Appendix I: Supplementary Homework and Practice Problems

I.1 Word Problems

Chapters 1–4

Exercise I.1. How many possible initials can be formed if each person has two given names and a last name? What if each person has at most two given names and a last name?

Exercise I.2. Three numbers are chosen at random with replacement from $0, 1, \ldots, 9$. Find the probabilities that the three are alike, that the three are distinct, and that exactly two are alike.

Exercise I.3. A, B, C, D are four independent events, each with probability .5. Find the probability that at least two of the four events occur.

Exercise I.4. The birthdays of five random people are known to fall in exactly three calendar months. Find the probability that exactly two of the five were born in January. State your assumptions.

Exercise I.5. There are two good bulbs and two bad bulbs in a package. These will be tested one by one in a random order. Find the probabilities that the second bad bulb is the second bulb tested, the third bulb tested, and the fourth bulb tested.

Exercise I.6. Suppose $B_1, B_2, \ldots$ are infinitely many events, and let B be their union. An event A is independent of each individual B_i. Prove, or give a counterexample, that A and B are independent.

Exercise I.7. A man seeks advice from three oracles on whether or not to accept a particular job offer. He acts according to the advice of the majority. The three oracles have probabilities .95, .9, .95 of giving the correct advice. Find the probability that the man will take the correct action. State your assumptions.

Exercise I.8. Three people, say A, B, C, take turns rolling a fair die. A rolls first, then B, and then C. The first to roll a five wins. Find the probabilities of each player winning.

Exercise I.9.
(a) A six-sided die is manipulated in such a way that the face with the number i has probability proportional to i. Find the probability that the die will produce an even number if it is rolled once.
(b) An n-sided die is manipulated in such a way that the face with the number i has probability proportional to i. Find the probability that the die will produce an even number if it is rolled once.

Exercise I.10. There are ten black, ten white, and ten blue balls in an urn. Ten of these are chosen at random without replacement. Find the probability that there is at least one ball of each color among the ten drawn.

Exercise I.11. There are ten black, ten white, and ten blue balls in an urn. Ten of these are chosen at random without replacement. Find the probability that the first blue ball is drawn on the sixth draw.

Exercise I.12. There are ten black, ten white, and ten blue balls in an urn. Ten of these are chosen at random without replacement. Find the probability that the second ball drawn is blue if the third ball drawn is known to be blue.

Exercise I.13. Box 1 has two good bulbs and two bad ones; box 2 has three good bulbs and two bad ones. One bulb is chosen at random from box 1 and transferred to box 2. Then, one bulb is chosen at random from box 2. It is found to be a good bulb. What is the probability that the bulb from box 1 that was transferred to box 2 was a good bulb?

Exercise I.14. Find the probability that a hand in bridge has four cards of one suit and three cards each of three other suits.

Exercise I.15. Which is more likely: that a bridge hand will contain one card of each denomination or that it will contain cards of only two suits?

Exercise I.16. An urn contains four black, four white, and four blue balls. Three balls are drawn at random from the urn. Is it more likely that the balls will all be of the same color if sampling is with replacement or without replacement?

Exercise I.17. Find the probability that a randomly selected bridge hand will be void in at least one suit.

Exercise I.18. Find the probability that a randomly selected bridge hand will contain exactly five cards of at least one suit.

Exercise I.19. Cards are taken out one at a time from a well-shuffled deck. What is the probability that it will take at least five and at most ten draws to take out the first club?

Exercise I.20. A fair coin is tossed six times. Given that there are at least three heads, what is the probability that there are exactly four heads?

Exercise I.21. A fair die is rolled 12 times. Given that there are exactly two ones, what is the probability that there are exactly two sixes?

Exercise I.22. A fair die is rolled twice. Compute the probability that the sum of the two rolls is 3, 5, 7, 9, 11, respectively, given that the sum is odd.

Exercise I.23. A fair die was rolled four times. The faces 1 and 2 never appeared. What is the probability that the other four faces each appeared exactly once?

Exercise I.24. Jeff, Jen, and Cathy shoot at a bull's eye. They can hit the bull's eye 70%, 80%, and 75% of the time, respectively. One of the three is known to have hit the bull's eye. Find the probability that it was Jen.

Exercise I.25. A fair coin is tossed ten times. Given that at least seven heads were obtained, what is the probability that the first toss was a head? That at least one of the first two tosses was a head? That the first two tosses were both heads?

Exercise I.26. From a town of 25 Republicans and 25 Democrats, pollsters A and B each sampled ten residents at random without replacement. Find the probability that the two polls contained exactly the same number of Republicans.

Exercise I.27. A, B, C are three events. If A is independent of B given C and if C is independent of B, are A and B independent events? Prove or give a counterexample.

Exercise I.28. A library patron has decided to try five libraries for a particular book. Each library has a 50% chance of having the book, and if a library has the book, there is a 20% chance that it will be checked out. Find the probability that the patron can find the book. State your assumptions.

Exercise I.29. An urn has two white and three green balls. A number is selected at random from $1, 2, 3, 4, 5$, and then that many balls are taken out from the urn. Find the probability that they are all green.

Exercise I.30. An urn has five white and five green balls. Five balls are drawn at random without replacement. Find the probability that in each odd-numbered draw a green ball is drawn.

Exercise I.31. On a table, there are two dice. One is a fair die, and the faces of the other die are 1, 1, 2, 2, 6, 6. One die is selected at random and rolled, and it gives a six. What is the probability that it was the fair die?

Exercise I.32. A number is chosen at random from $1, 2, \ldots, 200$. Find the probability that it is even if it is not divisible by 7.

Exercise I.33. Team X plays against team Y in a best of seven series. In each game, team X has a 70% chance of winning, and assume that the games are independent. Find the probabilities that X wins, that X wins within five games, and that the series ends within five games.

Exercise I.34. A, B, C are three pairwise independent events. Also, A and $B \cap C^c$ are independent. Show that A, B, C are mutually independent.

Exercise I.35. Suppose a discrete random variable X has the distribution $P(X = n) = 2^{-n}, n \geq 1$.

(a) Find the mean of X.
(b) Find all medians of X.
(c) Find the variance of X.
(d) Find $P(|X - \mu| \geq 2\sigma)$, and compare it with the bound of Chebyshev's inequality.

Exercise I.36. Suppose X has a finite variance. Does $|X|$ have the same, a smaller, or a larger variance than X?

Exercise I.37. Suppose a discrete random variable X has a distribution such that $P(X > n + 1 | X > n) = \frac{n+1}{n+2}$ for all $n \geq 1$. Find the probability mass function of X.

Exercise I.38. Cards are drawn one at a time, without replacement, from a deck of 52 cards until the first club card is obtained. Let X be the number of draws required.

(a) Find the mass function of X.
(b) Find the mean of X.

Exercise I.39. X is uniformly distributed on $\{1, 2, \ldots, n\}$, and Y is uniformly distributed on $\{2, 4, \ldots, 2n\}$; X and Y are independent variables.

(a) Find the variance of $X + Y$.
(b) Find the variance of XY.
(c) Find $P(Y > X)$.

Exercise I.40. Given positive numbers M, ϵ, show a random variable X such that $\sigma^2 \geq M$ but $P(|X - \mu| > .01) < \epsilon$.

Exercise I.41. Suppose X has a positive mean μ and that $E(X^2)$ is also equal to μ. Prove that $\text{Var}(X) \leq \frac{1}{4}$.

Exercise I.42. X takes the values $1, 2, 3, 4$, and we know that $P(X = 1) = P(X = 2) = 2P(X = 3) = 3P(X = 4)$. Find the distribution of X.

Exercise I.43. Three fair dice are continually rolled until a sum of 15 on the three dice is obtained. Find the expected number of times the three dice would have to be rolled.

Exercise I.44. Four distinguishable balls are distributed independently at random into three distinguishable cells. Let X be the number of balls that land in the first cell, Y the number of balls that land in the second cell, and Z the number of cells that remain empty.

(a) Find $E(X)$ and $E(Y)$.
(b) Find $\text{Var}(X)$ and $\text{Var}(Y)$.
(c) Find $P(Z = 0)$ and $P(Z = 2)$.
(d) Find $E(Z)$.

Exercise I.45. A fair die is rolled six times. Let X be the sum of the first four rolls and Y the sum of the last four rolls. Find the variance of $X - Y$.

Exercise I.46. A fair die is rolled six times, and let $X_1, X_2, \dots, X_6$ be the six rolls obtained, respectively. Find the mean and the variance of $\sum_{i=1}^{6}(-1)^{X_i} X_i$.

Exercise I.47. Twenty-five people will each toss a fair coin 20 times. Let X be the number of people among the 25 people who get exactly ten heads and ten tails. Find the mean and variance of X.

Exercise I.48. Give an example of a random variable such that $E(X) = 1$ and $\text{Var}(X) > 100$.

Exercise I.49. Give an example of a random variable such that $E(X) = 100$ and $\text{Var}(X) = 1$.

Exercise I.50. In bridge, find the expected number of players who receive no aces or no hearts.

Exercise I.51. Coupons are drawn, independently with replacement, from a set of ten coupons. Find the expected number of draws:

(a) until the first coupon drawn is drawn again;
(b) until a duplicate occurs.

Exercise I.52. A fair die is rolled one hundred times. Find the expected number of rolls such that it and the next roll show the same face.

