A
Appendix: Prerequisites

A.1 Measure

The concept of measure generalizes the familiar notions of length, area, and volume. A reader, familiar with stochastic processes, may have also encountered this term, since probability is defined as a certain measure.[1]

Intuitively speaking, we define the measure of a set $A \subset \mathbb{R}$ to be a real function $\mu(A)$, such that the following conditions (we call them the axioms of measure) are satisfied:

1) $\mu(\varnothing) = 0.$
2) If the sets A_j, $j \in \mathbb{N}$, are pairwise disjoint, then

$$\mu\left(\bigcup_{j=1}^{\infty} A_j\right) = \sum_{j=1}^{\infty} \mu(A_j) .$$

Unfortunately, it is impossible to define this function in such a way that would enable us to compute it for every subset of $\mathbb{R}$. For this reason we confine our attention only to sets for which such computation is possible. We term such sets measurable.

If for every measurable set *A*, it is true that $\mu(A) \geq 0$, the measure is called nonnegative, otherwise we call it signed. If for every measurable set *A* (including $\mathbb{R}$), $\mu(A) < \infty$, the measure is called finite. An example of a finite measure is probability: Let $\mu(A)$ be the probability that the output of some random number generator belongs to *A*. Then $\mu(\mathbb{R}) = 1$.

An important role in analysis is played by the Lebesgue measure, $\mu_L(A)$, which can be axiomatically defined by requiring that $\mu_L(a;b) = b - a$, i.e., the Lebesgue measure of an interval or a segment is its length. It can be shown (See, for example,

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[1] Readers who studied probability without ever hearing about measure may now find themselves in the position of Monsieur Jourdain from Moliere's *Le Bourgeois Gentilhomme*, who suddenly found out that he was speaking prose all his life without knowing it!

[129]) that this condition, together with the axioms of measure, uniquely specifies the function $\mu_L(A)$ for any set A for which the Lebesgue measure can be defined. Clearly, the Lebesgue measure is nonnegative, but not finite.

Obviously, the Lebesgue measure of a set consisting of only one point is zero. This implies that the Lebesgue measure of a countable set is also zero because any such set consists only of isolated points. The converse is false: There exists an uncountable set of Lebesgue measure zero.[2] Also, as noted before, there are sets for which Lebesgue (or any other) measure cannot be defined.[3]

If a certain property holds for all real numbers except those that belong to set of Lebesgue measure zero, it is customary to say that this property holds almost everywhere (commonly abbreviated a.e.), and we will do so from now on.

Another concept of measure that plays an important role is that of Stieltjes, which is defined as follows. Let $f(t)$ be a nondecreasing lower-semicontinuous function[4] of a scalar argument t. Define

$$\begin{aligned} &\mu(a;b) \triangleq f(b) - f(a+0); \\ &\mu[a;b] \triangleq f(b+0) - f(a); \\ &\mu(a;b] \triangleq f(b+0) - f(a+0); \\ &\mu[a;b) \triangleq f(b) - f(a). \end{aligned}$$

Similarly to the Lebesgue measure, it can be shown that these conditions, together with the axioms of measure, uniquely specify the function $\mu(A)$. The measure thus defined is called the Lebesgue-Stieltjes measure. It is easy to see that the Lebesgue-Stieltjes measure is nonnegative. The function $f(t)$ is called the generating function for the measure, which we denote μ_f. It can be shown that every nonnegative measure can be generated by a nondecreasing lower-semicontinuous function. Note that if we set $f(t) = t$, we obtain the Lebesgue measure.

If we relax the assumption that the function $f(t)$ is nondecreasing by requiring instead for it to be a difference of two nondecreasing functions (functions that satisfy this requirement are said to be of bounded variation[5]), the above formulas define a signed measure. Furthermore, every signed measure can be generated by a function of bounded variation.

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[2] Interested readers are encouraged to look up information on the so-called Cantor tertiary set.

[3] Interested readers may consult the book by Schilling [129] for more details.

