

Index

A

Aberdeen Proving Ground, 643
activity analysis, 236-238, 466
Add Constraint dialog box, 53
adjacent extreme points, 118
affine combination, 514
affine functions, 628
affine independence, 515
affine space, 107, 513-516
aggregation in activity analysis, 237
aggregation in general equilibrium, 463
aggregation in linear programs, 165
aircraft scheduling, 317
Allais, M., 256
angle between two vectors, 546-548
Anstreicher, K., vii
anti-cycling rule, 205-207
arbitrage, 397
arc (*see* directed arc)
Arrow, K., 27
artificial variable, 197, 422, 488
ascending bid auction, 447
assignment problem, 242, 317 (*see also*
Hungarian method)
AT&T, 213, 214
 a business unit, 213, 214
 patents on interior-point methods,
 213
 KORBX, 213, 214

B

base stock model, 251-253
 order up to quantity, 252
 safety stock, 253
 economy of scale, 253
basic feasible tableau, 124

basic solution, 74, 78
basic system, 74-76
basic variable, 71
basis, 78
 as a set of integers, 96
 as a set of variables, 78, 96
 as a set of vectors, 96
 found by Gauss-Jordan elimination,
 95
basis matrix, 370
 inverse of, 371
 Full Rank proviso, 371-373
Baumol, W., 256
Baumol/Tobin model, 256
Beale, E. M. L., 207
Bellman, R. E., 278, 279, 642
best response, 459, 474, 483, 510
bi-matrix game, 472, 473, 479
 almost complementary basis, 489
 artificial variable, 488
 best response, 483
 complementary basis, 492, 496
 complementary pivots, 487-492
 complementary solutions, 486
 complementary variables, 486
 dominant strategies, 481
 empathy, 483
 equilibrium, 481, 487, 496
 mansion, 492 (*see also* mansion)
 nondegeneracy hypothesis, 492
 randomized strategies, 481, 483, 484
 with side payments, 501-503
binding constraints, 141, 160, 623
binomial random variable, 245
 normal approximation to, 245
Bixby, R., 62

- Bland, R., 206, 215
 Bland's rule, 206, 441
 Bolzano, B., 550
 Bolzano-Weierstrass theorem, 551
 boundary, 561, 595
 relative, 610
 bounded feasible region, 118
 bounded linear program, 118
 bounded set of vectors, 549
 branch and bound, 427-435
 dual simplex pivot, 431-435
 incumbent, 429
 tree, 430
 Brouwer, L. E. J., 507, 537
 Brouwer's fixed-point theorem, 22, 462, 480, 508, 509
 computational issue in, 536
 fixed point, 535
 for n -person games, 509
 monotone labels, 534
 on a closed bounded convex set, 536, 537
 on a simplex, 535, 536
 Brown, D. J., vii
- C**
- California State University, Northridge, 642
 canonical form, 110, 622
 Carathéodory's theorem, 349, 350
 cash management, 253, 259
 chain, 298
 Charnes, A., 19, 28, 205, 215, 217, 392
 Chvátal, V., 124
 Clapton, Eric, 176, 189
 closed subset of \mathfrak{R}^n , 549
 column generation, 369
 complementary slackness, 388, 389, 620, 623
 in basic tableaus, 388
 in optimal solutions, 389, 630
 concave function, 584
 constraint, 4, 622
 binding, 141, 623
 nonbinding, 141, 623
 constraint qualification, 625-629
 essentiality of, 625-627
 global optimum, 634
 Hypotheses, 628, 637-639
 local optimum, 641
 necessity, 631-634, 640
 Slater conditions, 629, 654-656
 sufficiency, 630, 640
 consumers in an economy, 463
 continuous function, 549
 continuously differentiable function, 577
 contribution, 155
 convergent sequence of vectors, 548
 convex cone, 552-557
 non-polyhedral, 554
 polar, 554
 polyhedral, 553
 convex function, 582
 and decreasing marginal revenue, 584
 and increasing marginal cost, 584
 chords of, 583, 585-588
 composites of, 591, 592
 continuity of, 595-598
 epigraph of, 590
 on relative interior, 608-611
 once differentiable, 584, 591
 partial derivatives of, 606-608
 support of, 601-606
 twice differentiable, 584, 591
 unidirectional derivatives of, 595-601
 convex nonlinear program, 644
 convex set, 86, 589, 590, 630
 boundary of, 561
 Cooper, W. W., 19, 28, 392
 Cowles Foundation, 643
 CPM (*see* critical path method)
 critical path method, 281-289
 crashing in, 288, 292
 critical task and path, 286
 with workforce allocation, 661
 cross-over table, 384, 635-637
 Crusoe, Robinson, 175, 190