Exercise I.53. A random variable X takes the values $1, 0$ with probabilities p, $1 - p$. It has a variance equal to .16, and we know that $E(X - p)^3 > 0$. Find p.

Exercise I.54. Consider couples that have children until they have a girl. What is the expected proportion of boys in such families?

Chapters 5 and 6

Exercise I.55. A random variable X takes the values $0, \pm 1, \pm 2$ with the equal probability $\frac{1}{5}$. Find the mgf of X and $E(X)$. Verify that $E(X) = \psi'(0)$, ψ being the mgf.

Exercise I.56. Suppose $X \sim Bin(n, p)$, where $p = \frac{1}{2}$. Define Y as $Y = X$ if X is even and zero if X is odd. Find the mgf of Y and hence $E(Y)$.

Exercise I.57. Suppose $X \sim Ber(p)$ and $Y \sim Poi(\lambda)$, and assume that X and Y are independent. Find the mgf of XY.

Exercise I.58. Suppose X has a finite mean μ and a finite variance σ^2, and that its mgf $\psi(t)$ exists in some interval around zero. Show that $\sigma^2 = \xi''(0)$, where $\xi(t) = e^{-t\mu}\psi(t)$.

Exercise I.59. Suppose $X_i \overset{indep.}{\sim} Ber(p_i), i = 1, 2, \ldots, n$. Find the mgf of $X_1 + \cdots + X_n$, and hence the variance of $X_1 + \cdots + X_n$.

Exercise I.60. Suppose X has the mgf $\psi(t) = \cosh t, -\infty < t < \infty$. Find the distribution of X.

Exercise I.61. Suppose X has the mgf $\psi(t) = \frac{\sinh t}{t}$ for $t \ni 0$, and $\psi(0) = 1$. Find the distribution of X.

Exercise I.62. Suppose $X \sim Poi(1)$, and define $X_n = XI_{\{X \leq n\}}, n \geq 1$.

(a) Find $\psi_n(t)$, the mgf of X_n.
(b) Find $\lim_{n\to\infty} \psi_n(t)$.

Exercise I.63. Find the factorial moments of the $Bin(n, p)$ distribution.

Exercise I.64. Find the factorial moments of the $Poi(\lambda)$ distribution.

Exercise I.65. A random variable X has the generating function (pgf) $c + \frac{1}{2}s + \frac{1}{4}s^2 + \frac{1}{8}s^3$ for some c.

(a) Find c.
(b) Find the distribution of X.
(c) Find the mean of X.

Exercise I.66. Find the generating function of a general Poisson distribution.

Exercise I.67. Find the generating function of the $NB(r, p)$ distribution.

Exercise I.68. Is it possible that neither of two random variables X and Y has a finite mgf in any interval around zero but $X + Y$ does in all intervals around zero?

Exercise I.69. Suppose X has a finite mgf in some interval around zero. Does $|X|$ also have a finite mgf in some interval around zero?

Exercise I.70. A binomial random variable has mean 6 and variance 2.4. Evaluate $P(X < 5)$.

Exercise I.71. Harry's experience is that 7% of the parcels he mails do not reach their destination. He has bought two books for 25 dollars apiece and wants to mail them to his brother. If he sends them in one parcel, the postage is 6 dollars, while for separate parcels the postage is 4 dollars for each parcel. To minimize his expected total cost (possible loss of books + postage), should he send one or two parcels?

Exercise I.72. You are promised a reward if you obtain exactly ten heads by tossing a coin. How many times you toss the coin is up to you, but you have to announce this before starting. Assuming the coin is a fair coin, what is the best number of times to toss the coin?

Exercise I.73. Suppose I roll three dice. Those that show a six are rolled again. Let X be the number of resulting sixes. Find the distribution, mean, and variance of X.

Exercise I.74. Printing errors occur on any specific page in a book with probability $.01$. A certain book has 400 pages.

(a) Find the probability that the book has ten or more printing errors.
(b) Find the probability that the first hundred pages are error-free.
(c) Find the probability that page 90 is error-free.
(d) Find the probability that the first error occurs on page 91.
(e) Find the probability that there are exactly three errors in pages 1 to 200 and exactly three errors in pages 201 to 400.

Exercise I.75. A telephone operator receives 25 calls on average per hour. What is the probability that in two consecutive five minute intervals she receives no calls at all?

Exercise I.76. A Poisson random variable has the property that $\psi(0) = \psi(1)$, where ψ denotes its mgf. Find $P(X > 1)$.

Exercise I.77. Peter has a coin that gives heads with probability p in individual tosses. Paul has a coin that gives heads with probability θ in individual tosses. Both toss their coins repeatedly. Let Y be the first toss at which Paul obtains a head and X be the number of heads Peter obtains up to and including the Yth toss. Find the mean of X.

Exercise I.78. Suppose $X \sim Bin(20, .1)$. Compute $P(X \leq k)$ for $k = 1, 2, 3$ exactly and then by using the normal approximation with and without a continuity correction. Compare the approximations with the exact values.

Exercise I.79. Let X be the number of people who will want to buy the daily newspaper from a vendor on a given day.

(a) Suppose $X \sim Poi(\lambda)$ with $\lambda = 10$. If the vendor stocks 14 papers, what is the probability that the demand will exceed the supply?
(b) Suppose $X \sim Bin(n, p)$ with $n = 20$, $p = .5$. If the vendor stocks 14 papers, what is the probability that the demand will exceed the supply?

(c) In each case, find the minimum number of papers the vendor should stock so that the chance that the demand will exceed the supply is at most 5%.

Exercise I.80.

(a) Compute the exact probability that a bridge hand is void in spades.
(b) Compute the exact probability that in one hundred independent plays at least twice a player finds his hand to be void in spades.
(c) Compute the Poisson approximation to the probability above.

Exercise I.81. For each of $p = .05, .1, .25, .4$, find the smallest value of n such that the $Bin(n, p)$ distribution has a skewness $\leq .2$ and kurtosis $\leq .1$.

Exercise I.82. Peter has a coin that gives heads with probability .6 in individual tosses, and Paul has a fair coin. Both toss their coins repeatedly. Let X and Y be the first tosses at which they obtain the first heads, respectively. Find the distribution and the mean of $\max\{X, Y\}$.

Exercise I.83. Suppose X and Y are independent Poisson random variables with means λ, μ. Find the mgf of $X - Y$.

Exercise I.84. Suppose $X_1, X_2, \ldots, X_{10}$ are independent Bernoulli variables with the common parameter p. Find the mgf of $X_1 - X_2 + X_3 - X_4 + \cdots - X_{10}$.

Exercise I.85. Find the first four moments of a Poisson distribution with mean 2.

Exercise I.86. Find the first four moments of a $Bin(10, .5)$ distribution.

Exercise I.87. Suppose $X \sim Bin(10, .5)$. Compute $E(X - 5)^5$.

Exercise I.88. Cards are drawn one by one from a deck of 52 cards.

(a) Compute the expected number of draws necessary to draw the first ace.
(b) Compute the expected number of draws necessary to draw the second ace.

Exercise I.89. Suppose a couple will have children until they have at least one boy and at least one girl, but they will not have more than four children. Compute the expected number of children they will have.

Exercise I.90. A coin with probability p for heads is repeatedly tossed until r heads or r tails are obtained, whichever happens first. Find the mass function of the number of tosses necessary.

Exercise I.91. For $i = 1, 2, \ldots 10$, let X_i be a randomly selected number from $\{1, 2, \ldots, i\}$. Find the expected number of even numbers drawn; i.e., the expected value of the number of X_i that are even.

Exercise I.92. Suppose X and Y are independent Poisson random variables with means λ, μ. Can XY have a Poisson distribution for any λ, μ?

Exercise I.93. Suppose $X_1, X_2, \ldots, X_n$ are n independent random variables. Show that $\text{Var}(X_1 X_2 \ldots X_n) \geq \text{Var}(X_1)\text{Var}(X_2)\ldots\text{Var}(X_n)$.

Exercise I.94. Suppose a random variable X is such that $E(X) = 0, E(X^2) = 1, E(X^6) = 1$. Find and plot the CDF of X.

Exercise I.95. Find all medians of the number of aces in a bridge hand.

Exercise I.96. In a small town of 100 people, there are 90 right-handed and ten left-handed people. If ten tosses of a fair coin produce eight or more heads, a sample of 20 people with replacement will be taken. If the number of heads is less than eight, a sample of ten people without replacement will be taken. Find the expected number of left-handed people in the sample.

Chapters 7–10

Exercise I.97. A density function is verbally described as follows: it is zero for $x < 1$, rises linearly between 1 and 2 to $\frac{1}{3}$, remains constant between 2 and 4, decreases linearly to zero from 4 to 5, and remains zero thereafter.

(a) Plot the density function.
(b) Find the corresponding CDF and plot it.
(c) Find the mean of the distribution.
(d) Find $P(2.5 < X < 4.5)$.

Exercise I.98. A random variable X has the density cx for x between 0 and .5 and $c(1-x)$ for x between .5 and 1.

(a) Find the normalizing constant c.
(b) Let A, B, C be the three events $X < .5, X > .5, .25 < X < .75$. Find $P(A|B)$; $P(C)$; $P(C|A)$; $P(C|A \cap B)$.

Exercise I.99. Suppose we know that the following functions are valid CDFs. Find, for each case, the smallest number M such that $F(M) = 1$.

(a) $F(x) = x^2/4, x \geq 0$.
(b) $F(x) = \log x, x \geq 1$.
(c) $F(x) = \dfrac{1 - \cos ax}{2}, x, a > 0$.

