\ (f(t) \not\equiv 0)$ (§5)

$\frac{I}{s^n} = h^n = \left\{\frac{t^{n-1}}{(n-1)!}\right\}\quad (n = 1,2,\ldots)$ (§6)

$\frac{I}{s^\lambda} = h^\lambda = \left\{\frac{t^{\lambda-1}}{\Gamma(\lambda)}\right\}\quad (\text{Re } \lambda > 0)$ (§13)

$$h^{\gamma} = \frac{h^{\gamma+n}}{h^{n}} = \frac{\Gamma(\gamma+n)^{-1}t^{\gamma+n-1}}{\Gamma(n)^{-1}t^{n-1}} \quad \text{for complex numbers } \gamma$$

$$(\text{integer } n \geqq 1 \text{ such that } \operatorname{Re}(\gamma+n) > 1) \qquad (\S 19)$$

$$Dh^{\gamma} = -\gamma h^{\gamma+1} \qquad (\S 19)$$

$$D(I-\alpha h)^{\gamma} = \gamma(I-\alpha h)^{\gamma-1}\alpha h^{2}, \quad \text{where}$$

$$(I-\alpha h)^{\gamma} = \sum_{k=0}^{\infty} \binom{\gamma}{k}(-\alpha)^{k}h^{k} \qquad (\S 19)$$

$$D(s-\alpha I)^{\gamma} = D\,\frac{(I-\alpha h)^{\gamma}}{h^{\gamma}} = \gamma(s-\alpha I)^{\gamma-1} \qquad (\S 19)$$

$$\frac{I}{\sqrt{s}} = h^{1/2} = \left\{\frac{1}{\sqrt{\pi t}}\right\} \qquad (\S 13)$$

$$\frac{I}{s-\alpha} = \{e^{\alpha t}\} \qquad (\S 6)$$

$$\frac{I}{(s-\alpha)^{\lambda}} = \left\{\frac{t^{\lambda-1}}{\Gamma(\lambda)}\,e^{\alpha t}\right\} \quad (\operatorname{Re}\lambda > 0) \qquad (\S 13)$$

$$\frac{I}{\sqrt{s+\alpha}} = \left\{\frac{1}{\sqrt{\pi t}}\,e^{-\alpha t}\right\} \qquad (\S 13)$$

$$\frac{I}{s\sqrt{s+\alpha}} = \left\{\frac{1}{\sqrt{\alpha}}\operatorname{Erf}\sqrt{\alpha t}\right\} \quad (\alpha > 0) \qquad (\S 13)$$

$$s^{\gamma} = h^{-\gamma}, \text{ in particular } s^{0} = h^{0} = I \text{ and } Ds = I \qquad (\S 19)$$

$$(s-\alpha I)^{\gamma}(s-\alpha I)^{\delta} \text{ for complex numbers } \gamma \text{ and } \delta$$

$$= (s-\alpha I)^{\gamma+\delta} \qquad (\S 19)$$

$$h^{\gamma}h^{\delta} = h^{\gamma+\delta} \qquad (\S 19)$$

$$\frac{I}{(s-\alpha)^{2}+\beta^{2}} = \left\{\frac{1}{\beta}\,e^{\alpha t}\sin\beta t\right\}$$

$$\frac{I}{[(s-\alpha)^{2}+\beta^{2}]^{2}} = \left\{\frac{e^{\alpha t}}{2\beta^{2}}\left[\frac{1}{\beta}\sin\beta t - t\cos\beta t\right]\right\}$$

$$\frac{s-\alpha}{(s-\alpha)^{2}+\beta^{2}} = \{e^{\alpha t}\cos\beta t\} \qquad (\S 9)$$

$$\frac{I}{(s-\alpha)^{2}-\beta^{2}} = \left\{\frac{1}{\beta}\,e^{\alpha t}\sinh\beta t\right\}$$

$$\frac{s-\alpha}{(s-\alpha)^{2}-\beta^{2}} = \{e^{\alpha t}\cosh\beta t\}$$

$$\frac{I}{(s^2+\alpha^2)^{1/2}} = \{J_0(\alpha t)\}$$

$$\frac{I}{(s^2-\alpha^2)^{1/2}} = \{J_0(i\alpha t)\}$$

$$\frac{(s^2+\alpha^2)^{1/2}-s}{(s^2+\alpha^2)^{1/2}} = \{\alpha J_1(\alpha t)\} \qquad (\S 16)$$

