

# Appendix A

## Basic Concepts from Probability Theory

In this Appendix, we give a brief introduction to elementary probability theory, which is the basis of the mathematical approach to modelling failures. The presentation is non-rigorous. The objective is to develop an intuitive feel for the topic that forms the foundation for most models used in solving reliability-related problems.

### A.1 Scalar Random Variables

Consider an experiment whose outcome is not known in advance but is such that the set of all outcomes (called the “sample space”  $\mathfrak{S}$ ) is known. Any subset of the sample space  $\mathfrak{S}$  is called an event. A random variable is a function, which maps outcomes from the sample space  $\mathfrak{S}$  to  $\mathfrak{R}$  the space of real numbers. In other words, for every outcome  $\omega$  in the sample space  $\mathfrak{S}$ ,  $X(\omega)$  assigns a real number to  $\omega$ . It can be either discrete or continuous. A discrete random variable takes on at most a countable number of values [for example the set of non-negative integers), and a continuous random variable can take on values from a set of possible values, which is uncountable (for example values in the interval  $(-\infty, \infty)$ ].

Because the outcomes are uncertain, the value assumed by  $X$  is uncertain before the event occurs. Once the event occurs,  $X$  assumes a certain value. The standard convention used is as follows:  $X$  (upper case) represents the random variable before the event, and the value it assumes after the event is represented by  $x$  (lower case).

#### A.1.1 Distribution and Density Functions

The distribution function  $F(x; \theta)$  is defined as the probability that  $X \leq x$  and is given by

$$F(x; \theta) = P\{X \leq x\} \tag{A.1}$$

The domain of  $F(x; \theta)$  is  $(-\infty, \infty)$ , the range is  $[0, 1]$ , and  $\theta$  denotes the set of parameters of the distribution function. Often the parameters are omitted for

notational ease, so that one uses  $F(x)$  instead of  $F(x, \theta)$ . We will do this in the remainder of the Appendix.

$F(x)$  has the following properties:

- $F(x)$  is a non-decreasing function of  $x$ .
- $F(-\infty) = 0$  and  $F(\infty) = 1$
- For  $x_1 < x_2$ ,  $P\{x_1 < X \leq x_2\} = F(x_2) - F(x_1)$

When  $X$  is continuous valued and  $F(x)$  is differentiable, the density function  $f(x)$  is given by

$$f(x) = \frac{dF(x)}{dx} \quad (\text{A.2})$$

$f(x)$  may be interpreted as

$$P\{x < X \leq x + \delta x\} \approx f(x)\delta x + O(\delta x^2). \quad (\text{A.3})$$

When  $X$  takes on only values in a set  $(x_1, x_2, \dots, x_n)$ , with  $n$  being finite or infinite, the probability that  $X = x_i$  is given by

$$p_i = P\{X = x_i\}, \quad i = 1, 2, \dots, n \quad (\text{A.4})$$

In this case,  $X$  is called a *discrete* random variable, and the CDF is a step function with steps of height  $p_i$  at each of the possible values  $x_i$ . The probabilities  $p_i$  have the following properties: (i)  $p_i \geq 0$  and (ii)  $\sum_{i=1}^n p_i = 1$

### Moments of Random Variables

The  $j$ th *moment* of the random variable  $X$ ,  $M_j(\theta)$ , is given by<sup>1</sup>

$$M_j(\theta) = E[X^j] = \begin{cases} \int_0^{\infty} x^j f(x) dx, & \text{if } X \text{ is continuous} \\ \sum_x x^j P\{X = x\}, & \text{if } X \text{ is discrete} \end{cases} \quad (\text{A.5})$$

The first moment of  $X$  is called the *mean* and is usually denoted by  $\mu$ , so that

$$\mu = E[X] \quad (\text{A.6})$$

The  $j$ th *central moment* of  $X$ ,  $\mu_j$ , is given by

$$\mu_j = E[(X - \mu)^j] \quad (\text{A.7})$$

The second central moment of  $X$  is called the *variance* and is usually denoted by  $\sigma^2$ , so that

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<sup>1</sup> The parameters are omitted for notational ease, so that one uses  $M_j$  instead of  $M_j(\theta)$ .

$$\sigma^2 = E[(X - \mu)^2] \quad (\text{A.8})$$

$\sigma$  is called the *standard deviation*.

### A.1.2 Discrete Distributions

The following are some well-known discrete distributions that are useful in failure modelling<sup>2</sup>:

*Bernoulli Distribution*  $X$  assumes two possible values, 0 and 1, with probabilities given by

$$p_0 = p \quad \text{and} \quad p_1 = (1 - p) \quad (\text{A.9})$$

The parameter set is  $\theta = \{p\}$ , with  $0 \leq p \leq 1$ .

*Binomial Distribution*  $X$  assumes integer values from 0 to  $n$ , where  $n$  is a positive integer and  $p_i$ ,  $0 \leq i \leq n$ , is given by

$$p_i = \frac{n!}{i!(n-i)!} p^i (1-p)^{(n-i)} \quad (\text{A.10})$$

The parameter set is  $\theta = \{n, p\}$  with  $0 \leq p \leq 1$  and  $0 < n < \infty$ .

*Geometric Distribution*  $X$  assumes integer values from 0 to  $\infty$ , with probabilities  $p_i$ ,  $0 \leq i < \infty$ , given by

$$p_i = (1-p)^i p \quad (\text{A.11})$$

The parameter set is  $\theta = \{p\}$  with  $0 \leq p \leq 1$ .

*Poisson Distribution*  $X$  assumes integer values from 0 to  $\infty$ .  $p_i$ ,  $0 \leq i < \infty$ , is given by

$$p_i = \frac{e^{-\lambda} \lambda^i}{i!} \quad (\text{A.12})$$

The parameter set is  $\theta = \{\lambda\}$ , with  $\lambda > 0$ .

### A.1.3 Continuous Distributions

Continuous distribution functions useful in failure modelling can be grouped into three categories—(1) basic, (2) those derived from basic and (3) those involving two or more basic/derived distributions<sup>3</sup>:

<sup>2</sup> Most basic books on statistics and probability discuss some of the well-known distributions. (Johnson and Kotz 1969a, b) gives a more comprehensive coverage of many discrete distributions.

<sup>3</sup> Most basic books on statistics and probability discuss some of the well-known distributions. (Johnson and Kotz 1970a, b) give a more comprehensive coverage of many continuous distributions.

### A.1.3.1 Basic Distributions and Density Functions

*Exponential Distribution* The distribution function for the exponential distribution is given by

$$F(x; \theta) = 1 - e^{-\lambda x}, \quad x \geq 0. \quad (\text{A.13})$$

The parameter set is  $\theta = \{\lambda\}$ , with  $\lambda > 0$ .

*Gamma Distribution* The gamma density function is given by

$$f(x; \theta) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)}, \quad x \geq 0 \quad (\text{A.14})$$

The parameter set is  $\theta = \{\alpha, \beta\}$ , with  $\alpha > 0$  and  $\beta > 0$ . Here,  $\Gamma(\cdot)$  is the gamma function. Extensive tables can be found in Abramowitz and Stegun (1964).

*Weibull Distribution* The two-parameter Weibull distribution function is given by

$$F(x; \theta) = 1 - e^{-(x/\alpha)^\beta}, \quad x \geq 0. \quad (\text{A.15})$$

The parameter set is  $\theta = \{\alpha, \beta\}$ , with  $\alpha > 0$  and  $\beta > 0$ .

### A.1.3.2 Derived Distributions and Density Functions

The derived distributions given below are obtained by (1) transformation of the random variable from a basic distribution, (2) modification of the form of a basic distribution by introducing additional parameters (for example the exponentiated Weibull distribution) and (3) devising forms that involve two or more basic distribution functions (for example mixtures of distributions, competing risk models). We present some of each form of derived distribution.<sup>4</sup>

*Three-Parameter Weibull Distribution* This is an extension of the two-parameter Weibull distribution (A.16), and the distribution function is given by

$$F(x; \theta) = 1 - e^{-\{(x-\tau)/\alpha\}^\beta} x \geq \tau. \quad (\text{A.16})$$

The additional parameter is the location parameter  $\tau > 0$ .

*Exponentiated Weibull Distribution* The distribution function is given by

$$F(x) = [1 - \exp\{-(x/\alpha)^\beta\}]^v, \quad x \geq 0, \quad (\text{A.17})$$

with  $v \geq 0$ . The distribution reduces to the two-parameter Weibull (A.15) when  $v = 1$ .

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<sup>4</sup> For additional details with regard to the three types, see (Blischke and Murthy 2000) and (Murthy et al. 2003).

### A.1.3.3 Distributions Involving Two or More Basic/Derived Distributions

*Mixtures of Distributions* A *finite mixture* of distributions is a weighted average of distribution functions given by

$$F(x) = \sum_{i=1}^K p_i F_i(x) \quad (\text{A.18})$$

with  $p_i \geq 0$ ,  $i = 1, 2, \dots, K$ ,  $\sum_{i=1}^K p_i = 1$  and  $F_i(x) \geq 0$ ,  $i = 1, 2, \dots, K$  distribution functions (called the *components* of the mixture).

*Competing Risks* The distribution function is given by

$$F(x) = 1 - \prod_{i=1}^K (1 - F_i(x)) \quad (\text{A.19})$$

*Multiplicative* The distribution function is given by

$$F(x) = \prod_{i=1}^K F_i(x), \quad x \geq 0 \quad (\text{A.20})$$

## A.2 Vector Random Variables

We now give important probability results for the case where two or more random variables are needed to represent the outcomes of an uncertain event.

### A.2.1 Two Random Variables

Let the two continuous random variables be denoted as  $X$  and  $Y$ .

#### A.2.1.1 Joint, Marginal and Conditional Distributions and Density Functions

The *joint distribution function*  $F(x, y)$  is given by

$$F(x, y) = P\{X \leq x, Y \leq y\} \quad (\text{A.21})$$

The random variables are said to be jointly continuous if there exists a function  $f(x, y)$ , called the *joint probability density function*, such that

$$f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y} \quad (\text{A.22})$$

The *marginal distribution functions*  $F_X(x)$  and  $F_Y(y)$  are given by

$$F_X(x) = F(x, \infty) \quad \text{and} \quad F_Y(y) = F(\infty, y) \quad (\text{A.23})$$

The marginal density functions are given by

$$f_X(x) = \frac{dF_X(x)}{dx} \quad \text{and} \quad f_Y(y) = \frac{dF_Y(y)}{dy}. \quad (\text{A.24})$$

The *conditional distribution* of  $X$  given that  $Y = y$  is denoted  $F(x|y)$  and is given by

$$F(x|y) = P\{X \leq x | Y = y\} \quad (\text{A.25})$$

The conditional distribution of  $Y$  given that  $X = x$ ,  $F(y|x)$ , is defined similarly.

For jointly continuous random variables with a joint density function  $f(x, y)$ , the *conditional probability density function* of  $X$ , given  $Y = y$ , is given by

$$f(x|y) = \frac{f(x, y)}{f_Y(y)} \quad (\text{A.26})$$

Similarly,

$$f(y|x) = \frac{f(x, y)}{f_X(x)} \quad (\text{A.27})$$

The random variables  $X$  and  $Y$  are said to be *independent* (or *statistically independent*) if and only if

$$F(x, y) = F_X(x) F_Y(y) \quad (\text{A.28})$$

for all  $x$  and  $y$ .

The results are similar for discrete random variables, with summation replacing integration.

### A.2.1.2 Moments of Two Random Variables

The covariance of  $X$  and  $Y$  is defined as

$$\text{Cov}(X, Y) = E[\{X - E[X]\}\{Y - E[Y]\}] = E[XY] - E[X] E[Y] \quad (\text{A.29})$$

The correlation  $\rho_{XY}$  is defined as

$$\rho_{XY} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}, \quad (\text{A.30})$$

where  $\sigma_x$  and  $\sigma_y$  are the standard deviations of  $X$  and  $Y$ , respectively. The random variables  $X$  and  $Y$  are said to be *uncorrelated* if  $\rho_{XY} = 0$ . Note that independent random variables are uncorrelated but that the converse is not necessarily true.

### A.2.1.3 Conditional Expectation

$E[X|Y = y]$  is called the conditional expectation of  $X$  given that  $Y = y$ . The unconditional expectation of  $X$ , given by

$$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx, \tag{A.31}$$

is related to the conditional expectation by the relation

$$E[X] = \int_{-\infty}^{\infty} E[X|Y = y] f_Y(y) dy. \tag{A.32}$$

This is written symbolically as

$$E[X] = E[E[X|Y]] \tag{A.33}$$

### A.2.1.4 Sum of Two Independent Random Variables

Let  $X$  and  $Y$  be two independent random variables with density functions  $f_X(x)$  and  $f_Y(y)$ , respectively, and let  $Z = X + Y$ . Then, the density function for  $Z$ ,  $f_Z(Z)$ , is given by

$$f_Z(z) = \int_{-\infty}^{\infty} f_Y(t) f_X(z - t) dt \tag{A.34}$$

or

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(t) f_Y(z - t) dt \tag{A.35}$$

This operation is called the convolution operation indicated by the symbol “\*”.

Thus,

$$f_Z(z) = f_X(z) * f_Y(z) = f_Y(z) * f_X(z) \tag{A.36}$$

### A.2.2 *The General Case*

The  $k$  ( $>2$ ) random variables may be represented by the vector  $(X_1, X_2, \dots, X_k)$ . The approach is similar to the two random variable case, but involving an  $k$ -dimensional distribution function  $F(x_1, x_2, \dots, x_k)$ . We have  $k$  marginal distributions and several different conditional distributions, depending on how the  $k$ -variables are divided into two sets, with the distribution of the first-set conditioned on the values of the variables in the second. Similarly, there are many different correlation coefficients. Details can be found in Johnson and Kotz (1972).

#### A.2.2.1 Sums of Independent Random Variables

When  $Z$  is the sum of  $n$  independent variables,  $X_i$  ( $i = 1, 2, \dots, n$ ), with respective density functions  $f_i(x)$ , then the density function for  $Z$  is given by

$$f_Z(z) = f_1(z) * f_2(z) * \dots * f_n(z) \quad (\text{A.37})$$

## References

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# Appendix B

## Introduction to Stochastic Processes

In this Appendix, we give a brief introduction to stochastic processes and discuss some of the processes that are used in the book. Our presentation will be intuitive and non-rigorous and will highlight the important concepts. Readers interested in a deeper understanding of the underlying theory should consult the references given at the end.

