

Samples of Examination Problems

In the four-hour written examination, 15 interrelated problems are given. Within square brackets, we indicate the point value of each problem. These values are revealed to the students beforehand.

Variant 1

$$\ddot{x} = -\sin x + \varepsilon \cos t. \quad (1)$$

I. Let $\varepsilon = 0$.

- (1) Linearize at the point $x = \pi, \dot{x} = 0$, [1].
- (2) Is this equilibrium position stable [1]?
- (3) Find the Jacobian of the mapping of the phase flow at the point $x = \pi, \dot{x} = 0$ at time $t = 2\pi$ [3].
- (4) Find the derivative of the solution with initial condition $x = \pi, \dot{x} = 0$ with respect to the parameter ε at $\varepsilon = 0$ [5].
- (5) Draw the graph of the solution and its derivative with respect to t under the initial condition $x = 0, \dot{x} = 2$ [3].
- (6) Find this solution [3].

II. Let Eq. (2) be the linearized equation along the solution indicated in problem (5).

- (7) Does Eq. (2) have unbounded solutions [8]?
- (8) Does Eq. (2) have nonzero bounded solutions [8]?
- (9) Find the Wronskian of a fundamental system of solutions of Eq. (2), given that $W(0) = 1$ [5].
- (10) Write out Eq. (2) explicitly and solve it [10].
- (11) Find the eigenvalues and eigenvectors of the monodromy operator for the linearized equation along the solution with initial condition $x = \pi/2, \dot{x} = 0$ [16].
- (12) Prove that Eq. (1) has a 2π -periodic solution depending smoothly on ε and vanishing at $x = \pi$ for $\varepsilon = 0$ [6].
- (13) Find the derivative of this solution with respect to ε at $\varepsilon = 0$ [6].

III. Consider the equation $u_t + uu_x = -\sin x$.

(14) Write out the equation of characteristics [2].

(15) Find the largest value of t for which the solution of the Cauchy problem with $u|_{t=0} = 0$ can be extended to $[0, t)$ [8].

Variant 2

I. Let a vector field in three-dimensions have the origin as its singular point and let one of the eigenvalues of this singular point be equal to zero and the other two purely imaginary.

- (1) Reduce to normal form the terms of degree 1 of the Taylor series expansion at zero of the components of the field [1].
- (2) Do the same for terms of degree 2 [3].
- (3) Do the same for terms of any degree [8].
- (4) Average the system with respect to the fast rotation given by the linear part of the field [12].

II. Let us have a family of fields depending on a parameter and containing the field in part I for the zero value of the parameter.

- (5) Reduce a beginning segment of the Taylor series at zero of the fields of the family to the simplest possible form by a diffeomorphism depending smoothly on parameters varying in the neighborhood of zero [10].
- (6) Average the same system with respect to the fast rotation given by the linear part of the initial field [20].

III. In the space of 1-jets of vector fields in three-dimensions we consider the manifold of jets with one vanishing and two purely imaginary eigenvalues at the singular points.

- (7) Find the codimension of the indicated manifold [2].
- (8) Write the condition of transversality of the family, given in the form found in problem (5), to the indicated manifold [8].
- (9) Analyze the bifurcations of singular points in generic two-parameter families transversal to the indicated manifold [10].
- (10) Analyze the bifurcations of cycles from these singular points [15].
- (11) Study the existence and smoothness of a phase curve connecting these singular points [15].

IV. Let a straight line going through zero be distinguished in the plane. A diffeomorphism of the plane is said to be *distinguished* if it transforms the distinguished line into itself. A vector field is said to be distinguished if it is tangent to the distinguished line at all of its points. Let a distinguished field be given which has a singular point at zero with two vanishing eigenvalues.

- (12) Reduce a segment of the Taylor series of the field at zero to the simplest possible form by means of a distinguished diffeomorphism [12].

- (13) Reduce a family of distinguished fields, which is a deformation of the given field, to formal normal form by means of distinguished formal diffeomorphisms depending formally smoothly on the parameters varying in the neighborhood of zero [16].
- (14) Analyze the bifurcations of singular points in generic families obtained from the normal forms of problem (13) by omitting terms of high degree [18].
- (15) Apply the results of problems (12)–(14) to the study of bifurcations of the phase portrait of a field with one vanishing and two purely imaginary eigenvalues [25].

Additional Problems

- (1) Let $\dot{z} = \varepsilon z + Az|z|^2 + \bar{z}^3$. Prove that the number of limit cycles is not greater than 1 if $|\operatorname{Re} A| > 1$. *Hint*: Divide the field by $z\bar{z}$ and use the formula $\operatorname{div} P(z, \bar{z}) = 2 \operatorname{Re}(\partial P/\partial z)$.
- (2) Let $A = (3 + i)/\sqrt{2}$. Then, for $\arg \varepsilon = 5\pi/4$, the singular separatrix of every saddle-node coincides with the nonsingular separatrix of the next saddle-node. *Hint*: At the moment of saddle-node coalescence, the equation can be reduced to the form $\dot{w} = e^{i\theta} [Rw(|w|^2 - 1) + i(\bar{w}^2 - w^2)]$, where $A = (R - i)e^{i\theta}$. If $R = 2$ and $\theta = \pi/4$, the separatrices are straight lines.
- (3) Analyze the curves in the complex A -plane which divide domains where, when $\arg \varepsilon$ varies, singular points coalesce on the cycle, inside the cycle, and outside the cycle. *Hint*: If one varies θ , the field vectors rotate. The curves are located approximately like the four parabolas $a^2 = 2(\pm b \pm 1)$, $A = a + ib$.
- (4) For small $|\operatorname{Re} A|$ and $1 < |\operatorname{Im} A| < c \approx 4.11$, the equation of problem (1) has (for appropriate ε) two limit cycles, with nine singular point inside the inner cycle. For $|\operatorname{Im} A| > c$, only one cycle exists (Neiřtadt). The boundary between domains of existence of one or two cycles looks like an ellipse with the major axis $1 \leq |\operatorname{Im} A| \leq 4.11$ and the minor axis of length 2.
- (5) Prove that there are no limit cycles in the generalized system of Lotka–Volterra, $\dot{x} = x(\alpha + ax + by)$, $\dot{y} = y(\beta + cx + dy)$. *Hint*: At stability loss, the system has a first integral: a product of degrees of three linear functions (Bautin).

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