- current tableau, 357
 - multipliers for, 360
 - updating, 362
- cutting plane method, 435-440
 - cutting plane, 436-439
 - dual simplex pivots, 437, 438
 - strong cut, 438
- cycle, 271
- cycling, 134, 203-207
 - avoided by Bland's rule, 206
 - avoided by perturbation, 205
 - with Rule A, 205
- D**
- Dantzig, G. B., 21, 26, 27, 178, 183, 206, 207, 215, 238, 367, 462, 516
- data envelopment, 392-397
- decision variable, 5
- decreasing marginal benefit, 13, 184
- decreasing marginal cost, 235-240
 - and binary variables, 236
 - and integer programs, 236
- degenerate pivot, 133
- Denardo, E. V., 282, 369, 661
- derivative, 566, 569
- descending price auction, 448
- detached coefficient tableau, 78, 79
- Dialog box in Solver (*see* Solver dialog box)
- Dialog box in Premium Solver (*see* Premium Solver)
- dictionary, 74, 124, 650
- diet, 407
- differentiable function
 - at a point, 566, 569, 570
 - linear approximation to, 569
 - on a set, 566
- differentiability,
 - of convex functions, 606
- Dijkstra, E. W., 281
- Dijkstra's method, 281
- Dikin, I., 213
- directed arc, 270
 - forward and reverse orientations, 298
 - head and tail, 271, 298
 - length, 271
- directed network, 270
 - acyclic, 271
 - cyclic, 271
- directional derivative (*see* bidirectional derivative)
- Doig, A. G., 435
- Dorfman, R., 154
- dot product, 546
- dual linear program, 379
 - complementary constraints, 383-385
 - complementary variables, 383-385
 - cross-over table for, 383
 - recipe for, 383-387
- dual simplex method, 414-419
 - Bland's rule for, 441
 - cycling in, 441
 - relation to the simplex method, 419
- dual simplex pivot, 416
 - in branch-and-bound, 431-435
 - in parametric self-dual method, 422
 - in the cutting plane method, 437, 438
- duality, 22, 23, 179-183
 - for linear programs, 381
 - for closed convex cones, 558
 - from Farkas, 563, 564
 - in general equilibrium, 470
- Dutch auction, 448
- Dylan, Bob, 176, 189
- dynamic program, 274
 - embedding, 273
 - functional equation, 278
 - linking, 274
 - optimal policy, 276
 - optimality equation, 274
 - policy for, 276
 - principle of optimality, 276-278
 - solved by LP, 275
 - solved by reaching, 280, 281
 - solved by backwards optimization, 283-285
 - solved by forwards optimization, 287
 - states of, 273

