# Appendix A: Averaging Operators

The averaging operators used in Jackson [1] are presented. For further details on the form of averaged time and space derivatives, the reader is referred to [1].

## A.1 Fluid Average

The weighting factor of Eq. 3.1 can be used to define the volume fraction of the fluid at position $\mathbf{x}$ according to

$$\alpha_f(\mathbf{x}) = \int_{V_f} g(|\mathbf{x} - \mathbf{y}|)\mathrm{d}\mathbf{y}, \tag{A.1}$$

where the integration is over the entire fluid volume. The quantities are also time dependent, but this has been omitted for clarity. Similarly, the average of a general property $\psi$ (where $\psi$ is vector or scalar) is defined as

$$\alpha_f \langle \psi \rangle_f(\mathbf{x}) = \int_{V_f} \psi(\mathbf{y}) g(|\mathbf{x} - \mathbf{y}|)\mathrm{d}\mathbf{y}. \tag{A.2}$$

The average of space and time derivatives is slightly more involved, and the reader is once again referred to [125] for details.

## A.2 Solid Average

The solid averages are analogous to the fluid-phase averages and are constructed by integration over the entire volume occupied by the solid particulate phase. Thus, the volume fraction of the solid is given by

J. S. Shrimpton et al., *Statistical Treatment of Turbulent Polydisperse Particle Systems*, Green Energy and Technology,
DOI: 10.1007/978-1-4471-6344-2, © Springer-Verlag London 2014

$$\alpha_s(\mathbf{x}) = \sum_n \int_{V_s} g(|\mathbf{x}-\mathbf{y}|)\mathrm{d}\mathbf{y}, \tag{A.3}$$

and similarly, the average of a solid-phase property is defined as

$$\alpha_s\langle\psi\rangle_s(\mathbf{x}) = \sum_n \int_{V_s} \psi(\mathbf{y})g(|\mathbf{x}-\mathbf{y}|)\mathrm{d}\mathbf{y}. \tag{A.4}$$

## A.3 Mixture Average

A mixture or overall volume average $\langle\psi\rangle$ of a point property $\psi$ is defined as

$$\begin{aligned}\langle\psi\rangle_m(\mathbf{x}) &= \int_V \psi(\mathbf{y})g(|\mathbf{x}-\mathbf{y}|)\mathrm{d}\mathbf{y} \\ &= \alpha_f(\mathbf{x})\langle\psi\rangle_f + \alpha_s(\mathbf{x})\langle\psi\rangle_s,\end{aligned} \tag{A.5}$$

or alternatively, the mass weighted average is given as

$$\rho_m(\mathbf{x}) = \rho_f\alpha_f\langle\psi\rangle_f + \rho_s\alpha_s\langle\psi\rangle_s, \tag{A.6}$$

where $\rho_f$ and $\rho_p$ are the material densities of the fluid and particulate phases, respectively, and $\rho_m(\mathbf{x})$ is the mixture density.

## A.4 Particle Average

The particle phase is a discrete phase, so the averaged quantities require a different averaging method. The particles to be averaged can be completely described according to the 'point-particle' assumption [99]. Following this assumption, the motion of each particle is determined by the particle velocity and resultant force (Jackson also considers the moment acting on the particle). The details of the stress distribution within the particle are not required. The number of particles per unit volume $n$ at position $\mathbf{x}$ is defined as

$$n(\mathbf{x}) = \sum_{i=1}^{n_p} g(|\mathbf{x}-\mathbf{x}^{(i)}|), \tag{A.7}$$

where $\mathbf{x^{(i)}}$ is the position of the centre of an individual particle $i$. Similarly, the average of a property $\psi$ is given by

$$n(\mathbf{x})\langle\psi\rangle_{\mathbf{p}} = \sum_{i=1}^{n_p} \psi^{(i)} g(|\mathbf{x} - \mathbf{x}^{(i)}|). \tag{A.8}$$

## Reference

1. Jackson R (1997) Locally averaged equations of motion for a mixture of identical spherical particles and a Newtonian fluid. Chem Eng Sci 52:2457–2469

# Index




J. S. Shrimpton et al., *Statistical Treatment of Turbulent Polydisperse Particle Systems*, Green Energy and Technology,
DOI: 10.1007/978-1-4471-6344-2, © Springer-Verlag London 2014