### B.1 Stochastic Processes

In Appendix A, we defined a random variable,  $X(\omega)$ , as function that map outcomes from the sample space to real numbers. A stochastic process  $X(t, \omega)$ ,  $t \in T$ , where  $T$  is a set of non-negative numbers, can be viewed as an extension of  $X(\omega)$  in the following sense— $t$  represents a time instant in the set  $T$ , which may be either finite or infinite. For a fixed  $t \in T$ ,  $X(t, \omega)$  is a random variable in the usual sense. For a fixed  $\omega$  (outcome),  $X(t, \omega)$  can be viewed as a function of  $t$  and  $X(t, \omega)$  denotes the state of the process at time  $t$ . If  $T$  is countable, then  $X(t, \omega)$  is called a *discrete-time stochastic process*. If  $T$  is a continuum, then it is called a *continuous-time stochastic process*. Henceforth, we omit  $\omega$  and represent  $X(t, \omega)$  as simply  $X(t)$ .

Let  $t_i, i = 1, 2, \dots, n$ , denote  $n$  different time instants. The probabilistic characterisation of the process  $X(t)$  at these  $n$  points can be done through the joint probability distribution

$$F(t_1, x_1; t_2, x_2; \dots; t_n, x_n) = P\{X(t_1) \leq x_1; X(t_2) \leq x_2; \dots; X(t_n) \leq x_n\} \quad (\text{B.1})$$

As  $n$  increases, this function becomes cumbersome and is of limited use in modelling real world problems.

## B.2 Markov Property

A stochastic process  $X(t)$  is said to have the *Markov property* if

$$P\{X(t + \tau) \leq x | X(u) = x(u), -\infty < u \leq t\} = P\{X(t + \tau) \leq x | X(t) = x(t)\} \quad (\text{B.2})$$

In other words, the probabilistic characterisation of  $X(t + \tau)$  (a future event) given  $\{X(u) = x(u), -\infty < u \leq t\}$  (past history and present value of the process) depends only on the present value  $X(t)$  and not its past values. This simplifies the mathematical characterisation of the process considerably. Using conditional probability, we have, for an increasing sequence in  $t_i$ ,

$$P\{X(t_1) \leq x_1; X(t_2) \leq x_2; \dots; X(t_n) \leq x_n\} = P\{X(t_n) \leq x_n | X(t_{n-1}) \leq x_{n-1}\} \dots P\{X(t_2) \leq x_2 | X(t_1) \leq x_1\} P\{X(t_1) \leq x_1\} \quad (\text{B.3})$$

Thus, the joint probability distribution for  $X(t)$  at  $n$  different points along the time axis can be obtained in terms of the conditional distribution of  $X(t)$  involving two different values of  $t$ . In other words, the probabilistic characterisation of the process can be done as a function of sets of four variables  $F(t_i, x_i; t_j, x_j)$  with

$$F(t_i, x_i; t_j, x_j) = P\{X(t_i) \leq x_i \text{ and } X(t_j) \leq x_j\} \quad (\text{B.4})$$

for all  $t_i$  and  $t_j$  over the interval  $T$  and all  $x_i$  and  $x_j$  over the real line.

## B.3 Classification of Stochastic Processes

Stochastic processes can be divided into four categories depending on whether:

1. the values assumed by the process  $X(t)$  are discrete or continuous, and
2. the values assumed by the time variable  $t$  are discrete or continuous.

We briefly discuss each of these four categories.

### Discrete State/Discrete Time Process

Here, both  $X(t)$  and  $t$  assume only discrete values. Let the values assumed by  $X(t)$  be denoted by  $s_i$ ,  $i = 1, 2, \dots, r$ .  $r$  may be either finite or infinite. The values assumed by  $t_i$ ,  $i = 1, 2, \dots$ , form an increasing sequence. If the process is Markovian, then it is called a discrete-time Markov chain (DTMC).

### Discrete State/Continuous-Time Process

Here,  $X(t)$  assumes only discrete values with  $r$  either finite or infinite, and  $t$  assumes a continuous range of values in the interval  $(-\infty, \infty)$ . If the process is Markovian, it is called a continuous-time Markov chain (CTMC).

### Continuous State/Discrete-Time Process

In this case,  $X(t)$  assumes a continuous range of values and  $t$  assumes discrete values. If the process is Markovian, it is called a discrete-time Markov process.

### Continuous State/Continuous-Time Process

In this process, both  $X(t)$  and  $t$  assume continuous ranges of values. If the process is Markovian, it is called a continuous-time Markov process (or simply a Markov process).

A further subclassification is *stationary* and *non-stationary* stochastic processes. A stochastic process is said to be stationary if the joint distribution function is invariant under a shift in  $t$ , i.e. if

$$F(t'_1, x_1; t'_2, x_2; \dots; t'_n, x_n) = F(t_1, x_1; t_2, x_2; \dots; t_n, x_n) \quad (\text{B.5})$$

with  $t'_i = t_i + \tau$ , ( $i = 1, 2, \dots, n$ ) for all  $\tau$  and  $n$ .

## B.4 Point Processes

A *point process* is a continuous-time stochastic process characterised by events that occur randomly along the time continuum. An example, in the context of reliability, is an item being put into operation or an item failing. The theory of point processes is very rich, as a variety of such processes have been formulated and studied. Of particular interest to reliability modelling is the counting process.

### B.4.1 Counting Processes

A point process  $\{N(t), t \geq 0\}$  is a *counting process* if it represents the number of events that have occurred until time  $t$ . It must satisfy:

1.  $N(t) \geq 0$ .
2.  $N(t)$  is integer valued.
3. If  $s < t$ , then  $N(s) \leq N(t)$ .
4. For  $s < t$ ,  $\{N(t) - N(s)\}$  is the number of events in the interval  $(s, t]$ .

We shall confine ourselves to  $t \geq 0$ . The behaviour of  $N(t)$ , for  $t \geq 0$ , depends on whether or not  $t = 0$  corresponds to the occurrence of an event. The analysis of the case with  $t = 0$  corresponding to the occurrence of an event is simpler than the alternate case. Also, we assume that  $N(0) = 0$ .

A counting process  $\{N(t), t \geq 0\}$  is said to have *independent increments* if, for all choices  $0 \leq t_1 < t_2 < \dots < t_n$ , the  $(n - 1)$  random variables

$\{N(t_2) - N(t_1)\}, \{N(t_3) - N(t_2)\}, \dots, \{N(t_n) - N(t_{n-1})\}$  are independent. A counting process  $\{N(t), t \geq 0\}$  is said to have *stationary independent increments*

if, for each  $s > 0$ ,  $\{N(t_2 + s) - N(t_2)\}$  and  $\{N(t_1 + s) - N(t_1)\}$  have the same distribution function, i.e. if the distribution function of  $\{N(t + s) - N(t)\}$  does not depend on  $t$ .

Two special counting processes of particular importance to reliability modelling are (1) the Poisson process and (2) the renewal process.

## B.4.2 Poisson Processes

We first consider the stationary Poisson process and later discuss some extensions.

### B.4.2.1 Stationary Poisson Process

**Definition 1** A counting process,  $N(t)$ ,  $t \geq 0$ , is a *stationary or homogeneous Poisson process (HPP)* if

1.  $N(0) = 0$ .
2. The process has independent increments.
3. The number of events in any interval of length  $t$  is distributed according to Poisson distribution with parameter  $\lambda t$ , i.e.

$$P\{N(t + s) - N(s)\} = \frac{e^{-\lambda t} (\lambda t)^n}{n!} \quad (\text{B.6})$$

$n = 0, 1, 2, \dots$ , and for all  $s$  and  $t \geq 0$ .

It can be shown through simple analysis [see, e.g. Ross (1970)] that for an HPP, the times between events (also called inter-event times) are independent and identically distributed exponential random variables with mean  $(1/\lambda)$ . This is the basis of a second definition for an HPP.

**Definition 2** Consider a counting process. Let  $X_1$  denote the time instant of the first event occurrence, and for  $j \geq 2$ , let  $X_j$  denote the time interval between the  $(j - 1)$ st and  $j$ th events. The counting process is an HPP with parameter  $\lambda$  if the sequence  $X_j$ ,  $j \geq 1$ , are independent and identically distributed exponential random variables with mean  $(1/\lambda)$ .

We also have a third definition for an HPP [see Ross (1970)].

**Definition 3** A counting process  $\{N(t), t \geq 0\}$  is an HPP if

1. The probability of an event occurring in  $[t, t + \delta t)$  is  $\lambda \delta t + o(\delta t)$ .
2. The probability of two or more events occurring in  $[t, t + \delta t)$  is  $o(\delta t)$ .
3. The occurrence of an event in  $[t, t + \delta t)$  is independent of the number of events in  $[0, t)$ .

$\lambda$  is called the *intensity* of the process.

*Comment* The above discussion illustrates the point that there is more than one way of characterising a counting process. In the context of reliability modelling, a particular characterisation may be more appropriate than alternate, equivalent characterisations. For example in the case of non-repairable items, Definition 2 is more appropriate; in the case of repairable items with the item being subjected to minimal repair after each failure, Definition 3 is more appropriate.

*Expected Number of Events in  $[0, t]$*

Let  $M(t)$  denote the expected number of events in  $[0, t)$ . Since  $N(t)$  is distributed according to Poisson distribution with parameter  $\lambda t$ , we have

$$M(t) = E[N(t)] = \lambda t \tag{B.7}$$

**B.4.2.2 Non-stationary Poisson Process**

In an HPP, the probability of an event occurring in  $[t, t + \delta t)$  is  $\lambda \delta t + o(\delta t)$ , with  $\lambda$  a constant. A *non-stationary* or *non-homogeneous* Poisson process (NHPP) is a natural extension in which  $\lambda$  changes with time.

A counting process  $\{N(t), t \geq 0\}$  is an NHPP if

1.  $N(0) = 0$ .
2.  $\{N(t), t \geq 0\}$  has independent increments.
3.  $P\{N(t + \delta t) - N(t) = 1\} = \lambda(t)\delta t + o(\delta t)$ .
4.  $P\{N(t + \delta t) - N(t) \geq 2\} = o(\delta t)$ .  $\lambda(t)$  is called the *intensity function*. Let

$$\Lambda(t) = \int_0^t \lambda(x) dx \tag{B.8}$$

Then, it can be shown [see Ross (1970)] that

$$P\{N(t + s) - N(t) = j\} = \frac{e^{-\{\Lambda(t+s) - \Lambda(t)\}} \{\Lambda(t + s) - \Lambda(t)\}^j}{j!} \tag{B.9}$$

for  $j \geq 0$ .

This result may be used to define an NHPP in a manner similar to Definition 1 for an HPP.

*Expected Number of Events in  $[0, t]$*

Since the probability of  $j$  events ( $j \geq 0$ ) in  $[0, t)$  is given by

$$P\{N(t) = j\} = \frac{e^{-\Lambda(t)} \{\Lambda(t)\}^j}{j!} \tag{B.10}$$

the expected number of events in  $[0, t)$ ,  $M(t)$ , is given by

$$M(t) = E[N(t)] = \Lambda(t)$$

### B.4.3 Renewal Processes

We first consider the ordinary renewal process and then discuss some extensions.

#### B.4.3.1 Ordinary Renewal Processes

As indicated earlier, a counting process characterised in terms of inter-event times is a stationary Poisson process if these times are independent and identically distributed exponential random variables. A natural generalisation is one where the inter-event times are independent and identically distributed with an *arbitrary* distribution.

A counting process  $\{N(t), t \geq 0\}$  is an ordinary renewal process if

1.  $N(0) = 0$ .
2.  $X_1$ , the time to occurrence of the first event (from  $t = 0$ ) and  $X_j, j \geq 2$ , the time between the  $(j - 1)$ st and  $j$ th events, are a sequence of independent and identically distributed random variables with distribution function  $F(x)$ .
3.  $N(t) = \text{Sup}\{n : S_n \leq t\}$ , where

$$S_0 = 0, \quad S_n = \sum_{i=1}^n X_i, \quad n \geq 1 \quad (\text{B.12})$$

[*Note:* The HPP is a special case of the ordinary renewal process with  $F(x)$  an exponential distribution function.]

#### *Distribution of $N(t)$*

Note that  $S_n$  is the time instant for the  $n$ th renewal (or event) and is the sum of  $n$  independent and identically distributed random variables. Since the  $X_i$ 's are distributed with distribution function  $F(x)$ , from a result in Appendix A, the distribution of  $S_n$  is given by the  $n$ -fold convolution of  $F$  with itself—i.e.

$$P\{S_n \leq x\} = F^{(n)}(x) = F(x) * F(x) * \cdots * F(x) \quad (\text{B.13})$$

It is easily seen that  $N(t) \geq n$  if and only if  $S_n \leq t$ . As a result,

$$\begin{aligned} P\{N(t) = n\} &= P\{N(t) \geq n\} \\ &\quad - P\{N(t) \geq (n + 1)\} = P\{S_n \leq t\} - P\{S_{n+1} \leq t\} \end{aligned} \quad (\text{B.14})$$

for  $n = 0, 1, \dots$ , where  $S_0 \equiv 0$ . Since

$$P\{S_n \leq t\} = F^{(n)}(t) \quad (\text{B.15})$$

where  $F^{(0)} \equiv 1$ , we have

$$P\{N(t) = n\} = F^{(n)}(t) - F^{(n+1)}(t) \quad (\text{B.16})$$

From this, expressions for the moments of  $N(t)$  can be obtained. Of particular interest in reliability analysis is the first moment, the expected number of renewals in  $[0, t)$ .

*Expected Number of Renewals in  $[0, t)$*

The expected number of renewals  $M(t)$  is given by the integral equation

$$M(t) = E[N(t)] = \sum_{n=0}^{\infty} n P\{N(t) = n\} \quad (\text{B.17})$$

Using (B.16), this can be written as

$$M(t) = \sum_{n=0}^{\infty} n \{F^{(n)}(t) - F^{(n+1)}(t)\} = \sum_{n=1}^{\infty} F^{(n)}(t) \quad (\text{B.18})$$

Using Laplace transforms, it can be shown that

$$M(t) = F(t) + \int_0^t M(t-x)f(x)dx \quad (\text{B.19})$$

This equation for  $M(t)$  can also be derived, using conditional expectation, as follows. Conditioned on  $X_1$ , the time to first failure,  $M(t)$  can be written as

$$M(t) = \int_0^{\infty} E[N(t)|X_1 = x] dF(x) \quad (\text{B.20})$$

But,

$$E[N(t)|X_1 = x] = \begin{cases} 0, & \text{if } x > t \\ 1 + M(t-x), & \text{if } x \leq t \end{cases} \quad (\text{B.21})$$

Using (B.21) in (B.20) yields (B.19). *Comment* One is using the “renewal property” in deriving the above expression. If the first failure occurs at  $x \leq t$ , then the renewals over  $(t-x)$  occur according to an identical renewal process and hence the expected number of renewals over this period is  $M(t-x)$ .