E

Eaves, C., 538
economy, 463
 agents, 463
 consumers and producers, 463
 consumers' equilibrium, 468
 endowments, 463
 general equilibrium, 464, 470
 goods and technologies, 463
 market clearing, 466, 468
 producers' equilibrium, 467
edge, 117, 186
elementary row operations, 82
ellipsoid method, 212
English auction, 447
EOQ model, 253-256
 economy of scale, 255
 flat bottom, 256
 opportunity cost, 253
 the EOQ, 254
EOQ model with uncertain demand,
 256-260
 backorders, 257
 cycle stock, 258
 reorder point, 258
 reorder quantity, 258
 safety stock, 258
 with constant resupply intervals,
 263, 264
epigraph, 589-590
evolutionary Solver, 60-62, 241, 251
Excel, 33-65
 circular reference in, 46, 47
 for PCs, 34
 for Macs, 34
 formula bar, 37
Excel Add-Ins, 50
 Solver, 50-56
 Premium Solver, 50, 56-59
 OP_TOOLS, 02, 37
Excel array, 37
Excel array functions, 44-46
 matrix multiplication, 45, 46
 pivot, 62, 63

Excel cell, 35
 absolute address, of 37, 38
 entering functions in, 36
 entering numbers in, 35
 fill handle of, 35
 relative address of, 37, 38
 selecting an, 35
Excel commands
 copy and paste, 38
 drag, 43, 44
 format cells, 36, 37
Excel functions, 36
 ABS, 62
 error, 61
 ISSERROR, 646
 LN, 61
 MIN, 62
 MMULT, 339
 NL, 62, 248
 OFFSET, 241, 284
 SUMPRODUCT, 42, 43, 48, 49
Excel Solver Add-In, 50-56, 62-64
Excel 2008 (for MACs only) 34, 50
exchange operations, 81
extreme points, 117, 516
extreme value theorem, 551, 558

F

Farkas, G., 390, 557, 558
Farkas's lemma, 390-392
feasible basis, 123
feasible pivot, 127, 133
feasible region, 115
 bounded, 118
 edge of, 117
 extreme point of, 117
feasible solution, 114
Feinberg, E. A., 369
Ferraro, P., 176, 189
Fiacco, T., 213
Final Jeopardy, 477
financial economics, 397-404
 arbitrage opportunity, 399
 no-arbitrage tenet, 397

- risk-free asset, 397
- risk-neutral probability distribution, 403
- fixed cost, 155
- fixed point, 508
- Form 1, 119, 332
- Form 2, 208, 209
- Fox, B. L., 282
- free variables, 208
- Fulkerson prize, 212
- full rank proviso, 136, 344
- functional equation (*see* optimality equation)

- G**
- Gale, D., 27, 450
- game, 445
 - best response, 447, 459
 - dominant strategy, 446, 448, 455, 473
 - equilibrium strategies, 446, 460, 473
 - solution concepts, 446
 - stable strategies, 446, 449-454
 - win-win, 446
 - zero-sum, 446
- game theory (*see* game)
- Gaussian elimination, 98-103
 - back-substitution in, 101
 - lower pivots in, 98
 - small pivot elements, 103
 - sparsity, 102
- Gaussian operations, 68, 69
 - exchange, 353
 - with the pivot function, 80, 81
- Gauss-Jordan elimination, 75, 332
 - identical columns in, 76-78
 - work of, 75-77
- Gay, D., 211
- general equilibrium, 23, 446, 470, 513
 - budget constraint, 468
 - consumer's equilibrium, 468
 - market clearing, 468
 - producers' equilibrium, 467
 - production capacities, 476
 - via LP duality, 470
 - with decreasing marginal return, 472
 - with multiple consumers, 4727
- Generalized Reduced Gradient method (*see* GRG method)
- geometric mean, 613
- Gödel prize, 212
- Gomory, R. E., 439
- gradient of a function, 570
 - as direction of increase, 571, 572
 - as rate of change, 571
 - as vector of partial derivatives, 574
- GRG method, 643-654
 - improving direction, 646, 650
 - line search in, 646
 - local optimum, 644
 - pivots in, 651
 - reduced gradient, 653
 - the KKT conditions, 644
 - with constraints, 649-654
 - zigzagging in, 647-649
- GRG Solver, 251, 260, 643-646
 - aiming for a local optimum, 644
 - for a convex NLP, 644
 - starting close, 644
 - with continuous derivatives, 644
 - with continuous functions, 645
 - with Excel's ISERROR function, 646
 - with the multi-start feature, 645
- Gu, Zonghau, 62
- Gurobi software, 62

