References


References


Abel’s summation formula, 383
Absorbing state, 165
Acceptance function, 184
Adapted random variable, 255
Aperiodicity, 152
Arithmetic function, 375
   periodic, 385
Arithmetic-geometric mean inequality, 58
Asymptotic expansions, 298
   incomplete gamma function, 303
   Laplace’s method, 304–308, 410–411
   order statistic moments, 305
   Poincaré’s definition, 303
   Stieltjes function, 318
   Stirling’s formula, 306
   Taylor expansions, 299
Asymptotic functions, 299
   examples, 318–320
Azuma-Hoeffding bound, 264
Azuma-Hoeffding theorem, 260
Backtracking, 109
Backward equations, 189
Balance equation, 191
Barker’s function, 184
Bayes’ rule, 15
Bell numbers, 77, 119
Bernoulli functions, 308–310, 408–410
Bernoulli numbers, 409
   Euler-Maclaurin formula, in, 309
Bernoulli polynomials, 308–310, 321, 408–410
Bernoulli-Laplace model, 175
Bernstein polynomial, 67, 72
Bessel process, 275, 284
Beta distribution, 12, 90, 179
   asymptotics, 308
   mean, 39
Beta-binomial distribution, 29, 47
Biggest random gap, 358
Binary expansions, 316
Binomial distribution, 12, 51, 178
   factorial moments, 33
Biorthogonality, 164
Bipartite graph, 155
Birthday problem, 305, 357
Block matrix decomposition, 166
Bonferroni inequality, 82
Borel sets, 3
Borel-Cantelli lemma, 5, 324
partial converse of, 20
Bradley-Terry ranking model, 64
Branching process, 217, 340
convergence, 254
criticality, 229
irreducible, 229
martingale, 251
multitype, 229–231
Brownian motion, 270, 272–275
Buffon needle problem, 28

Campbell's moment formulas, 139–142
Cancer models, 167, 243–245
Cantelli's inequality, 71
Catalan numbers, 84–85
asymptotics, 308
Cauchy distribution, 17
Cauchy-Schwarz inequality, 66, 256
Cell division, 218
Central limit theorem, 316
Change of variables formula, 16
Chapman-Kolmogorov relation, 191
Characteristic function, 6
example of oscillating, 35
table of common, 12
Chebyshev's bound, 268
Chebyshev's inequality, 67
Chen's lemma, 41
Chen-Stein method, 355
proof, 363–368
Chernoff's bound, 67, 72
Chi-square distribution, 47, 51
Cholesky decomposition, 19
Circuit model, 196
Circulant matrix, 351
Cofinite subset, 20
Coin tossing, waiting time, 352
Compact set, 192
Composition chain, 213

Conditional probability, 6
Connected graph, 155
Convergence
almost sure, 314, 324
in distribution, 315, 324
in probability, 314, 324
Convex functions, 56–60
minimization, 61–63
Convex set, 56
Convolution
finite Fourier transform, 401
integral equation, 336
Coupling, 158–163
applications, 178–179
independence sampler, 171
Covariance, see Variance