Equation (B.19) is called the *renewal integral equation*, and  $M(t)$  is called the *renewal function* associated with the distribution function  $F(t)$ .  $M(t)$  plays an important role in reliability analysis. In general, it is difficult to obtain  $M(t)$  analytically.

The renewal density function,  $m(t)$ , is given by

$$m(t) = \frac{dM(t)}{dt} \quad (\text{B.22})$$

and satisfies the integral equation

$$m(t) = f(t) + \int_0^t m(t-x)f(x)dx \quad (\text{B.23})$$

where  $f(t)$  is the density function associated with  $F(t)$ .

#### B.4.3.2 Delayed Renewal Process

A counting process  $\{N(t), t \geq 0\}$  is a delayed renewal process if

1.  $N(0) = 0$ .
2.  $X_1$ , the time to the first event, is a non-negative random variable with distribution function  $F(x)$ .
3.  $X_j, j \geq 2$ , the time intervals between the  $j$ th and  $(j-1)$ st events, are independent and identically distributed random variables with a distribution function  $G(x)$  different from  $F(x)$ .
4.  $N(t) = \text{Sup}\{n: S_n \leq t\}$  where  $S_n$  is given by (B.13).

[Note that when  $G(x)$  equals  $F(x)$ , then the delayed renewal process reduces to an ordinary renewal process.]

*Expected Number of Renewals in  $[0, t)$*

$M_d(t)$ , the expected number of renewals over  $[0, t)$  for the delayed renewal process is given by

$$M_d(t) = F(t) + \int_0^t M_g(t-x)f(x) dx \quad (\text{B.24})$$

where  $M_g(t)$  is the renewal function associated with the distribution function  $G(t)$ .

#### B.4.3.3 Alternating Renewal Process

In an ordinary renewal process, the inter-event times are independent and identically distributed. In an alternating renewal process, the inter-event times are all independent but not identically distributed. More specifically, the odd numbered inter-event times  $X_1, X_3, X_5, \dots$  have a common distribution function  $F(x)$  and the even numbered ones  $X_2, X_4, X_6, \dots$  have a common distribution function  $G(x)$ .

## B.4.4 Additional Topics from Renewal Theory

### B.4.4.1 Renewal-Type Equation

A renewal-type equation is an integral equation of the form

$$g(t) = h(t) + \int_0^t g(t-x) dF(x) \quad (\text{B.25})$$

where  $h(\cdot)$  and  $F(\cdot)$  are known functions and  $g(\cdot)$  is the unknown function to be obtained as a solution to the integral equation. Then,  $g(t)$  given by

$$g(t) = h(t) + \int_0^t h(t-x) dM(x) \quad (\text{B.26})$$

where  $M(x)$  is the renewal function associated with  $F(x)$  is a solution of (B.19).

### B.4.4.2 Renewal Reward Theorem

Consider an ordinary renewal process with inter-arrival times  $X_1, X_2, X_3, \dots$ . Suppose that a reward of  $Z_i$  is earned at the time of the  $i$ th renewal. Then, the total reward earned by time  $t$  is given by

$$Z(t) = \sum_{i=1}^{N(t)} Z_i \quad (\text{B.27})$$

where  $N(t)$  is the number of renewals in  $[0, t)$ .  $Z(t)$  is a cumulative process with  $N(t)$  given by a renewal process. If  $E[|Y_i|]$  and  $E[X_i]$  are finite, then

1. with probability 1,  $\lim_{t \rightarrow \infty} \frac{Z(t)}{t} \rightarrow \frac{E[Z_i]}{E[X_i]}$ , and
2.  $\lim_{t \rightarrow \infty} E\left[\frac{Z(t)}{t}\right] \rightarrow \frac{E[Z_i]}{E[X_i]}$

For a proof, see Ross (1970).

## B.4.5 Marked Point Process

A marked point process is a point process with an auxiliary variable, called a mark, associated with each event. Let  $Y_i$ ,  $i \geq 1$ , denote the mark attached to the  $i$ th event.

For example in the case of a multicomponent item, failure of a component can cause induced failures of one or more of the remaining components. If the number of components that must be replaced at the  $i$ th failure of the item is a random variable, then it can be viewed as a mark attached to an underlying point process characterising item failures.

#### **B.4.5.1 A Simple Marked Point Process**

A **simple marked** point process is characterised by

1.  $\{N(t), t \geq 0\}$ , a stationary Poisson process with intensity  $\lambda$ .
2. A sequence of independent and identically distributed random variables  $\{Y_i\}$ , called marks, which are independent of the Poisson process. This point process is also called a *Compound Poisson* process. Various extensions (e.g. a non-stationary point process and marks constituting a dependent sequence, to name a few) yield more complex marked point processes.

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# Appendix C

## Deterministic Optimisation

In this Appendix, some basic results for deterministic optimisation problems are presented. We first consider static optimisation and present results for both the unconstrained and constrained cases. Following this, we look at dynamic optimisation, where both discrete-time (multistage) and continuous-time formulations are considered.

### C.1 Static Optimisation with a Scalar Objective Function

#### C.1.1 Single Variable Optimisation

In the simplest case, a decision maker (DM) has a single decision variable  $x$  that has to be selected optimally in order to maximise a scalar objective function  $J(x)$ <sup>5</sup>.  $J(x)$  is differentiable, and the possible values of  $x$  belong to the interval  $X = [a, b]$ , where  $a$  and  $b$  are non-negative real numbers.  $X$  is termed the feasible region. The objective function represents the pay-off or reward earned, and it will also contain parameters that are fixed and so cannot be controlled by the DM.

This DM's problem can be expressed as

$$\max J(x), \text{ subject to } x \in [a, b]. \tag{C.1}$$

A *local maximum* of  $J(x)$  occurs at the point  $x^* \in [a, b]$  if  $J(x^*) \geq J(x)$  for all  $x$  sufficiently close to  $x^*$ .  $J(x)$  has a *global maximum* at  $x^*$  if  $J(x^*) \geq J(x)$  for all  $x \in [a, b]$ . The global maximum is the optimal solution to (C.1).

A necessary condition for a local maximum at an interior point  $x^* \in [a, b]$  is that

$$J'(x^*) = \left. \frac{dJ(x)}{dx} \right|_{x=x^*} = 0. \tag{C.2}$$

---

<sup>5</sup> If the DM's problem is to minimise the function  $J(x)$ , then it is equivalent to maximising the function  $K(x)$  where  $K(x) = -J(x)$ . Hence, without loss of generality, we will confine our attention to maximisation.

Sufficient conditions for a local interior maximum at  $x = x^*$  are

$$J'(x^*) = 0 \text{ and } J''(x^*) = \left. \frac{d^2J(x)}{dx^2} \right|_{x=x^*} < 0 \quad (\text{C.3})$$

The end points  $a$  and  $b$  of the interval  $[a, b]$  also need to be checked. If  $J'(a) \leq 0$  then  $a$  is a local maximum and if  $J'(b) \geq 0$  then  $b$  is a local maximum.

The global maximum of  $J(x)$  is the particular local maximum that produces the largest value of the objective function. If  $J(x)$  is a *concave* function over the interval  $[a, b]$ , then the analysis becomes much simpler. The two end points cannot be local maxima so any interior point  $x^*$  with  $J'(x^*) = 0$  is automatically the global maximum. In most cases, the equation  $J'(x) = 0$  has to be solved numerically using a one-dimensional search procedure such as Binary search or Golden Section search. These numerical methods are discussed in Rao (2009).

### C.1.2 Multivariable Unconstrained Optimisation

The DM now has a vector of decision variables  $\underline{x} = (x_1, x_2, \dots, x_n)$  to select in order to maximise the scalar objective function  $J(\underline{x})$ .  $J(\underline{x})$  is assumed to be differentiable, and there is no constraint on any of the variables, so the feasible region  $X$  is  $n$ -dimensional Euclidean space  $R^n$ . The problem can be expressed as

$$\max J(\underline{x}), \text{ subject to } \underline{x} \in R^n. \quad (\text{C.4})$$

The optimal solution to (C.4) is the global maximum of  $J(\underline{x})$ . The necessary condition for a local maximum is that

$$\frac{\partial J(\underline{x})}{\partial x_i} = 0 \text{ at } \underline{x} = \underline{x}^*, \quad \text{for } i = 1, 2, \dots, n. \quad (\text{C.5})$$

If we define the *gradient* of the function  $J(\underline{x})$  to be

$$\nabla J(\underline{x}) = \left( \frac{\partial J}{\partial x_1}, \frac{\partial J}{\partial x_2}, \dots, \frac{\partial J}{\partial x_n} \right), \quad (\text{C.6})$$

then the above necessary condition can be stated more succinctly as

$$\nabla J(\underline{x}) = \underline{0} \text{ at } \underline{x} = \underline{x}^*. \quad (\text{C.7})$$

The *Hessian* is the  $n \times n$  matrix whose element in the  $i$ th row and  $j$ th column ( $1 \leq i, j \leq n$ ) is given by

$$[H(\underline{x})]_{ij} = \frac{\partial^2 J(\underline{x})}{\partial x_i \partial x_j} \quad (\text{C.8})$$

The  $k$ th *principal minor* of  $H(\underline{x})$  is the  $k \times k$  submatrix  $H_k(\underline{x})$  obtained by deleting the last  $n - k$  rows and columns of the Hessian. If  $\nabla J(\underline{x}^*) = \underline{0}$  and the

determinant of  $H_k(\underline{x}^*)$  for  $k = 1, 2, \dots, n$  has the same sign as  $(-1)^k$ , then  $\underline{x}^*$  is a local maximum.

The optimal solution to (C.4) is the global maximum of  $J(\underline{x})$  and this point  $\underline{x}^*$  is the particular local maximum that produces the largest value of the objective function. If the function  $J(\underline{x})$  is concave, then any local maximum is also a global maximum. To locate possible local maxima, the equation  $\nabla J(\underline{x}) = \underline{0}$  has to be solved. This can be done numerically using, for example, the multivariable gradient search (steepest ascent) procedure, Newton’s or Quasi-Newton methods. These techniques are discussed in Rao (2009).

### C.1.3 Multivariable Constrained Optimisation

We now consider maximisation problems where there are two types of constraints on the decision variables  $\underline{x} = (x_1, x_2, \dots, x_n)$ —equality and inequality constraints.

#### C.1.3.1 Equality Constraints

*Lagrange multipliers* can be used to solve problems with equality constraints which take the form

$$\max J(\underline{x}), \text{ s.t. } g_j(\underline{x}) = b_j \quad \text{for } j = 1, 2, \dots, m \tag{C.9}$$

We associate a *multiplier*  $\lambda_j$  with the  $j$ th constraint in (C.9) and construct the *Lagrangian*

$$L(\underline{x}, \underline{\lambda}) = J(x_1, x_2, \dots, x_n) + \sum_{j=1}^m \lambda_j [b_j - g_j(x_1, x_2, \dots, x_n)] \tag{C.10}$$

The necessary conditions for a constrained local maximum are obtained by setting  $\partial L / \partial x_i = 0$  for  $i = 1, 2, \dots, n$  and  $\partial \lambda / \partial x_i = 0$  for  $j = 1, 2, \dots, m$ , giving

$$\begin{aligned} \frac{\partial J}{\partial x_i} - \sum_{j=1}^m \left[ \lambda_j \frac{\partial g_j}{\partial x_i} \right] &= 0 \quad \text{for } i = 1, 2, \dots, n, \\ b_j - g_j(x_1, x_2, \dots, x_n) &= 0 \quad \text{for } j = 1, 2, \dots, m. \end{aligned} \tag{C.11}$$

The solution of this system of  $(n + m)$  equations in the  $(n + m)$  unknowns yields all the possible local maxima which satisfy the constraints and, in general, has to be obtained numerically. The optimal solution (constrained global maximum) will be among the local maxima. If the function  $J(\underline{x})$  is concave and each  $g_j(x_1, x_2, \dots, x_n)$  is a linear function, then any local maximum will also be a global maximum.

### C.1.3.2 Inequality Constraints

The general maximisation problem with inequality constraints, also called a *non-linear programming problem*, takes the form

$$\max J(\underline{x}), \text{ s.t. } g_j(\underline{x}) \leq b_j \quad \text{for } j = 1, 2, \dots, m, \quad (\text{C.12})$$

where the functions  $J(\underline{x})$  and  $g_1(\underline{x}), g_2(\underline{x}), \dots, g_m(\underline{x})$  must satisfy some regularity conditions (see Bazarra et al. 2006).

If  $\underline{x}^* = (x_1^*, x_2^*, \dots, x_n^*)$  is an optimal solution to (C.12), then it must satisfy the  $m$  inequality constraints and there must exist  $m$  multipliers  $\lambda_1, \lambda_2, \dots, \lambda_m$  which satisfy the *Karush-Kuhn-Tucker* (KKT) conditions

$$\begin{aligned} \frac{\partial J(\underline{x}^*)}{\partial x_i} - \sum_{j=1}^m \left[ \lambda_j \frac{\partial g_j(\underline{x}^*)}{\partial x_i} \right] &= 0 \quad \text{for } i = 1, 2, \dots, n, \\ \lambda_j [b_j - g_j(\underline{x}^*)] &= 0 \quad \text{for } j = 1, 2, \dots, m, \quad \lambda_j \geq 0 \quad \text{for } j = 1, 2, \dots, m. \end{aligned} \quad (\text{C.13})$$

If the function  $J(\underline{x})$  is concave and each  $g_j(x_1, x_2, \dots, x_n)$  is a convex function, then any point satisfying the conditions in (C.13) is an optimal solution to the problem given in (C.12).

## C.2 Static Optimisation with a Vector Objective Function

The DM may be involved in a multiobjective optimisation problem, wishing to find the value of the single decision variable  $x$  that maximises the  $k$  objective functions  $J_1(x), J_2(x), \dots, J_k(x)$ . In this case, the notion of optimality is not obvious because of the possible presence of conflicting objectives. In general, there will be no single optimal solution  $x^*$  for the DM that maximises all the objective functions simultaneously. A value of the decision variable  $x'$  is called *Pareto optimal* for the DM if there is no other value  $x$  that *dominates*  $x'$ , so there is no  $x$  such that  $J_i(x) > J_i(x')$ , for all  $i = 1, 2, \dots, k$  and  $J_j(x) > J_j(x')$ , for at least one  $j = 1, 2, \dots, k$ . When all the Pareto optimal solutions have been found, the DM has to identify the one that achieves the best compromise between all the conflicting objectives.