- H**
- Hansen, T., 538
- Harris, F. W., 256
- Hessian, 594
- Hoffman, A., 206, 207, 661
- Hölder's inequality, 613
- homogeneous system, 94
- homotopy, 421
- Howson, J. T., 500, 524, 538
- Hungarian method, 318-324
 - incremental shipment, 323
 - partial shipping plan, 320
 - reachable network, 320
 - revised shipping costs, 319, 324
 - speed of, 324
- hyperplane, 559

I

identity matrix, 333
 inconsistent equation, 72
 increasing marginal cost, 15
 inequality constraint, 4
 binding, 160
 nonbinding, 160
 infeasible linear program, 381
 initial tableau, 357
 Institute for Advanced Study, 462
 integer linear program (*see* integer program)
 integer nonlinear program, 240
 integer program 11, 236-240, 427
 binary variables in, 238
 mixed, 439
 no shadow prices for, 239
 pure, 435
 interior, 594, 595, 608
 interior-point methods, 212
 interval, 85
 invertible matrix, 345
 characterization of, 347
 computation of inverse, 46
 iso-profit line, 115

J

Jensen, J., 589
 Jensen's inequality, 588, 589
 John, F., 642

K

Kachian, L. G., 212
 Kantorovich, L. V., 25, 178
 Karmarkar, N., 212, 213
 Karush, W., 623, 641, 642
 Karush-Kuhn-Tucker conditions, (*see* KKT conditions)
 KKT conditions, 623, 635-637, 651
 constraint qualification, 625 (*see also* constraint qualification)
 cross-over table and, 635-637
 interpretation of, 625-627
 Klee, V., 211, 216
 Koopmans, T. C., 27, 178, 238, 468
 Kuhn, H. 27, 318, 538, 623, 642

Kuhn-Tucker conditions, 641 (*see also* KKT conditions)

L

Lagrange multiplier, 162, 178, 621, 623
 Land, A.H., 435
 Lemke, C., 419, 500, 524, 538, 539
 length of a vector, 546
 Leontief, W., 238
 lexicographic rule, 215, 218
 limit point, 548
 line, 86
 linear combination, 514
 linear constraint, 4
 linear expression, 4
 linear fractional program, 19
 linear independence, 515
 linear program, 4
 absolute value objective, 16
 bounded, 118
 bounded feasible region, 9
 feasible, 8
 feasible solution, 5
 feasible region, 5
 Form 1, 119
 Form 2, 208
 infeasible, 8
 maximin objective, 12
 minimax objective, 12
 optimal solution, 7
 optimal value, 7
 ratio constraint, 18
 standard format for, 158
 unbounded, 8, 119
 unintended option, 15
 linear program as a model, 165-167
 linear programming, 5
 load curve for electricity demand, 248
 longest path problem, 272
 loop, 298
 LP relaxation, 428