[4] A function $f(t)$ is called lower-semicontinuous if it is continuous from below, that is for any sequence of numbers $t_n<t_0$ and converging to the point t_0, the sequence $f(t_n)$ converges to $f(t_0)$. Notation $f(t+0)$ will be used to denote a jump that the function $f(t)$ may undergo at the point t_0.

[5] This is not the rigorous definition, but it will suffice for our purposes. An example of a function that is not of bounded variation is $t \sin(1/t)$.

Unlike the Lebesgue measure, the Lebesgue-Stieltjes measure of a set consisting of only one point is not necessarily zero. Suppose that $f(t)=0$ for $t \le 0$ and $f(t)=1$ for $t>0$. Then from the second axiom of measure we have $\mu_f[0;1)=\mu_f(\{0\})+\mu_f(0;1)$, which implies that $\mu_f(\{0\})=1$, since $\mu_f[0;1)=1$ and $\mu_f(0;1)=0$.

A.2 Lebesgue Integral

The reader undoubtedly remembers how the concept of integration was introduced in a basic calculus course. The interval of integration was partitioned and the so-called Riemann sums were defined. Then, the Fundamental Theorem of Calculus was proved, which asserted that the derivative with respect to the upper limit of the integral of the continuous function is the integrand.

This concept of integration is sufficient for all practical purposes. However, it has a few theoretical flaws. One of them is that the class of functions that can be integrated this way is "too narrow." What exactly we mean by this statement will become clear as we proceed.

In order to address these theoretical issues, Lebesgue introduced a more flexible concept of integration that involves partitioning the range, rather than the domain of a function.

First, we need to introduce the concept of a measurable function. We say that the function $f(t)$ is measurable if the set $A=\{t: f(t)<c\}$ is measurable for every real number *c*. Again, this is not the formal definition, but it will suffice for our purposes. Of course, whether a given function is measurable depends on the measure we use. However, the one used most frequently is the Lebesgue measure, and we will use it as well without stating the name explicitly. Practically speaking, any function one can reasonably think of is measurable.

Next, we consider a class of the so-called simple functions. A function $f(t)$ is called simple if its range is either a finite or a countable set. Let the values of this function be denoted by y_j. The simple function $f(t)$ is measurable if and only if all sets $A_j=\{t: f(t)=y_j\}$ are measurable. An arbitrary function $f(t)$ is measurable if and only if it is a limit of a uniformly convergent[6] sequence of simple measurable functions.

The integral of a simple function $f(t)$ with respect to a measure μ is defined by

$$\int_A f(t)d\mu(t)=\sum_j y_j\mu(A_j)\ ,\ A_j=\{t\in A: f(t)=y_j\}\,.$$

[6] Roughly speaking, uniform convergence of a sequence of functions $f_n(t)$ to the limit function $f(t)$ means that the rate of convergence does not depend on t.

The expression in the right-hand side of the integral may be either a finite sum or an infinite series. If the latter converges, the function $f(t)$ is called Lebesgue integrable or simply integrable. If the sum is finite, then the measurability of the function $f(t)$ is necessary and sufficient for it to be integrable.

This definition roughly means that for each of the possible values of the function $f(t)$, we compute the measure of the set on which the function takes this value and then add the results of such computations.

An arbitrary function $f(t)$ is integrable if it is a limit of a uniformly convergent sequence $f_n(t)$ of simple integrable functions. If so, we write

$$\int_A f(t)d\mu(t) = \lim_{n\to\infty} \int_A f_n(t)d\mu(t) .$$

It can be shown that this limit always exists for any uniformly convergent sequence $f_n(t)$ of simple integrable functions and, furthermore, for the given function $f(t)$, it does not depend on a choice of the sequence $f_n(t)$. Furthermore, if the function $f(t)$ is bounded on the set *A,* its measurability is necessary and sufficient for it to be integrable.

If the Lebesgue measure is used, which is the most common case, the integral we have just defined is called the Lebesgue integral, and *dt* is often written instead of $d\mu(t)$. From now on, all the integrals will be understood in this sense unless it is explicitly stated that some other measure is being used. If the set *A* is a segment, we may resort to the familiar notation involving lower and upper limits of integration.