The DM may have a vector of decision variables  $\underline{x} = (x_1, x_2, \dots, x_n)$  to select. In this case, the objective functions would be  $J_1(\underline{x}), J_2(\underline{x}), \dots, J_k(\underline{x})$  and the DM then has a more complex multivariable optimisation problem to solve.

Techniques for optimisation of vector objective functions are discussed in Steuer (1986) and Coello et al. (2007).

### C.3 Dynamic Optimisation with a Scalar Objective Function

#### C.3.1 Multistage (Discrete Time) Dynamic Optimisation

We now look at a dynamic optimisation problem where the DM has to make multiple decisions over time in order to maximise a specified objective function (e.g. total reward earned). The decisions may be made either at discrete-time points or continuously. We begin by looking at the discrete case where the time points or *stages* are denoted by  $t = 0, 1, 2, \dots, N$ , and  $N$  is the length of the *time horizon*.

The technique that we now describe to solve this type of problem is called dynamic programming (DP). There are two major concepts used in this approach at each stage of the process—*state variables* and *decision (or control) variables*. A state variable  $s_t$  provides all the information about the “current position” that the DM needs to know. The DM then makes a decision  $x_t$  in order to change the state, and this decision results in a pay-off/reward  $L(S_t, x_t)$  to the DM at this particular stage. There may be constraints on both variables at each stage.

The DM wants to determine the *policy* (sequence of decisions) that will maximise total reward earned over the  $N$  time periods which is given by

$$\sum_{t=0}^{N-1} L_t(s_t, x_t) + L_N(s_N), \quad (\text{C.14})$$

where  $L_N(s_N)$  denotes the possible reward earned by the DM at the end of the time horizon (terminal reward) if the process is in state  $s_N$ .

The state variables undergo a transformation represented by the equation

$$s_{t+1} = f_t(s_t, x_t) \quad \text{for } t = 0, 1, 2, \dots, N - 1. \quad (\text{C.15})$$

Bellman (1957) showed the dynamic optimisation problem described in (C.14) and (C.15) could be divided into a sequence of smaller problems. When the smaller problems have been solved, they are then combined to produce the solution to the complete problem. Bellman’s *principle of optimality* says that given the current state, the optimal decision for each of the remaining stages does not depend on the previously reached states or previously chosen decisions.

To solve the optimisation problem, we define the *optimal value function*  $V_t(s_t)$  as the maximum total reward earned by the DM using an optimal sequence of decisions for the remainder of the time horizon starting from state  $s_t$  at time (stage)  $t = 0, 1, \dots, N - 1, N$ . The optimal value function is a solution of the *functional equations*

$$\begin{aligned} V_t(s_t) &= \max_{x_t \in C_t} \{L_t(s_t, x_t) + V_{t+1}[f_t(s_t, x_t)]\}, \quad t = 0, 1, \dots, N - 1, \\ V_N(s_N) &= L_N(s_N). \end{aligned} \quad (\text{C.16})$$

$C_t$  is the set of possible decisions (feasible or constraint set) at time  $t$ . The optimal policy for the DM consists of the optimising decisions  $x_t^*$  which produce the optimal value function for each  $t = 0, 1, \dots, N - 1, N$ .

### C.3.2 Continuous-Time Dynamic Optimisation

We now focus on the situation where the DM has to make decisions continuously over a fixed time horizon of length  $T$ . The state variables and decision variables are now  $s(t)$  and  $x(t)$  for  $0 \leq t \leq T$ , and the *reward rate* earned by the DM at time  $t$  is  $L(s(t), x(t), t)$ . The DM wants to determine the policy  $x^*(t)$  that will maximise total reward earned over the time horizon which is given by

$$\int_0^T L(s(t), x(t), t) dt. \quad (\text{C.17})$$

The objective function in (C.17) can easily be modified to include a terminal reward at time  $T$ . The state of the process evolves according to the differential equation

$$\dot{s}(t) = \frac{ds(t)}{dt} = f(s(t), x(t), t), \quad (\text{C.18})$$

with the initial value  $s(0) = s_0$  specified. The decision variable will also usually be constrained, so  $x(t) \in C(t)$  for  $0 \leq t \leq T$ , where  $C(t)$  is the feasible set of decisions at time  $t$ .

This is a standard *optimal control* problem (see, for example Bryson and Ho 1975 or Sethi and Thompson 2000). To obtain its optimal solution, we introduce a *co-state* variable  $\lambda(t)$  and define the *Hamiltonian* function

$$H(s, x, t, \lambda) = L(s, x, t) + \lambda f(s, x, t). \quad (\text{C.19})$$

Note that, in the Hamiltonian, the  $t$  dependence in the functions  $s(t), x(t)$  and  $\lambda(t)$  has been suppressed. The first-order (necessary) conditions for an optimal solution to the DM's problem are due to Pontryagin et al. (1962) and are known as the *maximum principle*. They are

$$\max_x H(s, x, t, \lambda) \quad \text{for all } t \in [0, T] \quad (\text{C.20})$$

$$\dot{s} = \frac{\partial H}{\partial \lambda} \quad [\text{equation of motion for } s] \quad (\text{C.21})$$

$$\dot{\lambda} = -\frac{\partial H}{\partial s} \quad [\text{equation of motion for } \lambda] \quad (\text{C.22})$$

$$\lambda(T) = 0 \quad [\text{transversality condition}] \quad (\text{C.23})$$

Note that (C.20), the first-order condition with respect to the decision variable is not stated as a derivative. This is to allow for the possibility of “end-point” solutions. The condition may be stated alternatively as

$$H(s, x^*, t, \lambda) \geq H(s, x, t, \lambda) \quad \text{for all } t \in [0, T] \quad (\text{C.24})$$

The transversality condition (C.23) implies that, in this problem, there is no binding constraint on the terminal value of the state variable. Conditions (C.21) and (C.22) produce two first-order differential equations for  $s(t)$  and  $\lambda(t)$ , respectively. The general procedure is to first solve (C.22) with (C.23) as the required final condition. Then, (C.21) is solved with initial condition  $s(0) = s_0$ . Finally, the maximisation in (C.20) is performed.

Details of the derivation of the necessary conditions (C.20)–(C.23) along with the corresponding sufficient condition for an optimal solution can be found in Bryson and Ho (1975) or Kamien and Schwartz (1991). Note that, in the dynamic optimisation problems described in this section, only one state variable and one decision variable at each time  $t$  have been specified. The results that have been given can be generalised to the case of many state variables and decision variables and also where the number of decision variables need not equal the number of state variables.

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# Appendix D

## Illustrative EWs, MSCs and LCs

The material given in this Appendix was obtained from various websites on the Internet.

### D.1 Consumer Products [EWs and MSCs]

#### *D.1.1 Case 1 Manufacturer's EW for Electrical and Electronic Products [Sony]*

##### 1 Extended Warranty Services

1.1 The benefits given to you in Sony's Extended Warranty are in addition to other rights and remedies you have under a law in relation to the product. Sony products come with guarantees that cannot be excluded under the Australian Consumer Law. You are entitled to a replacement or refund for a major failure and for compensation for any other reasonably foreseeable loss or damage. You are also entitled to have the products repaired or replaced if the products fail to be of acceptable quality, and the failure does not amount to a major failure. A "major failure" to comply with a consumer guarantee applying to products (goods) has a defined meaning under the Australian Consumer Law. One example of a major failure is if a reasonable consumer fully acquainted with the nature and extent of the failure would not have acquired the product.

1.2 Subject always to section 1.1 and the rest of these Extended Warranty Terms, Sony will provide to you the following benefits during the Standard Warranty Term and Extended Warranty Term for the product:

- (a) When the product or any Sony accessory supplied with it does not perform in accordance with the manufacturer's specifications, Sony will repair or replace at Sony's cost the product or accessory.
- (b) For any claim under section 1.2(a), Sony will provide you with access to your own Sony Extended Warranty Liaison for specialist support and end-to-end management of your claim.

- (c) For any claim under section 1.2(a), Sony will provide onsite pick-up and delivery for the product or accessory, if the pick-up address is within 25 kms of the nearest Sony Authorised Service Centre.
- (d) If the claim under section 1.2(a) is for BRAVIA product, you can choose between an in-home service or onsite pick-up and delivery for the product, if the pick-up address is within 25 kms of the nearest Sony Authorised Service Centre. This means greater choice and more convenience for you.
- (e) Should you need to make a claim or require assistance with your Sony product, your claim will be handled and serviced only by Sony Authorised repairers and support staff.
- (f) For any claim under section 1.2(a), you will have access to a range of service bookings through the extended hours of operation from our Authorised Service Centres (subject to availability).

## 2 Making a claim

2.1 To make a claim under section 1.2(a) under the Extended Warranty, you will need to:

- (a) Contact Sony to notify Sony of your claim. Contact details are as follows:  
Ph: 1300 782 657  
**Service and Support Hotline**  
1300 13 SONY (7669)  
(Service Centre locations, Product Information, Spare Parts, Support)  
[www.sony.com.au](http://www.sony.com.au)
- (b) When making your claim, provide the plan number issued on your Extended Warranty Certificate. If you do not have your plan number, you may provide proof of purchase (e.g. Bill of sale, invoice or purchase receipt) with your claim.
- (c) For claims, complete the claim form Sony provides to you and send your claim form to Sony as set out on the form. Your claim will need to provide Sony with sufficient details so we understand the nature of the problem.
- (d) For claims, unless onsite service applies make the product available to Sony for onsite pick-up and delivery or provide the product to a Sony Authorised Service Centre (as applicable under these terms or as otherwise agreed with Sony). If you are required to return the product to a Sony Authorised Service Centre, Sony will provide details of the centre to you. Alternatively, to find the nearest Sony Authorised Service Centre, contact Sony using the above contact details. If service is in-home pick-up service, Sony will contact you to make arrangements for on-site service or pick-up.

2.2 The product will be at your risk while in transit to and from the Sony Authorised Service Centre, unless transported by Sony or its authorised representatives.

2.3 Sony and its Authorised Service Centres may seek reimbursement from you of any costs incurred by them when the product is found to be in good working order. To check if your product requires any type of service, please feel free to give our Customer Support line a call on 1300 13 SONY (7669) prior to going to any service centre.

### **3. Repairs**

3.1 Products presented for repair may be replaced by refurbished products of the same type rather than being repaired. Refurbished parts may be used to repair the products. Replacement of the product or a part does not extend or restart the Standard Warranty Term or Extended Warranty Term. If the product is replaced during the Extended Warranty Term, the Extended Warranty automatically terminates upon replacement and a Standard Warranty Term will apply to the replacement product. You can ask your Extended Warranty Liaison about any special offers on a new Extended Warranty plan for the replacement product at that time.

3.2 If the product presented for repair is capable of retaining user-generated data, you are advised that repair of the product may result in loss of the data. Sony's dedicated team of Sony technicians and support staff can assist you in backing up any data of this type prior to any servicing of your product.

### **4. Extended Warranty Term duration**

4.1 The Extended Warranty Term commences when Sony receives from you payment of the Extended Warranty fee, or when your Standard Warranty Term ends, whichever occurs last. The term of your cover will be listed in your Extended Warranty Certificate.

4.2 An Extended Warranty Certificate with a plan number will be issued to you within 28 days of the Extended Warranty fee being paid.

4.3 Sony is entitled to terminate the Extended Warranty by written notice to you if in Sony's opinion, the product is used contrary to its specifications, in which case Sony will pay a refund for the unexpired period of the Extended Warranty less administration expenses.

4.4 The Extended Warranty is transferable to a new owner, in the event of sale of the product, provided Sony is informed of the transfer in writing at the following address: Sony Warranty Support, Reply Paid 73765, NORTH RYDE BC, NSW 1670, AUSTRALIA.

### **5. Limitations and exclusions to Extended Warranty coverage**

5.1 To the full extent permitted by law, but subject always to section 1.1, you do not have a right to make a claim under section 1.2(a):

- (a) If the product has not been installed, operated, maintained or used in accordance with the manufacturer's instructions or specifications provided with the product.

- (b) If the factory-applied serial number has been altered or removed from the product.
- (c) For damage, malfunction or failure resulting from alterations, accident, misuse, abuse, fire, liquid spillage, mis-adjustment of customer controls, use on an incorrect voltage, power surges and dips, thunderstorm activity, acts of God, voltage supply problems, tampering or unauthorised repairs by any persons, use of defective or incompatible accessories, the operation of a computer virus of any kind, exposure to abnormally corrosive conditions or entry by any insect, vermin or foreign object in the product.
- (d) For damage arising during transportation, installation or while moving the product, or to any transportation costs of the product or any parts thereof to and from the owner, unless otherwise specified in these warranty conditions.
- (e) In relation to any third-party software or hardware not contained in the product as originally configured by the manufacturer.
- (f) For any failure, to the extent that the failure is not a failure of the product to perform in accordance with its manufacturer's specifications.
- (g) For replacement or repair of any (1) consumables (including batteries and cables), or (2) lost parts or accessories.
- (h) While the product is outside Australia or New Zealand.
- (i) For any wear and tear including to projector lamp or optical block assembly if the product is used in commercial, industrial, educational or rental applications.
- (j) For the normal incidence of off-coloured or dark pixels in LCD screens as described in the User Manual for the product. Sony will only repair or replace the product if there are (1) 8 or more dark pixels in the screen (unless 2 or more are adjoining dark pixels) or (2) 2 or more bright pixels in the screen.
- (k) If the product is a projector, to a claim for a replacement projector lamp or optical block assembly.

5.2 To the full extent permitted by law, but subject always to section 1.1:

- (a) Sony will not be liable for any loss, damage or alterations to (1) third-party hardware or software; or (2) programs, data or information stored on any media or any part of the product, no matter how occurring; or for any loss or damage arising from loss of use, loss of profits or revenue, or for any resulting indirect or consequential loss or damage.
- (b) Sony's aggregate liability in respect of all claims under the Extended Warranty shall not exceed the original purchase price of the product or, at Sony's option, the cost of replacing the product.
- (c) Sony excludes all other warranties, conditions, terms, representations and undertakings relating to the product other than those expressly identified in these Extended Warranty Terms.