M

MacKensie, T., 661
 Manhattan Project, 642
 mansion, 492, 523

- blue rooms, 492, 500, 523
- doors of, 493, 523
- doors to outside, 494, 498, 523
- green rooms, 492, 523
- labels on doors, 494
- path to blue room, 523
- marginal benefit, 23 (*see also* reduced cost)
- marginal profit, 125 (*see also* reduced cost)
- Markov decision model, 290
- Markowitz, H., 17, 235
- marriage problem, 453, 454
 - best strategies for men, 453
 - best strategies for women, 453
 - solution by DAP/M, 451
 - solution by DAP/W, 453
 - stable solutions to, 452
- matching, 450
- matrix, 89-93
 - column and row rank, 335
 - column space, 331
 - inverse, 345
 - multiplication, 90, 332
 - permutation, 346
 - rank, 97, 344
 - row space, 331
 - transpose of, 93
- matrix game, 455-462
 - an historic conversation, 462
 - constant sum, 505
 - duality in, 460
 - equilibrium for, 460
 - maximin formulation, 459
 - minimax formulation, 460
 - minimax theorem for, 462
 - randomized strategy in, 456-462
 - value of, 455
 - zero-sum, 455
- McCormick, G., 213
- mean value theorem, 568
- Mellon, B. 28
- Merrill, O. H., 538
- Minty, G. J., 211, 216
- Morgenstern, O., 513
- Moore's law, 28
- multipliers, 173, 178, 360, 621 (*see also* shadow price)
 - as break-even prices, 363-367
 - as shadow prices, 365, 366
 - in current tableau, 360
 - in the simplex method, 367
 - updating, 362
- Muzino, S., 214
- N**
- Nash, J., 513
- Nash equilibrium, 446, 513 (*see also* equilibrium)
- Nautilus submarine, 289
- neighborhood, 548, 595, 608
- network (*see* directed network)
- network flow model, 234-236, 300-304
 - integer-valued data, 235, 304
 - integrality theorem, 225, 304
 - solved by the simplex method, 306
 - unseen node in, 300
- New York University, 643
- Nobel Prize, 27, 178, 235, 238, 448, 513
- nonbinding constraint, 623
- nondecreasing function, 590
- nondegenerate pivot, 133
- nonlinear program, 11, 621, 622
 - binding constraint, 623
 - convex, 644
 - feasible region, 621
 - feasible solution, 621
 - global optimum, 622, 634
 - KKT conditions, 623 (*see also* KKT conditions)
 - local optimum, 622
 - nonbinding constraint, 623
- norm of a vector, 546
- normal loss function, 247, 250
- normal random variable, 245-248
 - sum of, 246
- O**
- objective value, 7
- objective vector, 116

one-sided directional derivative
 (see unidirectional derivative)
 open halfspace, 559
 open set, 548
 opportunity cost, 23, 173-179
 and marginal benefit, 175
 difficulties with, 176-178
 opposite columns, 106
 optimal solution, 116
 optimal value, 116
 optimality conditions
 for a linear program, 620
 for a nonlinear program, 630-635
 optimization and computation with
 evolutionary software, 62
 LP quadratic software, 60
 GRG nonlinear software, 62
 Gurobi software, 62
 Orchard-Hayes, W., 367

P

parametric self-dual method, 419-427
 as a homotopy, 421
 dual simplex pivots in, 422
 simplex pivots in, 423
 partial derivative, 574
 as an entry in the gradient, 574
 continuous, 575-577
 path, 271
 path following method, 214
 path length, 272
 as longest arc length, 291, 292
 as sum of arc lengths, 272
 as sum of node lengths, 286
 PERT, 289
 perturbation theorem, 166
 perturbed RHS values, 142
 optimal basis and, 144
 shadow prices for, 142
 petroleum industry, 28, 224
 Phase I, 123, 196-203
 fast start, 203
 for infeasible LP, 202
 simplex pivot, 200
 simplex tableau, 199

Phase II, 123
 pivot, 69, 70
 admissible, 357
 feasible, 127, 133
 pivot matrix, 335-342, 361, 362
 portfolio, 229
 efficient, 230
 efficient frontier in, 231
 risk in, 230 (see also risk)
 Premium Solver, 25, 50-56, 162, 233
 from the ribbon, 233
 from the tools menu, 56-58, 163
 modal or modeless, 58
 primitive set, 527
 border condition, 529
 completely labeled, 529, 533
 distinguished points, 526
 entering facet, 529, 531
 leaving facet, 530, 531
 nondegeneracy hypothesis, 526
 pivot scheme, 532-533
 proper labeling of, 528
 subdivision of simplex by, 526-533
 Princeton University, 641, 642
 principle of optimality, 276-278
 prisoner's dilemma, 472 (see also
 bi-matrix game)
 dominant strategies, 473
 equilibrium, 473
 producers in an economy, 462
 profit, 155 (see also contribution)
 Project SCOOP, 27
 Pulleyblank, W., 661