It is important to note that if the function $f(t)$ is integrable using the familiar Riemann sum procedure (we call such functions Riemann integrable), then both approaches give the same values. All the integration techniques learned in a standard calculus course are applicable to Lebesgue integrals as well.

There is a theorem that states that a function is Riemann integrable on a certain interval if and only if it is continuous a.e. on this interval. A sequence of uniformly convergent Riemann integrable functions is also Riemann integrable. Without the requirement for the convergence to be uniform, this statement is false. From the theoretical point of view, this restriction turns out to be too severe, and this is the reason for using the Lebesgue approach. From the practical point of view, however, there is relatively little loss in thinking about integrals in terms of the ordinary Riemann definition.

A.3 Lebesgue-Stieltjes Integral

Another definition of integral that is sometimes used in this book is that of Lebesgue-Stieltjes. As the term suggests, this definition involves the use of Lebesgue-Stieltjes measures and functions that generate them.

Let $\vartheta(t)$ be a function of bounded variation that generates the signed measure μ_ϑ. Then, we introduce the following integral expression:

$$\int_A f(t)d\vartheta(t) = \int_A f(t)d\mu_\vartheta(t) .$$

The integral expression in the left-hand side is called the Stieltjes integral. Because we defined it using the concept of measure, we call it the Lebesgue-Stieltjes integral. It is thus distinguished from the original Stieltjes approach which used the procedure similar to the Riemann sums. The latter proved to be unsatisfactory, because it requires continuity of the function $f(t)$ on the interval of integration.

The reason for using the Stieltjes notation is that sometimes we need to emphasize certain properties of the generating function as opposed to the measure. Let us elaborate this idea a little further. In order to do this, we need to define some classes of functions.

From the standard calculus course, it is known that if the function $f(t)$ is continuous on a segment $[a;b]$, then, the following equality holds for every $t \in [a;b]$:

$$\int_a^t f'(t)dt = f(t) - f(a),$$

where the integral is understood in the sense of Riemann.

When the Lebesgue integral is considered, this statement is no longer true in general. In order to make it true, a narrower class of functions, called absolutely continuous, is introduced. A function $f(t)$ is called absolutely continuous on a segment $[a;b]$ if for every $\varepsilon > 0$ there exists $\delta > 0$ such that

$$\sum_{j=1}^{n} \left| f(\alpha_j) - f(\beta_j) \right| < \varepsilon$$

for every finite collection of intervals $(\alpha_j;\beta_j) \subset [a;b]$ with

$$\sum_{j=1}^{n} \left| \alpha_j - \beta_j \right| < \delta .$$

The crucial property of absolutely continuous functions is that they are differentiable a.e. This statement is generally false for continuous functions. In fact, there are functions continuous on a segment but not differentiable at any point of this segment![7] Another important property of absolutely continuous functions is that they are of bounded variation.

[7] Such functions are often called nowhere-differentiable. See, for example, [99, p. 113].

Furthermore, it is possible to show that for Lebesgue integrals, the equality

$$\int_a^t f'(t)dt = f(t) - f(a)$$

completely characterizes the class of absolutely continuous functions. In addition, the Lebesgue integral of any integrable function over a segment is an absolutely continuous function of the upper limit of integration.

Let us now introduce the concept of a saltus function constructed as follows. Define within a segment $[a;b]$ a finite or a countable set of points denoted by t_j. Suppose further that each point t_j has a finite number h_j associated with it, and furthermore, $\sum_j h_j < \infty$. Let

$$h(t) = \sum_{t_j < t} h_j .$$

The function $h(t)$ thus defined is called a saltus function.

For every function $f(t)$ of bounded variation, we have $f(t) = g(t) + h(t)$, where $g(t)$ is a continuous function and $h(t)$ is a saltus function. If the function $g(t)$ is not absolutely continuous, we define a function $\phi(t)$ by

$$\phi(t) = \int_a^t g'(t)dt .$$

Since the function $g(t)$ is of bounded variation, it is differentiable a.e., which means that the integral in the right-hand side exists.

