

# Appendix

This appendix presents the continuous-time designs of model reference adaptive control using state feedback for state tracking, state feedback for output tracking, or output feedback for output tracking (and its discrete-time version), and multivariable design, as well as that of adaptive pole placement control. Key issues such as *a priori* system knowledge, controller structure, plant-model matching, adaptive laws, and stability are addressed.

## A.1 Model Reference Adaptive Control

There are three types of model reference adaptive control (MRAC) designs: state feedback for state tracking; state feedback for output tracking; and output feedback for output tracking, all described in this section.

### A.1.1 MRAC: State Feedback for State Tracking

Consider a linear time-invariant plant described by

$$\dot{x}(t) = Ax(t) + bu(t), \quad x(t) \in R^n, \quad u(t) \in R \quad (\text{A.1})$$

where  $A \in R^{n \times n}$ ,  $b \in R^n$  are unknown constant parameter matrices, and the state vector  $x(t)$  is available for measurement.

The control objective is to design a *state feedback* control signal  $u(t)$  that ensures that all signals in the closed-loop system are bounded and the plant state vector  $x(t)$  asymptotically tracks a given reference state vector  $x_m(t)$  generated from the reference model system

$$\dot{x}_m(t) = A_m x_m(t) + b_m r(t), \quad x_m(t) \in R^n, \quad r(t) \in R \quad (\text{A.2})$$

where  $A_m \in R^{n \times n}$ ,  $b_m \in R^n$  are known constant matrices such that all eigenvalues of  $A_m$  are in the open left-half complex plane, and  $r(t)$  is bounded.

To meet this control objective, we assume

(A.1-1) There exist a constant vector  $k_1^* \in R^n$  and a constant scalar  $k_2^* \in R$  such that

$$A + bk_1^{*T} = A_m, \quad bk_2^* = b_m \quad (\text{A.3})$$

(A.1-2)  $\text{sign}[k_2^*]$ , the sign of the parameter  $k_2^*$ , is known.

While Assumption (A.1-2) is needed for implementing an adaptive law, Assumption (A.1-1) is the so-called matching condition such that if the parameters of  $A$  and  $b$  are known and (A.3) is satisfied, then the control law

$$u(t) = k_1^{*T}x(t) + k_2^*r(t) \quad (\text{A.4})$$

achieves the desired control objective that the closed-loop system becomes

$$\dot{x}(t) = Ax(t) + b(k_1^{*T}x(t) + k_2^*r(t)) = A_mx(t) + b_mr(t) \quad (\text{A.5})$$

whose state vector  $x(t)$  is bounded, and so is the control  $u(t)$  in (A.4), and that the tracking error  $e(t) = x(t) - x_m(t)$  satisfies

$$\dot{e}(t) = A_me(t), \quad e(0) = x(0) - x_m(0) \quad (\text{A.6})$$

which indicates that  $\lim_{t \rightarrow \infty} e(t) = 0$  exponentially. It is clear that Condition (A.3) is also necessary for the control law (A.3) to achieve the stated control objective, even if the plant parameters  $A$  and  $b$  are known.

In the adaptive control problem, the parameters of  $A$  and  $b$  are unknown so that the control law (A.4) cannot be used for control. In this case, the following adaptive controller structure is used:

$$u(t) = k_1^T(t)x(t) + k_2(t)r(t) \quad (\text{A.7})$$

where  $k_1(t)$  and  $k_2(t)$  are the estimates of  $k_1^*$  and  $k_2^*$ , respectively. The adaptive control design task now is to choose adaptive laws to update these estimates so that the stated control objective is still achievable.

Defining the parameter errors as

$$\tilde{k}_1(t) = k_1(t) - k_1^*, \quad \tilde{k}_2(t) = k_2(t) - k_2^* \quad (\text{A.8})$$

and using (A.1), (A.3), and (A.7), we obtain

$$\begin{aligned} \dot{x}(t) &= Ax(t) + b(k_1^T(t)x(t) + k_2(t)r(t)) \\ &= A_mx(t) + b_mr(t) + b_m \left( \frac{1}{k_2^*} \tilde{k}_1^T(t)x(t) + \frac{1}{k_2^*} \tilde{k}_2(t)r(t) \right). \end{aligned} \quad (\text{A.9})$$

Substituting (A.2) in (A.9), we have the tracking error equation

$$\dot{e}(t) = A_m e(t) + b_m \left( \frac{1}{k_2^*} \tilde{k}_1^T(t) x(t) + \frac{1}{k_2^*} \tilde{k}_2(t) r(t) \right). \quad (\text{A.10})$$

Consider the positive definite function

$$V(e, \tilde{k}_1, \tilde{k}_2) = e^T P e + \frac{1}{|k_2^*|} \tilde{k}_1^T \Gamma^{-1} \tilde{k}_1 + \frac{1}{|k_2^*|} \tilde{k}_2^2 \gamma^{-1} \quad (\text{A.11})$$

as a measure of the system error signals  $e(t)$ ,  $\tilde{k}_1(t)$ , and  $\tilde{k}_2(t)$ , where  $P \in R^{n \times n}$  is constant,  $P = P^T > 0$  and satisfies

$$P A_m + A_m^T P = -Q \quad (\text{A.12})$$

for some constant  $Q \in R^{n \times n}$  such that  $Q = Q^T > 0$ ,  $\Gamma \in R^{n \times n}$  is constant and  $\Gamma = \Gamma^T > 0$ , and  $\gamma > 0$  is a constant scalar.

The time derivative of  $V(e, \tilde{k}_1, \tilde{k}_2)$  is

$$\begin{aligned} \dot{V} &= \frac{d}{dt} V = \left( \frac{\partial V}{\partial e} \right)^T \dot{e}(t) + \left( \frac{\partial V}{\partial \tilde{k}_1} \right)^T \dot{\tilde{k}}_1(t) + \frac{\partial V}{\partial \tilde{k}_2} \dot{\tilde{k}}_2(t) \\ &= 2e^T(t) P \dot{e}(t) + \frac{2}{|k_2^*|} \tilde{k}_1^T(t) \Gamma^{-1} \dot{\tilde{k}}_1(t) + \frac{2}{|k_2^*|} \tilde{k}_2(t) \gamma^{-1} \dot{\tilde{k}}_2(t). \end{aligned} \quad (\text{A.13})$$

Substituting (A.10) and (A.12) in (A.13), we have

$$\begin{aligned} \dot{V} &= -e^T(t) Q e(t) + e^T(t) P b_m \frac{2}{k_2^*} \tilde{k}_1^T(t) x(t) + e^T(t) P b_m \\ &\quad + \frac{2}{k_2^*} \tilde{k}_2(t) r(t) + \frac{2}{|k_2^*|} \tilde{k}_1^T(t) \Gamma^{-1} \dot{\tilde{k}}_1(t) + \frac{2}{|k_2^*|} \tilde{k}_2(t) \gamma^{-1} \dot{\tilde{k}}_2(t). \end{aligned} \quad (\text{A.14})$$

To make  $\dot{V} \leq 0$ , we choose the adaptive laws for  $k_1(t)$  and  $k_2(t)$  as

$$\dot{k}_1(t) = -\text{sign}[k_2^*] \Gamma x(t) e^T(t) P b_m \quad (\text{A.15})$$

$$\dot{k}_2(t) = -\text{sign}[k_2^*] \gamma r(t) e^T(t) P b_m \quad (\text{A.16})$$

with  $\Gamma = \Gamma^T > 0$ ,  $\gamma > 0$ ,  $k_1(0)$  and  $k_2(0)$  being arbitrary.

Indeed, with this choice of  $\dot{k}_1(t)$  and  $\dot{k}_2(t)$ , (A.14) becomes

$$\dot{V} = -e^T(t) Q e(t) \leq 0. \quad (\text{A.17})$$

From (A.17), we conclude that  $x(t)$ ,  $k_1(t)$ , and  $k_2(t)$  are all bounded, and so are  $u(t)$  in (A.7) and  $\dot{e}(t)$  in (A.10), and that  $e(t) \in L^2$ .<sup>1</sup>

<sup>1</sup> A *continuous-time* vector signal  $x(t) \in R^n$  belongs to  $L^2$  if

$$\sqrt{\int_0^\infty (x_1^2(t) + \dots + x_n^2(t)) dt} < \infty$$

To show that  $\lim_{t \rightarrow \infty} e(t) = 0$ , we see that for  $e = [e_1, \dots, e_n]^T$ ,  $e_i(t) \in L^2$  and  $\dot{e}_i(t) \in L^\infty$ ,  $i = 1, 2, \dots, n$ . It follows that

$$\int_0^t e_i^2(\tau) |\dot{e}_i(\tau)| d\tau \leq \sup_{t \geq 0} |\dot{e}_i(t)| \int_0^\infty e_i^2(\tau) d\tau < \infty \quad (\text{A.18})$$

for any  $t \geq 0$ . This implies that  $\lim_{t \rightarrow \infty} \int_0^t e_i^2(\tau) |\dot{e}_i(\tau)| d\tau$  exists and is finite, and, therefore,  $\lim_{t \rightarrow \infty} \int_0^t e_i^2(\tau) \dot{e}_i(\tau) d\tau$  exists and is finite. From the identity

$$e_i^2(t) = |e_i^3(t)|^{\frac{2}{3}} = |3 \int_0^t e_i^2(\tau) \dot{e}_i(\tau) d\tau + e_i^3(0)|^{\frac{2}{3}} \quad (\text{A.19})$$

we have that  $\lim_{t \rightarrow \infty} e_i^2(t)$  exists and is zero as  $e_i(t) \in L^2$ . This proves that  $\lim_{t \rightarrow \infty} e_i(t) = 0$ ,  $i = 1, 2, \dots, n$ , so that  $\lim_{t \rightarrow \infty} e(t) = 0$ .

In summary, we have the following result.