### ***D.1.2 Case 2 Retailer's EW for Electrical and IT Products [Harvey Norman]***

Take advantage of our comprehensive extended warranty and purchase those electrical and IT products you have always wanted! Prolong the life of your new purchase and enjoy the satisfaction and peace of mind of up to 5 year parts and labour coverage

#### **No More Worries about Parts and Labour Costs**

If a part is going to break down, undoubtedly it will be just after the warranty period has expired. No longer will this be a problem for you as all parts and labour costs are covered by the extended warranty

#### **No More Worries about Beyond Repair Products**

Harvey Norman is dedicated to providing a quality service. If for some reason there is a problem with your product that cannot be fixed, a replacement product will be provided to you

#### **Extended Warranty Period**

You can choose to cover your product for 3, 4 or 5 years. That will be 2, 3 or 4 years on top of the manufacturer's warranty

#### **Can't Decide?**

At Harvey Norman, we realise that major purchase requires some thought, so we are glad to extend up to 14 days after your purchase to decide whether you would like to take advantage of our Extended Warranty

#### **Note:**

Limit of liability (the sum of all repairs and/or replacement) shall not exceed the original purchase price of the product.

Harvey Norman Extended Warranty is for residential use only.

Harvey Norman Extended Warranty is not transferable.

Terms and Conditions apply.

### D.1.3 Case 3 Manufacturer’s EW for Cars [Chrysler]

Powertrain Care	Powertrain Care Plus	Added Care Plus	Maximum Care
■ Engine	■ Engine	■ Engine	■ Engine
■ Transmission	■ Transmission	■ Transmission	■ Transmission
■ Driveline	■ Driveline	■ Driveline	■ Driveline
 Our "Basic" Protection	<b>PLUS</b> ■ Steering ■ Air Conditioning  Our "Good" Protection. <i>Available on 3/36 warranties only.</i>	<b>PLUS</b> ■ Steering ■ Air Conditioning <b>PLUS</b> ■ Engine Cooling and Fuel ■ Front Suspension ■ Rear Suspension ■ Electrical ■ Expanded Electrical ■ Instrumentation ■ Brakes ■ Anti-Lock Brakes ■ Power Group ■ Luxury Group	<b>PLUS</b> ■ Steering ■ Air Conditioning <b>PLUS</b> ■ Engine Cooling and Fuel ■ Front Suspension ■ Rear Suspension ■ Electrical ■ Expanded Electrical ■ Instrumentation ■ Brakes ■ Anti-Lock Brakes ■ Power Group ■ Luxury Group <b>PLUS</b> ■ Complete Mechanical Coverage for up to 5007+ components. <i>"If it's Mechanical... It's Covered!"</i>  Our "Best" Protection.

## D.2 Commercial and Industrial Products [EWs and MSCs]

### D.2.1 Case 4 Computer Servers [Hewlett Packard]

The HP service contracts in the USA contain the following elements

1. Support Services<sup>6</sup>

<sup>6</sup> Support Services include the following:

- Constant monitoring and alerting on network components
- Full remote control and diagnostics of server equipment
- Immediate alert and response to all events
- Remote diagnostic and repair for all incidents and failures

2. Customer
3. Charges
4. Eligible Products
5. Limitations of Liability and Remedies
6. Timeliness of Action
7. Limitations of Service
8. Supported Software Versions
9. Non-HP Products
10. Customer Responsibilities
11. Off-Site Support and Exchange Services
12. On-Site Support for HP Network Connectivity Products
13. Maximum Use Limitations
14. Transfer of Service
15. Post Warranty Agreement Services
16. Term
17. Termination
18. Governing Laws
19. Entire Agreement

## ***D.2.2 Case 5 Diesel Engines [Wartsila]***

### **MSC-I: Supply Agreement**

With supply agreement status, you get access to our global parts distribution network and are able to order and receive spare parts 24/7, including reconditioned components, wherever your facility is located and with the shortest possible lead time. We can also guarantee the availability of a global network of trained and skilled service professionals with the right tools and onboard/ on-site manpower to assist them.

#### *Parts*

24/7 global logistics of spare parts

Shortening of lead time

Correct spare parts

#### *Information*

Online services

#### *Manpower*

Availability to a global network of trained and skilled service professionals with right tools

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(Footnote 6 continued)

- On-site hot swap exchange of failure devices
- Remote repair and fix, even of 'hung servers'
- Full incident logging and reporting
- 100 % cover for network server, clients and users.

On board/Onsite manpower supply  
*Workshop services*  
 Global component drops for reconditioning

### **MSC-II: Technical Maintenance Agreement**

A long-term service agreement covering maintenance planning and service crews wherever and whenever needed through the local and global presence of Wärtsilä's networks. We provide fixed prices for inspection, technical support, spare parts, training and maintenance work. Our dynamic maintenance concept leads to a better prediction of maintenance needs and the system's overall function.

Performance guarantee available.

*Inspection*

Regular inspections expert assistance and monthly reporting

*Spare parts*

Exchange programme

Reconditioning

### **MSC-III: Maintenance Agreement**

A long-term service agreement covering maintenance planning and service crews wherever and whenever needed through the local and global presence of Wärtsilä's networks. We provide fixed prices for inspection, technical support, spare parts, training and maintenance work. Our dynamic maintenance concept leads to a better prediction of maintenance needs and the system's overall function.

Performance guarantee available.

*Power plant agreement*

Long-term service agreements with fixed fees for the duration of the agreements

Spare parts and/or labour supply for maintenance work

There can be performance guarantees

Inspection, technical support, spare parts, training/competence

*Marine agreement*

Long-term service agreements with fixed fees for the duration of the agreements

Spare parts and/or labour supply for maintenance work

There can be performance guarantees

Global agreements

Risk management

### **MSC-IV: Asset Management Agreement**

More comprehensive than a maintenance agreement, an asset management agreement gives us full responsibility for performance and equipment life, so that you can concentrate on your core business. Our asset management agreements typically include full operation, management and maintenance services (O&M) as well as performance guarantees.

*Power plant agreement*

Full Operation Management and Maintenance services

Supply of required manpower, parts and knowledge to be able to take full responsibility for the operation of a plant

Performance guarantee

Responsibility for unscheduled maintenance and breakdowns

Risk management

*Marine agreement*

Wärtsilä provides manpower as a part of the crew

Supply of required manpower, parts and knowledge to be able to take full responsibility for the operation of the engine room

Performance guarantee

Responsibility for unscheduled maintenance and breakdowns

Risk management

### D.3 Infrastructures [MSCs]

#### *D.3.1 Case 5: New Zealand Transport Authority*

**Contract Agreement [Contract for NZTA1234, Highway Maintenance Example]**

**THIS AGREEMENT** is made on (“date”) **BETWEEN** (“the Contractor”)

**AND**

**The NZ Transport Agency**, a Crown entity, established on 1 August 2008 by Section 93 of the Land Transport Management Act 2003 (“the Principal”)

**IT IS AGREED** as follows:

1. **THE** Contractor shall carry out the obligations imposed on the Contractor by the Contract Documents.
2. **THE** Principal shall pay the Contractor the sum of \$ \_\_\_\_\_ or such greater or lesser sum as shall become payable under the Contract Documents together with Goods and Services Tax at the times and in the manner provided in the Contract Documents.
3. **EACH** party shall carry out and fulfil all other obligations imposed on that party by the Contract Documents.
4. **THE** Contract Documents are this Contract Agreement and the following which form part of this agreement:
  - (a) The Conditions of Tendering
  - (b) Notices to Tenderers (give details with dates):
  - (c) The Contractor’s tender;
  - (d) The notification of acceptance of tender;

- (e) The General Conditions of Contract, NZS 3910:2003
- (f) The Special Conditions of Contract;
- (g) Specifications issued prior to the Date of Acceptance of Tender;
- (h) Drawings issued prior to the Date of Acceptance of Tender;
- (i) The Schedule of Prices
- (j) The following additional documents: (*Identify any additional documents to be included for example agreed correspondence*)

## D.4 Lease Contracts

### D.4.1 Case 6: Automobile [*British Columbia Transit*]

THIS VEHICLE LEASE AGREEMENT dated the \_\_\_\_ day of \_\_\_\_\_, 19\_\_ BETWEEN:

**BRITISH COLUMBIA TRANSIT**, a corporation incorporated pursuant to the British Columbia

Transit Act,  
(hereinafter called “BC Transit”)

OF THE FIRST PART

AND:

**OPERATING COMPANY**  
(hereinafter called “Operating Company”)

OF THE SECOND PART

- A. **WHEREAS** the Operating Company is desirous of leasing and/or subleasing from BC Transit the vehicle (s) and equipment described in the list attached hereto as Schedule “A” (hereinafter collectively called the “equipment”).
- B. **AND WHEREAS** BC Transit either owns the equipment or is itself leasing the equipment (or part thereof as the case may be) from a third-party pursuant to the Head Lease Agreement (hereinafter called the “Head Lease Agreement”).

**THIS AGREEMENT WITNESSETH** that for good and valuable consideration, the parties hereto agree and covenant is as follows:

1. **DEFINITIONS** In this indenture, the words “Lease” and “Lease Agreement” shall be deemed to mean, refer to and include the words “Sub-Lease” and “Sub-Lease Agreement”, if applicable and as the context of this Lease Agreement so requires, as between BC Transit (Sublessor) and the Operating Company (Sublessee).

2. **LEASE** BC Transit hereby agrees to lease to the Operating Company and the Operating Company hereby agrees to lease from BC Transit the equipment, together with all accessories, additions, repairs and replacement parts affixed thereto, now or in the future.
3. **RENT** The Operating Company agrees to pay to BC Transit the sum of One Dollar (\$1.00) forthwith and such payment shall be the rental charges payable by the Operating Company to BC Transit in respect of the equipment.
4. (A) **TERM** The term of this Lease Agreement shall commence on the date hereof and shall be terminated on that date of the following events first to occur:
  - (a) The termination date provided for in the Annual Operating Agreement made pursuant to the BC Transit Act to which BC Transit and the Operating Company are party thereto which is to effect at the time this Lease is enacted, and/or any successor Annual Operating Agreement (hereinafter collectively called the Annual Operating Agreement); or
  - (b) That date being 2 weeks after BC Transit has delivered to the Operating Company written notice of its intention to terminate this Lease Agreement; or
  - (c) That date which BC Transit and the Operating Company mutually agree shall be an effective date of termination of this Lease Agreement.
4. (B) **TERMINATION OF HEAD LEASE** Notwithstanding the provisions of Section 4A herein, if any item of equipment is the subject of a Head Lease and if for any reason such Head Lease is terminated, then at the option of BC Transit, this Lease Agreement shall terminate with respect to such item of equipment.
5. **ACCEPTANCE** The Operating Company acknowledges that it has inspected the equipment and accepts the equipment as being in a good state of repair, except to the extent that the Operating Company notifies BC Transit in writing within 10 days of delivery (manufacturer's latent defects included).
6. **USE** The Operating Company shall use the equipment only for those purposes set out in the Annual Operating Agreement. The Operating Company shall not use the equipment for pleasure or any other business not contemplated in the Annual Operating Agreement. The Operating Company shall observe and adhere to the operating procedures and guidelines as issued by BC Transit and which relate to the use of the equipment.
7. **HEAD LEASE** BC Transit covenants with the Operating Company to perform and observe the covenants on its part contained in the Head Lease Agreement, if any. The Operating Company covenants with BC Transit to perform and observe the covenants on the part of BC Transit to be performed or observed under the provisions of the Head Lease, if any, other than the covenant to pay rent.
8. **LOCATION** The Operating Company shall cause the equipment to remain situate in the transit service area as designated in the Annual Operating Agreement, and the Operating Company shall not remove the

equipment from the said transit service area without the prior written consent of BC Transit.

9. **OWNERSHIP** Title to and ownership of the equipment, subject to the provisions of any Head Lease Agreement, shall at all times be and remain in the name of BC Transit and the Operating Company shall have no right of property therein, except the right to use the equipment in accordance with the terms of this Lease Agreement.
10. **OPERATING COSTS** The Operating Company shall pay all operating costs whatsoever of the equipment, including without limiting the generality of the foregoing, the cost of fuel, oil, insurance as prescribed in the Annual Operating Agreement, licences pursuant to the Motor Carrier Act, licence and registration fees pursuant to the Motor Vehicle Act, municipal licences and motor vehicle inspections fees (where applicable).
11. **REPAIRS** The Operating Company shall maintain and keep the equipment in good condition and repair to the satisfaction of BC Transit, adhering to the BC Transit Preventive Maintenance Program. The Operating Company further covenants that as component parts of the equipment either wear out or become otherwise inoperative, to replace the same with either parts which are approved by the manufacturer of the equipment or such substitute parts as BC Transit may from time to time permit.
12. **INSPECTION** BC Transit shall have the right to inspect the equipment, without prior notice, at all reasonable times during the term of this Lease Agreement.
13. **ALTERATION** The Operating Company shall not alter or add or allow any other party to alter or add to the equipment in any way without the prior written approval of BC Transit. Any alterations, or additions to the equipment which are approved by BC Transit shall become and remain the property of BC Transit. The Operating Company shall affix on the equipment, any labels or insignias supplied by BC Transit. The Operating Company shall not permit any advertising to be posted on the exterior or the interior of the equipment, save and except as provided for in the Annual Operating Agreement.
14. **RECORDS** The Operating Company shall keep for each item of equipment and deliver to BC Transit as specified or upon request the following records:
  - (a) Any record as required by the Annual Operating Agreement to be provided by the Operating Company.
  - (b) Vehicle Daily Report Card (Form M098).
  - (c) Preventative Maintenance Inspection Guide (Form M299).
  - (d) Record of Preventive Maintenance Bus Inspections (Form M300).
  - (e) Road Call Analysis (Form M301).
  - (f) Monthly Bus Fuel Consumption Report (Form M307).
  - (g) Unit Change Record (Form M310).
  - (h) Accident/Incident Report (Form M318).