It can be shown that the derivative of the difference $g(t) - \phi(t)$ is zero a.e. A function with the derivative equal to zero a.e. is called singular. It is possible that the derivative of a singular function at points where it is not zero may not exist.

Therefore, every function of bounded variation can be represented as a sum of three functions (we call them components): an absolutely continuous function, a saltus function, and a singular function. This is called the canonical decomposition.

With this in mind, let us consider some special cases of the Lebesgue-Stieltjes integral. If the function $\vartheta(t)$ is absolutely continuous, it can be shown that

$$\int_a^b f(t)d\vartheta(t) = \int_a^b f(t)\vartheta'(t)dt .$$

In other words, the Lebesgue-Stieltjes integral reduces to the Lebesgue integral.

Next, consider the case when $\vartheta(t)$ is a saltus function having jumps equal to h_j at points t_j. Then we find

$$\int_a^b f(t)d\vartheta(t) = \sum_j f(t_j)h_j,$$

i.e., the integral degenerates into a finite sum or infinite series.

Putting the above considerations together, we conclude that if the function $\vartheta(t)$ has only the absolutely continuous and the saltus components, the Lebesgue-Stieltjes integral can be reduced to the sum of the Lebesgue integral and the series. If the singular component is present, such reduction is impossible. This fact will play an important role when we compare some of the stability criteria proved in this book with those obtained in earlier papers on the subject.

A.4 Norms and L^p Spaces

The reader has most likely encountered the notion of a vector space either in a linear algebra or state-space methods course. It might have been mentioned in passing that certain functions form various vector spaces. This concept will now be discussed in some more detail.

One of the concepts often associated with a vector space is the norm, which is a natural generalization of the length of a vector. The function $\rho(x)$ of a vector *x* is called a norm if it satisfies the following three properties:

1) For any vector *x*, $\rho(x) \geq 0$ with $\rho(x) = 0$ if and only if *x*=0;
2) For any real or complex number α and any vector *x*, $\rho(\alpha x) = |\alpha| \rho(x)$;
3) For any two vectors *x* and *y*, $\rho(x+y) \leq \rho(x) + \rho(y)$.

For usual finite-dimensional vector spaces $\mathbb{R}^m$, the most commonly used norm is Euclidean, denoted by the single bars and defined by

$$|x| = \sqrt{\sum_{j=1}^{m} x_j^2} .$$

An important point about the Euclidean norm is that it can also be defined using the product of a vector with itself:

$$|x| = \sqrt{x^* x} ,$$

where the star denotes the vector transpose operation.

Another vector norm that is sometimes used is the maximum of the absolute values of all the components. It is denoted by single bars with the index infinity:

$$|x|_\infty = \max_j |x_j| .$$

The space $L^1(a;b)$ is defined as a class of vector functions in $\mathbb{R}^m$ whose components have finite Lebesgue integral on the interval $(a;b)$ and either endpoint can be either positive or negative infinity. The norm in this vector space is given by

$$\|x(\cdot)\|_1 = \int_a^b |x(t)|\, dt .$$

The subscript next to double bars is used to indicate that we are considering the $L^1(a;b)$ function space. We are using the dot in place of the argument of the function to indicate that we are considering this function itself as an element of a vector space as opposed to the value of a specific function at a specific point, which is the case in the right-hand side.

In a similar way we define the spaces $L^p(a;b)$ for $p > 1$ with the norm given by

$$\|x(\cdot)\|_p = \sqrt[p]{\int_a^b |x(t)|^p\, dt} .$$

In addition, we define the space $L^\infty(a;b)$ to include all functions with absolute values not exceeding a certain finite number except on a set of measure zero. This number is called an essential supremum and will be defined to be the L^∞-norm of the function:

$$\|x(\cdot)\|_\infty = \operatorname{ess\,sup} |x(t)|.$$

Similarly, the norm in each of the L^p spaces is called the L^p-norm. The L^2-norm is usually called the Euclidean norm of a (possibly vector) function. Since it will occur most frequently, we will often omit the index and use the notation $\|\cdot\|$. This norm must be distinguished from the Euclidean norm of a value $x(t)$ of a vector function in a finite dimensional space, which, as stated above, is denoted by single bars.