**Theorem A.1.** *The adaptive controller (A.7), with the adaptive laws (A.15) and (A.16), and applied to the plant (A.1), ensures that all closed-loop signals are bounded and the tracking error  $e(t) = x(t) - x_m(t)$  goes to zero as  $t \rightarrow \infty$ .*

### A.1.2 MRAC: State Feedback for Output Tracking

Consider a linear time-invariant plant in the state variable form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (\text{A.20})$$

for some unknown constant matrices  $A \in R^{n \times n}$ ,  $b \in R^{n \times 1}$ , and  $C \in R^{1 \times n}$ , with  $n > 0$ . The input-output description of this plant is

$$y(s) = C(sI - A)^{-1}bu(s) = \frac{Z(s)}{P(s)}u(s) \quad (\text{A.21})$$

where  $P(s) = \det(sI - A)$  and

$$Z(s) = z_m s^m + \dots + z_1 s + z_0, \quad z_m \neq 0, \quad m < n \quad (\text{A.22})$$

with  $s$  being the Laplace transform variable or the time differentiation operator:  $s[x](t) = \dot{x}(t)$ ,  $t \in [0, \infty)$ , as the case may be.

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and it belongs to  $L^\infty$  if

$$\sup_{t \geq 0} \|x(t)\|_\infty = \sup_{t \geq 0} \max_{1 \leq i \leq n} |x_i(t)| < \infty.$$

A signal  $x(t)$  is bounded if it belongs to  $L^\infty$ .

To design an adaptive *state feedback* model reference controller for generating the plant input  $u(t)$ , which ensures closed-loop signal boundedness and asymptotic tracking of a given reference signal  $y_m(t)$  by the plant output  $y(t)$ , we need the following assumptions:

- (A.2-1)  $(A, b, C)$  is stabilizable and detectable,
- (A.2-2)  $Z(s)$  is a stable polynomial,
- (A.2-3) The degree  $m$  of  $Z(s)$  is known, and
- (A.2-4)  $\text{sign}[z_m]$ , the sign of  $z_m$ , is known.

Assumption (A.2-1) is needed for output matching, Assumption (A.2-2) is needed for model reference control, and Assumption (A.2-3) is needed for constructing a reference model system, while Assumption (A.2-4) is used for designing an adaptive parameter update law.

The reference model, independent of the dynamics of (A.20), is chosen as

$$y_m(t) = W_m(s)[r](t), \quad W_m(s) = \frac{1}{P_m(s)} \tag{A.23}$$

where  $P_m(s)$  is a desired stable polynomial of degree  $n - m$ , and  $r(t)$  is a bounded reference input signal.

The adaptive model reference controller structure is

$$u(t) = k_1^T(t)x(t) + k_2(t)r(t) \tag{A.24}$$

where  $k_1(t) = [k_{11}(t), k_{12}(t), \dots, k_{1n}(t)]^T \in R^n$  and  $k_2(t) \in R$  are the adaptive estimates of the unknown parameters  $k_1^* = [k_{11}^*, k_{12}^*, \dots, k_{1n}^*]^T \in R^n$  and  $k_2^*$ , which satisfy the matching equation

$$\det(sI - A - bk_1^{*T}) = P_m(s)Z(s)\frac{1}{z_m}, \quad k_2^* = \frac{1}{z_m} \tag{A.25}$$

that is, all zeros of  $\det(sI - A - bk_1^{*T})$  are stable [118].

With this definition of  $k_1^*$  and  $k_2^*$ , the fixed version of (A.24) is

$$u(t) = k_1^{*T}x(t) + k_2^*r(t) \tag{A.26}$$

which would lead to the desired closed-loop system:  $y(s) = W_m(s)r(s)$ .

In the adaptive control problem when  $A$ ,  $b$ , and  $z_m$  are all unknown parameters, we need to develop adaptive laws to update the parameter estimates  $k_1(t)$  and  $k_2(t)$ . With the controller (A.24), the closed-loop system is

$$\begin{aligned} \dot{x}(t) &= (A + bk_1^{*T})x(t) + bk_2^*r(t) + b((k_1(t) - k_1^*)^T x(t) + (k_2(t) - k_2^*)r(t)) \\ y(t) &= Cx(t). \end{aligned} \tag{A.27}$$

From (A.25) it follows that

$$C(sI - A - bk_1^{*T})^{-1}bk_2^* = \frac{Z(s)k_2^*}{\det(sI - A - bk_1^{*T})} = \frac{1}{P_m(s)} = W_m(s). \quad (\text{A.28})$$

In view of (A.23), (A.27), and (A.28), the tracking error equation is

$$\begin{aligned} e(t) &= y(t) - y_m(t) \\ &= \rho^* W_m(s)[(k_1 - k_1^*)^T x + (k_2 - k_2^*)r](t) + C e^{(A+bk_1^{*T})t} x(0) \end{aligned} \quad (\text{A.29})$$

where  $\rho^* = z_m$ , and  $\lim_{t \rightarrow \infty} C e^{(A+bk_1^{*T})t} x(0) = 0$  exponentially.

To derive an estimation error equation, we define

$$\theta(t) = [k_1^T(t), k_2(t)]^T \quad (\text{A.30})$$

$$\theta^* = [k_1^{*T}, k_2^*]^T \quad (\text{A.31})$$

$$\omega(t) = [x^T(t), r(t)]^T \quad (\text{A.32})$$

$$\zeta(t) = W_m(s)[\omega](t) \quad (\text{A.33})$$

$$\xi(t) = \theta^T(t)\zeta(t) - W_m(s)[\theta^T \omega](t) \quad (\text{A.34})$$

$$\epsilon(t) = e(t) + \rho(t)\xi(t) \quad (\text{A.35})$$

where  $\rho(t)$  is an estimate of  $\rho^* = z_m$ . Then, from (A.29)–(A.35), ignoring the exponentially decaying term  $C e^{(A+bk_1^{*T})t} x(0)$ , we have

$$\epsilon(t) = \rho^*(\theta(t) - \theta^*)^T \zeta(t) + (\rho(t) - \rho^*)\xi(t) \quad (\text{A.36})$$

which is linear in the parameters errors  $\theta(t) - \theta^*$  and  $\rho(t) - \rho^*$ .

We then choose the adaptive laws for  $\theta(t)$  and  $\rho(t)$  as

$$\dot{\theta}(t) = -\frac{\Gamma \text{sign}[z_m] \zeta(t) \epsilon(t)}{1 + \zeta^T(t)\zeta(t) + \xi^2(t)} \quad (\text{A.37})$$

$$\dot{\rho}(t) = -\frac{\gamma \xi(t) \epsilon(t)}{1 + \zeta^T(t)\zeta(t) + \xi^2(t)} \quad (\text{A.38})$$

where  $\Gamma = \Gamma^T > 0$  and  $\gamma > 0$  are adaptation gains.

This adaptive control scheme has the following properties [55], [95], [118].

**Lemma A.1.** *The adaptive laws (A.37) and (A.38) ensures that  $\theta(t) \in L^\infty$ ,  $\rho(t) \in L^\infty$ ,  $\epsilon(t)/m(t) \in L^2 \cap L^\infty$ ,  $\dot{\theta}(t) \in L^2 \cap L^\infty$ , and  $\dot{\rho}(t) \in L^2 \cap L^\infty$ , where*

$$m(t) = \sqrt{1 + \zeta^T(t)\zeta(t) + \xi^2(t)}. \quad (\text{A.39})$$

**Proof:** Consider the positive definite function

$$V(\tilde{\theta}, \tilde{\rho}) = \frac{1}{2}(|\rho^*| \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \gamma^{-1} \tilde{\rho}^2) \quad (\text{A.40})$$

where

$$\tilde{\theta}(t) = \theta(t) - \theta^*, \quad \tilde{\rho}(t) = \rho(t) - \rho^*. \quad (\text{A.41})$$

The time derivative of  $V(\tilde{\theta}, \tilde{\rho})$ , along the trajectories of (A.37) and (A.38), is

$$\dot{V} = -\frac{\epsilon^2(t)}{m^2(t)}. \quad (\text{A.42})$$

This, together with (A.36)–(A.40), leads to the results of the lemma.  $\nabla$

**Theorem A.2.** *All signals in the closed-loop control system, with the plant (A.20), the reference model (A.23), and the controller (A.24) updated by the adaptive law (A.37) and (A.38), are bounded, and the tracking error  $e(t) = y(t) - y_m(t)$  satisfies*

$$\lim_{t \rightarrow \infty} (y(t) - y_m(t)) = 0 \quad (\text{A.43})$$

$$\int_0^\infty (y(t) - y_m(t))^2 dt < \infty. \quad (\text{A.44})$$

The proof of Theorem A.2 can be found in [118].

### A.1.3 MRAC: Output Feedback for Output Tracking

Consider the linear time-invariant plant

$$y(t) = G(s)[u](t) \quad (\text{A.45})$$

where  $G(s) = k_p(Z(s)/P(s))$ ,  $k_p$  is a constant high frequency gain, and  $Z(s)$  and  $P(s)$  are monic polynomials of degrees  $n$  and  $m$ , respectively.

Given a reference model system

$$y_m(t) = W_m(s)[r](t) \quad (\text{A.46})$$

where  $W_m(s)$  is stable and  $r(t)$  is bounded, the control objective is to find an *output feedback* control  $u(t)$  such that all closed-loop signals are bounded and the plant output  $y$  tracks the reference output  $y_m$  asymptotically.

In this case, the plant state variables are not needed for control.

To meet this objective, we make the following assumptions:

(A.3-1)  $Z(s)$  is a stable polynomial,

(A.3-2) The degree  $n$  of  $P(s)$  is known,

(A.3-3) The relative degree  $n^* = n - m$  of  $G(s)$  is known,

(A.3-4) The sign of  $k_p$  is known, and

(A.3-5)  $W_m(s) = 1/P_m(s)$  for a stable polynomial  $P_m(s)$  of degree  $n^*$ .