$$((s^2+\alpha^2)^{1/2}-s)^n = \left\{\frac{n\alpha^n}{t} J_n(\alpha t)\right\} \qquad (n = 1,2,\ldots)$$

$$\frac{((s^2+\alpha^2)^{1/2}-s)^n}{(s^2+\alpha^2)^{1/2}} = \{\alpha^n J_n(\alpha t)\} \qquad (n = 0,1,2,\ldots)$$

$$\frac{I}{s} e^{-\lambda/s} = \{J_0(2\sqrt{\lambda t})\}$$

$$\frac{I}{s^2} e^{-\lambda/s} = \{\sqrt{(t/\lambda)}\; J_1(2\sqrt{\lambda t})\} \qquad (\S 16)$$

$$\frac{I}{\sqrt{s}} e^{-\lambda/s} = \left\{\frac{1}{\sqrt{\pi t}} \cos\ 2\sqrt{\lambda t}\right\}$$

$$\frac{I}{\sqrt{s}} e^{\lambda s} = \left\{\frac{1}{\sqrt{\pi t}} \cosh(2\sqrt{\lambda t})\right\}$$

$$e^{-\lambda s} = s^2\{h_1(\lambda,t)\} = s^2\left\{\int_0^t H_\lambda(u)\,du\right\} = s^2\left\{\begin{matrix} 0, & 0 \leqq t < \lambda \\ t-\lambda, & 0 \leqq \lambda < t \end{matrix}\right\} \qquad (\S 22)$$

$$\exp(-\lambda\sqrt{s}) = \left\{\frac{\lambda}{2\sqrt{\pi t^3}} \exp(-\lambda^2/4t)\right\}$$

$$\frac{1}{\sqrt{s}} \exp(-\lambda\sqrt{s}) = \left\{\frac{1}{\sqrt{\pi t}} \exp(-\lambda^2/4t)\right\} \qquad (\S 27)$$

$$\frac{1}{s} \exp(-\lambda\sqrt{s}) = \{\mathrm{Cerf}(\lambda/2\sqrt{t})\}$$

$$\exp(\lambda[s-(s^2+\alpha^2)^{1/2}]) = I - \left\{\frac{\lambda}{\sqrt{t^2+2\lambda t}} \alpha J_1(\alpha\sqrt{t^2+2\lambda t})\right\}$$

$$\frac{\exp(\lambda[s-(s^2+\alpha^2)^{1/2}])}{(s^2+\alpha^2)^{1/2}} = \left\{J_0(\alpha\sqrt{t^2+2\lambda t})\right\} \qquad (\S 28)$$

$$\exp(\lambda[s-(s^2-\alpha^2)^{1/2}]) = I - \left\{\frac{\lambda}{\sqrt{t^2+2\lambda t}} i\alpha J_1(i\alpha\sqrt{t^2+2\lambda t})\right\}$$

$$\frac{\exp(\lambda[s-(s^2-\alpha^2)^{1/2}])}{(s^2-\alpha^2)^{1/2}} = \left\{J_0(i\alpha\sqrt{t^2+2\lambda t})\right\}$$

$$\exp(\lambda[s-(s^2+2\alpha s)^{1/2}]) = e^{-\alpha\lambda} - \frac{\lambda}{\sqrt{t^2+2\lambda t}}\, e^{-\alpha(\lambda+t)} i\alpha J_1(i\alpha\sqrt{t^2+2\lambda t})$$