If the interval $(a;b)$ is finite, then $p < q$ implies that $L^p(a;b) \subset L^q(a;b)$. The statement is not true if either endpoint is infinite. However, the following statement is true for any interval, finite or infinite: $L^1 \cap L^\infty \subset L^p$ for any $p > 1$.

Let us consider some elementary examples.

1) The function $f_1(t) \equiv 1$ belongs to the space $L^\infty(0;\infty)$ but not to any other of the $L^p(0;\infty)$ spaces.

2) The function

$$f_2(t) = \frac{1}{1+t}$$

belongs to both $L^2(0;\infty)$ and $L^\infty(0;\infty)$ but not to the $L^1(0;\infty)$ space.

3) The function

$$f_3(t) = \frac{1}{1+t}\frac{1+\sqrt[4]{t}}{\sqrt[4]{t}}$$

belongs to the $L^2(0;\infty)$ space but not to either the $L^1(0;\infty)$ or $L^\infty(0;\infty)$ space.

4) The function $f_4(t) = a^{-t}$ belongs to both the $L^1(0;\infty)$ and $L^\infty(0;\infty)$ spaces. Hence, it also belongs to all the other $L^p(0;\infty)$ spaces with $p \geq 1$, which can be easily verified.

5) The function

$$f_5(t) = \frac{1}{1+t^2}\frac{1+\sqrt[4]{t}}{\sqrt[4]{t}}$$

belongs to both the $L^1(0;\infty)$ and $L^2(0;\infty)$ space but not to $L^\infty(0;\infty)$.

6) The function

$$f_6(t) = \frac{1}{1+t^2}\frac{1+\sqrt{t}}{\sqrt{t}}$$

belongs to the $L^1(0;\infty)$ space, but not to either $L^2(0;\infty)$ or $L^\infty(0;\infty)$.

More information about L^p spaces and corresponding norms can be found in the book by Desoer and Vidyasagar [44].

Of all the L^p spaces, the most crucial role throughout this book is played by the L^2. The reason for this lies in the fact that $L^2(a,b)$ is the only L^p space in which we can define the product of two vectors:

$$\langle x(\cdot), y(\cdot)\rangle = \int_a^b \left| x^*(t)\, y(t) \right| dt\,.$$

The norm (which for this reason will be called the Euclidean norm) can then be defined by

$$\|x(\cdot)\|_2 = \sqrt{\langle x(\cdot), x(\cdot)\rangle} .$$

Furthermore, the most crucial properties of the Fourier transform, which is discussed in the next section, namely the convolution property and the Plancherel theorem, are valid only for the functions that belong to the $L^2(-\infty,+\infty)$ space.

A.5 Two Facts about Fourier Transforms

Assuming that the function $f(t)$ is defined only for $t \geq 0$ (usually the case), its Fourier transform $\hat{f}(i\omega)$ can be obtained from its Laplace transform by substituting $i\omega$ for the Laplace variable s. More formal definition, which does not require the restriction of $t \geq 0$, is

$$\hat{f}(i\omega) = \int_{-\infty}^{+\infty} f(t)e^{i\omega t}dt .$$

If the two functions $f(\cdot)$ and $g(\cdot)$ both belong to the space $L^2(0,\infty)$, then the following two properties are valid and play a crucial role throughout the book.

The first property is the convolution theorem. Define the convolution of these two functions by

$$h(t) = f(t) * g(t) = \int_0^t f(\tau)g(t-\tau)d\tau .$$

The Fourier transform of the convolution is the product of the Fourier transforms of these two functions:

$$\hat{h}(i\omega) = \hat{f}(i\omega)\hat{g}(i\omega) .$$

The second property is known as the Plancherel[8] theorem. It is stated as follows:

$$\int_0^{\infty} f(t)g(t)dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f^*(i\omega)g(i\omega)d\omega .$$

The star in this equation denotes the complex conjugation.