The desired model reference controller structure is

$$u(t) = \theta_1^T \omega_1(t) + \theta_2^T \omega_2(t) + \theta_{20} y(t) + \theta_3 r(t) \quad (\text{A.47})$$

where

$$\omega_1(t) = \frac{a(s)}{\Lambda(s)}[u](t), \quad \omega_2(t) = \frac{a(s)}{\Lambda(s)}[y](t) \quad (\text{A.48})$$

with  $a(s) = [1, s, \dots, s^{n-2}]^T$ ,  $\theta_1, \theta_2 \in R^{n-1}$ ,  $\theta_{20}$ , and  $\theta_3 \in R$ , and  $\Lambda(s)$  being a monic stable polynomial of degree  $n - 1$ .

It can be shown [55], [118] that with  $\theta_3^* = k_p^{-1}$ , there exist constant parameters  $\theta_1^*, \theta_2^* \in R^{n-1}$ , and  $\theta_{20}^* \in R$  such that

$$\begin{aligned} & \theta_1^{*T} a(s) P(s) + (\theta_2^{*T} a(s) + \theta_{20}^* \Lambda(s)) k_p Z(s) \\ &= \Lambda(s) (P(s) - k_p \theta_3^* Z(s) P_m(s)) \end{aligned} \quad (\text{A.49})$$

and that the controller (A.47) implemented with  $\theta_1^*, \theta_2^*, \theta_{20}^*$ , and  $\theta_3^*$ , that is,

$$u(t) = \theta_1^{*T} \omega_1(t) + \theta_2^{*T} \omega_2(t) + \theta_{20}^* y(t) + \theta_3^* r(t) \quad (\text{A.50})$$

ensures that all the closed-loop signals are bounded and the output tracking is achieved:  $y(t) = y_m(t) + \eta_0(t)$ , for some exponentially decaying  $\epsilon_0(t)$  that depends on system initial conditions.

For the adaptive control problem when  $G(s)$  is unknown, an adaptive version of the controller (A.47) is implemented with  $\theta_1 = \theta_1(t)$ ,  $\theta_2 = \theta_2(t)$ ,  $\theta_{20} = \theta_{20}(t)$ , and  $\theta_3 = \theta_3(t)$ , where  $\theta_1(t)$ ,  $\theta_2(t)$ ,  $\theta_{20}(t)$ , and  $\theta_3(t)$  are the estimates of  $\theta_1^*, \theta_2^*, \theta_{20}^*$ , and  $\theta_3^*$ , to be updated from an adaptive law.

To derive an error equation, operating both sides of (A.49) on  $y(t)$  and using (A.45):  $P(s)[y](t) = k_p Z(s)[u](t)$ , we have

$$\begin{aligned} & \theta_1^{*T} a(s) k_p Z(s)[u](t) + (\theta_2^{*T} a(s) + \theta_{20}^* \Lambda(s)) k_p Z(s)[y](t) \\ &= \Lambda(s) k_p Z(s)[u](t) - \Lambda(s) Z(s) P_m(s)[y](t). \end{aligned} \quad (\text{A.51})$$

This equality, with  $Z(s)$  and  $\Lambda(s)$  stable and the effect of initial conditions ignored, leads to the parametrized plant model

$$u(t) = \theta_1^{*T} \frac{a(s)}{\Lambda(s)}[u](t) + \theta_2^{*T} \frac{a(s)}{\Lambda(s)}[y](t) + \theta_{20}^* y(t) + \theta_3^* W_m^{-1}(s)[y](t). \quad (\text{A.52})$$

Introducing  $\rho^* = 1/\theta_3^* = k_p$  and

$$\omega(t) = [\omega_1^T(t), \omega_2^T(t), y(t), r(t)]^T \quad (\text{A.53})$$

$$e(t) = y(t) - y_m(t), \quad \tilde{\theta}(t) = \theta(t) - \theta^* \quad (\text{A.54})$$

where

$$\theta(t) = [\theta_1^T(t), \theta_2^T(t), \theta_{20}(t), \theta_3(t)]^T \quad (\text{A.55})$$

$$\theta^* = [\theta_1^{*T}, \theta_2^{*T}, \theta_{20}^*, \theta_3^*]^T \quad (\text{A.56})$$

substituting (A.52) in (A.47) with adaptive parameter estimates, ignoring the effect of the initial conditions, and using (A.55) and (A.56), we obtain the tracking error equation

$$\begin{aligned} e(t) &= \rho^* W_m(s) [\tilde{\theta}^T \omega](t) \\ &= -\rho^* (\theta^{*T} W_m(s) [\omega](t) - W_m(s) [\theta^T \omega](t)). \end{aligned} \quad (\text{A.57})$$

Since both  $\theta^*$  and  $\rho^*$  are unknown, the second equality of (A.57) suggests that we define the estimation error

$$\epsilon(t) = e(t) + \rho(t)\xi(t) \quad (\text{A.58})$$

where  $\rho(t)$  is the estimate of  $\rho^*$ , and

$$\xi(t) = \theta^T(t)\zeta(t) - W_m(s) [\theta^T \omega](t) \quad (\text{A.59})$$

$$\zeta(t) = W_m(s) [\omega](t). \quad (\text{A.60})$$

For  $\tilde{\rho}(t) = \rho(t) - \rho^*$ , using (A.57), (A.59), and (A.60), we express  $\epsilon(t)$  as

$$\epsilon(t) = \rho^* \tilde{\theta}^T(t)\zeta(t) + \tilde{\rho}(t)\xi(t) \quad (\text{A.61})$$

which is linear in the parameter errors  $\tilde{\theta}(t)$  and  $\tilde{\rho}(t)$ .

Finally, we choose the adaptive laws for  $\theta(t)$  and  $\rho(t)$ :

$$\dot{\theta}(t) = \frac{-\text{sign}[k_p] \Gamma \epsilon(t)\zeta(t)}{m^2(t)} \quad (\text{A.62})$$

$$\dot{\rho}(t) = \frac{-\gamma \epsilon(t)\xi(t)}{m^2(t)} \quad (\text{A.63})$$

where  $\Gamma = \Gamma^T > 0$  and  $\gamma > 0$  are constant adaptation gains, and

$$m(t) = \sqrt{1 + \zeta^T(t)\zeta(t) + \xi^2(t)}. \quad (\text{A.64})$$

This adaptive controller has the following desired properties

[55], [95], [118].

**Lemma A.2.** *The adaptive update law (A.62) and (A.63) ensures that  $\theta(t) \in L^\infty$ ,  $\rho(t) \in L^\infty$ ,  $\epsilon(t)/m(t) \in L^2 \cap L^\infty$ , and  $\dot{\theta}(t) \in L^2 \cap L^\infty$ .*

**Theorem A.3.** *All signals in the closed-loop control system with the plant (A.45), reference model (A.46), and controller (A.47) updated by the adaptive laws (A.62) and (A.63) are bounded, and*

$$y(t) - y_m(t) \in L^2, \quad \lim_{t \rightarrow \infty} (y(t) - y_m(t)) = 0. \quad (\text{A.65})$$

The proof of Lemma A.2 is similar to that of Lemma A.1, and the proof of Theorem A.2 can be found in [118], [125].

**Discrete-Time Design.** While both state feedback for output tracking and output feedback for output tracking designs can be developed in discrete time, here we only illustrate how to develop a discrete-time adaptive output feedback control design for output tracking. Such a design can be obtained in a way similar to that for the continuous-time case: the same controller structure (A.47), the same parametrization (A.49)–(A.57), and the same estimation error (A.58), but in (A.45)–(A.61), we need to replace  $s$  by  $z$  which, for the discrete-time case, is the  $z$ -transform variable or the time advance operator:  $z[x](k) = x(k+1)$ ,  $k \in \{0, 1, 2, 3, \dots\}$ . In particular, a stable polynomial of degree  $n_p$  can be chosen as  $z^{n_p}$ , for example,  $\Lambda(z) = z^{n-1}$  for (A.47) and  $W_m(z) = z^{-n^*}$  for (A.46). However, the adaptive law for updating the controller parameter vector  $\theta(k)$  in (A.47) is different:

$$\theta(k+1) = \theta(k) - \frac{\text{sign}[k_p] \Gamma \epsilon(k) \omega(k - n^*)}{m^2(k)} \quad (\text{A.66})$$

$$\rho(k+1) = \rho(k) - \frac{\gamma \epsilon(k) \xi(k)}{m^2(k)} \quad (\text{A.67})$$

where the adaptation gains  $\Gamma$  and  $\gamma$  satisfy

$$0 < \Gamma = \Gamma^T < \frac{2}{k_p^0} I_{2n}, \quad 0 < \gamma < 2 \quad (\text{A.68})$$

for a known constant  $k_p^0 \geq |k_p|$ .

The adaptive laws (A.66) and (A.67) ensures that  $\theta(k) \in L^\infty$ ,  $\rho(k) \in L^\infty$ ,  $\epsilon(k)/m(k) \in L^2 \cap L^\infty$ , and  $\theta(k+i_0) - \theta(k) \in L^2$  for any finite integer  $i_0 > 0$ .<sup>2</sup>

<sup>2</sup> A *discrete-time* vector signal  $x(k) \in R^n$  belongs to  $L^2$  if

$$\sqrt{\sum_{k=0}^{\infty} (x_1^2(k) + \dots + x_n^2(k))} < \infty$$

The proof of this result is similar to that for the continuous-time case. The time increment of the positive definite function

$$V(\tilde{\theta}, \tilde{\rho}) = |\rho^*| \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \gamma^{-1} \tilde{\rho}^2 \quad (\text{A.69})$$

along the trajectories of (A.66) and (A.67) is

$$\begin{aligned} & V(\tilde{\theta}(k+1), \tilde{\rho}(k+1)) - V(\tilde{\theta}(k), \tilde{\rho}(k)) \\ &= - \left( 2 - \frac{|k_p| \omega^T(k - n^*) \Gamma \omega(k - n^*) + \gamma \xi^2(k)}{m^2(k)} \right) \frac{\epsilon^2(k)}{m^2(k)} \\ &\leq -\alpha_1 \frac{\epsilon^2(k)}{m^2(k)} \end{aligned} \quad (\text{A.70})$$

for some constant  $\alpha_1 > 0$ . This implies that  $\theta(k) \in L^\infty$ ,  $\rho(k) \in L^\infty$ , and  $\epsilon(k)/m(k) \in L^2$ . From (A.61), we have  $\epsilon(k)/m(k) \in L^\infty$ , and from (A.66), we have  $\theta(k+1) - \theta(k) \in L^2$ . Finally, using the inequality

$$\|\theta(k+i_0) - \theta(k)\|_2 \leq \sum_{i=0}^{i_0-1} \|\theta(k+i+1) - \theta(k+i)\|_2 \quad (\text{A.71})$$

we have that  $\theta(k+i_0) - \theta(k) \in L^2$  for any finite integer  $i_0$ .

