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# Abbreviations

a.a.	almost all
a.e.	almost everywhere
a.s.	almost surely
i.i.d.	independent and identically distributed
l.s.c.	lower semicontinuous
u.s.c.	upper semicontinuous
p.m.	probability measure
i.p.m.	invariant probability measure
AC	average cost
ACOE	Average Cost Optimality Equation
ACOI	Average Cost Optimality Inequality
ADO	asymptotic discount optimality
DC	discounted cost
DCOE	discounted cost optimality equation
DP	dynamic programming
ETC	expected total cost
IFS	iterated function system
LCSM	locally compact separable metric
LP	linear programming
LLN	Law of Large Numbers
MCM	Markov control model
MCP	Markov control process
O.O.	overtaking optimality
P.E.	Poisson equation

PI	policy iteration
PIA	policy iteration algorithm
RH	rolling horizon
VI	value iteration

# Glossary of notation

- end of proof or example or remark  
 := equality by definition  
 $I_B$  indicator function of a set  $B$ , defined as

$$I_B(x) := \begin{cases} 1 & \text{if } x \in B, \\ 0 & \text{otherwise.} \end{cases}$$

$$r^+ := \max(r, 0), \quad r^- := -\min(r, 0)$$

## Chapter 7

### Section 7.1

- $X$  Borel (state) space  
 $\mathcal{B}(X)$  Borel  $\sigma$ -algebra

$\|\mu\|_{TV}$  total variation norm of a measure  $\mu$

$\|\mu\|_w$   $w$ -norm of a measure  $\mu$

$\mathbb{M}(X)$  Banach space of signed measures  $\mu$  on  $X$  with  $\|\mu\|_{TV} < \infty$

### Section 7.2

- $w$  weight function  
 $\|u\|$  sup norm of a function  $u$   
 $\|u\|_w$   $w$ -norm of a function  $u$   
 $\mathbb{B}(X)$  Banach space of bounded measurable functions on  $X$   
 $\mathbb{B}_w(X)$  Banach space of measurable functions on  $X$  with finite  $w$ -norm

$\mathbb{M}_w(X)$  Banach space of signed measures  $\mu$  on  $X$  with  $\|\mu\|_w < \infty$

$Q(B|x)$  signed kernel on  $X$

$$Qu(x) := \int_X u(y)Q(dy|x)$$

$$\mu Q(B) := \int_X Q(B|x)\mu(dx)$$

$\|Q\|_w$   $w$ -norm of a signed kernel  $Q$

$\delta_x$	Dirac measure at $x \in X$	$\varphi^\infty$	randomized stationary policy, $\varphi \in \Phi$
$QR$	composition of the signed kernels $Q$ and $R$	$f^\infty$	deterministic stationary policy, $f \in F$
$Q^n := QQ^{n-1}$		$\Pi$	set of all control policies
$Q^0(B x) := \delta_x(B)$		$\Pi_{RM}$	set of randomized Markov policies
<b>Section 7.3</b>		$\Pi_{RS}$	set of randomized stationary policies $\varphi^\infty$
$P(B x)$	transition probability function of a Markov chain	$\Pi_D$	set of deterministic policies
$P^t(B x)$	$t$ -step transition probability	$\Pi_{DM}$	set of deterministic Markov policies
$\tau_B$	hitting time of the set $B$	$\Pi_{DS}$	set of deterministic stationary policies $f^\infty$
$\eta_B$	occupation time of the set $B$	$c(x, \varphi) \equiv c_\varphi(x) := \int_A c(x, a)\varphi(da x)$	for $\varphi \in \Phi$
$L(x, B) := P_x(\tau_B < \infty)$		$c(x, f) \equiv c_f(x) := c(x, f(x))$	for $f \in F$
$U(x, B) := E_x(\eta_B)$		$Q(\cdot x, \varphi) \equiv Q_\varphi(\cdot x) := \int_A Q(\cdot x, a)\varphi(da x)$	for $\varphi \in \Phi$
$\nu \ll \mu$	the measure $\nu$ is absolutely continuous with respect to the measure $\mu$	$Q(\cdot x, f) \equiv Q_f(\cdot x) := Q(\cdot x, f(x))$	for $f \in F$
$\lambda$	maximal irreducibility measure	$(\Omega, \mathcal{F})$	canonical measurable space
$B(X)^+$	family of sets $B \in \mathcal{B}(X)$ with $\lambda(B) > 0$	$P_\nu^\pi$	p.m. on $(\Omega, \mathcal{F})$ determined by the policy $\pi$ and the initial distribution $\nu$
$\int \nu d\mu \equiv \mu(\nu) \equiv \langle \mu, \nu \rangle$		$P_x^\pi := P_\nu^\pi$	if $\nu = \delta_x$
$C_b(X)$	Banach space of continuous bounded functions on $X$	$E_\nu^\pi$	expectation with respect to $P_\nu^\pi$
		$E_x^\pi := E_\nu^\pi$	if $\nu = \delta_x$