15. **MOTOR VEHICLE INSPECTION** The Operating Company shall be responsible for ensuring that the equipment is maintained in compliance with the British Columbia Motor Vehicle Act and Regulations, including the Commercial Vehicle Inspection Program. The Operating Company shall be responsible for ensuring the equipment is submitted for inspections pursuant to the provisions of Motor Vehicle Act, if so required by said provisions.
16. **LOSS OR DAMAGE** The Operating Company assumes and shall bear the entire risk of loss or damage to the equipment. No loss or damage to the equipment or any part thereof shall affect or impair any of the obligations of the Operating Company hereunder, and this Lease Agreement shall continue in full force and effect notwithstanding such loss or damage to the equipment. The Operating Company shall insure the equipment according to the laws in force and effect in the Province of British Columbia and in accordance with the provisions of the Annual Operating Agreement, and such provisions shall be incorporated into the terms and conditions of this Lease Agreement. The Operating Company shall punctually pay all insurance premiums when due in respect of any policies of insurance required to be purchased by it pursuant to the Annual Operating Agreement and the Operating Company shall provide BC Transit with copies of certificates of such insurance policies. In the event of loss or damage of any kind whatsoever to the equipment, the Operating Company shall forthwith comply with the reporting procedures in respect of such loss or damage as established by BC Transit. BC Transit at its sole discretion may either replace the lost or damaged equipment or alternatively direct the Operating Company to repair the damaged equipment, and the Operating Company shall comply with such direction.
17. **SURRENDER** Upon the termination of this Lease Agreement, the Operating Company shall forthwith return the equipment to BC Transit in good condition and repair, ordinary wear and tear resulting from the proper use of the equipment excepted, and the Operating Company shall, at its cost, return the equipment to BC Transit at a destination designated by BC Transit in the transit service areas as defined in the Annual Operating Agreement, and if the Operating Company fails to so deliver the equipment within 1 week from the termination of this Lease Agreement, BC Transit shall have the right to enter upon the premises where the equipment may be, and take possession of and remove it at the Operating Company's expense, all without legal process. The Operating Company covenants that, upon termination of this lease or upon surrender of the equipment for any other reason:
- (a) The equipment shall be in good condition and repair, in compliance with the BC Transit Maintenance Program;
  - (b) The records for mechanical repairs listed in Section 14 of this agreement shall accompany each vehicle;
  - (c) Average tire tread depth for all tires shall not be less than 8 mm (10/32").

- (d) A vehicle transfer form shall be executed by the Operating Company where applicable, and shall accompany each vehicle, and
  - (e) The Operating Company shall maintain insurance coverage in accordance with the provisions of Section 22 herein during the period of time that the equipment is being transferred to BC Transit, notwithstanding that this Lease Agreement may be terminated.
18. **LIENS AND CHARGES** The Operating Company shall, at all times, keep the equipment free from all levies, liens and encumbrances whatsoever and shall pay all licence fees, registration fees and assessments, charges and taxes, in accordance with the Annual Operating Agreement, which may be now or hereafter imposed directly upon the ownership, leasing, rent, possession or use of the equipment. If the Operating Company fails to pay any such levies, liens, encumbrances, assessments, charges or taxes, BC Transit may pay the same and in such event the costs thereof, together with interest calculated monthly at a rate equivalent to the prime rate established by The Royal Bank of Canada on the first day of each month, plus 2 % per annum, shall forthwith be due and payable by the Operating Company to BC Transit. Non-payment of such costs by the Operating Company to BC Transit forthwith upon demand by BC Transit shall be deemed to be a default under this Lease Agreement.
19. **WARRANTIES** The Operating Company acknowledges that BC Transit makes no warranties, either express or implied, as to any matter whatsoever, including without limiting the generality of the foregoing, the condition of the equipment nor its merchantability nor its fitness for any particular purpose.
20. **ASSIGNMENT, SUBLEASE** The Operating Company shall not transfer, deliver up possession of, or sublet the equipment, and the Operating Company's interest in this Lease Agreement shall not be assignable by the Operating Company without prior written consent of BC Transit; but nothing herein contained shall prevent BC Transit from assigning, pledging, mortgaging, transferring or otherwise disposing, either in whole or in part, of BC Transit's right hereunder. If the Operating Company is a corporation, then any sale or transfer of shares in the capital of the Operating Company shall be deemed to be an assignment under this Lease Agreement, and the written consent of BC Transit to such a sale or transfer shall be first had and obtained.
21. **INDEMNIFICATION** The Operating Company shall indemnify BC Transit against and hold BC Transit harmless from any and all claims, actions, suits, proceedings, costs, expenses, damages and liabilities including the costs arising out of, connected with or resulting from the equipment including without limitation the installation, possession, use, operation or return of the equipment or otherwise on account of any personal injury or death or damage to property occasioned by the operation of the said equipment during the term hereby granted.

22. **ANNUAL OPERATING AGREEMENT** The Operating Company covenants and agrees with BC Transit to perform each and every one of the conditions, terms, covenants and provisos contained in the Annual Operating Agreement, which on the part of the Operating Company are to be observed and performed.
23. **DEFAULT** Notwithstanding Section 4, the Operating Company covenants and agrees with BC Transit that BC Transit shall have the right to cancel and terminate this Lease Agreement forthwith by reason of any one or more of the following events:
- (a) If the Operating Company fails to observe and perform any of the terms, conditions, covenants and provisos contained in the Annual Operating Agreement, which on its part are to be observed and performed.
  - (b) If the Operating Company fails to perform any of the terms, conditions, covenants and provisos contained in this Lease Agreement which on its part are to be observed and performed.
  - (c) If a petition under any bankruptcy law shall be filed by or against the Operating Company or the Operating Company shall make any assignment for the benefit of its creditors or the Operating Company shall suffer or permit the appointment of any trustee or receiver or receiver-manager for the Operating Company's business or assets or any part thereof or otherwise becomes financially insolvent or if the Operating Company shall make or suffer any assignment, voluntary or involuntary, of the Operating Company's interest in any of the equipment included in this Lease Agreement or suffer any lien, attachment or levy of execution to become attached thereto.
  - (d) If the Operating Company uses any equipment included in this Lease Agreement unreasonably or abusively resulting in damage to such equipment or an abnormal reduction in the life of the equipment or any part thereof.
24. **TERMINATION** Upon the termination of this Lease Agreement, the Operating Company shall forthwith return to BC Transit all items of equipment as referred to herein and the Operating Company shall be liable to BC Transit for damages and costs which BC Transit may sustain by reason of the Operating Company's default of this Lease Agreement, including, without limiting the generality of the foregoing, all legal fees and other expenses incurred by BC Transit in attempting to enforce the provisions of this Lease Agreement or to recover damages for default under this Lease Agreement, or to recover any equipment not forthwith returned by the Operating Company to BC Transit.
25. **WAIVER** No covenant or proviso contained in this Lease Agreement to be performed by the Operating Company may be waived by BC Transit, except by prior written consent of BC Transit, and any forbearance or indulgence by

BC Transit in this regard shall not constitute its waiver of such covenant or proviso to be performed by the Operating Company.

26. **REGULATIONS** Nothing in this Lease Agreement shall preclude BC Transit from setting “lease fees” chargeable in connection with the public transportation system operated by the Operating Company pursuant to the aforementioned Annual Operating Agreement. (Note: Lease fees are based upon the capital cost of vehicles and are covered by the Annual Operating Agreement budget).
27. **TIME OF THE ESSENCE** Time is to be of the essence of this Lease Agreement and each and all of its provisions.
28. **INTERPRETATION** It is hereby agreed by and between the parties hereto that wherever the singular or masculine is used throughout this Lease Agreement, the same shall be construed as meaning the plural or the feminine or body corporate or politic, respectively, and vice versa, where the context or the parties hereto so require and in the case where more than one Operating Company is a party hereto, the liability of each Operating Company shall be joint and several.
29. **GOVERNING, LAW** This Lease Agreement shall be interpreted and enforced in accordance with the laws of the Province of British Columbia.
30. **EXECUTORS, ADMINISTRATORS AND ASSIGNS** This Lease Agreement shall ensure to the benefit of and be binding upon the parties hereto, and their respective heirs, executors, administrators, successors and permitted assigns.

**IN WITNESS WHEREOF** the parties hereto set their hands and seals and where a party is a corporate entity, the seal of such party has been affixed hereto in the presence of its duly authorised officers, the day, month and year first above written.

**BRITISH COLUMBIA TRANSIT**

**per:**

**The Corporate Seal of THE OPERATOR  
was hereunto affixed**

**Authorised Signatory (ies)**

### D.4.2 Case 7: Automobile [Ford Company in USA]

1-800-727-7000	<b>Motor Vehicle Lease Agreement</b>			Lease Date:																																								
Lessee (and Co-Lessee) - Name and Address (including County):																																												
Lessor - Name and Address:																																												
<p>"Ford Credit" is Ford Motor Credit Company. The "Holder" is _____ and its assigns.                  By signing "You", (Lessee and Co-Lessee) agree to lease this Vehicle according to the terms on the front and back of this lease.</p>																																												
New/Used/Demo	Mileage at Delivery	Year/Make/Model	GVW # Truck (lbs.)	Vehicle ID #	Vehicle Use																																							
<b>1. Amount Due At Lease Signing or Delivery</b> Below \$	<b>2. Monthly Payments</b> Your first monthly payment of \$ _____ is due on _____ followed by payments of \$ _____ due on the _____ day of each month. The total of Your monthly payments is \$ _____	<b>3. Other Charges</b> (not part of Your monthly payment) Disposition fee (if You do not purchase the Vehicle) Total \$ _____	<b>4. Total of Payments</b> (The amount You will have paid by the end of the lease) \$ _____																																									
<b>Itemization of Amount Due at Lease Signing or Delivery</b>																																												
<b>5. Amounts Due At Lease Signing or Delivery:</b> a. Capitalized cost reduction \$ _____ b. First monthly payment _____ c. Refundable security deposit _____ d. Title fees _____ e. Registration fees _____ f. _____ g. _____ h. _____ i. _____ <b>Total \$ _____</b>			<b>6. How the Amount Due At Lease Signing or Delivery will be paid:</b> a. Net trade-in allowance \$ _____ b. Rebates and noncash credits \$ _____ c. Amount to be paid in cash \$ _____ d. _____ <b>Total \$ _____</b>																																									
<b>7. Your monthly payment is determined as shown below.</b>																																												
a. <b>Gross capitalized cost.</b> The agreed upon value of the Vehicle (\$) and any items You pay over the lease term (such as service contracts, insurance, and any outstanding prior credit or lease balance) (Itemized below)** \$ _____ b. <b>Capitalized cost reduction.</b> The amount of any net trade-in allowance, rebate, noncash credit, or cash that You pay that reduces the gross capitalized cost _____ c. <b>Adjusted capitalized cost.</b> The amount used in calculating Your base monthly payment _____ d. <b>Residual value.</b> The value of the Vehicle at the end of the lease used in calculating Your base monthly payment _____ e. <b>Depreciation and any amortized amounts.</b> The amounts charged for the Vehicle's decline in value through normal use and for other items paid over the lease term _____ f. <b>Rent charge.</b> The amount charged in addition to the depreciation and any amortized amounts _____ g. <b>Total of base monthly payments.</b> The depreciation and any amortized amounts plus the rent charge _____ h. <b>Lease payments.</b> The number of payments in Your lease _____ i. <b>Base monthly payment</b> _____ j. <b>Monthly sales / use tax</b> _____ k. _____ l. _____ m. <b>Total monthly payment</b> _____ n. <b>Lease term in months.</b> _____																																												
\$	<b>Early Termination.</b> You may have to pay a substantial charge if You end this lease early. The charge may be up to several thousand dollars. The actual charge will depend on when the lease is terminated. The earlier You end the lease, the greater this charge is likely to be																																											
<b>8. Excess Wear and Use.</b> You may be charged for excessive wear based on our standards for normal use. At the scheduled end of this lease, unless You purchase the Vehicle, <b>You must pay to Lessor _____ cents per mile for each mile in excess of _____ miles shown on the odometer.</b> See Items 19 and 23 on back for additional excess wear and use terms.																																												
<b>9. Extra Mileage Option Credit.</b> At the scheduled end of this lease, You will receive credit of _____ cents per unused mile for the number of unused miles between _____ and _____ miles, less any amounts You owe under this lease. You will not receive any credit if the Vehicle is destroyed, if You terminate Your lease early, exercise any purchase option, are in default or the credit is less than <b>\$1.00</b> .																																												
<b>10. Purchase Option at End of Lease Term</b> \$ _____ plus official fees and taxes is <b>Your lease end purchase option price.</b> You have the option to purchase the Vehicle from Lessor in cash for the purchase option price at the end of this lease term if You are not not default.																																												
<b>Other Important Terms.</b> See Your lease documents for additional information on early termination, purchase option and maintenance responsibilities, warranties, late and default charges, insurance, and any security interests, if applicable.																																												
<b>11. WARRANTY</b> The Vehicle is covered by any warranty, extended warranty or service contract indicated below:  <input type="checkbox"/> Standard new Vehicle warranty provided by the manufacturer or distributor of the Vehicle.  <input type="checkbox"/> _____  If the Vehicle is of a type normally used for personal use and the Lessor, or the Vehicle's manufacturer, extends a written warranty or service contract covering the Vehicle within 90 days from the date of this lease, You get implied warranties of merchantability and fitness for a particular purpose covering the Vehicle. Otherwise, You understand and agree that there are no such implied warranties, except as otherwise required by state law.			<b>15. LIFE, DISABILITY AND OTHER INSURANCE</b> These coverages are not required to enter into this lease and will not be provided unless You sign below. If insurance is to be obtained by Lessor, the coverages are shown in a notice given to You this date and are for the term of this lease.  <table style="width:100%; border:none;"> <tr> <td style="width:40%;"><b>Life Insurance</b></td> <td style="width:20%; text-align:right;">Insurer</td> <td style="width:20%; text-align:right;">\$ _____</td> <td style="width:20%; text-align:right;">Initial Coverage Amount</td> </tr> <tr> <td></td> <td></td> <td style="text-align:right;">\$ _____</td> <td style="text-align:right;">Premium</td> </tr> <tr> <td></td> <td></td> <td></td> <td style="text-align:right;">Insured's Signature _____</td> </tr> <tr> <td><b>Disability Insurance</b></td> <td style="text-align:right;">Insurer</td> <td style="text-align:right;">\$ _____</td> <td style="text-align:right;">Monthly Coverage</td> </tr> <tr> <td></td> <td></td> <td style="text-align:right;">\$ _____</td> <td style="text-align:right;">Premium</td> </tr> <tr> <td></td> <td></td> <td></td> <td style="text-align:right;">Insured's Signature _____</td> </tr> <tr> <td><b>Other Insurance</b></td> <td style="text-align:right;">Insurer</td> <td style="text-align:right;">\$ _____</td> <td style="text-align:right;">Monthly Coverage</td> </tr> <tr> <td></td> <td></td> <td style="text-align:right;">\$ _____</td> <td style="text-align:right;">Premium</td> </tr> <tr> <td></td> <td></td> <td></td> <td style="text-align:right;">Insured's Signature _____</td> </tr> <tr> <td></td> <td></td> <td></td> <td style="text-align:right;">Total Premiums \$ _____</td> </tr> </table>		<b>Life Insurance</b>	Insurer	\$ _____	Initial Coverage Amount			\$ _____	Premium				Insured's Signature _____	<b>Disability Insurance</b>	Insurer	\$ _____	Monthly Coverage			\$ _____	Premium				Insured's Signature _____	<b>Other Insurance</b>	Insurer	\$ _____	Monthly Coverage			\$ _____	Premium				Insured's Signature _____				Total Premiums \$ _____
<b>Life Insurance</b>	Insurer	\$ _____	Initial Coverage Amount																																									
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			Total Premiums \$ _____																																									
<b>12. OFFICIAL FEES AND TAXES \$ _____</b> The estimated total amount You will pay for official and license fees, registration, title and taxes over the term of Your lease, whether included with Your monthly payments or assessed otherwise. The actual total of fees and taxes may be higher or lower depending on the tax rates in effect or the value of the leased property at the time a fee or tax is assessed.																																												
<b>13. LESSOR SERVICES</b> (See Item 18 on back)																																												
<b>14. LATE PAYMENTS</b> You will pay a late charge on each payment that is not received within 10 days after it is due. The charge is 7.5% of the full amount of the scheduled payment or \$50.00 whichever is less.																																												