[8] Some authors refer to this property as the Parseval identity. However, according to the *Mathematical Encyclopedia* [140] and the book by Kolmogorov and Fomin [70], the attribution to Plancherel is correct.

If in the above discussion the functions are components of a vector or a matrix, then all the products are defined in the appropriate vector space.

A.6 Quadratic and Hermitian Forms

Given a real symmetric $n \times n$ matrix A, the quadratic form for a vector $x \in \mathbb{R}^n$ is the expression $\mathcal{F}(x) = x^* Ax$. Note that there is no loss of generality in requiring the matrix A to be symmetric, because if it is not, the identical expression is obtained by replacing it with the symmetric matrix $(A^* + A)/2$.

Consider now the case with $x \in \mathbb{C}^n$ and let A be an $n \times n$ matrix with complex components. If this matrix is Hermitian, that is, $A^* = A$ (Hereafter the star will denote the transpose operation followed by taking the conjugate of the complex number. For real vectors or matrices, this reduces to just the transpose operation), the expression $\mathcal{F}(x) = x^* Ax$ is real and is called a Hermitian form. In a general case, the Hermitian form is the expression $\mathcal{F}(x) = \operatorname{Re} x^* Ax$. In other words, the value of the Hermitian form is always a real number.

If this value is positive (respectively, nonnegative, nonpositive, negative) for all $x \neq 0$, the form is called positive-definite (respectively, positive-semidefinite, negative-semidefinite, negative-definite). We call the symmetric (or Hermitian) matrix positive-definite, and write $A > 0$, if the corresponding quadratic (or Hermitian) form is positive-definite. In a similar manner we define positive-semidefinite, negative-semidefinite, and negative-definite symmetric or Hermitian matrices.

Every quadratic form can be extended to a Hermitian form by formally allowing the components of the real vector to be complex and taking the real part of the resulting expression. This process and the resulting expression will be called a Hermitian extension of a quadratic form.

Furthermore, for any Hermitian form there exists a Hermitian matrix B, such that $\operatorname{Re} x^* Ax = x^* Bx$. It is not difficult to find that this matrix B is given by

$$B = \frac{1}{2}(A + A^*) .$$

The expression in the right-hand side will be called the real part of the matrix A and denoted $\operatorname{Re} A$. From now on, when we write this, we will not be concerned whether the matrix A is real or complex and use the last expression as a formal definition of $\operatorname{Re} A$.

Let us illustrate these concepts with a simple example. Consider the quadratic form:

$$\mathcal{F}(x) = x_1 (x_2 - 3x_1) .$$

In the vector-matrix notation $x^* Ax$, we have

$$A = \begin{bmatrix} -3 & 1 \\ 0 & 0 \end{bmatrix}$$

The corresponding Hermitian form $\operatorname{Re} x^* Ax = x^* Bx$ will have the matrix:

$$B = \operatorname{Re} A = \frac{1}{2}(A + A^*) = \begin{bmatrix} -3 & 1/2 \\ 1/2 & 0 \end{bmatrix}.$$

If the components of the vector x are functions of a real variable t, then by Plancherel theorem we have

$$\int_0^\infty \mathcal{F}(x(t))dt = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \tilde{\mathcal{F}}(\hat{x}(i\omega))d\omega,$$

where the tilde denotes the Hermitian extension of the quadratic form. This equality means that the Fourier transform preserves the sign-definite property of the quadratic form.