**Chapter 8**

**Section 8.2**

$\mathcal{M} := (X, A, \{A(x) x \in X\}, Q, c)$	Markov control model
$\mathbb{K}$	set of feasible state-action pairs
$\mathbb{F}$	set of decision functions (or selectors)
$\Phi$	set of randomized decision functions
$H_t$	family of admissible histories up to time $t$
$\pi$	control policy

**Section 8.3**

$V(\pi, x)$	$\alpha$ -discounted cost ( $0 < \alpha < 1$ ) when using the policy $\pi$ , given the initial state $x$
$V^*(x)$	$\alpha$ -discount value function
$V_n(\pi, x)$	$\alpha$ -discounted $n$ -stage expected cost
$v_n(x)$	$\alpha$ -value iteration ( $\alpha$ -VI) function, $n = 1, 2, \dots$
$\ Q\ _w$	$w$ -norm of the transition law $Q$

$T_\alpha$  DP operator

**Section 8.4**

$\mathbb{F}_n$  set of  $\alpha$ -VI decision function,  $n = 1, 2, \dots$

$\mathbb{F}$  set of  $\alpha$ -discount optimal decision functions

$D(x, a)$   $\alpha$ -discount discrepancy function

$A_*(x)$   $\alpha$ -discount optimal control actions in the state  $x$

$A_n(x)$   $\alpha$ -VI optimal control actions in the state  $x$ ;  $n = 1, 2, \dots$

$D_n(x, a)$   $\alpha$ -VI discrepancy function,  $n = 1, 2, \dots$

**Section 8.5**

$\mathbb{L}(X)$  family of l.s.c. functions on  $X$

$\mathbb{L}_w(X) := \mathbb{L}(X) \cap \mathbb{B}_w(X)$

$C(X)$  family of continuous functions on  $X$

$C_w(X) := C(X) \cap \mathbb{B}_w(X)$

$C_b(X)$  family of continuous bounded functions on  $X$

**Chapter 9**

**Section 9.1**

$V_1(\pi, x)$  expected total cost (ETC) when using the policy  $\pi$ , given the initial state  $x$

$V_1^*(x)$  ETC value function

**Section 9.2**

$\bar{\mathbb{R}}$  extended real numbers

$r^+ := \max(r, 0)$

$r^- := \max(-r, 0) = -\min(r, 0)$

**Section 9.3**

$J_n(\pi, x)$   $n$ -stage ETC when using the policy  $\pi$ , given the initial state  $x$

$J_n^*(x)$   $n$ -stage optimal ETC

$V_\alpha(\pi, x) \equiv V(\pi, x)$   $\alpha$ -discounted cost,  $0 < \alpha < 1$

$V_\alpha^*(x) \equiv V^*(x)$   $\alpha$ -discount value function

$V_1^{(+)}(\pi, x) := E_x^\pi \left( \sum_{t=0}^{\infty} c_t^+ \right)$

$V_1^{(-)}(\pi, x) := E_x^\pi \left( \sum_{t=0}^{\infty} c_t^- \right)$

$V_1^n(\pi, x)$  ETC from time  $n$  onwards when using the policy  $\pi$ , given the initial state  $x_0 = x$

**Section 9.4.**

$V_1(\pi, \nu)$  ETC when using the policy  $\pi$ , given the initial distribution  $\nu$

$\mu_\nu^\pi$  ETC-expected occupation measure on  $X \times A$  when using the policy  $\pi$ , given the initial distribution  $\nu$