16. Itemization of Gross Capitalized Cost						
Agreed Upon Value of the Vehicle \$	Sales/Use Tax & Other Applicable Taxes \$	Title Fees \$	License & Registration Fees \$	Extended Warranty & Service Contract \$	Lessor Services \$	Acquisition Fee \$
Documentation Fee \$	Life Insurance Premium \$	Disability Insurance Premium \$				<b>Total Gross Capitalized Cost \$</b>
SIGNATURES AND IMPORTANT NOTICES						
<b>Modification:</b> This lease sets forth all of the agreements of Lessor and You for the lease of the Vehicle. There is no other agreement. Any change in this lease must be in writing and signed by You and Ford Credit.						
Lessee	By			Title		
Co-Lessee	By			Title		
<b>NOTICE: (1) Do not sign this lease before You read it or if it has any blank space to be filled in. (2) You have the right to get a filled-in copy of this lease. You state that You have been given a filled-in copy of this lease at the time You sign it and notice of an assignment of this lease by the Lessor to Holder.</b>						
Lessee	By			Title		
Co-Lessee	By			Title		
YOU ACKNOWLEDGE THAT YOU HAVE READ AND AGREE TO BE BOUND BY THE ARBITRATION PROVISION ON THE REVERSE SIDE OF THIS CONTRACT						
Lessor is hereby notified that Holder has assigned to "Intermediary," as defined in the Red Carpet Lease Assignment, its rights (but not its obligations) with respect to the purchase of this Vehicle and the sale of this Vehicle at lease termination.						
Lessor accepts this lease and assigns it to Holder under the terms of the Red Carpet Lease - WOR Plan Agreement between Lessor and Holder unless otherwise indicated here: <input type="checkbox"/> LEV GUARRANTY						
Lessor	By			Title		
VEHICLE MAINTENANCE, INSURANCE AND USE						
<p><b>17. VEHICLE USE AND SUBLEASING</b> You will not use, or permit others to use the Vehicle (a) in violation of any law, (b) contrary to the provisions of any insurance policies covering the Vehicle, (c) outside the state where first titled or registered for more than 30 days without Ford Credit's written consent, (d) outside the United States, except for less than 30 days in Canada or (e) as a private or public carrier. You will keep this lease and Vehicle free of all liens and encumbrances. <b>You will not assign or sublease any interest in the Vehicle or lease without Ford Credit's written consent.</b></p> <p><b>18. VEHICLE MAINTENANCE AND OPERATING COSTS</b> Proper Vehicle maintenance is Your responsibility. You must maintain and service the Vehicle at Your own expense, using materials that meet the manufacturer's specifications. This includes following the owner's manual and maintenance schedule, documenting maintenance performed, and making all needed repairs. You are also responsible for all operating costs such as gas and oil. Lessor will provide the service(s), if any, identified in the Lessor Services section under the terms of a separate agreement. The manufacturer will invalidate warranty coverage on parts affected by a failure to maintain the Vehicle as required by the manufacturer. (See Lessor Services on the front of lease.)</p> <p><b>19. DAMAGE REPAIR</b> You are responsible for repairs of <b>All Damage</b> which are not a result of normal wear and use. These repairs include, but are not limited to, those necessary to return the Vehicle to its pre-accident condition, including repairs to <b>Exterior Sheet Metal and Plastic Components</b>, and to <b>Vehicle Safety Systems</b>, including air bag, seat belt and bumper system components. Replacement of Sheet Metal must be made with Original Equipment Manufacturer Sheet Metal. All other repairs must be made with Original Equipment Manufacturer parts or those of equal quality. Discuss this requirement with Your insurance company prior to signing a collision repair estimate or before authorizing any collision repair work.</p> <p>If You have not had the repairs made before the Vehicle is returned at the scheduled end of this lease, You will pay the estimated costs of such repairs, even if the repairs are not made prior to Holder's sale of the Vehicle.</p>			<p><b>20. VEHICLE INSURANCE</b> You must insure the Vehicle during this lease. This insurance must be acceptable to Ford Credit and protect You and Holder with (a) comprehensive fire and theft insurance with a maximum deductible amount of <b>\$1,000</b>; and (b) collision and upset insurance with a maximum deductible of <b>\$1,000</b>; and (c) automobile liability insurance with minimum limits for bodily injury or death of <b>\$25,000</b> for any one person and <b>\$50,000</b> for any one accident, and <b>\$10,000</b> for property damage. If the state in which You title/register the Vehicle establishes or changes the minimum automobile liability insurance limits greater than those listed above for bodily injury or death and property damage insurance, You must insure the Vehicle and the Holder at the higher minimum limits established by the state. <b>These amounts may not be sufficient to cover all Your liabilities. You may wish to consult Your insurance advisor about obtaining additional coverage.</b> You will list the loss payee and additional insured as requested by Lessor. You must give Ford Credit evidence of this insurance.</p> <p>You authorize Ford Credit, on Your behalf, to receive and endorse checks or drafts, and settle or release any claim under the insurance related to Holder's ownership of the Vehicle. You also assign to Holder any other insurance proceeds related to this lease or Holder's interest in the Vehicle.</p> <p>If You or Ford Credit obtain a refund for amounts paid to third parties for insurance, service contracts, or any other amount paid to a third party included in the Gross Capitalized Cost of this lease, You must pay to the Holder the entire amount of the refund and You authorize the Holder to subtract the refund from the amount You owe under this lease.</p> <p style="text-align: center;"><b>LESSOR IS NOT PROVIDING VEHICLE INSURANCE OR LIABILITY INSURANCE</b></p> <p>If you title/register the Vehicle in, or change the garage location of the Vehicle to a state where Ford Credit has established minimum automobile liability insurance limits greater than those listed above for bodily injury or death and property damage insurance, You must insure the Vehicle and the Holder at the higher minimum limits established by Ford Credit.</p>			
ENDING YOUR LEASE						
<p><b>21. TERMINATION</b> This lease will terminate (end) upon (a) the end of the term of this lease, (b) the return of the Vehicle to Lessor, and (c) the payment by You of all amounts owed under this lease. Ford Credit may cancel this lease if You default.</p> <p><b>22. RETURN OF VEHICLE</b> If You do not buy the Vehicle, at lease end You must return it to Lessor unless Ford Credit specifies another place. If You fail to return the Vehicle, You must continue to pay the monthly payments plus other damages to Ford Credit, including amounts payable under default. Payment of these amounts will not allow You to keep the Vehicle.</p> <p><b>23. STANDARDS FOR EXCESS WEAR AND USE</b> You are responsible for all repairs to the Vehicle that are not the result of normal wear and use. These repairs include, but are not limited to those necessary to repair or replace: (a) <b>Tires</b> which are unmatched, unsafe or have less than 1/8 inch of remaining tread in any place; (b) <b>Electrical or Mechanical defects or malfunctions</b>; (c) <b>Glass, Paint, Body Panels, Trim and Grill Work</b> that are broken, mismatched, chipped, scratched, pitted, cracked, or if applicable, dented or rusted; (d) <b>Interior rips, stains, burns or worn areas</b>; and (e) <b>All Damage</b> which would be covered by collision or comprehensive insurance whether or not such insurance is actually in force. Replacement of Sheet Metal must be made with Original Equipment Manufacturer Sheet Metal. All other repairs must be made with Original Equipment Manufacturer parts or those of equal quality. Your use or repair of the Vehicle must not invalidate any warranty.</p> <p>If You have not had the repairs made before the Vehicle is returned at the scheduled end of this lease, You will pay the estimated costs of</p>			<p><b>25. VOLUNTARY EARLY TERMINATION AND RETURN THE VEHICLE</b> You may terminate this lease early, if You are not in default, by returning the Vehicle to Lessor and paying the following: (a) an early termination fee of <b>\$200</b>, plus (b) the difference, if any, between the Unpaid Adjusted Capitalized Cost and the Vehicle's Fair Market Wholesale Value, plus (c) all other amounts then due under this lease. You will never pay more than the sum of the remaining unpaid lease payments, plus any excess wear and use and mileage charges, and all other amounts then due under this lease.</p> <p><b>VOLUNTARY EARLY TERMINATION AND PURCHASE THE VEHICLE</b> You may purchase the Vehicle from Lessor at any time for the sum of the remaining payments, less any unearned Rent Charges, plus the purchase option price and all other amounts then due under this lease.</p> <p>Unpaid Adjusted Capitalized Cost is reduced on each payment due date. It is calculated by reducing the Adjusted Capitalized Cost each month by the difference between the Base Monthly Payment and the part of the Rent Charges earned in that month on an <b>actuarial basis</b>. Rent Charges are earned when due. Lessor or Ford Credit will provide You with a written explanation of the actuarial method upon Your request.</p> <p><i>Fair Market Wholesale Value</i>, at Your option, will be: (a) an amount agreed to by You and the Lessor, or (b) the value which could be realized at the wholesale sale of the Vehicle, as determined by a professional appraisal obtained by You at Your expense within 10 days from termination from an independent third party agreeable to Ford Credit, or (c) if not established by agreement or appraisal, the net amount received by Ford Credit upon the sale of the Vehicle at wholesale.</p> <p>Please contact Ford Credit at 1-800-727-7000 or</p>			

<p>such repairs, even if the repairs are not made prior to Holder's sale of the Vehicle.</p> <p><b>24. ODOMETER STATEMENT</b> Federal law requires You to complete a statement of the Vehicle's mileage at the end of this lease.</p>	<p>www.fordcredit.com if You have any questions regarding terminating Your Red Carpet Lease.</p>
<p><b>DEFAULT AND LOSS OF VEHICLE</b></p>	
<p><b>26. DEFAULT</b> You will be in default if (a) You fail to make any payment when due, or (b) a bankruptcy petition is filed by or against You, or (c) any governmental authority seizes the Vehicle and does not promptly and unconditionally release the Vehicle to You, or (d) You have provided false or misleading material information when applying for this lease, or (e) You fail to keep any other agreement in this lease.</p> <p>If You are in default, Ford Credit may cancel this lease, take back the Vehicle and sell it at a public or private sale. You also give Ford Credit the right to go on Your property to peacefully retake the Vehicle. Even if Ford Credit retakes the Vehicle, You must still pay at once: (a) the difference, if any, between the Unpaid Adjusted Capitalized Cost and the value which could be realized at the sale of the Vehicle, <b>plus</b> (b) all other amounts then due under this lease. The value which could be realized at the sale of the Vehicle at Your option will be: (a) the net amount received by Ford Credit upon the sale of the vehicle at wholesale, or (b) as determined by a professional appraisal obtained by You at Your expense within 10 days from default, from an independent third party agreeable</p>	<p>to Ford Credit. You must also pay all expenses, including reasonable attorney's fees, payable by Ford Credit to obtain, hold and sell the Vehicle, collect amounts due and enforce Holder's rights under this lease. You authorize Ford Credit to cancel Your insurance and apply any proceeds to Your obligation.</p> <p><b>27. LOSS OR DESTRUCTION OF VEHICLE</b> If the Vehicle is stolen or destroyed, You will pay to Ford Credit: (a) the Unpaid Adjusted Capitalized Cost, <b>plus</b> (b) all other amounts then due under this lease, <b>minus</b> (c) any insurance proceeds received by Ford Credit. Gap Waiver: If You had in effect the insurance required under this lease and Ford Credit receives the full proceeds, You will pay to Ford Credit: (a) any past due monthly lease payments, <b>plus</b> (b) the amount of the applicable insurance deductible, <b>plus</b> (c) all other amounts then due under this lease. Even if the Vehicle is insured, until Ford Credit receives the appropriate amount above, You are responsible for the scheduled monthly payments.</p>
<p><b>ADDITIONAL INFORMATION</b></p>	
<p><b>28. ASSIGNMENT AND ADMINISTRATION</b> When You and Lessor sign this lease, Lessor will assign it to Holder. Ford Credit or a substitute will administer this lease. You must then pay all amounts due under this lease to Ford Credit.</p> <p>If Ford Credit is not the Holder of this lease, Holder has appointed Ford Credit as its agent. As agent for Holder, Ford Credit has the power to act on Holder's behalf to administer, enforce, and defend this lease. If Lessor has agreed to repair or maintain the Vehicle, obtain any insurance or perform any other service, You will look only to the Lessor for these services.</p> <p><b>29. TAXES</b> You will promptly pay all fees, charges, and taxes relating to the lease or Vehicle (except for Lessor's or Holder's income taxes). You will pay these amounts even if they are assessed after lease end.</p> <p><b>30. TITLING</b> The Vehicle will be titled in the name of Holder. You will register the Vehicle as directed by Ford Credit. You will pay all license, title and registration costs.</p> <p><b>31. LIFE INSURANCE</b> If Ford Credit receives the benefits paid under any life insurance described on the reverse side, this lease will continue if there is a Co-Lessee. Any Co-Lessee will pay when due all amounts not paid by the insurance. If there is no Co-Lessee, Ford Credit will accept a reasonable replacement designated by Your estate who agrees in writing to perform Your obligations not covered by the insurance.</p>	<p><b>32. INDEMNITY</b> You will indemnify and hold harmless Lessor, Ford Credit and Holder and their assigns from any loss or damage to the Vehicle and its contents and from all claims, losses, injuries, expenses and costs related to the use, maintenance, or condition of the Vehicle. You will promptly pay all fines and tickets imposed on the Vehicle or its driver. If You do not pay, You will reimburse Ford Credit and pay a \$20 administration fee, unless prohibited by law, for every such fine, ticket, or penalty that must be paid on Your behalf.</p> <p><b>33. SECURITY DEPOSIT</b> Your security deposit may be used by Ford Credit to pay all amounts that You fail to pay under this Lease. You will not receive any interest, profits or other earnings on Your security deposit(s).</p> <p><b>34. CONSUMER REPORTS:</b> You authorize Ford Credit to obtain consumer credit reports from consumer reporting agencies (credit bureaus) for any reason and at any time in connection with this lease.</p> <p><b>35. GENERAL</b> Except as otherwise provided by the law of the state where You reside, the law that will apply to this lease is the law of the state where the Lessor's place of business is, as set forth on the front of the lease. If that law does not allow any of the agreements in this lease, the ones that are not allowed will be void. The rest of this lease will still be good.</p>
<p><b>READ THIS ARBITRATION PROVISION CAREFULLY AND IN ITS ENTIRETY</b></p>	
<p style="text-align: center;"><b>ARBITRATION</b></p> <p>Arbitration is a method of resolving any claim, dispute, or controversy (collectively, a "Claim") without filing a lawsuit in court. Either you or Lessor ("us" or "we") (each, a "Party") may choose at anytime, including after a lawsuit is filed, to have any Claim related to this contract decided by arbitration. Such Claims include but are not limited to the following: 1) Claims in contract, tort, regulatory or otherwise; 2) Claims regarding the interpretation, scope, or validity of this clause, or arbitrability of any issue; 3) Claims between you and us, our employees, agents, successors, assigns, subsidiaries, or affiliates; 4) Claims arising out of or relating to your application for credit, this contract, or any resulting transaction or relationship, including that with the dealer, or any such relationship with third parties who do not sign this contract.</p> <p><b>RIGHTS YOU AND WE AGREE TO GIVE UP</b> If either you or we choose to arbitrate a Claim, then you and we agree to waive the following rights:</p> <ul style="list-style-type: none"> <li>• RIGHT TO A TRIAL, WHETHER BY A JUDGE OR JURY</li> <li>• RIGHT TO PARTICIPATE AS A CLASS REPRESENTATIVE OR A CLASS MEMBER IN ANY CLASS CLAIM YOU MAY HAVE</li> <li>• AGAINST US WHETHER IN COURT OR IN ARBITRATION</li> <li>• BROAD RIGHTS TO DISCOVERY AS ARE AVAILABLE IN A LAWSUIT</li> <li>• RIGHT TO APPEAL THE DECISION OF AN ARBITRATOR</li> <li>• OTHER RIGHTS THAT ARE AVAILABLE IN A LAWSUIT</li> </ul> <p><b>Rights You And We Do Not Give Up:</b> If a Claim is arbitrated, you and we will continue to have the following rights, without waiving this arbitration provision as to any Claim: 1) Right to file bankruptcy in court; 2) Right to enforce the ownership interest in the vehicle, whether by repossession or through a court of law; 3) Right to take legal action to enforce the arbitrator's decision; and 4) Right to request that a court of law review whether the arbitrator exceeded its authority.</p> <p>Either Party must contact any association below and the other Party to start arbitration. The applicable rules (the "Rules") may be obtained from the association.</p> <ul style="list-style-type: none"> <li>• American Arbitration Association ("AAA"), at 1-800-778-7879, or www.adr.org;</li> <li>• J.A.M.S./Endispute, at 1-800-448-1660, or www.jamsadr.com;</li> <li>• National Arbitration Forum, at 1-800-474-2371, or www.artb-forum.com.</li> </ul> <p>If there is a conflict between the Rules and this contract, this contract shall govern. This contract is subject to the Federal Arbitration Act (9 U.S.C. § 1 et seq.) and the Federal Rules of Evidence. The arbitration decision shall be in writing with a supporting opinion. We will pay your total reasonable arbitration fees and expenses (not including attorney fees, except where applicable law otherwise provides) in excess of \$125. We will pay the whole filing fee if we demand arbitration first. Any portion of this arbitration clause that is unenforceable shall be severed, and the remaining provisions shall be enforced.</p>	