Based on this result, we can also establish the result of Theorem A.3 for the discrete-time adaptive control scheme [118].

## A.2 Multivariable MRAC

Consider the linear time-invariant plant

$$y(t) = G(s)[u](t) \quad (\text{A.72})$$

where  $y(t) \in R^M$ ,  $u(t) \in R^M$ ,  $G(s) = Z(s)P^{-1}(s)$  is strictly proper and has full rank, and  $Z(s)$  and  $P(s)$  are  $M \times M$  right co-prime polynomial matrices with  $P(s)$  column proper [95]. The symbol  $s$  is used as the Laplace transform variable or the time differentiation operator:  $s[x](t) = \dot{x}(t)$ ,  $t \in [0, \infty)$ . The following development of multivariable MRAC is applicable to the discrete-time case, with different system stability conditions (which lead to the special choice of adaptation gains in parameter adaptive laws).

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and it belongs to  $L^\infty$  (i.e., it is bounded) if

$$\sup_{k \geq 0} \|x(k)\|_\infty = \sup_{k \geq 0} \max_{1 \leq i \leq n} |x_i(k)| < \infty.$$

The control objective is to find a feedback control  $u(t)$  for the plant (A.72) with unknown  $G(s)$  such that all signals in the closed-loop system are bounded and  $y(t)$  asymptotically tracks the reference output

$$y_m(t) = W_m(s)[r](t), \quad W_m(s) = \xi_m^{-1}(s) \quad (\text{A.73})$$

where  $\xi_m(s)$  is a modified interactor matrix of  $G(s)$  [32], [95], [123], and  $r(t)$  is a bounded signal. The modified interactor matrix  $\xi_m(s)$  has a stable inverse and represents the zero structure at infinity of  $G(s)$ . The high frequency gain matrix  $K_p$  of  $G(s)$  is defined as  $K_p = \lim_{D \rightarrow \infty} \xi_m(s)G(s)$ , finite and nonsingular. To design MRAC schemes, we assume

- (A.4-1) All zeros of  $G(s)$  (that is, zeros of  $\det[Z(s)]$ ) are stable,
- (A.4-2)  $\bar{\nu} \geq \nu$ , the observability index of  $G(s)$ , is known,
- (A.4-3) For some known  $S_p \in R^{M \times M}$ ,  $\Gamma_p = K_p^T S_p^{-1} = \Gamma_p^T > 0$ , and
- (A.4-4) A modified interactor matrix  $\xi_m(s)$  of  $G(s)$  is known.

Note that Assumption (A.4-3) may be relaxed, as all leading principal minors of  $K_p$  are nonzero and their signs are known (see Section 5.3.4).

**Controller Structure.** The MRAC controller structure for (A.72) is

$$u(t) = \Theta_1^T \omega_1(t) + \Theta_2^T \omega_2(t) + \Theta_{20} y(t) + \Theta_3 r(t) \quad (\text{A.74})$$

where  $\Theta_1 = [\Theta_{11}, \dots, \Theta_{1\bar{\nu}-1}]^T$ ,  $\Theta_2 = [\Theta_{21}, \dots, \Theta_{2\bar{\nu}-1}]^T$ ,  $\Theta_{20}$ ,  $\Theta_3$ ,  $\Theta_{ij} \in R^{M \times M}$ ,  $i = 1, 2$ ,  $j = 1, \dots, \bar{\nu} - 1$ , and  $\omega_1(t) = F(s)[u](t)$ ,  $\omega_2(t) = F(s)[y](t)$ ,  $F(s) = A(s)/\Lambda(s)$ , with  $A(s) = [I, DI, \dots, D^{\bar{\nu}-2}I]^T$ , and  $\Lambda(s)$  being a monic stable polynomial of degree  $\bar{\nu} - 1$ .

**Error Model.** With the specification of  $\Lambda(s)$ ,  $\xi_m(s)$ ,  $P(s)$ , and  $Z(s)$ , there exist  $\Theta_1^*$ ,  $\Theta_2^*$ ,  $\Theta_3^* = K_p^{-1}$  such that  $I - \Theta_1^{*T} F(s) - \Theta_2^{*T} F(s)G(s) - \Theta_{20}^* G(s) - \Theta_3^* W_m^{-1}(s)G(s)$  [32], from which, for any  $u(t)$ , we have

$$\begin{aligned} & K_p (u(t) - \Theta_1^{*T} \omega_1(t) - \Theta_2^{*T} \omega_2(t) - \Theta_{20}^* y(t) - \Theta_3^* r(t)) \\ &= \xi_m(s)[y - y_m](t). \end{aligned} \quad (\text{A.75})$$

With the controller (A.74), we write (A.75) as

$$\xi_m(s)[y - y_m](t) = K_p \tilde{\Theta}^T(t) \omega(t) \quad (\text{A.76})$$

where  $\tilde{\Theta}(t) = \Theta(t) - \Theta^*$  with  $\Theta(t)$  the estimate of  $\Theta^* = [\Theta_1^{*T}, \Theta_2^{*T}, \Theta_{20}^*, \Theta_3^*]^T$ , and  $\omega(t) = [\omega_1^T(t), \omega_2^T(t), y^T(t), r^T(t)]^T$ .

Let  $e(t) = y(t) - y_m(t)$  and introduce  $\bar{e}(t) = h(s)\xi_m(s)[e](t)$  for  $h(s) = 1/f(s)$  with  $f(s)$  being a monic and stable polynomial such that  $h(s)\xi_m(s)$  is proper and stable. Define the estimation error

$$\epsilon(t) = \bar{e}(t) + \Psi(t)\xi(t) \quad (\text{A.77})$$

where  $\Psi(t)$  is the estimate of  $K_p$ , and

$$\xi(t) = \Theta^T(t)\zeta(t) - h(s)[\Theta^T\omega](t) \quad (\text{A.78})$$

$$\zeta(t) = h(s)[\omega](t). \quad (\text{A.79})$$

It then follows from (A.76) (filtered by  $h(s)\xi_m(s)$ ) and (A.77)–(A.79) that

$$\epsilon(t) = K_p\tilde{\Theta}^T(t)\zeta(t) + \tilde{\Psi}(t)\xi(t) \quad (\text{A.80})$$

where  $\tilde{\Psi}(t) = \Psi(t) - \Psi^*$ . The development containing (A.73)–(A.80) can be done for the discrete-time case, using discrete-time transfer functions.

**Adaptive Laws.** We choose the *continuous-time* adaptive laws as

$$\dot{\Theta}^T(t) = -\frac{S_p\epsilon(t)\zeta^T(t)}{m^2(t)} \quad (\text{A.81})$$

$$\dot{\Psi}(t) = -\frac{\Gamma\epsilon(t)\xi^T(t)}{m^2(t)} \quad (\text{A.82})$$

where  $K_pS_p = (K_pS_p)^T > 0$  (see Assumption (A.4-3)),  $\Gamma = \Gamma^T > 0$ , and

$$m(t) = \sqrt{1 + \zeta^T(t)\zeta(t) + \xi^T(t)\xi(t)}. \quad (\text{A.83})$$

Then the time derivative of the positive definite function

$$V = \text{tr}[\tilde{\Theta}(t)\Gamma_p\tilde{\Theta}^T(t)] + \text{tr}[\tilde{\Psi}^T(t)\Gamma^{-1}\tilde{\Psi}(t)] \quad (\text{A.84})$$

with  $\Gamma_p = \Gamma_p^T > 0$  (see Assumption (A.4-3)), along (A.81) and (A.82), is

$$\dot{V} = -\frac{2\epsilon^T(t)\epsilon(t)}{m^2(t)} \leq 0. \quad (\text{A.85})$$

Similarly, we choose the *discrete-time* adaptive laws as

$$\Theta^T(t+1) = \Theta^T(t) - S_p\epsilon(t)\zeta^T(t) \quad (\text{A.86})$$

$$\Psi(t+1) = \Psi(t) - \Gamma\epsilon(t)\xi^T(t) \quad (\text{A.87})$$

where  $2I > K_pS_p = (K_pS_p)^T > 0$  and  $2I > \Gamma = \Gamma^T > 0$ .

Then, the time increment of

$$V = \text{tr}[\tilde{\Theta}(k)\Gamma_p\tilde{\Theta}^T(k)] + \text{tr}[\tilde{\Psi}^T(k)\Gamma^{-1}\tilde{\Psi}(k)] \quad (\text{A.88})$$

with  $\Gamma_p = K_p^T S_p^{-1} = \Gamma_p^T > 0$ , along (A.86) and (A.87), is

$$\begin{aligned} & V(\tilde{\Theta}(t+1), \tilde{\Psi}(t+1)) - V(\tilde{\Theta}(t), \tilde{\Psi}(t)) \\ &= -\frac{\epsilon^T(t)}{m^2(t)} \left( 2I - \frac{\zeta^T(t)\zeta(t)S_p^T K_p^T + \xi^T(t)\xi(t)\Gamma}{m^2(t)} \right) \epsilon(t) \leq 0. \end{aligned} \quad (\text{A.89})$$

From (A.85) or (A.89), the closed-loop system stability and asymptotic tracking properties can be proved [123].