Q

quadratic function, 592, 593, 614
 convex, 593
 lower pivots, 614
 positive semi-definite, 593

R

Ramo-Wooldrige Corporation, 642
 RAND Corporation, 278, 279
 Random variable, 40-43
 expectation, 41
 mean absolute deviation, 42

- standard deviation, 41
 - variance, 41
 - rank of a matrix, 344
 - reaching, 280-282
 - as Dijkstra's method, 281
 - with buckets and pruning, 282
 - reduced cost, 121, 162
 - allowable increase and decrease, 161
 - differing sign conventions for, 163
 - of free variables, 189
 - reduced gradient, 162, 653
 - redundant constraint, 115
 - relative boundary, 610
 - relative cost, 178 (*see also* reduced cost)
 - relative interior, 610
 - relative neighborhood, 610
 - relative opportunity cost, 168-175
 - and multipliers, 173-175
 - and shadow prices, 172
 - full rank proviso, 172
 - of basic variables, 171
 - of nonbasic variables, 169, 170
 - relaxation, 428
 - Renegar, 214
 - revised simplex method (*see* simplex method with multipliers)
 - Rhodes, E. 392
 - Rickover, Adm. H., 289
 - risk, 234
 - expected downside, 235
 - MAD, 235
 - variance, 235
 - Rockafellar, R. T., 552
 - Rolle, M., 567, 577
 - Rolle's theorem, 567
 - Roth, A. E., 454
 - Rothenberg, E., 62
 - Rothblum, U. vi, 369
 - row space, 93
- S**
- Samuelson, P., 27, 175
 - Scarf, H., vii, 538, 539
 - Schwartz's inequality, 563
 - SEAC (an early computer), 27
 - sealed bid auction (*see* Vickery auction)
 - self-dual homogeneous method, 214
 - self-dual linear program, 409
 - Sensitivity Report, 161
 - with Premium Solver, 365, 366
 - with Solver, 366, 367
 - separating hyperplane, 559-561, 563
 - shadow price, 23, 137, 161
 - allowable increase and decrease, 139, 161
 - as a break-even price, 140, 162
 - differing sign conventions for, 163
 - large changes, 183, 184
 - most favorable, 184
 - sign of, 140, 141
 - Shapley, L., 450
 - shortest path problem, 272
 - Simon, H., 27
 - simple cycle, 271
 - simple loop, 299
 - simplex, 516-518
 - face of, 517
 - facet of, 517
 - unit, 519
 - vertex of, 517
 - simplex method, 123-132, 516
 - anti-cycling rules, 205-207
 - cycling, 203
 - economic interpretation, 140
 - integer-valued optima, 215, 304
 - Phase I, 196
 - Phase II, 123
 - speed of, 210-215
 - simplex method with multipliers, 367
 - column generation in, 369
 - lower pivots in, 368
 - product form of inverse in, 368
 - simplex pivot, 123-132
 - entering variable, 127
 - feasibility of, 127
 - leaving variable, 128
 - pivot row, 128
 - ratio, 127
 - Rule #1, 128