Exercise I.100. Annual rainfall in a desert town is zero with probability .9, and if it rains in some year, then the amount is exponential with mean 2 in. Plot the CDF of the amount of rainfall in this town.

Exercise I.101. The pth quantiles for $p = .1, .2, \ldots, .9$ are called the deciles of a distribution. Compute approximately the deciles of the exponential distribution with mean 1, a Beta distribution with parameters 2 and 1, and a standard normal distribution.

Exercise I.102. Suppose X has the standard double exponential density. Compute each of the following probabilities:

(a) X is a prime number;
(b) X is an irrational number;
(c) $X^3 - X^2 - X - 2 > 0$;
(d) $|X| + |X - 3| > 3$;
(e) $|X|e^{-|X|} > e^{-1}$.

Exercise I.103. Suppose $X \sim U[0, 1]$. Find the density of e^{-X^2}.

Exercise I.104. Suppose $X \sim Exp(1)$. Find the density of $2e^{-X}$.

Exercise I.105. Suppose $X \sim Exp(1)$. Define a function $g(X)$ as $g(X) = X$ if $X < 1$ and $g(X) = \frac{1}{X}$ if $X > 1$. Find the density of $Y = g(X)$.

Exercise I.106. Suppose X has the density $\frac{1}{x^2}$ for $x \geq 1$. Define a function $g(X)$ as $g(X) = 2X$ for $X \leq 2$ and $g(X) = X^2$ for $X > 2$. Find the density of $Y = g(X)$.

Exercise I.107. Suppose $Z \sim U[-1, 1]$ and X takes values ± 1 with probability $\frac{1}{2}$ each. We know that X and Z are independent.

(a) Find the CDF of $Y = ZX$.
(b) Find the density of $Y = ZX$.

Exercise I.108. Household incomes in a town have a Pareto distribution with $\theta = 10$; the value of the α parameter is not explicitly given. We know that the mean income is 40,000 dollars.

(a) Find the value of α.
(b) What percentage of the families earn more than 50,000 dollars?

Exercise I.109. It is known that the shortest interval containing 95% of the total area in a normal distribution is $[2, 8]$. Find:

(a) the mean and variance of this normal distribution;
(b) the 90th percentile of this normal distribution;
(c) the area between 5 and 10 in this normal distribution.

Exercise I.110. Find the shortest interval with probability $\geq$ .5 under the $N(0, 1)$, $U[-1, 1]$, and $C(0, 1)$ distributions, simultaneously.

Exercise I.111. Let $Z \sim N(0, 1)$. Evaluate $P(\Phi(Z)\Phi(-Z) > .1)$, where Φ denotes the standard normal CDF.

Exercise I.112. Suppose X has a normal distribution and $g(X)$ is a strictly increasing nonlinear function of X. Show that $g(X)$ cannot be normally distributed.

Exercise I.113. Suppose X has the Gamma density with parameters $\lambda = 1$, $\alpha = 2$. Find the expectation of the integer part and the fractional part of X.

Exercise I.114. Suppose $X_1, X_2, \ldots, X_n$ are n iid standard exponential variables. Find the mean, median, and variance of the minimum of $X_1, X_2, \ldots, X_n$.

Exercise I.115. Suppose X is uniformly distributed on $[0, 2\pi]$. Find $P(-.5 < \sin X < .5)$.

Exercise I.116. The diameter of a circular disk cut by a machine has the CDF $F(x) = \frac{(x-1)^3}{64}, 1 \leq x \leq 5$. Find the average diameter of disks coming from this machine.

Exercise I.117. Suppose $X \sim C(0, 1)$. Explicitly find a function $g(X)$ such that $Y = g(X) \sim Exp(1)$.

Exercise I.118. Let $f(x) = cx \sin x, 0 < x < \pi$.

(a) Evaluate a c that makes f a density function.
(b) Find the mean of this density function.

Exercise I.119. The waiting time at a teller's window in a bank has the density $f(x) = \frac{1}{3}e^{-x/3}, x > 0$.

(a) Find the average waiting time.
(b) Find the standard deviation of the waiting time.
(c) Find the probability that you have to wait longer than three minutes.
(d) Find a time such that the probability that you have to wait even longer than that time is only 5%.
(e) Find the probability that you have to wait at least three more minutes if you have already waited for three minutes.
(f) Interpret your result in part (e).

Exercise I.120. A square is to be constructed by choosing the common side length to be exponentially distributed with mean one inch. Find the expected area of the square.

Exercise I.121. A circle is to be constructed by choosing the radius of the circle to have the distribution of the absolute value of a standard normal. Find the expected perimeter of the circle.

Exercise I.122. A sphere is to be constructed by choosing the radius of the sphere such that it has a Beta distribution with both parameters equal to 3. Find the expected volume of the sphere.

Exercise I.123. Weights of individuals in some population are normally distributed with a mean of 150 lbs. and a standard deviation of 25 lbs. At least how many people must be sampled from this population if with a 90% probability we want at least one person in our sample who weighs more than 250 lbs.?

Exercise I.124. Suppose $X \sim N(0, 1)$. For what values of a, b, c is $E(e^{aX^2+bX+c}) < \infty$?

Exercise I.125. Suppose $X \sim N(0, 1)$. Find an expression for $P(|X| < 2a \mid |X| > a)$. Plot it as a function of a, and find the minima and the maxima.

Exercise I.126. Suppose $X \sim C(0, 1)$. Find an expression for $P(|X| < 2a \mid |X| > a)$. Plot it as a function of a, and find the minima and the maxima.

Exercise I.127. Explicitly exhibit a density function $f(x)$ whose hazard rate has the bathtub shape; i.e., at first decreasing, then constant, and eventually increasing.

Exercise I.128. Suppose a positive continuous random variable has a finite mean. Write an expression for the mean in terms of the hazard rate function of the random variable.

Exercise I.129. $X_1, X_2, \ldots, X_{10}$ are ten iid $U[0, 1]$ variables. Let m denote their minimum and M their maximum. Find $P(.05 < m < M < .95)$.

Exercise I.130. $X_1, X_2, \ldots, X_{10}$ are ten iid $N(0, 1)$ variables. Let m denote their minimum and M their maximum. Find $P(-2 < m < M < 2)$.

Exercise I.131. Suppose X has a lognormal distribution with parameters $\mu = 0$, $\sigma = 1$. Find the deciles of X.

Exercise I.132. Suppose $X_1, X_2, \ldots, X_n$ are n independent lognormal variables. What is the name of the distribution of their product $X_1 X_2 \ldots X_n$?

Exercise I.133. The 25th, 50th, and 75th percentiles of a distribution are -1, 0, and 1.5. Can this be a normal distribution?

Exercise I.134. The 10th, 90th, and 95th percentiles of a distribution are 2, 5, and 8. Can this be a normal distribution?

Exercise I.135. Suppose we want to construct a confidence interval for a normal mean assuming that the variance σ^2 is known. What is the minimum n required for the margin of error of the confidence interval to be at most .1 if we want a 90% confidence interval? A 95% confidence interval? A 99% confidence interval? A 99.99% confidence interval?

Exercise I.136. Weights of adult males in some population are normally distributed with mean 160 lbs. and standard deviation 30 lbs. Weights of adult females in the same population are normally distributed with mean 130 lbs. and standard deviation 25 lbs. Find the probability that the weights of one randomly selected male and one randomly selected female differ by more than 50 lbs.

Exercise I.137. Suppose $Z \sim N(0, 1)$. Find the mean, median, and mode of Z^5, $|Z|$, and $|Z - 1|$.

Exercise I.138. A fair die is tossed 100 times. Approximate the probability that the sum of the rolls is between 300 and 400 inclusive. Next, suppose a fair die is tossed 1000 times. Approximate the probability that the sum of the rolls is between 3400 and 3600.

Exercise I.139. $X_1, X_2, \ldots, X_n$ are iid from a density $f(x)$ that equals $\frac{1}{3}$ on $[-1, 0]$ and equals $\frac{2}{3}$ on $[0, 1]$. Sketch the approximate density of $X_1 + X_2 + \cdots + X_n$ for $n = 50$.

Exercise I.140. Shipments of some equipment to a factory come in boxes of 1000 items. From past experience, the factory knows that (about) 1% of the items are defective. It returns a shipment if a sample of 50 items from the box contains two or more defective items.

(a) Approximate the probability that a shipment will be rejected.
(b) Suppose that on one occasion a bad shipment arrived with 5% defective items. Approximate the probability that the shipment will be rejected.

Exercise I.141. In approximately how many tosses of a fair coin is the probability of getting more than 52% heads at most .01?

Exercise I.142. In approximately how many tosses of a fair coin is the probability of getting more than 52% or less than 48% heads at most .01?

Exercise I.143. A certain congenital birth defect is found in some geographic region at the average rate of one a year. Approximate the probability that 60 or more people with this birth defect will be found in the next 50 years. State your assumptions.

Exercise I.144. A random variable X has the density $\frac{1}{x^2}$ for $x \geq 1$ and zero otherwise. An iid sample of size 100 is available from this density. Can we use a normal approximation to approximate the distribution of their sum? If so, sketch such an approximate normal density. If not, explain why we cannot do a normal approximation here.

Exercise I.145. A gambler repeatedly plays a game in which his earnings are iid $U[0, 1]$ in dollars. After each play, he tips the manager an amount equal to the square of the amount he just won. Approximate the probability that if he plays and tips 600 times, then his total winnings minus his total tip will exceed 105 dollars.