$\widehat{\mu}_\nu^\pi$  marginal of  $\mu_\nu^\pi$  on  $X$

$\mu_{\nu,t}^\pi$  distribution of  $(x_t, a_t)$  when using the policy  $\pi$ , given the initial distribution  $\nu$ ,  $t = 0, 1, \dots$

$\widehat{\mu}_{\nu,t}^\pi$  marginal of  $\mu_{\nu,t}^\pi$  on  $X$

$\pi^{(1)}$  1-shift policy determined by  $\pi$

**Section 9.5**

$T := T_1$  dynamic programming operator (when  $\alpha = 1$ )

$\mathcal{U}$  see Definition 9.5.1

$D_1(x, a)$  ETC-discrepancy function

$\pi^{(n)}$   $n$ -shift policy determined by  $\pi$  ( $n = 0, 1, \dots$ )

$M_n^*$  see (9.5.12)



**Section 9.6**

$Q_\varphi(\cdot|x) \equiv Q(\cdot|x, \varphi)$   
 $\quad := \int_A Q(\cdot|x, a)\varphi(da|x)$   
 $Q_t(\cdot|x) := Q_{\varphi_t}(\cdot|x)$   
 $Q_\pi^t := Q_0 Q_1 \cdots Q_{t-1}$  for  $t = 1, 2, \dots$   
 $\|Q_\varphi\|_w$   $w$ -norm of  $Q_\varphi$

**Chapter 10**

**Section 10.1**

$OC(\pi, x)$  opportunity cost of policy  $\pi$ , given the initial state  $x$   
 $D(\pi, x)$  Dutta's criterion  
 $J(\pi, x)$  expected average cost (AC) when using the policy  $\pi$ , given the initial state  $x$   
 $J^*(x)$  optimal expected AC

**Section 10.2**

$Q_f^t(\cdot|x) \equiv Q^t(\cdot|x, f)$   $t$ -step transition probability,  $t = 0, 1, \dots$   
 $\|Q_f\|_w$   $w$ -norm of  $Q_f$   
 $\mu_f$  i.p.m. of  $Q_f$   
 $L_1(\mu) := L_1(X, \mathcal{B}(X), \mu)$   
 $\mu_f(u) := \int_X u d\mu_f$   
 $c_f(x) \equiv c(x, f) := c(x, f(x))$   
 $J(f) := \mu_f(c_f)$   
 $h_f$  bias function of  $f \in \mathbb{F}$

**Section 10.3**

$\mathbb{F}_{AC}$  family of AC-optimal decision functions  
 $\mathbb{F}_{ca}$  family of canonical decision functions  
 $\mathbb{F}_{bias}$  family of bias-optimal decision functions  
 $(\rho^*, h^*)$  solution of the ACOE  
 $J_n(\pi, x, h)$   $n$ -stage ETC with terminal cost function  $h$   
 $J_n^*(x, h)$  value function for  $J_n(\pi, x, h)$

$\lambda$  irreducibility measure  
 $\widehat{h}(x)$  optimal bias function  
 $A^*(x)$  AC-canonical control actions at state  $x$

**Section 10.4**

$u_\alpha(\cdot) := V_\alpha^*(\cdot) - V_\alpha^*(z)$   
 $\rho(\alpha) := (1 - \alpha)V_\alpha^*(z)$

**Section 10.7**

$\mathcal{M}_{bias}$  see (10.7.3)

**Chapter 11**

**Section 11.1**

$J_n^0(\pi, \nu)$   $n$ -stage sample-path cost when using the policy  $\pi$ , given the initial distribution  $\nu$   
 $J^0(\pi, \nu)$  long-run sample-path AC  
 $J^I(\pi, \nu)$  limit-infimum expected AC  
 $\text{Var}(\pi, \nu)$  limiting average variance when using the policy  $\pi$ , given the initial distribution  $\nu$   
 $\rho_{\min}$  see (11.1.12)  
 $\widehat{\rho}$  see (11.1.15)  
 $\rho^*$  see (11.1.23)  
 $\mathcal{P}(X)$  set of probability measures on  $X$