### A.3 Adaptive Pole Placement Control

Consider the linear time-invariant plant described by

$$y(t) = G(s)[u](t) \quad (\text{A.90})$$

where  $y(t) \in R$  and  $u(t) \in R$  are the measured plant input and output, respectively, and  $G(s) = Z(s)/P(s)$  with

$$P(s) = s^n + p_{n-1}s^{n-1} + \cdots + p_1s + p_0 \quad (\text{A.91})$$

$$Z(s) = z_{n-1}s^{n-1} + z_{n-2}s^{n-2} + \cdots + z_1s + z_0 \quad (\text{A.92})$$

for some unknown but constant parameters  $p_i, z_i, i = 0, 1, \dots, n-1$ .

The control objective is to find an output feedback control signal  $u(t)$  such that all closed-loop signals are bounded and  $y(t)$  asymptotically tracks a reference output  $y_m(t)$  satisfying

$$Q(s)[y_m](t) = 0 \quad (\text{A.93})$$

where  $Q(s)$  is a monic polynomial of degree  $n_q$  with no zeros in  $\text{Re}[s] > 0$  and only nonrepeated zeros on the  $j\omega$ -axis so that  $y_m(t)$  is bounded.

For examples,  $Q(s) = 1$  for  $y_m(t) = 0$ ;  $Q(s) = s$  for  $y_m(t) = a \neq 0$ ;  $Q(s) = s^2 + \sigma^2$  for  $y_m(t) = a \sin(\sigma t) + b \cos(\sigma t)$  with  $\sigma \neq 0$  and not both  $a, b$  being zero;  $Q(s) = (s^2 + \sigma_1^2)(s^2 + \sigma_2^2)$  for  $y_m(t) = a_1 \sin(\sigma_1 t) + a_2 \sin(\sigma_2 t) + b_1 \cos(\sigma_1 t) + b_2 \cos(\sigma_2 t)$  with  $\sigma_1 \neq 0, \sigma_2 \neq 0, \sigma_1 \neq \sigma_2$ , not both  $a_1, b_1$  being zero, and not both  $a_2, b_2$  being zero; and  $Q(s) = s(s^2 + \sigma^2)$  for  $y_m(t) = a \sin(\sigma t) + b \cos(\sigma t) + c$  with  $c \neq 0, \sigma \neq 0$  and not both  $a, b$  being zero.

The following assumptions are needed for this control problem:

(A.5-1)  $Q(s)P(s)$  and  $Z(s)$  are co-prime, and

(A.5-2) The order  $n$  of  $P(s)$  is known.

**Design Procedure.** We use the following indirect approach to design an adaptive controller: First estimate the plant parameters in

$$\theta_p^* = [p_0, p_1, \dots, p_{n-1}]^T \in R^n, \theta_z^* = [z_0, z_1, \dots, z_{n-1}]^T \in R^n \quad (\text{A.94})$$

and then use a design equation to calculate the parameters of an output feedback controller from the adaptive estimates of  $\theta_p^*$  and  $\theta_z^*$ .

**Parameter estimation.** Let the estimates of  $\theta_p^*$  and  $\theta_z^*$  be

$$\theta_p = [\hat{p}_0, \hat{p}_1, \dots, \hat{p}_{n-1}]^T \in R^n, \theta_z = [\hat{z}_0, \hat{z}_1, \dots, \hat{z}_{n-1}]^T \in R^n \quad (\text{A.95})$$

and choose a monic stable polynomial  $\Lambda(s)$  as

$$\Lambda(s) = s^n + \lambda_{n-1}s^{n-1} + \dots + \lambda_1s + \lambda_0. \quad (\text{A.96})$$

Then, operating both sides of (A.90) by the stable filter  $1/\Lambda(s)$  and using

$$\theta^* = [\theta_z^{*T}, (\theta_\lambda - \theta_p^*)^T]^T, \theta_\lambda = [\lambda_0, \dots, \lambda_{n-1}]^T \in R^n \quad (\text{A.97})$$

$$\phi(t) = \left[ \frac{1}{\Lambda(s)}[u](t), \frac{s}{\Lambda(s)}[u](t), \dots, \frac{s^{n-2}}{\Lambda(s)}[u](t), \frac{s^{n-1}}{\Lambda(s)}[u](t), \right. \\ \left. \frac{1}{\Lambda(s)}[y](t), \frac{s}{\Lambda(s)}[y](t), \dots, \frac{s^{n-2}}{\Lambda(s)}[y](t), \frac{s^{n-1}}{\Lambda(s)}[y](t) \right]^T \in R^{2n} \quad (\text{A.98})$$

we express the plant (A.90) as

$$y(t) = \frac{Z(s)}{\Lambda(s)}[u](t) + \frac{\Lambda(s) - P(s)}{\Lambda(s)}[y](t) = \theta^{*T}\phi(t). \quad (\text{A.99})$$

Denote  $\theta(t)$  as the estimate of  $\theta^*$ :

$$\theta(t) = [\theta_z^T(t), (\theta_\lambda - \theta_p(t))^T]^T \quad (\text{A.100})$$

and define the estimation error

$$\epsilon(t) = \theta^T(t)\phi(t) - y(t). \quad (\text{A.101})$$

Using (A.99) and (A.101), we have

$$\epsilon(t) = \tilde{\theta}^T(t)\phi(t), \tilde{\theta}(t) = \theta(t) - \theta^*. \quad (\text{A.102})$$

Based on the error model (A.102), we can use some adaptive algorithms to update  $\theta(t)$ . For example, the normalized gradient algorithm is

$$\dot{\theta}(t) = -\frac{\Gamma\phi(t)\epsilon(t)}{m^2(t)}, \theta(0) = \theta_0, \Gamma = \Gamma^T > 0 \quad (\text{A.103})$$

where  $m(t) = \sqrt{1 + \alpha\phi^T(t)\phi(t)}$  with  $\alpha > 0$ .

This algorithm guarantees that  $\theta(t)$ ,  $\dot{\theta}(t)$ , and  $\epsilon(t)/m(t)$  are bounded, and  $\dot{\theta}(t)$  and  $\epsilon(t)/m(t)$  belong to  $L^2$ , as verified using the positive definite function  $V(\tilde{\theta}) = \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$  whose time derivative is  $\dot{V} = -2\epsilon^2(t)/m^2(t)$ .

**Control design.** Let the estimates of  $P(\lambda)$  and  $Z(\lambda)$  be

$$\hat{P}(\lambda, \theta_p) = \lambda^n + \hat{p}_{n-1}\lambda^{n-1} + \cdots + \hat{p}_1\lambda + \hat{p}_0 \quad (\text{A.104})$$

$$\hat{Z}(\lambda, \theta_z) = \hat{z}_{n-1}\lambda^{n-1} + \hat{z}_{n-2}\lambda^{n-2} + \cdots + \hat{z}_1\lambda + \hat{z}_0 \quad (\text{A.105})$$

such that  $Q(\lambda)\hat{P}(\lambda, \theta_p)$  and  $\hat{Z}(\lambda, \theta_z)$  are co-prime polynomials.<sup>3</sup>

Choose a monic desired closed-loop characteristic polynomial  $A^*(s)$  of degree  $2n + n_q - 1$  which has all its zeros in  $\text{Re}[s] < 0$ .

Find the polynomials  $C(\lambda, \psi_c)$  and  $D(\lambda, \psi_d)$ :

$$C(\lambda, \psi_c) = \lambda^{n-1} + c_{n-2}\lambda^{n-2} + \cdots + c_1\lambda + c_0 \quad (\text{A.106})$$

$$D(\lambda, \psi_d) = d_{n_q+n-1}\lambda^{n_q+n-1} + \cdots + d_1\lambda + d_0 \quad (\text{A.107})$$

where  $\psi_c = [c_0, c_1, \dots, c_{n-2}]^T \in R^{n-1}$  and  $\psi_d = [d_0, d_1, \dots, d_{n+n_q-1}]^T \in R^{n+n_q}$ , by solving the Diophantine equation<sup>4</sup>

$$C(\lambda, \psi_c)Q(\lambda)\hat{P}(\lambda, \theta_p) + D(\lambda, \psi_d)\hat{Z}(\lambda, \theta_z) = A^*(\lambda) \quad (\text{A.108})$$

and obtain  $C(s, \psi_c)$  and  $D(s, \psi_d)$  by the substitution

$$C(s, \psi_c) = C(\lambda, \psi_c)|_{\lambda=s} = s^{n-1} + \cdots + c_1s + c_0 \quad (\text{A.109})$$

$$D(s, \psi_d) = D(\lambda, \psi_d)|_{\lambda=s} = d_{n_q+n-1}s^{n_q+n-1} + \cdots + d_1s + d_0. \quad (\text{A.110})$$

Note that as an operator,  $c_i s^i$ ,  $i = 1, \dots, n-1$ , is different from  $s^i c_i$  if  $c_i = c_i(t)$  is a time-varying signal, and so is  $d_j s^j$ ,  $j = 1, \dots, n_q + n - 1$ , different from  $s^j d_j$  if  $d_j = d_j(t)$  is a time-varying signal.

Then the pole placement controller structure is

$$u(t) = (A_1(s) - C(s, \psi_c)Q(s)) \frac{1}{A_1(s)} [u](t) + D(s, \psi_d) \frac{1}{A_1(s)} [y_m - y](t) \quad (\text{A.111})$$

where  $A_1(s) = A_0(s)A_c(s)$ , with  $A_0(s)$  and  $A_c(s)$  being some monic and stable polynomials of degrees  $n_q - 1$  and  $n$ , respectively.

Under the condition that  $Q(\lambda)\hat{P}(\lambda, \theta_p)$  and  $\hat{Z}(\lambda, \theta_z)$  are co-prime, this adaptive control scheme ensures that all signals in the closed-loop system are bounded and that  $\lim_{t \rightarrow \infty} (y(t) - y_m(t)) = 0$ .

<sup>3</sup> The adaptive estimate  $\theta(t)$  from (A.103) does not automatically have this property. Special modifications of such  $\theta(t)$  are needed for this property [55].