Exercise I.146. Suppose $X_1, X_2, \ldots, X_n$ are iid standard exponentials. For $n = 8$, sketch the exact density, the CLT approximation, and the first-order Edgeworth approximation for the density of their sum.

Exercise I.147. Suppose $X \sim Bin(n, p)$. For $n = 50, 100, 250$, plot the Berry-Esseen bound, as given in the text, as a function of p. Identify the peak value in the plot for each n.

Chapters 11–13

Exercise I.148. A fair die is rolled twice, and X and Y are the two rolls.

(a) Write the joint mass function of $X + Y$ and $\frac{X}{X+Y}$.
(b) From this, find the marginal pmf of $\frac{X}{X+Y}$.
(c) From this, find $E(\frac{X}{X+Y})$. Was the answer obvious to begin with?
(d) Find the conditional expectation of $\frac{X}{X+Y}$ given $X + Y = t, t = 2, 3, \ldots, 12$.
(e) By inspecting the numerical values in part (d), write a formula for $E(\frac{X}{X+Y} \,|X + Y = t)$. Was the answer obvious to begin with?

Exercise I.149. A fair coin is tossed 20 times. Let X be the number of heads in the first 15 tosses and Y the number of heads in the last 15 tosses. Find a formula for $E(Y \,|X = x)$.

Exercise I.150. Suppose X and Y are two random variables such that $E(Y|X) = X$. Assuming that the variances exist, prove that $\text{Var}(Y) \geq \text{Var}(X)$.

Exercise I.151. X, Y, Z have the joint pmf $p(x, y, z) = \frac{1}{8}$ for $x = \pm 1, y = \pm 1, z = \pm 1$.

(a) Find the marginal pmfs of each of X, Y, Z.
(b) Find the joint pmfs of each of $(X, Y), (Y, Z), (X, Z)$.
(c) Find the pairwise correlations between X and Y, Y and Z, and X and Z.
(d) Find the correlation between $X + Y$ and $Y + Z$.

Exercise I.152. A fair coin is tossed n times, and suppose X heads are obtained. Given $X = x$, a Poisson random variable Y with mean x is generated. Here, a Poisson with zero mean is the constant zero.

(a) Find the variance of the marginal distribution of Y.
(b) Evaluate the limit

$$\lim_{n\to\infty} P\left(|Y - \frac{n}{2}| > n^{\frac{3}{4}}\right).$$

Exercise I.153. Midterm grades in a class of 40 students are normally distributed with mean 50 and variance 100. The cutoffs for A, B, C, D are 70, 60, 40, 30, and a grade less than 30 is an F.

By recognizing it as a suitable multinomial distribution problem, calculate the probability that the number of students receiving each of the five letter grades is eight.

Exercise I.154. Suppose X, Y, Z are three independent Poisson variables with means λ, μ, η. Prove that the conditional distribution of (X, Y, Z) given $X + Y + Z = t$ is a trivariate multinomial distribution. Identify all the parameters of this multinomial distribution.

Exercise I.155. Suppose X has a discrete uniform distribution on $\{-n, -n+1, \ldots, 0, 1, \ldots, n-1, n\}$. Find the conditional expectation of X given $X^2 = t$ for a general t.

Exercise I.156. A fair coin is tossed repeatedly until the first head is obtained. Let X be the first toss at which the first head is obtained, and let $Y = \min(X, k)$, for a general $k \geq 1$.

(a) Find $E(Y)$.
(b) Find $E(Y|X = x)$.
(c) Find the correlation between X and Y in as simple a form as you can.
(d) Where does this correlation converge as $k \to \infty$?

Exercise I.157. From an urn with N balls numbered $1, 2, \ldots, N$, two balls are taken out without replacement. Let X, Y denote the numbers on the first ball chosen and the second ball chosen, respectively.

(a) Find $E(X), E(Y)$.
(b) Find $E(Y|X = n), n = 1, 2, \ldots, N$.
(c) Find $\text{Cov}(X, Y)$.
(d) Find the correlation between X and Y as a function of N.
(e) Compute the correlation for $N = 2, 3, 5, 10$.
(f) Find the limit of the correlation as $N \to \infty$. Is the answer what you would intuitively expect?

Exercise I.158. Let X be the number of kings and Y the number of hearts in a hand in bridge. Find the correlation between X and Y.

Exercise I.159. A fair die is rolled three times. Let X, Y, Z be the three individual rolls. Define $U = X, V = \max(X, Y), W = \max(X, Y, Z)$.

(a) Find $P(U = V)$.
(b) Find $P(V = W)$.
(c) Find $P(U = W)$.

Exercise I.160. Suppose (X_1, X_2, X_3) is jointly multinomially distributed with parameter vector (n, p_1, p_2, p_3). By using the joint mgf, find $E(X_1 X_2 X_3)$.

Exercise I.161. Suppose (X_1, X_2, X_3) is jointly multinomially distributed with parameter vector (n, p_1, p_2, p_3). Find the correlation between $X_1 + X_2$ and $X_2 + X_3$.

Exercise I.162. In bridge, find the conditional expectation of the number of aces in the hands of South given that North has k aces in his hand, $k = 0, 1, \ldots, 4$. Does your answer make intuitive sense?

Exercise I.163. Consider the joint density function

$$f(x, y) = cx^2y^2, 0 < x, y; x + y < 1.$$

(a) Find the normalizing constant c.
(b) Find the marginal densities of X, Y.
(c) Prove or disprove that X and Y are independent.

Exercise I.164. Consider again the joint density function

$$f(x, y) = cx^2y^2, 0 < x, y; x + y < 1,$$

as in the problem above.

(a) Find a formula for $E(X \mid Y = y)$.
(b) Find a formula for $E(Y \mid X = x)$.
(c) Find $E(XY)$.
(d) Find $E(X^2Y^2)$.

Exercise I.165. Suppose X, Y are iid standard normal variables. Find

(a) $P(|X + Y| < |X - Y|)$.
(b) $E(XI_{\{Y<c\}})$.
(c) $E(XI_{\{\max(X,Y)<c\}})$.
(d) $P(X < Y < 2X)$.

Exercise I.166. Suppose X, Y are iid $U[0, 1]$. Find the density of $X - Y$.

Exercise I.167. A foot-long stick is broken at a random point, and then the longer of the two pieces is again broken at a random point. Find the probability that a triangle can be made with these three pieces.

Exercise I.168. X, Y, Z are iid $U[0, 1]$. Find the probability that the largest of the three is larger than the sum of the other two.

Exercise I.169. X, Y, Z are iid standard exponential. Find the joint density of (X, XY, XYZ).

Exercise I.170. X, Y, Z are iid $U[0, 1]$. Find the joint density of (X, XY, XYZ).

Exercise I.171. Suppose X, Y, Z are iid *Exp*(1). Define $U_1 = \sqrt{X_1X_2}, U_2 = \sqrt{X_2X_3}, U_3 = \sqrt{X_1X_3}$.

(a) Find the mean and variance of each U_i.
(b) Let $T = \frac{U_1+U_2+U_3}{3}$. Find the mean and variance of T.

Remark. U_1, U_2, U_3 are not independent.

Exercise I.172. Suppose X_1, X_2 are iid standard normal variables. Show that:

(a) $X_1 + X_2$ and $X_1 - X_2$ are independent.
(b) $X_1 + X_2$ and $|X_1 - X_2|$ are independent.
(c) $X_1^2 + X_2^2$ and $\frac{X_1}{X_2}$ are independent.

Exercise I.173. Suppose (X, Y) has a bivariate normal distribution with means equal to zero, standard deviations equal to 1, and a correlation .5.

(a) Find the mean and variance of XY.
(b) Find the mean of X^2Y.
(c) Find the correlation between X and XY.
(d) Find a constant c such that $X + Y$ and $X + cY$ are independent.

Exercise I.174. The heights of husbands and wives in some population are jointly distributed as bivariate normal, with means 71 in. and 66 in. and standard deviations 2 in. and 1 in., respectively. Furthermore, the correlation between the heights of the husband and wife is .7. Find the probability that, for a randomly selected couple, the wife is taller than the husband.

Exercise I.175. Suppose (X, Y) is jointly uniformly distributed inside the unit circle in two dimensions.

(a) Find $P(X^2 + Y^2 < .5)$.
(b) For general $0 < r < s < 1$, find $P(r \leq X^2 + Y^2 \leq s)$.
(c) Find $E(e^{-X^2-Y^2})$.

Exercise I.176. Suppose X, Y are iid $U[0, 1]$. Let $U = \max(X, Y), V = \min(X, Y)$. Find $P(U > 2V)$.

Exercise I.177. Suppose X, Y are iid standard exponentials. Let $U = \max(X, Y)$, $V = \min(X, Y)$. Find $P(U > 2V)$.

Exercise I.178. Suppose X, Y are iid standard normal variables. Let $R = \frac{X}{Y}$ and $U = \sqrt{|R|}$. Is $E(U) < \infty$? If it is, find its value.

Exercise I.179. Suppose X, Y are iid random variables with the common density function $f(x) = \frac{c}{1+x^4}, -\infty < x < \infty$, where c is a normalizing constant. Show that $R = \frac{X}{Y}$ has the standard Cauchy distribution.

Exercise I.180. Let X be standard normal and Y independent of X.