$\mathcal{P}_\delta(X) := \{\delta_x | x \in X\}$

**Section 11.2**

$P^{(n)}(\cdot|x)$  expected average occupation measure  
 $\sigma^2(c, x)$  limiting average variance for a Markov chain  
 $\sigma_c^2$  see (11.2.11)  
 $\psi(x)$  see (11.2.12)  
 $Y_t$  see (11.2.17)

$$M_n := \sum_{t=1}^n Y_t$$

$$\mathcal{F}_t := \sigma\{x_0, \dots, x_t\}$$

**Section 11.3**

- $\mathcal{P}_w(X)$  see (11.3.2)
- $(\rho^*, h_*)$  solution of the ACOE
- $\underline{J}^0(\pi, x)$  lim-inf sample path AC
- $\psi(x, a)$  see (11.3.5)
- $\psi_f(x) := \psi(x, f(x))$
- $(\sigma_*^2, V_*)$  see (11.3.17)
- $v(x) := w(x)^{1/2}$
- $\mathcal{F}_t(\pi, x)$  see (11.3.23)
- $Y_t(\pi, x)$  see (11.3.24)
- $M_n(\pi, x)$  see (11.3.25)
- $\widehat{D}(x, a)$  AC-discrepancy function
- $\Psi_f(X)$  see (11.3.35)
- $\sigma^2(f) := \text{Var}(f^\infty, \cdot)$
- $\mathcal{M}\text{var}$  MCM for the variance minimization problem

**Chapter 12**

**Section 12.2**

- $(\mathcal{X}, \mathcal{Y})$  dual pair of vector spaces  $\mathcal{X}, \mathcal{Y}$
- $\sigma(\mathcal{X}, \mathcal{Y})$  weak topology on  $\mathcal{X}$
- $\sigma(\mathcal{Y}^*, \mathcal{Y})$  weak\* topology on  $\mathcal{Y}^*$
- $C_0(S)$  Banach space of continuous functions vanishing at infinity
- $\langle \mu, u \rangle := \int u d\mu$
- $G^*$  adjoint of linear map  $G$
- $w_0$  weight function on  $X$
- $w$  weight function on  $X \times Y$
- $\mathbb{M}_w(X)_+$  positive cone in  $\mathbb{M}_w(X)$
- $\mathbb{B}_w(X)_+$  positive cone in  $\mathbb{B}_w(X)$
- $K^*$  dual cone of the positive cone  $K$
- $\widehat{\mu}$  marginal on  $X$  of the measure  $\mu$  on  $X \times Y$

- $\mathbb{P}$  primal linear program
- $\text{inf } \mathbb{P}$  value of  $\mathbb{P}$
- $\text{min } \mathbb{P}$  optimal value of  $\mathbb{P}$
- $\mathbb{P}^*$  dual of  $\mathbb{P}$
- $\text{sup } \mathbb{P}^*$  value of  $\mathbb{P}^*$
- $\text{max } \mathbb{P}^*$  optimal value of  $\mathbb{P}^*$

**Section 12.3**

- $w(x, a)$  weight function on  $\mathbb{K}$ ; see (12.3.5)
- $w_0(x)$  weight function on  $X$ ; see (12.3.5)
- $L_0\mu$  see (12.3.12)
- $L_1\mu$  see (12.3.13)
- $L\mu$  see (12.3.14)
- $L^*(\rho, \mu)$  see (12.3.15)
- (P) AC-related primal linear program
- (P\*) dual of (P)
- $\mathcal{L}$  perturbation of  $L$

**Section 12.5**

- $\mathcal{C}(X)$  a countable dense subset of  $C_0(X)$
- $\{\mathcal{C}_k\}$  increasing sequence of finite sets  $\mathcal{C}_k \uparrow \mathcal{C}(X)$
- $\mathbb{P}(\mathcal{C}_k)$  aggregation of constraints of (P)
- $\{\varepsilon_k\}$  sequence of numbers  $\varepsilon_k \downarrow 0$
- $\mathbb{P}(\mathcal{C}_k, \varepsilon_k)$  aggregation-relaxation of (P)
- $D$  a countable dense subset of  $\mathbb{K}$
- $\{D_n\}$  increasing sequence of finite sets  $D_n \uparrow D$
- $\Delta_n := \mathcal{P}(D_n)$
- $\Delta := \bigcup_{n=1}^\infty \Delta_n$
- $\mathbb{P}(\mathcal{C}_k, \varepsilon_k, \Delta_n)$  aggregation-relaxation-inner approximation of (P)

**Section 12.6**

$$E_j := \{(x, a) \in \mathbb{K} | c(x, a) \leq j\}$$

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