<sup>4</sup> If  $Q(\lambda)\hat{P}(\lambda, \theta_p)$  and  $\hat{Z}(\lambda, \theta_z)$  are co-prime, then for any  $A^*(\lambda)$  of degree  $n_q + n - 1$ , this equation has a unique solution  $(C(\lambda, \psi_c), D(\lambda, \psi_d))$ .

## References

1. F. Ahmed-Zaid, P. Ioannou, K. Gousman, and R. Rooney, "Accommodation of failures in the F-16 aircraft using adaptive control," *IEEE Control Systems Magazine*, vol. 11, no. 1, pp. 73–78, 1991.
2. E. G. Alcorta and P. M. Frank, "Deterministic nonlinear observer-based approaches to fault diagnosis: A survey," *Control Engineering Practice*, vol. 5, no. 5, pp. 663–700, 1997.
3. A. M. Annaswamy, F. P. Skantze, and A.P. Loh, "Adaptive control of continuous-time systems with convex/concave parametrization," *Automatica*, vol. 14, no. 1, pp. 33–49, 1998.
4. P. J. Antsaklis and A. N. Michel, *Linear Systems*, McGraw-Hill, New York, 1997.
5. Z. Artstein, "Stabilization with relaxed controls," *Nonlinear Analysis*, pp. 1163–1173, 1983.
6. K. J. Åström and B. Wittenmark, *Adaptive Control*, 2nd ed., Addison-Wesley, Reading, MA, 1995.
7. E.-W. Bai and S. Sastry, "Discrete-time adaptive control utilizing prior information," *IEEE Trans. on Automatic Control*, vol. 31, no. 8, pp. 779–782, 1986.
8. D. S. Bayard, "A modified augmented error algorithm for adaptive noise cancellation in the presence of plant resonances," *Proc. of the 1998 American Control Conference*, pp. 137–141, Philadelphia, PA, 1998.
9. M. Blanke, "Fault-tolerant control systems," in *Advances in Control, Highlights of ECC99*, pp. 171–196, Springer-Verlag, New York, P. M. Frank, ed., 1999.
10. M. Blanke, R. Izadi-Zamanabadi, R. Bogh, and Z. P. Lunan, "Fault-tolerant control systems—a holistic view," *Control Engineering Practice*, vol. 5, no. 5, pp. 693–702, 1997.
11. M. Blanke, M. Staroswiecki, and N. E. Wu, "Concepts and methods in fault-tolerant control," *Proc. of the 2001 American Control Conference*, pp. 2606–2620, Arlington, VA, 2001.
12. M. Bodson, "Performance of an adaptive algorithm for sinusoidal disturbance rejection in high noise," *Automatica*, vol. 37, no. 7, pp. 1133–1140, 2001.
13. M. Bodson and J. E. Groszkiewicz, "Multivariable adaptive algorithms for re-configurable flight control," *IEEE Trans. on Control Systems Technology*, vol. 5, no. 2, pp. 217–229, 1997.

14. M. Bodson, J. Jensen, and S. Douglas, "Active noise control for periodic disturbances," *IEEE Trans. on Control Systems Technology*, vol. 9, no. 1, pp. 200–205, 2001.
15. J. D. Boskovic, S. Li, and R. K. Mehra, "Intelligent control of spacecraft in the presence of actuator failures," *Proc. of the 38th IEEE Conference on Decision and Control*, pp. 4472–4477, Phoenix, AZ, December 1999.
16. J. D. Boskovic and R. K. Mehra, "A multiple model-based reconfigurable flight control system design," *Proc. of the 37th IEEE Conference on Decision and Control*, pp. 4503–4508, 1998.
17. J. D. Boskovic and R. K. Mehra, "Stable multiple model adaptive flight control for accommodation of a large class of control effector failures," *Proc. of the 1999 American Control Conference*, pp. 1920–1924.
18. J. D. Boskovic and R. K. Mehra, "An adaptive scheme for compensation of loss of effectiveness of flight control effectors," *Proc. of the 40th IEEE Conference on Decision and Control*, pp. 2448–2453, Orlando, FL, 2001.
19. J. D. Boskovic, S.-H. Yu, and R. K. Mehra, "A stable scheme for automatic control reconfiguration in the presence of actuator failures," *Proc. of the 1998 American Control Conference*, pp. 2455–2459.
20. J. D. Boskovic, S.-H. Yu, and R. K. Mehra, "Stable adaptive fault-tolerant control of overactuated aircraft using multiple models, switching and tuning," *Proc. of the 1998 AIAA Guidance, Navigation and Control Conference*, vol. 1, pp. 739–749, 1998.
21. A. J. Calise, S. Lee, and M. Sharma, "Development of a reconfigurable flight control law for tailless aircraft," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 25, no. 5, pp. 896–902, 2001.
22. D. G. Chen and B. Paden, "Nonlinear adaptive torque-ripple cancellation for step motors," *Proc. of the 29th IEEE Conference on Decision and Control*, pp. 3319–3324, Honolulu, HI, 1990.
23. H. F. Chen and L. Guo, *Identification and Stochastic Adaptive Control*, Birkhauser, Boston, MA, 1991.
24. J. Chen and R. Patton, *Robust Model-based Fault Diagnosis for Dynamic Systems*, Kluwer, New York, 1998.
25. S. H. Chen, G. Tao and S. M. Joshi, "Adaptive actuator failure compensation for a transport aircraft model," *Proc. of the 2001 American Control Conference*, pp. 1827–1832, Arlington, VA, 2001.
26. S. H. Chen, G. Tao, and S. M. Joshi, "On matching conditions for adaptive state tracking control of systems with actuator failures," *Proc. of the 40th IEEE Conference on Decision and Control*, pp. 1479–1484.
27. A. Datta and P. A. Ioannou, "Performance improvement versus robust stability in model reference adaptive control," *IEEE Trans. on Automatic Control*, vol. 39, no. 12, pp. 2370–2388, 1994.

28. M. De Mathelin and M. Bodson, "Frequency domain conditions for parameter convergence in multivariable recursive identification," *Automatica*, vol. 26, no. 4, pp. 757–767, 1990.
29. M. A. Demetriou and M. M. Polycarpou, "Incipient fault diagnosis of dynamical systems using online approximators," *IEEE Trans. on Automatic Control*, vol. 43, no. 11, pp. 1612–1617, November 1998.
30. T. E. Duncan and B. Pasik-Duncan, "Adaptive control of continuous-time linear stochastic systems," *Mathematics of Control, Signals, and Systems*, vol. 3, pp. 45–60, 1990.
31. D. C. G. Eaton, "An overview of structural acoustics and related high-frequency-vibration activities," *ESA Bulletin*, no. 92, November 1997.
32. H. Elliott and W. A. Wolovich, "A parameter adaptive control structure for linear multivariable systems," *IEEE Trans. on Automatic Control*, vol. AC-27, no. 2, pp. 340–352, 1982.
33. A. F. Filippov, "Differential equations with discontinuous right hand sides," *American Mathematical Society Translations*, vol. 42, pp. 199–231, 1964.
34. T. E. Fortmann and K. L. Hitz, *An Introduction to Linear Control Systems*, Marcel Dekker, New York, 1977.
35. P. M. Frank, "Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy—A survey and some new results," *Automatica*, vol. 26, no. 3, pp. 459–474, 1990.
36. P. M. Frank, "Enhancement of robustness in observer-based fault detection," *International Journal of Control*, vol. 59, no. 4, pp. 955–981, 1994.
37. G. F. Franklin, J. D. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems*, 3rd ed., Addison-Wesley, Reading, MA, 1994.
38. J. P. Gao, B. Huang, Z. D. Wang, and D. G. Fisher, "Robust reliable control for a class of uncertain nonlinear systems with time-varying multistate time delays," *International Journal of Systems Science*, vol. 32, no. 7, pp. 817–824, 2001.
39. S. S. Ge, T. H. Lee, and C. J. Harris, *Adaptive Neural Network Control of Robot Manipulators*, World Scientific, River Edge, NJ, 1998.
40. S. S. Ge, C. C. Hang, T. H. Lee, and T. Zhang, *Stable Adaptive Neural Network Control*, Kluwer Academic, Boston, 2001.
41. J. J. Gertler, *Fault Detection and Diagnosis in Engineering Systems*, Marcel Dekker, New York, 1998.
42. G. C. Goodwin and R. S. Long, "Generalization of results on multivariable adaptive control," *IEEE Trans. on Automatic Control*, vol. AC-25, no. 6, pp. 1241–1245, 1980.
43. G. C. Goodwin, P. J. Ramadge, and P. E. Caines, "Discrete-time multivariable adaptive control," *IEEE Trans. on Automatic Control*, vol. AC-25, no. 6, pp. 449–456, 1980.