(a) Show that the density of $X + Y$ is uniformly bounded. Give such an explicit bound.
(b) Is the density of XY necessarily uniformly bounded? Prove it, or give a counterexample.

Exercise I.181. Let X be standard normal and Y independent of X. Find the density of $X + Y$ for each of the following cases:

(a) $Y \sim Bin(2, .5)$.
(b) $Y \sim U[a, b]$.
(c) $Y \sim Exp(\lambda)$.
(d) $Y \sim Gamma(\alpha, \lambda)$ with $\alpha = 2$.

Exercise I.182. Suppose $X_1, X_2, \ldots$ are iid $U[0,1]$. Let $U = \sum_{i=1}^{\infty} \frac{X_i}{10^i}$ and $V = \sum_{i=1}^{\infty} \frac{X_i}{2^i}$. Find the expectation of $|U - V|$.

Exercise I.183. Suppose $X \sim Geo(p), Y \sim Geo(\theta)$, and that X and Y are independent. Find $P(X > Y)$.

Exercise I.184. A number N is chosen according to a Poisson distribution with mean 10. One hundred balls are then distributed completely at random into $N + 1$ cells. What are the mean and the variance of the number of balls received by the first cell?

Exercise I.185. A number N is chosen according to a Poisson distribution with mean 10. A fair coin is then tossed until $N + 1$ heads are obtained. What is the expected number of tosses it will take to stop the experiment?

I.2 True-False Problems

For each of the following questions, answer whether the statement is true (T) or false (F).

Chapters 1–4

1. A and B are two events such that $P(A) = .5, P(B) = .25$. Then, $P(A \cup B) \leq .75$.
2. A, B, C are three events such that $P(A) = P(B) = .5$, $P(A \cap B) = .25$, and if either A or B occurs, then C also occurs. Then, $P(C) < .75$.
3. A, B, C are three events such that A and B are independent, and if both A and B occur, then C cannot occur. Furthermore, $P(A) = P(B) = P(C) = .5$. Then, P(Either A and C both occur or B and C both occur) = .75.
4. Ten numbers are drawn without replacement from $1, 2, \ldots, 100$. The probability that the second number drawn will be an even number is .5.
5. The six letters in the word CHEESE are rearranged in a random manner. The probability that it will still spell CHEESE is less than .5.
6. Two calculus and two history books are placed on a shelf in random order. The probability that the two calculus books will be placed next to each other is less than .5.
7. A fair die is rolled three times. It is more likely that the sum will be 16 or more than that two or more of the rolls will be a six.
8. It is possible for the total number of events in an experiment with probabilities strictly between 0 and 1 to be 62.
9. In bridge, it is more likely that North has no spades than that he has no aces.
10. If three distinguishable balls are distributed completely at random into three distinguishable cells, then it is more likely that no cell will remain empty than that only one cell remains nonempty.

11. Tim chose one number at random from $1, 2, \ldots, 10$, and Tom chose one number at random from $1, 2, \ldots, 10$. They chose independently. The probability that they happened to choose the number is less than 5%.
12. If A and B are independent and B and C are also independent, then A, B, C are mutually independent.
13. If $P(A|B) = .5$, then for $P(B|A)$ also to be .5, $P(A)$ and $P(B)$ must both be .5.
14. $P(A|A^c \cap B)$ is always zero.
15. $P(A|A^c \cup B)$ is always zero.
16. If $P(A) > P(B)$, then $P(A|B) > P(B|A)$.
17. Among five people in a room, two are twins and the other three are three random people. The probability that there are three or more people in the room with the same birthday is less than 5%.
18. A fair die is rolled three times. The probability that at least two of the rolls are even if we know that at least one of the rolls is even is $\frac{2}{3}$.
19. If $P(A) = P(B) = .8$, then $P(B|A)$ cannot be .6.
20. If $P(A|B)$ and $P(B|C)$ are both strictly positive, then $P(A|C)$ is also strictly positive.
21. Tim and Doug shoot simultaneously at the bull's eye. Tim misses 80% of the time, and Doug hits 80% of the time. We know that one of the two shots hit the eye and the other missed. The probability that it was Doug who hit is .8.
22. A random variable X has a CDF $F(x)$ such that $F(x) - F(x-) = .2$ at $x = 1, 2, 3, 4, 5$. Then $F(2.5) = .4$.
23. A random variable X takes values 0, .5, and 1 and has mean .5. Then $P(X = 0)$ and $P(X = 1)$ are equal.
24. A discrete random variable X assumes the values $0, 1, 2, 3, \ldots$. Then, $E(X^2) = \sum_{n=0}^{\infty} P(X > \sqrt{n})$.
25. A fair coin is tossed 20 times. Then the expected number of times that a head is followed by four or more heads is larger than .25.
26. A couple wants to have at least two boys or at least two girls, whichever happens first. Then the expected number of children they will have is 2.5.
27. If X, Y, Z are independent random variables, then $\text{Var}(XYZ) \geq \text{Var}(X)\text{Var}(Y)$ $\text{Var}(Z)$.
28. If X, Y, Z are independent random variables, then $\text{Var}(XYZ)$ cannot be equal to $\text{Var}(X)\text{Var}(Y)\text{Var}(Z)$.
29. An urn contains three green and three red balls. Four of them are taken out at random, without replacement, one at a time. Let X be the first draw at which a green ball is taken out. Then $E(X) < 3$.
30. A fair coin is tossed repeatedly until both a head and a tail are obtained. Let X be the number of tosses it will take. Then $E(X) = 3$.
31. A nonnegative random variable X has variance 100. Then $P(X > 20)$ cannot be zero.
32. It is not possible that neither X nor Y has a finite variance but $X + Y$ does.
33. It is not possible for a random variable X to be such that both $E(X)$ and $E(\frac{1}{X})$ are strictly larger than 1.

34. If $X_1, X_2, \ldots, X_{100}$ are 100 independent variables, and if $\text{Var}(X_1 + X_2 + \cdots + X_{100}) = 100$, then it cannot be true that $\text{Var}(X_i) < 1$ for each i.
35. X and Y are two random variables such that $E(X + Y) = 2$. Then at least one of $E(|X|)$ and $E(|Y|)$ must be ≥ 1.
36. A fair coin is tossed repeatedly until the first head is obtained. If we know that two tosses did not suffice, then the expected value of the number of tosses it actually took to obtain the first head is larger than 3.5.
37. X_1 and X_2 are iid random variables with the common pmf $p(x) = \frac{1}{2}$, $x = \pm 1$, and $p(x) = 0$ otherwise. If we define $X_3 = X_1 X_2$, then X_1, X_2, X_3 are mutually independent.
38. If X and Y are independent random variables with a finite variance, then necessarily $E(X^2 Y^2) = E(X^2)E(Y^2)$.
39. If X and Y are iid random variables with mean 1 and a finite and nonzero variance, then necessarily $E(X - Y)^2 > E(X - 1)^2$.
40. For any random variable X with a finite variance, $\text{Var}(|X|) \leq \text{Var}(X)$.
41. A random variable X has finite variance and another random variable Y takes only the values ± 1 with probability $\frac{1}{2}$ each; X and Y are independent. Then X and XY have the same variance.

Chapters 5–9

42. A nonnegative integer-valued random variable X has a finite mgf at some $t > 0$. Another random variable Y equals X if $X > 1$ but is zero if $X = 0$ or 1. Then Y also has a finite mgf at that t.
43. If X and Y are independent random variables and each has a finite mgfs for $-1 < t < 1$, then $X + Y$ and $X - Y$ also have finite mgfs for $-1 < t < 1$.
44. X, Y, Z are three iid random variables. Then XY and YZ are necessarily equal; i.e., $P(XY = YZ) = 1$.
45. X, Y, Z are three iid random variables. Then XY and YZ necessarily have the same distribution.
46. A certain positive random variable X does not have a finite mgf at any $t > 0$. However, $Y = Xe^{-X}$ must still have a finite mgf at all $t > 0$.
47. X is a standard normal variable. Then $Y = 2\Phi(x) - 1$ is distributed uniformly on $[-1, 1]$.
48. X is a Bernoulli random variable with parameter $p = .5$. Let $F(x)$ be the CDF of X. Then $2F(X) - 1$ is also a Bernoulli random variable.
49. X is a standard normal variable. Then no integer power of X can be normally distributed.
50. X is a standard Cauchy variable. Then no strictly monotone function of X can also be a standard Cauchy variable.
51. If X_1 and X_2 are iid random variables and all their moments exist, then all odd moments of $X_1 - X_2$ must also exist and be zero.
52. A continuous random variable X has all odd moments equal to zero. Then the density of X is symmetric about zero.

