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words they are psychic in origin. Events in the world, causal or otherwise, can be assigned a tao or meaning. Synchronicity is an acausal juxtaposition of two meaningfully related events. In this category come coincidences, predictive dreams and possibly astrology. Synchronicitic events being at a more primitive or deeper level are beyond space and time. For example if I say that someone is born under Mars and is therefore warlike my statement may have no rational basis but may be statistically true.

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