Density
as likelihood, 13
conditional, 15
marginal, 15
table of common densities, 12
Detailed balance, 153, 192
Hasting-Metropolis algorithm, in, 169
Diagonally dominant matrix, 337, 353
Differentiable functions, 57
Differential, see Jacobian
Diffusion process, 270–272
first passage, 282
moments, 280
numerical method, 343–347
Dirichlet distribution, 44
as sum of gammas, 52
variance and covariance, 52
Dirichlet product, 379
Distribution, 9
and symmetric densities, 22
continuous, 10
convolution of, 13
discrete, 9
marginal, 15
of a random vector, 14
of a transformation, 16
table of common, 12
DNA sequence analysis, 115
Doob’s martingale, 250
Ehrenfest diffusion, 156, 180, 211
Eigenvalues and eigenvectors, 163–165, 172, 207
Epidemics, 219
Equilibrium distribution, 152, 190
existence of, 160
Ergodic theorem, 153
ESP example, 50
Euclidean norm, 57
Euler’s constant, 88, 310
Euler’s totient function, 81, 377
Euler-Maclaurin formula, 308–311
Eulerian numbers, 120, 324
Ewens’ sampling distribution, 91
Exchangeable random variable, 80
Expectation, 4
and subadditivity, 114–117
conditional, 6, 29–31
sum of i.i.d. random variables, 7
differentiation and, 5
of a random vector, 14
Exponential distribution, 12
bilateral, 49
convolution of gammas, 32
lack of memory, 129
Exponential integral, 302
Extinction, 221
Extinction probability, 217, 224
genetic distribution, 223
Faà di Bruno’s formula, 91
Family name survival, 219
Family planning model, 31, 135
Fast Fourier transform, 402–403
applications, 331–335
Fatou’s lemma, 4
Fejér’s theorem, 406
Fibonacci numbers, 77, 119
asymptotics, 312
Filter, 248
Finite Fourier transform
computing, see Fast Fourier transform
definition, 401
examples, 350–353
inversion, 401
properties, 402
Finnish population growth, 348
Flux, 271
Formation of polymers, 238
Forward equations, 200
Four-color theorem, 184
Fourier coefficients, 320–321, 407
Fourier inversion, see Inversion formula
Fourier series, 332, 406
Bernoulli polynomials, 408
pointwise convergence, 407
Fourier transform
definition, 403
function pairs, table of, 404
inversion, 405
Riemann-Lebesgue lemma, 405
Fractional linear transformation, 236
Fubini’s theorem, 9
Fundamental theorem of arithmetic, 374, 397
Gambler’s ruin, 258
Gamma distribution, 12, 49, 91
as convolution, 32
characteristic function, 32
inverse, 38
Gamma function, 60
asymptotic behavior, 306
Gauss-Seidel algorithm, 329–331
block version, 331
Gaussian, see Normal distribution
Gaussian elimination, 328
Generalized hyperbolic functions, 145
Generating function, see Progeny
 generating function
coin toss wait time, 352
convolution, 332
jump counting, 336
Genetic drift, see Wright-Fisher model
Geometric distribution, 12
Geometric progeny, 220
Gibbs prior, 133
Gibbs sampling, 170
Gillespie’s algorithm, 340
Graph coloring, 108–112
Group homomorphism, 236
Hölder’s inequality, 69
Hadamard product, 23
Hamming distance, 362
Harmonic series, 310
Hastings-Metropolis algorithm, 168–171
aperiodicity, 184
Gibbs sampler, 170
independence sampler, 169
convergence, 171–172
random walk sampling, 169
Heron’s formula, 70
Hessian matrix, 58
Hitting probability, 165
matrix decomposition, 166
Hitting time, 165–167
expectation, 166
HIV
new cases of AIDS, 138
viral reproduction, 231
Huffman bit string, 106
Huffman coding, 106–108
string truncation, 108
vowel tree, 106
Huntington’s disease, 227
Hurwitz’s zeta function, 385
Hyperbolic trigonometric functions, 146
Hypergeometric distribution, 7, 178
Immigration, 225–229
Importance ratio, 169, 171
Inclusion-exclusion formula, 78–83
Incomplete gamma function, 303
Independence, 8
Independence sampler, 169
convergence, 171–172
Indicator random variable, 4
sums of, 25
Inequality, 66–69
arithmetic-geometric mean, 58
Cantelli’s, 71