53. X has a Poisson distribution. Then no function of X can be normally distributed.
54. X and Y are independent Poisson random variables. Then $\max\{X, Y\}$ is also Poisson distributed.
55. X and Y are independent Poisson random variables. Then $\min\{X, Y\}$ is also Poisson distributed.
56. X and Y are iid Poisson random variables with mean 1. Then $\frac{X+Y}{2}$ is also Poisson with mean 1.
57. If X and Y are independent continuous random variables and each has a density symmetric about zero, then $X + Y$ also has a density symmetric about zero.
58. If X and Y are independent continuous random variables and each has a density symmetric about zero, then XY also has a density symmetric about zero.
59. If a continuous random variable X has zero mean, then its density $f(x)$ has to be strictly positive at zero.
60. If a continuous random variable X has zero mean, then its density $f(x)$ has to be finite at zero.
61. A continuous random variable X has a density symmetric about zero, and X^2 has a chi-square distribution with one degree of freedom. Then X must be standard normal.
62. If X has a Pareto distribution with $\theta = 1$, then $\frac{1}{X}$ has a Beta distribution.
63. The variance of a Beta distribution cannot be 2.
64. If X has a lognormal distribution, then $E(\frac{1}{X})$ must exist.
65. If X, Y, Z are three independent lognormal variables, then XY^2Z^3 is another lognormal variable.
66. If X, Y, Z, W are four iid standard normal variables, then $\frac{X}{Y} + \frac{Z}{W}$ is a Cauchy variable.
67. If X has a standard double exponential density, then $|X|$ has an exponential density.
68. If X is a positive random variable and $E(X^2) = E(X^6) = 1$, then X is constantly equal to 1.
69. If $f(x)$ is a density function on [0, 1], then $\int_0^1 f^2(x)dx < \infty$.
70. If $f(x)$ is a density function on [0, 1], then $\int_0^1 \sqrt{f(x)}dx < \infty$.
71. If X is a positive random variable and $E[g(X)] < \infty$, then $E[g(-X)]$ must also be finite.

Chapter 10

72. The sum of 50 independent Poisson variables with mean 1 and the sum of 50 independent exponential variables with mean 1 have approximately the same distribution.
73. One hundred numbers are chosen at random independently with replacement from $1, 2, \ldots, 9$. Their sum should be 500 ± 50 with about a 95% probability.

74. One hundred numbers are chosen independently from the unit interval $[0, 1]$ according to a uniform distribution. Their sum should be 50 ± 5 with about a 92% probability.
75. If a fair coin is tossed 500 times, the probability that exactly 250 heads will be obtained is about 4%.
76. If a fair coin is tossed 5000 times, the probability that exactly 2500 heads will be obtained is about 1%.
77. The length of an approximate 95% confidence interval for a Poisson mean increases with the data value X.
78. The center of an approximate 95% confidence interval for a Poisson mean moves to the right with the data value X.
79. The sum of the squares of 90 iid $U[0, 1]$ variables should be approximately normal with mean 30 and variance 7.5.
80. The sum of the squares of 90 iid $U[0, 1]$ variables should be approximately normal with mean 30 and variance 8.
81. X is a Poisson variable with mean λ. If $P(X \leq 10) \approx .95$, then $\lambda \approx 6$.

Chapters 11–13

82. If X and Y are discrete random variables with the joint pmf $p(x, y) = \frac{1}{9}$, $1 \leq x \leq 3, 1 \leq y \leq 3$, then X and Y are independent random variables.
83. If X and Y are discrete random variables with the joint pmf $p(x, y) = \frac{1}{6}$, $1 \leq x \leq y \leq 3$, then $E(Y - X) > 0$.
84. A fair coin is tossed eight times. X is the number of heads in the first four tosses and Y the number of tails in the last four tosses. Then, $E[(Y - 2)^2 \mid X = 2] > 1$.
85. Given a positive random variable X, let $Y = e^{X \log X}$. Then $E(Y \mid X = 1) = 1$.
86. Given a positive random variable X, let $Y = e^{X \log X}$. Then $\text{Var}(Y \mid X = 1) > 1$.
87. X and Y are independent random variables and $E(Y) = 0$. Then $E(XY \mid X = 1) = 0$.
88. It is not possible that $\text{Var}(Y) > 0$ but $\text{Var}(Y \mid X = x) = 0$ for some particular x and some particular random variable X.
89. If the correlation between X and Y is strictly positive, then the correlation between X^2 and Y is also strictly positive.
90. Always, $\text{Var}(X) \geq E_Y[\text{Var}(X \mid Y = y)]$.
91. If $E(X \mid Y = y)$ exists for every y, then $E(X)$ also exists.
92. If X and Y are independent, then the correlation between $\sin X$ and $\cos Y$ is zero.
93. If 50 balls are distributed independently and with equal probability into ten cells, then the correlation between the number of balls that are allocated to the first cell and the number of balls that are allocated to the tenth cell is $< -.1$.
94. A fair die is rolled repeatedly. X is the first roll where a five is obtained, and Y is the first roll where a six is obtained. Then $E(Y \mid X = x) = x$.

95. If X and Y are two random variables with finite variances, then $\sigma_{X+Y} \leq \sigma_X + \sigma_Y$.
96. If X, Y, Z are three random variables with finite variances, then $\sigma_{X+Y+Z} \leq \sigma_X + \sigma_Y + \sigma_Z$.
97. A fair die is rolled repeatedly. X is the first roll where a six is obtained, and Y is the roll where the second six is obtained. Then $E(Y \mid X = 4) = 10$.
98. If X and Y are continuous random variables with joint density $f(x, y) = 2$, $x, y \geq 0, x + y \leq 1$, then marginally X and Y are both $U[0, 1]$.
99. If X and Y are both marginally $U[0, 1]$, then the joint density must be $f(x, y) = 1, x, y \in [0, 1]$.
100. If X and Y have a joint uniform density in the unit circle $C = \{(x, y) : x^2 + y^2 \leq 1\}$, then each of $E(X), E(Y), E(XY)$, and $E(XY^2)$ is zero.
101. One thousand observations are generated independently according to a uniform distribution in the ten dimensional unit cube. The number of observations among these 1000 observations that fall inside the inscribed sphere of the cube has an expected value of only about 2.
102. If X and Y have a joint uniform density in the unit circle $C = \{(x, y) : x^2 + y^2 \leq 1\}$, then $P(X^2 + Y^2 \leq .5) = .5$.
103. If X and Y are iid $U[0, 1]$ random variables, then $E[\min\{X, Y\}] = 1 - E[\max\{X, Y\}]$.
104. Whatever the joint distribution of two positive random variables X and Y, if $E(X) = 2$ and $E(Y) = 1$, then $E(|X - Y|) \geq 1$.
105. $X \sim N(\mu, \sigma^2)$. Let $Y = I_{\{X>0\}}$. Then $|X|, Y$ are independent if and only if $\mu = 0$.
106. If X and Y are random variables such that $\frac{X}{Y}$ has a standard Cauchy distribution, then X and Y must be independent standard normal.
107. If $X_1, \ldots, X_n$ are iid $U[0, 1]$ random variables and $X_{(1)}$ and $X_{(n)}$ are the smallest and the largest order statistics, then $\rho_{X_{(1)}, X_{(n)}} \to 0$ as $n \to \infty$.
108. If $X_1, \ldots, X_n$ are iid $U[0, 1]$ random variables and $X_{(1)}$ and $X_{(n)}$ are the smallest and the largest order statistics, then $P(X_{(n)} - X_{(1)} > .99) \to 1$ as $n \to \infty$.
109. If $X_1, \ldots, X_n$ are iid $U[0, 1]$ random variables and $X_{(1)}$ and $X_{(n)}$ are the smallest and the largest order statistics, then $X_{(n)} + X_{(1)}$ and $X_{(n)} - X_{(1)}$ are uncorrelated.
110. If $X_1, \ldots, X_n$ are iid $U[0, 1]$ random variables and $X_{(1)}$ and $X_{(n)}$ are the smallest and the largest order statistics, then $X_{(n)} + X_{(1)}$ and $X_{(n)} - X_{(1)}$ are independent.
111. If $X_1, \ldots, X_5$ are iid standard normal variables, then $E[X_{(5)} + X_{(4)} + X_{(3)} + X_{(2)} + X_{(1)}] = 0$.
112. If $X_1, \ldots, X_n$ are iid $U[0, 1]$ random variables, then the density of $X_{(i)}$ is unimodal for any $i, 1 \leq i \leq n$.
113. If $X_1, \ldots, X_n$ are iid standard exponential variables, then $\frac{E(X_{(n)})}{\log n} \to 1$ as $n \to \infty$.
114. If X and Y are jointly bivariate normal with marginal variance 1 and $\mathrm{Var}(X \mid Y = 0) = .36$, then $\rho_{X,Y} = +.8$.

115. If X and Y are jointly bivariate normal, then $\frac{d^2}{dx^2}E(Y \mid X = x) = 0$ at any x.
116. If X has a t distribution, then X^2 has an F distribution.
117. If X and Y are iid standard exponentials, than $\text{Var}(\frac{X}{X+Y}) < .1$.
118. If X and Y are iid standard exponentials, than $E(X + Y \mid \frac{X}{X+Y} = .5) = 2$.
119. If X and Y are iid standard normal, then $P(\frac{X}{Y} < 1) = P(X < Y)$.
120. If X and Y have the joint density $f(x, y) = \frac{1}{4}e^{-|x|-|y|}, x, y \in \mathcal{R}$, then the polar coordinates r, θ are not independent.
121. If X and Y have the joint density $f(x, y) = \frac{c}{(1+x^2+y^2)^{5/2}}, x, y \in \mathcal{R}$, where c is a normalizing constant, then the polar coordinates r, θ are independent.