### ***D.4.3 Case 8: WENDT EQUIPMENT LEASING TERMS AND CONDITIONS***

**ARTICLE 1. THE PARTIES.** Wendt, LLP, (“Lessor”) agrees to lease to the customer (the “Lessee”) identified on the front page or order form of this lease agreement (the “Lease”) the equipment as described on the front page or order form of this Lease (the “Equipment”).

**ARTICLE 2. THE RENTAL PERIOD.** The rental period extends from the time the Equipment leaves the Lessor’s yard until it is returned to the Lessor’s yard in satisfactory working condition. On out-of-town shipments of Equipment, the date of the bill of lading is the beginning of the rental period and it ends on the date the Equipment is returned to the Lessor’s yard or siding, or on the date of return bill of lading, if stipulated by the Lessor.

**ARTICLE 3. RENT.** The rental rates are set forth on the face of this Lease. Rental rates are based on 8 h per day, 5 days per week and 22 eight-hour days in any 30 consecutive day period. Should the Equipment be used longer, the overtime rates, set forth in Article 3, shall apply. The Lessee shall pay rent for the entire period on each piece of Equipment. Rent is not subject to any deductions on account of non-working time. The monthly rates are not subject to deductions on account of non-working time. Fractions of the month at the beginning or the end of the rental period shall be at the monthly rental rate, pro-rated, but only after one full month of rental. If Lessee fails to take possession of the Equipment reserved for it or cancels this Lease, the Lessee agrees to pay a cancellation fee to the Lessor in the amount of 2 % of the value of the Equipment as noted on the face of this Lease and 4 % of the value of the Equipment if it has been loaded for transit to the Lessee.

**ARTICLE 4. OVERTIME RATE BASIS.** One of the following schedules of overtime charges should be agreed upon, and noted on the front page or order form of this Lease.

*Schedule A:* On the daily rate, add 1/8th of the daily rate for each hour worked in excess of 8 h in any one day; 1/40th of the weekly rate for each hour worked in excess of 40 h in any one week; and 1/176th of the monthly rate for each hour in excess of 176 h worked in any 30 consecutive day period.

*Schedule B:* On the daily rate, for each hour over 8 h, 1/16th of the daily rate shall be charged. On the weekly or monthly rate, two shifts are charged at 11/2 times the single shift, and three shifts are charged at 2 times the single shift rate. If no overtime rate schedule is referenced on the front page or order form of this Lease. Schedule “A” shall apply. Lessee agrees to state in writing the number of excess hours the Equipment is used and to pay the Lessor the appropriate rent amount.

**ARTICLE 5. TERMS OF PAYMENT.** Rentals shall be paid on the 15th of the month following the first use of the Equipment unless otherwise stipulated on the first page or order form of this Lease. Lessor shall be entitled to reimbursement of all costs and expenses, including court costs and attorneys fees, incurred in

collecting payment from Lessee. Any past due accounts shall have interest accruing at a rate of 2 % per month. Any payments made on past due accounts shall first be applied to collection costs and expenses, then late payment fees, then to interest, then to rent. Payment of late payment fees and interest shall not waive the Lessor's right to terminate this Lease as hereinafter provided.

In addition to any other rights available to Lessor under this Lease, if any rent is not paid within 30 days of due date, the Lessee shall be in breach of the terms of this Lease. If the Lessee is in breach of this Lease or becomes subject to any of bankruptcy, receivership or insolvency proceeding, the Lessor may, without notice, declare the entire amount of rent under this Lease due and payable, terminate this Lease without court order and take possession of the Equipment without being in breach of this Lease or liable to Lessee for trespass. Lessee will be responsible for any and all legal and transportation costs incurred by Lessor in any such repossession.

**ARTICLE 6. LOADING AND FREIGHT CHARGES.** The Equipment is rented F.O.B. to the Lessor's yard or siding. Any additional charges incurred in loading, unloading, erection, dismantling, are the responsibility of the Lessee. If the Lessee does not furnish shipping instructions, the Lessor will select the means of conveyance for Lessor.

**ARTICLE 7. NOTICE OF RETURN OR RECALL.** The Lessor may recall any or all Equipment upon 30 days written notice to the Lessee. The Lessee may return any or all Equipment to the Lessor upon 30 days written notice to Lessor.

**ARTICLE 8. SUBLEASING.** No Equipment listed herein may be subleased by the Lessee. The Lessee further agrees not to assign or transfer any interest in this Lease without written consent of the Lessor.

**ARTICLE 9. RELOCATION EQUIPMENT.** Lessee agrees not to move the Equipment to another location without the express written consent of the Lessor.

**ARTICLE 10. REPAIRS AND MAINTENANCE.** The Lessor is required to supply the Equipment in good operating condition. The Lessee acknowledges by signing this Lease that it has carefully examined the Equipment and accepts the Equipment as being in good operating condition. The Lessee agrees that it will pay all cost of repairs during the rental period, including labour, material, parts and other items, except for normal wear and tear. Rent continues until the Equipment is returned to Lessor with all necessary repairs made to the Equipment and with it in normal operating condition. "Normal wear and tear" is defined as use of the Equipment under normal work conditions, with qualified personnel providing proper operation, maintenance and service. If repairs exceeding the normal wear and tear are necessary upon return of the Equipment, Lessor is authorised to make such repairs and Lessee agrees to pay Lessor the reasonable costs of such repairs to the Equipment and rent while such repairs are being made. Lessee agrees not to cover, alter, substitute, or remove any identifying insignia displayed on the Equipment. Lessee will not permit the Equipment to be abused, overloaded, and used beyond its capacity. Lessee will not alter the Equipment in any fashion and shall use and operate the Equipment in accordance with all applicable laws and the manufacturer's operating manual. The Equipment furnished is standard from

manufacturer only. Any modification or additions or optional equipment to be added to the Equipment shall be at an additional cost to Lessee. Equipment to be used by Lessee under normal working conditions as designed and specified by manufacturer. Unusual or abnormal working conditions, requiring work in rock, excessive mud, abrasives, etc., or tying down, towing, demolition, adding additional or excessive weight will be billed to the Lessee as additional wear and tear and/or cost of repairs as provided herein.

**ARTICLE 11. INSPECTION.** Before shipment is made, the Lessee may require inspection of the Equipment. If it is not in substantially the condition required by this Lease, the cost of inspection will be paid by the Lessor, and Lessee may cancel the Lease at its option or require the Lessor to supply Equipment in normal operating condition. The Lessor will have the right at any time to inspect Equipment and will be given free access by Lessee to it and the necessary facilities to accomplish the inspection.

**ARTICLE 12. INSURANCE & INDEMNIFICATION.** Lessee, at its own expense, shall carry and maintain in force at all times during the term of this Lease insurance of the type and minimum coverage limits as follows:

- (1) Worker's Compensation—Statutory amount under the laws of the state where the Lessee is operating the Equipment.
- (2) Commercial General Liability—\$1,000,000 per occurrence.
- (3) Property/Casualty insurance—with coverage limits sufficient to cover the full replacement cost of the Equipment.

All such insurance shall be in form and with companies reasonably satisfactory to the Lessor. Evidence of adequate insurance shall be delivered to Lessor within 10 days after execution of this Lease, and thereafter certificates of renewal policies shall be delivered to Lessor within 10 days prior to the expiration of the term of such policy. Any policies of insurance carried by the Lessee shall provide that as against Lessor, the Lessee and insurers shall waive any rights of subrogation, set off, counterclaim or any other deduction, whether by attachment or otherwise.

Notwithstanding Lessee's responsibility for insurance hereunder, Lessee shall defend, indemnify and hold the Lessor harmless from and against any and all loss or damage to the Equipment or liability through use of the Equipment during the term of this Lease. If the Equipment is destroyed through fire, flood, explosion, or any other cause, the Lessee will repay the Lessor the full replacement cost of the Equipment. Rent shall continue to accrue through the date Lessor receives payment in full of the replacement cost of the Equipment.

**ARTICLE 13. TITLE.** Title to the Equipment shall at all times remain vested in the Lessor. The Lessee agrees to keep the Equipment free and clear of any claims, liens or encumbrances. Lessee further agrees to use the Equipment in accordance with all applicable government regulations, ordinances or laws. The Lessee shall give the Lessor immediate notice in case any Equipment is levied upon or becomes subject to seizure.

**ARTICLE 14. TAXES.** Lessee agrees to pay all government taxes or other assessments against this Equipment except as provided on the first page or order form of this Lease.

**ARTICLE 15. WAIVERS.** No waivers of any part or article of this Lease shall be construed to be a waiver of any other part or article or be recognised unless it is in writing and signed by both parties.

**ARTICLE 16. LIMITED LIABILITY:** LESSEE AGREES LESSOR DOES NOT AND CANNOT GUARANTEE OR WARRANT THE SUCCESS OR FAILURE OF THE USE OF ANY EQUIPMENT LEASED HEREUNDER. BECAUSE LESSOR CANNOT GUARANTEE OR WARRANT THE OUTCOME FROM ANY USE OF EQUIPMENT LEASED HEREUNDER, LESSEE AGREES IT SHALL RENT THE EQUIPMENT FROM LESSOR ON THE BASIS THAT SUCH EQUIPMENT MAY BE WHOLLY INEFFECTIVE AT THE INTENDED PURPOSE FOR WHICH IT HAS BEEN LEASED. BASED ON THE FOREGOING, LESSOR DISCLAIMS ALL WARRANTIES, EXPRESS OR IMPLIED, INCLUDING ANY WARRANTIES OF MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE. LESSEE FURTHER AGREES THAT LESSOR SHALL NOT BE LIABLE TO LESSEE, OR ANY OF ITS AGENTS, EMPLOYEES, CUSTOMERS OR CONTRACTORS FOR ANY LOSS OR INJURY ARISING OUT OF, IN WHOLE OR IN PART, THE EQUIPMENT LEASED HEREUNDER. NOTWITHSTANDING THE FOREGOING AND BASED UPON THE NEGOTIATED RENT FOR THE EQUIPMENT LEASED HEREUNDER, LESSOR'S MAXIMUM LIABILITY FOR ANY CLAIM BROUGHT AGAINST IT HEREUNDER SHALL BE THE LESSER OF: (I) THE AMOUNT OF RENT PAID BY LESSEE TO LESSOR FOR THE EQUIPMENT AT ISSUE, OR (II) ONE MONTH'S RENT FOR THE EQUIPMENT AT ISSUE. UNDER NO CIRCUMSTANCES SHALL LESSOR BE RESPONSIBLE FOR ANY BUSINESS INTERRUPTION DAMAGES INCURRED BY LESSEE OR ANY OTHER THIRD PARTY RELATING IN ANY MANNER TO THIS LEASE OR THE EQUIPMENT THAT IS THE SUBJECT OF THIS LEASE.

**ARTICLE 17. INDEMNITY.** Lessee agrees to indemnify, defend and hold harmless Lessor, its affiliates, employees, successors and assigns (all referred to as "Lessor") from and against any losses, damages, claims, fines, penalties and expenses (including reasonable attorneys fees) that arise out of or result from injuries or death to persons or damage to property in any way arising out of or caused or alleged to have been caused by services or Equipment provided by Lessor.

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