- simplex tableau, 124
 - degenerate and nondegenerate, 133
 - optimality condition, 131, 132
 - shadow prices, 137
 - unboundedness condition, 136
 - simplicial subdivision, 518-526 (*see also* primitive sets)
 - border condition, 522
 - completely labeled subsimplex, 523
 - in 4-space, 524-526
 - labeling vertices of, 522
 - mansion, 523 (*see also* mansion)
 - Slater, M., 629, 643
 - Slater conditions, 629, 643, 654-656
 - necessity of, 656
 - nondifferentiability, 654
 - sufficiency of, 655
 - Solow, 27
 - Solver, 25, 50-56, 156-162
 - installing and activating, 50-52
 - repeated use of, 232
 - SolverSensitivity Report, 161, 166, 175
 - Solver dialog box
 - in Excel 2007 and earlier, 52-54
 - in Excel 2010 and later, 54-56
 - Sotomayor, O., 454
 - spanning tree, 299
 - speed of the simplex method, 210-215
 - atypical behavior, 211
 - expected behavior, 211
 - Klee-Minty examples, 211, 216
 - typical behavior, 210, 211
 - Sperner, E., 538
 - Sperner's lemma, 538
 - Spielman, D., 212
 - standard format for linear systems, 49
 - standard format for linear programs, 158
 - stationary independent increments, 257
 - strict inequalities
 - in data envelopment, 397
 - in financial economics, 403
 - in strong complementary slackness, 404
 - via Farkas's lemma, 391
 - strong complementary slackness, 404-406
 - strong duality, 381, 382
 - Strum, J., 124
 - supporting hyperplane theorem, 562
 - Swersey, A. J., vii
 - Systems Development Corporation, 642
- T**
- tailored spreadsheet, 223
 - Takayama, A., 642
 - Talman, D., 539
 - Tang, S.-H., 212
 - Taylor, L., 176, 189
 - theorem of the alternative, 347 (*see also* Farkas)
 - for closed convex cones, 555
 - for data envelopment, 392, 396, 397
 - for linear systems, 348
 - for nonnegative solutions, 391
 - in financial economics, 401
 - recipe for, 391
 - Tobin, J., 256
 - Todd, M. J., 214
 - transportation problem, 306-318
 - basis as spanning tree, 311
 - degeneracy in, 316
 - demand nodes in, 307
 - dummy demand node in, 308
 - entering variable in, 314
 - Hungarian method for (*see* Hungarian method)
 - leaving variable, 315
 - loop, 314
 - multipliers for, 312, 313
 - northwest corner rule for, 309
 - simplex pivots in, 310-318
 - supply nodes in, 307
 - worst-case behavior, 318
 - traveling salesperson problem, 240-244, 265
 - an assignment problem with side constraints, 242
 - evolutionary Solver for, 241
 - optimal solution to, 244

subtour, 243
 subtour elimination constraint, 243
 trite equation, 72
 tree, 271
 from a node, 271
 to a node, 271
 TRW Corporation 642
 two-person game (*see* bi-matrix game)
 equilibrium of, 510
 stable distributions for, 512
 two-sided directional derivative (*see*
 bidirectional derivative)
 two-sided market, 449
 matching in a, 449-454
 medical
 Tucker, A. W., 27, 623, 641

U

unidirectional derivative, 573
 unit simplex, 519
 UNIVAC I (an early computer), 27
 University of Chicago, 27, 235, 642
 University of Kentucky, 643

V

Vanderbei, R., 211
 Van der Heyden, L. vii
 variable cost, 155
 vectors, 83-87
 addition of, 83
 convex combination of, 85,

 linear combination of, 88
 linear independence of, 88
 linearly dependent, 89
 scalar multiplication of, 83
 vector space, 87, 513
 basis for, 89
 dimension of, 98, 335
 Vickrey, W., 448
 Vickrey auction, 448
 dominant strategy in, 448
 reservation price in, 448
 Vickrey, W, 448
 von Neumann, J., 13, 24, 455, 462
 von Neumann Prize, 27
 von Wieser, F., 175

W

Wagner, H. M., vii
 Walras, L, 468
 weak duality, 379-381
 Weierstrass, K., 550
 Wilson, C. E., 279
 Wilson, R.W., 256

Y

Yale University, vii
 Ye. Y., 214

Z

Zadeh, N., 318