Appendix II: Symbols and Formulas

II.1 Glossary of Symbols

$n!$	$n(n-1)\cdots 1$
$\binom{n}{k}$	$\frac{n!}{k!(n-k)!}$
$a_n \sim b_n$	$0 < \liminf \frac{a_n}{b_n} \leq \limsup \frac{a_n}{b_n} < \infty$
$a_n = O(b_n)$	$\lvert a_n\rvert \leq Kb_n$ for some finite positive constant K
$a_n = o(b_n)$	$\lim \frac{a_n}{b_n} = 0$
$a_n \approx b_n$	$\lim \frac{a_n}{b_n} = 1$
$\mathcal{R}$	real line
$\mathcal{R}^d$	d-dimensional Euclidean space
f'	first derivative of f
f''	second derivative of f
$\frac{\partial}{\partial x}$	partial derivative
$\Gamma(\alpha)$	Gamma function
$B(\alpha, \beta)$	Beta function
${}_1F_1$	hypergeometric function
I_z	Bessel function
H_j	Hermite polynomials
log	natural logarithm
$\log_b$	Logarithm to the base b
$\lfloor x \rfloor$	Integer part
$\{\}$	fractional part
ω	sample point
Ω	sample space
$P(A)$	probability of A
$P(A \mid B)$	conditional probability of A given B
A^c	complement of A
$\cup_{i=1}^n A_i$	union of $A_1, \cdots, A_n$
$\cap_{i=1}^n A_i$	intersection of $A_1, \cdots, A_n$
$G_X(s), G(s)$	generating function
$\psi_X(t), \psi(t)$	moment generating function

iid	independent and identically distributed
A, B, C, D	events
X, Y, Z, U, V, W	random variables
I_A	indicator function of A
sgn, sign	signum function
x_+, x^+	$\max\{x, 0\}$
max, min	maximum, minimum
sup, inf	supremum, infimum
$MN(n, p_1, \ldots, p_k)$	multinomial distribution with these parameters
$N(\mu, \sigma^2)$	normal distribution
$t_n, t_{(n)}$	Student's t distribution with n degrees of freedom
$Ber(p), Bin(n, p)$	Bernoulli and binomial distributions
$Poi(\lambda)$	Poisson distribution
$Geo(p)$	geometric distribution
$NB(r, p)$	negative binomial distribution
$Hypergeom(n, D, N)$	hypergeometric distribution
$Exp(\lambda)$	exponential distribution with mean λ
$Gamma(\alpha, \lambda)$	Gamma distribution with shape parameter α and scale parameter λ
$\chi^2_n, \chi^2_{(n)}$	chi-square distribution
$C(\mu, \sigma)$	Cauchy distribution
$U[a, b]$	uniform distribution
$Be(\alpha, \beta)$	Beta distribution
$Pa(\theta, \alpha)$	Pareto distribution
$\phi(x)$	standard normal density
$\Phi(x)$	standard normal CDF
$R(x)$	Mills ratio
$f(x)$	general density
$F(x)$	general CDF
$\bar{F}(x)$	survival function
$F^{-1}(p), Q(p)$	quantile function
$p(x)$	general pmf
$p(x, y)$	bivariate pmf
$f(x, y)$	bivariate density
$p(x_1, \cdots, x_n)$	multivariate pmf
$f(x_1, \cdots, x_n)$	multidimensional density
$F(x_1, \cdots, x_n)$	multidimensional CDF
$p(x\|y)$	conditional pmf
$f(x\|y)$	conditional density
f_X, f_Y	marginal densities
F_X, F_Y	marginal cdfs
$F_{X\|Y}$	conditional CDF
$E(X), \mu$	expected value
Var, σ^2	variance

μ_k	$E(X-\mu)^k$
κ_r	rth cumulant
β	skewness
γ	kurtosis
Cov	covariance
ρ	correlation
r, θ	polar coordinates in two dimensions
J	Jacobian matrix
$\|J\|$	determinant of J
$X_{(1)}, X_{(2)}, \cdots, X_{(n)}$	order statistics
W_n	sample range
p_{ij}	transition probabilities in a Markov chain
P	one-step transition probability matrix
$P^{(n)}$	n-step transition probability matrix
S	state space of a Markov chain
T_i, T_{ij}, T_{iD}	first-passage times in a Markov chain
π	stationary distribution of a Markov chain
$x_{(n)}$	$x(x-1)\cdots(x-n+1)$
$s(n,k)$	Stirling numbers of the first kind
$S(n,k)$	Stirling numbers of the second kind

II.2 Formula Summaries

II.2.1 Moments and MGFs of Common Distributions

Discrete Distributions

Distribution	$p(x)$	Mean	Variance	Skewness	Kurtosis	MGF
Uniform	$\frac{1}{n}, x = 1, \cdots, n$	$\frac{n+1}{2}$	$\frac{n^2-1}{12}$	0	$-\frac{6(n^2+1)}{5(n^2-1)}$	$\frac{e^{(n+1)t}-e^t}{n(e^t-1)}$
Binomial	$\binom{n}{x} p^x (1-p)^{n-x}, x = 0, \cdots, n$	np	$np(1-p)$	$\frac{1-2p}{\sqrt{np(1-p)}}$	$\frac{1-6p(1-p)}{np(1-p)}$	$(pe^t + 1 - p)^n$
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, \cdots$	λ	λ	$\frac{1}{\sqrt{\lambda}}$	$\frac{1}{\lambda}$	$e^{\lambda(e^t-1)}$
Geometric	$p(1-p)^{x-1}, x = 1, 2, \cdots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{2-p}{\sqrt{1-p}}$	$6 + \frac{p^2}{1-p}$	$\frac{pe^t}{1-(1-p)e^t}$
Negative Binomial	$\binom{x-1}{r-1} p^r (1-p)^{x-r}, x \geq r,$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\frac{2-p}{\sqrt{r(1-p)}}$	$\frac{6}{r} + \frac{p^2}{r(1-p)}$	$\left(\frac{pe^t}{1-(1-p)e^t}\right)^r$
Hypergeometric	$\frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$	$n\frac{D}{N}$	$n\frac{D}{N}(1-\frac{D}{N})\frac{N-n}{N-1}$	Complex	Complex	Complex
Benford	$\frac{\log(1+\frac{1}{x})}{\log 10}, x = 1, \cdots, 9$	3.44	6.057	.796	2.45	$\sum_{x=1}^{9} e^{tx} p(x)$

Continuous Distributions

Distribution	$f(x)$	Mean	Variance	Skewness	Kurtosis
Uniform	$\frac{1}{b-a}, a \le x \le b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	0	$-\frac{6}{5}$
Exponential	$\frac{e^{-x/\lambda}}{\lambda}, x \ge 0$	λ	λ^2	2	6
Gamma	$\frac{e^{-x/\lambda}x^{\alpha-1}}{\lambda^\alpha \Gamma(\alpha)}, x \ge 0$	$\alpha\lambda$	$\alpha\lambda^2$	$\frac{2}{\sqrt{\alpha}}$	$\frac{6}{\alpha}$
χ^2_m	$\frac{e^{-x/2}x^{m/2-1}}{2^{m/2}\Gamma\left(\frac{m}{2}\right)}, x \ge 0$	m	$2m$	$\sqrt{\frac{8}{m}}$	$\frac{12}{m}$
Weibull	$\frac{\beta}{\lambda}\left(\frac{x}{\lambda}\right)^{\beta-1}e^{-\left(\frac{x}{\lambda}\right)^\beta}, x > 0$	$\lambda\Gamma(1+\frac{1}{\beta})$	$\lambda^2\Gamma(1+\frac{2}{\beta}) - \mu^2$	$\frac{\lambda^3\Gamma\left(1+\frac{3}{\beta}\right)-3\mu\sigma^2-\mu^3}{\sigma^3}$	Complex
Beta	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, 0 \le x \le 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$\frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{\sqrt{\alpha\beta}(\alpha+\beta+2)}$	Complex
Normal	$\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}, x \in \mathcal{R}$	μ	σ^2	0	0
lognormal	$\frac{1}{\sigma\sqrt{2\pi}x}e^{-\frac{(\log x-\mu)^2}{2\sigma^2}}, x > 0$	$e^{\mu+\sigma^2/2}$	$(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$	$e^{\sigma^2+2}\sqrt{e^{\sigma^2}-1}$	Complex
Cauchy	$\frac{1}{\sigma\pi(1+(x-\mu)^2/\sigma^2)}, x \in \mathcal{R}$	None	None	None	None
t_m	$\frac{\Gamma\left(\frac{m+1}{2}\right)}{\sqrt{m\pi}\Gamma\left(\frac{m}{2}\right)}\frac{1}{(1+x^2/m)^{(m+1)/2}}, x \in \mathcal{R}$	$0\ (m > 1)$	$\frac{m}{m-2}(m > 2)$	$0\ (m > 3)$	$\frac{6}{m-4}(m > 4)$
F	$\frac{\left(\frac{\beta}{\alpha}\right)^\beta x^{\alpha-1}}{B(\alpha,\beta)\left(x+\frac{\beta}{\alpha}\right)^{\alpha+\beta}}, x > 0$	$\frac{\beta}{\beta-1}(\beta > 1)$	$\frac{\beta^2(\alpha+\beta-1)}{\alpha(\beta-2)(\beta-1)^2}(\beta > 2)$	Complex	Complex
Double Exponential	$\frac{e^{-\lvert x-\mu\rvert/\sigma}}{2\sigma}, x \in \mathcal{R}$	μ	$2\sigma^2$	0	3
Pareto	$\frac{\alpha\theta^\alpha}{x^{\alpha+1}}, x \ge \theta > 0$	$\frac{\alpha\theta}{\alpha-1}(\alpha > 1)$	$\frac{\alpha\theta^2}{(\alpha-1)^2(\alpha-2)}(\alpha > 2)$	$\frac{2(\alpha+1)}{\alpha-3}\sqrt{\frac{\alpha-2}{\alpha}}(\alpha > 3)$	Complex
Gumbel	$\frac{1}{\sigma}e^{\left(-e^{-\frac{x-\mu}{\sigma}}\right)}e^{-\frac{x-\mu}{\sigma}}, x \in \mathcal{R}$	$\mu + \gamma\sigma$	$\frac{\pi^2}{6}\sigma^2$	$\frac{12\sqrt{6}\zeta(3)}{\pi^3}$	$\frac{12}{5}$

Note: For the Gumbel distribution, $\gamma \approx .577216$ is the Euler constant and $\zeta(3)$ is Riemann's zeta function $\zeta(3) = \sum_{n=1}^{\infty}\frac{1}{n^3} \approx 1.20206$.