44. G. C. Goodwin and D. Q. Mayne, "A parameter estimation perspective of continuous time model reference adaptive control," *Automatica*, vol. 23, no. 1, pp. 57–70, 1987.
45. G. C. Goodwin and K. S. Sin, *Adaptive Filtering Prediction and Control*, Prentice-Hall, Englewood Cliffs, NJ, 1984.
46. M. Gopinathan, J. D. Boskovic, R. K. Mehra, and C. Rago, "A multiple model predictive scheme for fault-tolerant flight control design," *Proc. of the 37th IEEE Conference on Decision and Control*, pp. 1376–1381, 1998.
47. H. Hammouri, M. Kinnaert, and E. H. El Yaagoubi, "Observer-based approach to fault detection and isolation for nonlinear systems," *IEEE Trans. on Automatic Control*, vol. 44, no. 10, pp. 1879–1884, 1999.
48. L. Hsu, R. Costa, A. Imai, and P. Kokotović, "Lyapunov based adaptive control of MIMO systems," *Proc. of the 2001 American Control Conference*, pp. 4808–4813, 2001.
49. M. Idan, M. Johnson, and A. J. Calise, "A hierarchical approach to adaptive control for improved flight safety," *Proc. of the 2001 AIAA Guidance, Navigation, and Control Conference*, Montreal, Canada.
50. M. Idan, M. Johnson, A. J. Calise, and J. Kaneshige, "Intelligent aerodynamic/propulsion flight control for flight safety: A nonlinear adaptive approach," *Proc. of the 2001 ACC*, pp. 2918–2923.
51. F. Ikhouane and M. Krstić, "Robustness of the tuning functions adaptive backstepping design for linear systems," *Proc. of the 34th IEEE CDC*, pp. 159–164, New Orleans, LA, 1995.
52. A. K. Imai, R. Costa, L. Hsu, G. Tao, and P. Kokotović, "Multivariable MRAC using high frequency gain matrix factorization," *Proc. of the 40th IEEE Conference on Decision and Control*, pp. 1193–1198, 2001.
53. P. A. Ioannou and P. V. Kokotović, *Adaptive Systems with Reduced Models*, Springer-Verlag, Berlin, 1983.
54. P. A. Ioannou and P. V. Kokotović, "Instability analysis and improvement of robustness of adaptive control," *Automatica*, vol. 20, no. 5, pp. 583–594, 1984.
55. P. A. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice-Hall, Upper Saddle River, NJ, 1996.
56. P. A. Ioannou and K. Tsakalis, "A robust direct adaptive controller," *IEEE Trans. on Automatic Control*, vol. AC-31, no. 11, pp. 1033–1043, 1986.
57. P. A. Ioannou and K. Tsakalis, "Robust discrete time adaptive control," in *Adaptive and Learning Systems: Theory and Applications*, K. S. Narendra, ed., Plenum Press, New York, 1986.
58. R. Iserman, "On the applicability of model based fault detection for technical process," *Control Engineering Practice*, vol. 2, no. 3, pp. 439–450, 1997.
59. R. Iserman and P. Balle, "Trends in the application of model-based fault detection and diagnosis of technical processes," *Control Engineering Practice*, vol. 5, pp. 709–719, 1997.

60. A. Isidori, *Nonlinear Control Systems*, 3rd ed., Springer-Verlag, New York, 1995.
61. C. A. Jacobson and C. N. Nett, "An integrated approach to controls and diagnostic using the four parameter controller," *IEEE Control Systems Magazine*, vol. 11, pp. 22–29, 1991.
62. Z. P. Jiang, "A combined backstepping and small-gain approach to adaptive output feedback control," *Automatica*, vol. 35, no. 6, pp. 1131–1139, 1999.
63. Z. P. Jiang, "Decentralized and adaptive nonlinear tracking of large-scale systems via output feedback," *IEEE Trans. on Automatic Control*, vol. 45, no. 11, pp. 2122–2128, 2000.
64. P. Kabore and H. Wang, "On the design of fault diagnosis filters and fault tolerant control," *Proc. of the American Control Conference*, San Diego, CA, June 1999.
65. T. Kailath, *Linear Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1980.
66. I. Kanellakopoulos, P. V. Kokotović, and A. S. Morse, "Systematic design of adaptive controllers for feedback linearizable systems," *IEEE Trans. on Automatic Control*, vol. 36, no. 11, pp. 1241–1253, 1991.
67. H. K. Khalil, *Nonlinear Systems*, 2nd ed., Prentice Hall, Upper Saddle River, NJ, 1996.
68. H. K. Khalil, "Adaptive output feedback control of nonlinear systems represented by input-output model," *IEEE Trans. on Automatic Control*, vol. 41, no. 2, pp. 177–188, 1996.
69. B. S. Kim and A. J. Calise, "Nonlinear flight control using neural networks," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 20, no. 1, pp. 26–33, 1997.
70. M. Krstić, I. Kanellakopoulos, and P. V. Kokotović, "Nonlinear design of adaptive controllers for linear systems," *IEEE Trans. on Automatic Control*, vol. AC-39, no. 4, pp. 738–752, 1994.
71. M. Krstić, I. Kanellakopoulos, and P. V. Kokotović, *Nonlinear and Adaptive Control Design*, John Wiley & Sons, New York, 1995.
72. Y. D. Landau, *Adaptive Control: The Model Reference Approaches*, Marcel Dekker, New York, 1979.
73. Y. D. Landau, R. Lozano, and M. M'Saad, *Adaptive Control*, Springer-Verlag, London, 1998.
74. J. Leitner, A. J. Calise, and J. V. R. Prasad, "Analysis of adaptive neural networks for helicopter flight controls," *AIAA Journal of Guidance, Control, and Dynamics*, vol. 20, no. 5., pp. 972–979, September–October 1997.
75. F. L. Lewis, S. Jagannathan, and A. Yesildirek, *Neural Network Control of Robot Manipulators and Nonlinear Systems*, Taylor and Francis, Philadelphia, PA, 1999.

76. Y.-W. Liang, D.-C. Liaw, and T. C. Lee, "Reliable control of nonlinear systems," *IEEE Trans. on Automatic Control*, vol. 45, no. 4, pp. 706–710, 2000.
77. F. Liao, J. L. Wang, and G.-H. Yang, "Reliable robust flight tracking control: An LMI approach," *IEEE Trans. on Control Systems Technology*, vol. 10, no. 1, pp. 76–89, 2002.
78. W. Lin and C. Qian, "Adaptive regulation of high-order lower-triangular systems: Adding a power integrator technique," *Systems and Control Letter*, vol. 39, pp. 353–364, 2000.
79. W. Lin and C. Qian, "Adaptive control of nonlinearly parametrized systems: Smooth feedback domination design," submitted to *IEEE Trans. on Automatic Control*.
80. X. C. Lou, A. S. Willsky, and G. C. Verghese, "Optimal robust redundancy relations for failure detection in uncertain systems," *Automatica*, vol. 22, no. 3, pp. 333–344.
81. B. Lui and J. Si, "Fault isolation filter design for linear time-invariant systems," *IEEE Trans. on Automatic Control*, vol. 42, no. 5, pp. 704–706, 1997.
82. J. M. Maciejowski, "Reconfigurable control using constrained optimization," *Proc. of ECC97*, pp. 107–130, Brussels, Belgium, 1997.
83. R. Mangoubi, *Robust Estimation and Failure Detection: A Concise Treatment*. Springer-Verlag, New York, 1998.
84. R. Marino and P. Tomei, *Nonlinear Control Design: Geometric, Adaptive and Robust*, Prentice-Hall, Englewood Cliffs, NJ, 1995.
85. M. A. Massoumnia, G. C. Verghese, and A. S. Willsky, "Failure detection and identification," *IEEE Trans. on Automatic Control*, vol. 34, no. 3, pp. 316–321, 1989.
86. R. H. Miller, "On-line detection of aircraft icing: An application of optimal fault detection and isolation," Ph.D. dissertation, University of Michigan, Ann Arbor, 2000.
87. R. H. Miller and B. R. William, "The effects of icing on the longitudinal dynamics of an icing research aircraft," *The 37th Aerospace Sciences Conference*, AIAA, 1999.
88. A. S. Morse, "Global stability of parameter adaptive control systems," *IEEE Trans. on Automatic Control*, vol. 25, no. 6, pp. 433–439, 1980.
89. A. S. Morse, "A gain matrix decomposition and some of its applications," *Systems and Control Letters*, vol. 21, no. 1, pp. 1–10, 1993.
90. A. S. Morse, D. Q. Mayne, and G. C. Goodwin, "Applications of hysteresis switching in parameter adaptive control," *IEEE Trans. on Automatic Control*, vol. AC-37, no. 9, pp. 1343–1354, 1992.
91. W. D. Morse and K. A. Ossman, "Model-following reconfigurable flight control system for the AFTI/F16," *Journal of Guidance, Control and Dynamics*, vol. 14, no. 6, pp. 969–976, 1990.

92. L. H. Mutuel and J. L. Speyer, "Fault-tolerant estimation," *Proc. of the 2000 ACC*, pp. 3718–3722, Chicago, IL, 2000.
93. "Linearized Dynamic Models of Boeing 737-100 Transport Aircraft," NASA Langley Research Center, 2000.
94. S. M. Naik, P. R. Kumar, and B. E. Ydstie, "Robust continuous-time adaptive control by parameter projection," in *Foundations of Adaptive Control*, P. V. Kokotović, ed., pp. 153–199, Springer-Verlag, Berlin, 1991.
95. K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1989.
96. K. S. Narendra and J. Balakrishnan, "Adaptive control using multiple models," *IEEE Trans. on Automatic Control*, vol. 42, no. 2, pp. 171–187, February 1997.
97. H. Noura, D. Sauter, F. Hamelin, and D. Theilliol, "Fault-tolerant control in dynamic systems: application to a winding machine," *IEEE Control Systems Magazine*, vol. 20, pp. 33–49, 2000.
98. R. D. Nussbaum, "Some remarks on a conjecture in parameter adaptive control," *Systems & Control Letters*, vol. 3, pp. 243–246, 1983.
99. R. Ortega, L. Hsu, and A. Astolfi, "Adaptive control of multivariable systems with reduced prior knowledge," *Proc. of the 40th IEEE Conference on Decision and Control*, pp. 4198–4203, Orlando, FL, 2001.
100. R. Ortega and Y. Tang, "Robustness of adaptive controllers: A survey," *Automatica*, vol. 25, no. 5, pp. 651–677, 1989.
101. Z. Pan and T. Basar, "Adaptive controller design for tracking and disturbance attenuation in parametric strict-feedback nonlinear systems," *IEEE Trans. on Automatic Control*, vol. 43, no. 8, pp. 1066–1083, 1998.
102. R. J. Patton, "Fault tolerant control: The 1997 situation," *Proc. of IFAC Symp. on SAFEPROCESS*, pp. 1033–1055, Hull, UK, 1997.
103. R. J. Patton and Kanguette, "Robust fault diagnosis using eigenstructure assignment of observers," in *Fault Diagnosis in Dynamic Systems: Theory and Application*, R. J. Patton, P. M. Frank, and R. N. Clark, eds., pp. 99–154, Prentice-Hall, Englewood Cliffs, NJ, 1989.
104. M. M. Polycarpou, "Fault accommodation for a class of multivariable nonlinear dynamical systems using a learning approach," *IEEE Trans. on Automatic Control*, vol. 46, no. 5, pp. 736–742, May 2001.
105. M. Polycarpou and P. A. Ioannou, "On the existence and uniqueness of solutions in adaptive control systems," *IEEE Trans. on Automatic Control*, vol. 38, no. 3, pp. 474–479, 1993.
106. M. M. Polycarpou and A. B. Trunov, "Learning approach to nonlinear fault diagnosis: detectability analysis," *IEEE Trans. on Automatic Control*, vol. 45, no. 4, pp. 806–812, April 2000.
107. M. M. Polycarpou and A. T. Vemuri, "Learning approaches to fault tolerant control: An overview," *Proc. of the IEEE Intl. Symp. on Intelligent Control*, pp. 157–162, Gaithersburg, MD, 1998.