(§37)

$$\frac{\exp(\lambda[s-(s^2+2\alpha s)^{1/2}])}{(s^2+2\alpha s)^{1/2}} = e^{-\alpha(\lambda+t)} J_0(i\alpha\sqrt{\lambda^2+2\lambda t})$$

$$\exp(-\lambda(s^2+\alpha^2)^{1/2}) = e^{-\lambda s} - \begin{cases} 0, & 0 \leqq t < \lambda \\ \dfrac{\lambda}{\sqrt{t^2-\lambda^2}}\alpha J_1(\alpha\sqrt{t^2-\lambda^2}), & 0 \leqq \lambda < t \end{cases}$$

$$\frac{\exp(-\lambda(s^2+\alpha^2)^{1/2})}{(s^2+\alpha^2)^{1/2}} = \begin{cases} 0, & 0 \leqq t < \lambda \\ J_0(\alpha\sqrt{t^2-\lambda^2}), & 0 \leqq \lambda < t \end{cases}$$

(§28)

$$\exp(-\lambda(s^2-\alpha^2)^{1/2}) = e^{-\lambda s} - \begin{cases} 0, & 0 \leqq t < \lambda \\ \dfrac{\lambda}{\sqrt{t^2-\lambda^2}}\, i\alpha J_1(i\alpha\sqrt{t^2-\lambda^2}), & 0 \leqq \lambda < t \end{cases}$$

$$\frac{\exp(-\lambda(s^2-\alpha^2)^{1/2})}{(s^2-\alpha^2)^{1/2}} = \begin{cases} 0, & 0 \leqq t < \lambda \\ J_0(i\alpha\sqrt{t^2-\lambda^2}), & 0 \leqq \lambda < t \end{cases}$$

$$\exp(-\lambda(s^2+2\alpha s)^{1/2}) = e^{-\alpha\lambda}e^{-\lambda s} - \begin{cases} 0, & 0 \leqq t < \lambda \\ \dfrac{\lambda}{\sqrt{t^2-\lambda^2}}\, e^{-\alpha t} i\alpha J_1(i\alpha\sqrt{t^2-\lambda^2}), & 0 \leqq \lambda < t \end{cases}$$

$$\frac{\exp(-\lambda(s^2+2\alpha s)^{1/2})}{(s^2+2\alpha s)^{1/2}} = \begin{cases} 0, & 0 \leqq t < \lambda \\ e^{-\alpha t} J_0(i\alpha\sqrt{t^2-\lambda^2}), & 0 \leqq \lambda < t \end{cases}$$

$$\exp(\lambda[s-((s-\alpha)^2+\beta^2)^{1/2}]) = e^{\alpha\lambda} - \left\{\frac{\lambda}{\sqrt{t^2+2\lambda t}}\, e^{\alpha(\lambda+t)}\beta J_1(\beta\sqrt{t^2+2\lambda t})\right\}$$

(§37)

$$\frac{\exp(\lambda[s-((s-\alpha)^2+\beta^2)^{1/2}])}{((s-\alpha)^2+\beta^2)^{1/2}} = \left\{e^{\alpha(\lambda+t)} J_0(\beta\sqrt{t^2+2\lambda t})\right\}$$

$$\exp(-\lambda[(s-\alpha)^2+\beta^2]^{1/2}) = e^{\alpha\lambda}e^{-\lambda s} - \begin{cases} 0, & 0 \leqq t < \lambda \\ \dfrac{\lambda}{\sqrt{t^2-\lambda^2}}\, e^{\alpha t}\beta J_1(\beta\sqrt{t^2-\lambda^2}), & 0 \leqq \lambda < t \end{cases}$$

$$\frac{\exp(-\lambda[(s-\alpha)^2+\beta^2]^{1/2})}{((s-\alpha)^2+\beta^2)^{1/2}} = \begin{cases} 0, & 0 \leqq t < \lambda \\ e^{\alpha t} J_0(\beta\sqrt{t^2-\lambda^2}, & 0 \leqq \lambda < t \end{cases}$$

(There are also those obtained by substituting $\sqrt{-1}\,\beta$ for β in the last two formulas above.)

$$e^{-\lambda\sqrt{s}} \leqq \left[\frac{3}{\lambda^2}\left(\sqrt{6/\pi e^3}\right)\right] h \qquad (\lambda > 0)$$

(§36)

$$\frac{I}{\sqrt{s}}\, e^{-\lambda\sqrt{s}} \leqq \left[\left(\sqrt{2/\pi e}\right)\frac{1}{\lambda}\right] h \qquad (\lambda > 0)$$

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