Cauchy-Schwarz, 66
Chebyshev’s, 67
Hölder’s, 69
Jensen’s, 68
Markov’s, 66
Minkowski’s, 73
Schlömilch’s, 68
Infinitesimal generator, 190
Infinitesimal mean, 270
Infinitesimal transition matrix, see
Infinitesimal generator
Infinitesimal transition probabilities, see Transition intensity
Infinitesimal variance, 270
Inner product in \( \mathbb{R}^n \), 112
Integrable function, 404
Integration by parts, 302–303
Intensity, 188
Intensity leaping, 339–343
Inversion formula, 11
Involution, 96
Irreducibility, 153
Ising model, 170
Jacobi algorithm, 329–331
block version, 331
Jacobian, 16
Jensen’s inequality, 68
Kendall’s birth-death-immigration process, 200–206, 215, 276, 342
Kimura’s model of DNA substitution, 193, 199, 210, 339
Kirchhoff’s laws, 197
Kolmogorov’s circulation criterion, 154
Kolmogorov’s forward equation, 272, 344
Laplace transform, 34, 37, 254, 320, 337, 388
Laplace’s method, 304–308, 320, 410–411
Law of rare events, 360
Least absolute deviation, 65
Left-to-right maximum, 88
Liapunov function, 191
Light bulb problem, 176
Likelihood, 13
Lindeberg’s condition, 316
Liouville’s arithmetic function, 393
Lipschitz condition, 262
Logarithmic distribution, 301
Logistic distribution, 21
Longest common subsequence, 115, 263
Longest increasing subsequence, 93
Lotka’s surname data, 223
Möbius function, 236, 380
Ménage problem, 356
Marking and coloring, 138–139
Markov chain, 151–154
  continuous time
    equilibrium distribution, 328
counting jumps, 336–339
  ergodic assumptions, 152, 174
  intensity leaping, 339
  stationary distribution, 152
  finite state, 160
  transition matrix, 151
Markov chain Monte Carlo, 168–172
  Gibbs sampling, 170
  Hastings-Metropolis algorithm, 168–170
  simulated annealing, 173–174
Markov property, 20
Markov’s inequality, 66, 261
Martingale, 247–251
  convergence, 251
  large deviations, 260
Master equations, 341
Matrix exponentials, 197–199
Maximum likelihood estimates, 13, 65
MCMC, see Markov chain Monte Carlo
Median finding, 118
Minkowski’s triangle inequality, 73
MM algorithm, 63–66
Moment, 11
  asymptotics, 305, 318
  factorial, 33
  generating function, 11
  polynomials on a sphere, 43
Moment inequalities, 66–69
Monotone convergence theorem, 4
Moran’s genetics model, 353
Multinomial sampling, 134
Mutant gene survival, 219
Negative binomial distribution, 31, 51, 179
Negative multinomial distribution, 148
Neuron firing, see Ornstein-Uhlenbeck
Neutron chain reaction, 218
Newton’s method, 237
Normal distribution, 12
  affine transforms of, 18
  characteristic function, 31
  characterization of, 36
  distribution function
    asymptotic expansion, 303
    multivariate, 17
  maximum likelihood, 62
NP-completeness, 173
Null recurrence, 158
Number-theoretic density, 3
O-notation, see Order relations
Optional stopping theorem, 255, 257
Order relations, 298–299
examples, 318
Order statistics, 83–84
distribution function of, 83
from an exponential sample, 130
moments, 305–306
Ornstein-Uhlenbeck process, 277, 290
Oxygen in hemoglobin, 193
Pareto distribution, 21
Pascal’s triangle, 76
Pattern matching, 26
Permutation cycles, 87, 317
Permutation inversions, 317
Pigeonhole principle, 93–94
Planar graph, 109
Point sets with acute angles, 112–113
Poisson distribution, 12, 124, 179
birthday problem, 305
factorial moments, 33
Poisson process, 124
from given intensity function, 126
inhomogeneous, 202–206, 341
one-dimensional, 127
restriction, 137
superposition, 137
transformations, 136–138
transformed expectations, 137
waiting time, 127
waiting time paradox, 130
Polar coordinates, 138
Polya’s model, see Urn model
Polynomial
multiplication, 332
Polynomial on $S_{n-1}$, 44
Positive definite quadratic forms, 58
Power method, 328–331
Powers of integers, sum of, 322
Prime integer, 374