108. R. T. Rysdyk and A. J. Calise, "Nonlinear adaptive flight control using neural networks," *IEEE Control Systems Magazine*, vol. 18, no. 6, December 1998.
109. S. Sastry and A. Isidori, "Adaptive control of linearizable systems," *IEEE Trans. on Automatic Control*, vol. 34, no. 11, pp. 1123–1131, 1989.
110. S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence, and Robustness*, Prentice-Hall, Englewood Cliffs, NJ, 1989.
111. L. V. Schmidt, *Introduction to Aircraft Flight Dynamics*, American Institute of Aeronautics and Astronautics, Reston, VA, 1998.
112. M. Staroswiecki and A.-L. Gehin, "From control to supervision," *Annual Review in Control*, vol. 25, pp. 1–11, 2001.
113. B. L. Stevens and F. L. Lewis, *Aircraft Control and Simulation*, John Wiley & Sons, New York, 1992.
114. X. D. Tang, G. Tao, and S. M. Joshi, "Adaptive control of parametric-strict-feedback nonlinear systems with actuator failures," *Proc. of the 40th IEEE Conference on Decision and Control*, pp. 1613–1614, Orlando, FL, 2001.
115. X. D. Tang, G. Tao, and S. M. Joshi, "Adaptive actuator failure compensation control of parametric strict-feedback systems with zero dynamics," *Proc. of the 40th IEEE Conference on Decision and Control*, pp. 2031–2036, Orlando, FL, 2001.
116. G. Tao, "Inherent robustness of MRAC schemes," *Systems and Control Letters*, vol. 29, no. 3-11, pp. 165–174, November 1996.
117. G. Tao, "Friction compensation in the presence of flexibility," *Proc. of the 1998 American Control Conference*, pp. 2128–2132, Philadelphia, PA.
118. G. Tao, *Adaptive Control Design and Analysis*, John Wiley and Sons, New York, 2003.
119. G. Tao, S. H. Chen, and S. M. Joshi, "An adaptive control scheme for systems with unknown actuator failures," *Proc. of the 2001 American Control Conference*, pp. 1115–1120.
120. G. Tao, S. H. Chen, and S. M. Joshi, "An adaptive actuator failure compensation controller using output feedback," *Proc. of the 2001 American Control Conference*, pp. 3085–3090.
121. G. Tao, S. H. Chen, and S. M. Joshi, "An adaptive control scheme for systems with unknown actuator failures," Technical report UVA-ECE-ASC-01-03-01, Department of Electrical and Computer Engineering, University of Virginia, Charlottesville, VA, March 2001.
122. G. Tao and P. A. Ioannou, "Robust model reference adaptive control for multivariable plants," *Intl. J. Adaptive Control and Signal Process*, vol. 2, no. 3, pp. 217–248.
123. G. Tao and P. A. Ioannou, "Stability and robustness of multivariable model reference adaptive control schemes," in *Advances in Robust Control Systems Techniques and Applications*, Academic Press, C. T. Leondes, ed., vol. 53, pp. 99–123, 1992.

124. G. Tao, S. M. Joshi, and X. L. Ma, "Adaptive state feedback and tracking control of systems with actuator failures," *IEEE Trans. on Automatic Control*, vol. 46, no. 1, pp. 78–95, January 2001.
125. G. Tao and P. V. Kokotović, *Adaptive Control of Systems with Actuator and Sensor Nonlinearities*, John Wiley & Sons, New York, 1996.
126. G. Tao, X. L. Ma, and S. M. Joshi, "Adaptive output tracking control of systems with actuator failures," *Proc. of the 2000 American Control Conference*, pp. 2654–2658, Chicago, IL, 2000.
127. G. Tao, X. L. Ma, and S. M. Joshi, "Adaptive state feedback control of systems with actuator failures," *Proc. of 2000 American Control Conference*, pp. 2669–2673, Chicago, IL, 2000.
128. G. Tao, X. D. Tang, and S. M. Joshi, "Output tracking actuator failure compensation control," *Proc. of the 2001 American Control Conference*, pp. 1821–1826, Arlington, VA, 2001.
129. M. Tian and G. Tao, "Adaptive dead-zone compensation for out-feedback canonical systems," *International Journal of Control*, vol. 67, pp. 791–812, 1997.
130. M. Tian and G. Tao, "Adaptive control of a class of nonlinear systems with unknown dead-zones," *Proc. of the 13th World Congress of IFAC*, vol. E, pp. 209–213, San Francisco, CA, July 1996.
131. K. Tsakalis, "Robustness of model reference adaptive controllers: An input-output approach," *IEEE Trans. on Automatic Control*, vol. 37, no. 5, pp. 556–565, 1992.
132. K. Tsakalis and P. A. Ioannou, *Linear Time Varying Systems: Control and Adaptation*, Prentice-Hall, Englewood Cliffs, NJ, 1993.
133. K. S. Tsakalis, "Model reference adaptive control of linear time-varying plants: The case of 'jump' parameter variations," *International Journal of Control*, vol. 56, no. 6, pp. 1299–1345.
134. V. I. Utkin, *Sliding Modes in Control Optimization*, Springer-Verlag, Berlin, 1991.
135. A. T. Vemuri and M. M. Polycarpou, "Robust nonlinear fault diagnosis in input-output systems," *International Journal of Control*, vol. 68, no. 2, pp. 343–360, 1997.
136. R. J. Veillette, "Reliable linear-quadratic state-feedback control," *Automatica*, vol. 31, no. 1, pp. 137–143, 1995.
137. R. J. Veillette, J. V. Medanic, and W. R. Perkins, "Designs of reliable control systems," *IEEE Trans. on Automatic Control*, vol. 37, no. 3, pp. 290–304.
138. M. Vidyasagar, *Nonlinear Systems Analysis*, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1993.
139. H. Wang and S. Daley, "Actuator fault diagnosis: An adaptive observer-based technique," *IEEE Trans. on Automatic Control*, vol. 41, no. 7, pp. 1073–1078, 1996.

140. H. Wang, Z. J. Huang, and S. Daley, "On the use of adaptive updating rules for actuator and sensor fault diagnosis," *Automatica*, vol. 33, pp. 217–225, 1997.
141. S. R. Weller and G. C. Goodwin, "Hysteresis switching adaptive control of linear multivariable systems," *IEEE Trans. on Automatic Control*, vol. 39, no. 7, pp. 1360–1375, 1994.
142. K. Wise, J. S. Brinker, A. J. Calise, D. F. Enns, and M. R. Elgersma, "Direct adaptive reconfigurable flight control for a tailless advanced fighter aircraft," *Intl. J. Robust and Nonlinear Control*, vol. 9, pp. 999–1009, 1999.
143. N. E. Wu and T. J. Chen, "Feedback design in control reconfigurable systems," *International Journal of Robust and Nonlinear Control*, vol. 6, pp. 560–570, 1996.
144. N. E. Wu, Y. Zhang, and K. Zhou, "Detection, estimation, and accommodation of loss of control effectiveness," *Intl. Journal of Adaptive Control and Signal Processing*, vol. 14, pp. 775–795, 2000.
145. N. E. Wu, K. Zhou, and G. Salomon, "Reconfigurability in linear time-invariant systems," *Automatica*, vol. 36, pp. 1767–1771, 2000.
146. H. Xu and M. Mirmirani, "Robust adaptive sliding control for a class of MIMO nonlinear systems," *Proc. of the 2001 AIAA Guidance, Navigation and Control Conference*, Montréal, Québec, Canada, 2001.
147. G-H. Yang, J. L. Wang, and Y. C. Soh, "Reliable  $H_\infty$  controller design for linear systems," *Automatica*, vol. 37, pp. 717–725, 2001.
148. Y. Yang, G.-H. Yang, and Y. C. Soh, "Reliable control of discrete-time systems with actuator failures," *IEE Proceedings: Control Theory and Applications*, vol. 147, no. 4, pp. 424–432, 2000.
149. B. Yao and M. Tomizuka, "Adaptive robust control of MIMO nonlinear systems in semi-strict-feedback forms," *Automatica*, vol. 37, no. 9, pp. 1305–1321, 2001.
150. G. Yen and L. Ho, "Fault tolerant control: An intelligent sliding model control strategy," *Proc. of the American Control Conference*, pp. 4204–4208, Chicago, IL, June 2000.
151. Y. M. Zhang and J. Jiang, "Design of proportional-integral reconfigurable control systems via eigenstructure assignment," *Proc. of the American Control Conference*, pp. 3732–3736, Chicago, IL, June 2000.
152. Y. M. Zhang and J. Jiang, "Integrated active fault-tolerant control using IMM approach," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 37, no. 4, pp. 1221–1235, 2001.
153. Y. M. Zhang and J. Jiang, "Bibliographical review on reconfigurable fault-tolerant control systems," *Proc. of the 5th IFAC Symp. on SAFEPROCESS*, pp. 265–276, Washington, DC, 2003.
154. Q. Zhao and J. Jiang, "Reliable state-feedback control systems design against actuator failures," *Automatica*, vol. 30, no. 10, pp. 1267–1272, 1998.

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