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Index

A

absolute stability 1, 2, 4, 5, 8–13, 15–17, 19–21, 25–27, 30, 31, 46, 47, 51, 57, 61, 68, 72, 82, 85, 87, 98, 103
  definition 27
absolutely continuous function 51, 53, 55, 56, 58, 59, 61, 66, 67, 71, 121–123
  definition 121
Aizerman Conjecture 6, 7
Aizerman problem 2

B

bounded continuation 31, 79, 80, 85, 97, 98, 102, 103
  definition 31

C

circle criterion 5, 10, 11, 13, 70, 110, 112, 115
convolution theorem 126

D

dichotomy 31, 106

F

flying vehicle 18
Fourier transform 27, 44, 71, 73, 126, 128
  definition 126
frequency condition 10, 12, 15, 29–31, 39, 46–54, 57, 59, 60, 65, 66, 72, 73, 82, 84–86, 92, 93, 97, 99, 102–104, 106, 110, 114
function of bounded variation 55, 118, 121, 122

H

Hermitian extension of a quadratic form 127, 128
Hermitian form 17, 28, 36, 127, 128
Hermitian matrix 10, 12, 18, 28, 36, 127
Hill equation 111
hydraulic regulator 21
hysteresis-type nonlinearity 17

I

inners 5, 21
input-output stability 1, 15, 16, 25
integrable function 26, 120, 122
integral-quadratic constraints 3, 17, 38, 51, 66
  delay- 26, 31, 32, 44, 46, 55, 81, 82

K

Kalman Conjecture 2, 3, 13
Kalman-Yakubovich Lemma 3, 10–12

L

Lebesgue integrable function 120
Lebesgue integral 119–124
Lebesgue-Stieltjes integral 58, 120–123
linear matrix inequalities (LMIs) 5, 72
linear periodic systems 104
Lipatov plot 70, 86–89, 110, 111, 115
L^p Spaces 123
Lyapunov function 3, 10, 11, 17

M

Mathieu equation 112
measurable
  function 119

set 32, 44, 45, 51, 83, 117, 119
measure 29, 36, 47, 49, 53, 55, 64–68, 82, 117–121, 124
definition 117
finite, defintion 117
Lebesgue 117–120
Lebesgue-Stieltjes 118, 119
nonegative 29, 30, 45–50, 52, 54, 57, 58, 60–62, 64, 66, 118
signed 47, 49, 53, 55, 58, 61, 66, 118, 121
MIMO nonlinearity 62, 69, 72, 90
minimal stability 31, 46, 47, 53, 57, 61, 65, 66, 79, 92, 97, 99
definition 30
monotone Lipschitz nonlinearities 2, 82, 94, 100
multiplier 13, 15, 16, 69, 72, 73, 115
Barabanov 14
for time periodic systems 83
matrix realization 72
Zames-Falb 3, 51

N

norm 14, 123, 124, 126
euclidean 123, 124
Nyquist plot 5, 11, 13, 15, 70, 72

O

off-axis circle criterion 13, 15, 70
output regulation problem 68

P

parabola criterion 9
Plancherel Theorem 17, 126, 128
Popov criterion 5, 9–13, 55, 58, 70, 73
local analogue 12
modification 9
Popov plot 7–9
Positive Real Lemma. See Kalman-Yakubovich Lemma

Q

quadratic criterion 17, 26, 27, 30–32, 35
quasimonotone nonlinearities 94, 100, 101
in the sense of Barabanov 95

R

regularity conditions 26, 31, 45, 48, 51, 55, 58, 59, 61, 62, 83, 90, 95, 101, 105, 108, 113
definition 26
retarded second-order systems 2, 6

S

saltus function 122, 123
sector condition 1, 6, 11, 12, 17, 55, 56, 95, 96, 101, 102, 105, 107, 113, 115
semimonotone function 34, 72
simple function
definition 119
SISO nonlinearities 43, 50, 63–66, 69, 72, 82, 94, 110, 113
slope restricted nonlinearities 59
slope restriction 2, 12, 43, 45, 46, 48–52, 55, 58–62, 82, 90
S-procedure 4, 11, 12, 15, 17
stability of periodic solutions 93
stable continuation 30, 31, 80, 85
definition 30
Stieltjes integral 121

T

time delays 3, 18
time periodic nonlinearities 95, 100

Y

Yakubovich criterion 13, 51, 59, 70
for linear time periodic systems 104, 110