Prime number theorem, 386
Probabilistic embedding, 89
Probability measure, 2
Probability space, 2
Product measure formula, 42
Progeny generating function, 218
Proposal distribution, 168
QR decomposition, 18
Quick sort, 104–106
average-case performance, 105
median finding, 118
promotion process, 104
Random circles
in $\mathbb{R}^2$, 148
in $\mathbb{R}^3$, 149
Random deviates, generating
logistic, 21
Pareto, 21
Weibull, 21
Random permutation, 26
and card shuffling, 158
and Poisson distribution, 80
and Sperner’s theorem, 113
cycles in, 87
fixed points, 46, 80
successive, 259
Random sums, 33, 49
Random variables
correlated, coupling, 158
definition, 3
measurability of, 3
Random walk, 85
as a branching process, 236
coupling, 178
equilibrium, 156, 330
eventual return, 182
first return, 97
hitting probability, 183, 210, 267
hitting time, 183, 210, 267
martingales, 258, 267
on a graph, 155, 176
renewal theory, 158
sampling, 169
self avoiding, 121
Reaction channel, 340
Recessive gene equilibrium, 290
Recurrence relations, 31
average-case quick sort, 105
Bernoulli numbers, 409
Bernoulli polynomials, 408
family planning model, 47
Relatively prime integers, 374
Renewal equation, 333–335
Renewal process, 157
Repeated uniform sampling, see Uniform distribution
Residual, 66
Reversion of sequence, 402
Riemann’s zeta function, 70, 374
Riemann-Lebesgue lemma, 404
Right-tail probability, 36
Runs in coin tossing, 323, 335
Sampling without replacement, 27
Scheffe’s lemma, 20
Schlömilch’s inequality, 68, 256
Schrödinger’s method, see Multinomial sampling
Self-adjointness condition, 182
Self-avoiding random walk, 121
Sequential testing, 259
Simulated annealing, 173–174
Skorokhod representation theorem, 315
Smoothing, 350
Socks in the laundry, 89–91
asymptotics, 307
Somatic cell hybrid panels, 360
Sperner’s theorem, 113–114
Splitting entry, 104
Squares of integers, sum of, 322
Starlight intensity, 141
Stationary distribution, see Equilibrium distribution
Stein’s lemma, 40
Stieltjes function
asymptotic expansion, 318
Stirling numbers, 86–89
first kind, 87
second kind, 86
Stirling’s formula, 94, 306
Euler-Maclaurin formula, derived from, 310
Stochastic domination, 178–179
Stochastic simulation, 339–343
Stone-Weierstrass theorem, 407
Stopping time, 255
Stretching of sequence, 402
Strong law of large numbers, 253
Strong stationary time, 162
Subadditive sequence, 114
Sudoku puzzle, 185
Summation by parts, 301, 383
Superadditive sequence, 114
Superposition process, 90
Surface integral, 42
Surrogate function, 64
Symmetric difference, 20
Tauberian lemma, 412
Taylor expansion, 299–300
Temperature, 173
Top-in shuffling, 162
Total variation norm, 159
binomial-Poisson, 180
Chen-Stein method, 355–356
Ehrenfest process, 211
stopping time, 162
Tower property, 8, 248, 250
Transient state, 165
Transition intensity, 188
Transition probabilities, 188
Translation of sequence, 401
Transmission tomography, 131–134
loglikelihood, 146
Traveling salesman problem, 116, 173, 263
Triangle
in random graph, 99
random points on, 100
Turing’s morphogen model, 353
Uniform distribution, 12
  continuous, 3
  discrete, 2
  on surfaces, 43
  products of, 30
  sums of, 34
Uniform process, 89
Uniformization, 199
Urn model, 89–91

Variance, 12
  as inner product, 12
  of a product of independent
    random variables, 22
von Mangoldt function, 379
Von Mises distribution, 319

Waiting time
  insurance claim, 149
  paradox, 130
  train departures, 149
Wald’s identity, 256
Watson’s lemma, 307–308, 412
Weibull distribution, 21
Weierstrass’s approximation theorem, 67
  and coupling, 159
Weighted mean, 68, 73
Wright-Fisher model, 156, 251, 257,
  278, 283
  numerical solution, 347

X-linked disease, 239

Yeast cell reproduction, 239

Zipf’s probability measure, 374