Table of MGFs of Continuous Distributions

Distribution	$f(x)$	MGF
Uniform	$\frac{1}{b-a}, a \le x \le b$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
Exponential	$\frac{e^{-x/\lambda}}{\lambda}, x \ge 0$	$(1-\lambda t)^{-1} (t < 1/\lambda)$
Gamma	$\frac{e^{-x/\lambda} x^{\alpha-1}}{\lambda^{\alpha}\Gamma(\alpha)}, x \ge 0$	$(1-\lambda t)^{-\alpha} (t < 1/\lambda)$
χ^2_m	$\frac{e^{-x/2} x^{m/2-1}}{2^{m/2}\Gamma(\frac{m}{2})}, x \ge 0$	$(1-2t)^{-m/2} (t < \frac{1}{2})$
Weibull	$\frac{\beta}{\lambda}(\frac{x}{\lambda})^{\beta-1} e^{-(\frac{x}{\lambda})^{\beta}}, x > 0$	$\sum_{n=0}^{\infty} \frac{(\lambda t)^n}{n!}\Gamma(1+\frac{n}{\beta})$
Beta	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, 0 \le x \le 1$	${}^1F_1(\alpha, \alpha+\beta, t)$
Normal	$\frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}, x \in \mathcal{R}$	$e^{t\mu + t^2\sigma^2/2}$
lognormal	$\frac{1}{\sigma\sqrt{2\pi}x} e^{-\frac{(\log x-\mu)^2}{2\sigma^2}}, x > 0$	None
Cauchy	$\frac{1}{\sigma\pi(1+(x-\mu)^2/\sigma^2)}, x \in \mathcal{R}$	None
t_m	$\frac{\Gamma(\frac{m+1}{2})}{\sqrt{m\pi}\Gamma(\frac{m}{2})} \frac{1}{(1+x^2/m)^{(m+1)/2}}, x \in \mathcal{R}$	None
F	$\frac{(\frac{\beta}{\alpha})^{\beta} x^{\alpha-1}}{B(\alpha,\beta)(x+\frac{\beta}{\alpha})^{\alpha+\beta}}, x > 0$	None
Double Exponential	$\frac{e^{-\lvert x-\mu\rvert/\sigma}}{2\sigma}, x \in \mathcal{R}$	$\frac{e^{t\mu}}{1-\sigma^2 t^2} (\lvert t\rvert < 1/\sigma)$
Pareto	$\frac{\alpha\theta^{\alpha}}{x^{\alpha+1}}, x \ge \theta > 0$	None
Gumbel	$\frac{1}{\sigma} e^{(-e^{-\frac{x-\mu}{\sigma}})} e^{-\frac{x-\mu}{\sigma}}, x \in \mathcal{R}$	$e^{t\mu}\Gamma(1-t\sigma)\ (t < 1/\sigma)$

II.2.2 Useful Mathematical Formulas

$1+2+\cdots+n=\frac{n(n+1)}{2}$;
$1^2+2^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$;
$1^3+2^3+\cdots+n^3=\left(\frac{n(n+1)}{2}\right)^2$;
$1^4+2^4+\cdots+n^4=\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$;
$\binom{n}{0}+\binom{n}{1}+\cdots+\binom{n}{n}=2^n$;
$\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\cdots=\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\cdots=2^{n-1}$;
$\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}+\cdots=0$;
$\binom{n}{0}^2+\binom{n}{1}^2+\cdots+\binom{n}{n}^2=\binom{2n}{n}$;
$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\cdots+b^n$;
$1+x+x^2+\cdots+x^n=\frac{1-x^{n+1}}{1-x}$;
$1+x+x^2+x^3+\cdots=\frac{1}{1-x},-1<x<1$;
$x+2x^2+3x^3+4x^4+\cdots=\frac{x}{(1-x)^2},-1<x<1$;
$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$;
$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots,-1<x\le 1$;
$\frac{1}{1-x}=1+x+x^2+x^3+\cdots,-1<x<1$;
$\sum_{n=1}^{\infty}\frac{1}{n}=\infty$;
$\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$;
$\sum_{n=1}^{\infty}\frac{1}{n(n+1)}=1$;
$\lim_{n\to\infty}[1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\log n]=\gamma$(Euler's constant);
$n!\approx e^{-n}n^{n+\frac{1}{2}}\sqrt{2\pi},n\to\infty$(Stirling's approximation);
$\arcsin x=\arccos\sqrt{1-x^2}$; $\arccos x=\arcsin\sqrt{1-x^2}$;
$\arctan x=\arcsin\frac{x}{\sqrt{1+x^2}}$;
$\arctan x+\arctan y=\pi-\arctan\frac{x+y}{xy-1},x,y>0,xy>1$;
$\sin 2x=2\sin x\cos x$; $\sin 3x=3\sin x-4\sin^3 x$;
$\cos 2x=2\cos^2 x-1=\cos^2 x-\sin^2 x$; $\cos 3x=4\cos^3 x-3\cos x$;
$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$; $\tan\frac{x}{2}=\frac{\sin x}{1+\cos x}$;
$\Gamma(\alpha)=\int_0^{\infty}e^{-x}x^{\alpha-1}dx,\alpha>0$; $Be(\alpha,\beta)=\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$;
$\Gamma(n)=(n-1)!,n=1,2,3,\cdots$; $\Gamma(x)=x\Gamma(x-1),x>1$; $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$;
$\Gamma(2x)=\frac{2^{2x-1}\Gamma(x)\Gamma(x+\frac{1}{2})}{\sqrt{\pi}}$(Gamma duplication formula);
area of triangle $=\sqrt{s(s-a)(s-b)(s-c)},s=\frac{a+b+c}{2},a,b,c$ the side lengths;
area of circle = $\pi r^2,r$ the radius; volume of sphere in three dimensions $=\frac{4}{3}\pi r^3$;
volume of unit sphere in n dimensions $=V_n=\frac{\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2}+1)}$; surface area of unit sphere in n dimensions $=nV_n$;
volume of circular cylinder $=\pi r^2h$; volume of circular cone $=\frac{1}{3}\pi r^2h$.

II.2.3 Useful Calculus Facts

$(fg)' = f'g + fg'$; $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$; $\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$; $(\log f)' = \frac{f'}{f}$;
$(e^f)' = f'e^f$; $(f \circ g)' = f'(g)g'$(chain rule); $(fg)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)} g^{(n-k)}$;
$(\int_a^x f(t)dt)' = f(x)$; $(\int_x^a f(t)dt)' = -f(x)$.

Basic derivatives and indefinite integrals

$f(x)$	Derivative	Indefinite integral
$x^a, a \neq -1$	ax^{a-1}	$\frac{x^{a+1}}{a+1}$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\log \lvert x \rvert$
$\log x$	$\frac{1}{x}$	$x \log x - x$
e^{tx}	te^{tx}	$\frac{e^{tx}}{t}$
xe^{tx}	$(1 + tx)e^{tx}$	$\frac{e^{tx}}{t^2}(tx - 1)$
$\sin ax$	$a \cos ax$	$-\frac{1}{a} \cos ax$
$\cos ax$	$-a \sin ax$	$\frac{1}{a} \sin ax$
$x \sin ax$	$ax \cos ax + \sin ax$	$\frac{1}{a^2}[\sin ax - ax \cos ax]$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$x \arcsin x + \sqrt{1 - x^2}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$	$x \arccos x - \sqrt{1 - x^2}$
$\frac{1}{a^2x^2+c^2}$	$-\frac{2a^2x}{(a^2x^2+c^2)^2}$	$\frac{1}{ac} \arctan \frac{ax}{c}$
$\frac{x}{a^2x^2+c^2}$	$\frac{c^2-a^2x^2}{(a^2x^2+c^2)^2}$	$\frac{1}{2a^2} \log \lvert a^2x^2 + c^2 \rvert$

II.3 Tables

II.3.1 Normal Table

Standard normal probabilities $P(Z \leq t)$ and standard normal percentiles

Quantity tabulated on the next page is $\Phi(t) = P(Z \leq t)$ for a given $t \geq 0$, where $Z \sim N(0, 1)$. For example, from the table, $P(Z \leq 1.52) = .9357$. For any positive t, $P(-t \leq Z \leq t) = 2\Phi(t) - 1$ and $P(Z < -t) = P(Z > t) = 1 - \Phi(t)$. Selected standard normal percentiles z_α are given below. Here, the meaning of z_α is $P(Z > z_\alpha) = \alpha$.

α	z_α
.25	.675
.2	.84
.1	1.28
.05	1.645
.025	1.96
.02	2.055
.01	2.33
.005	2.575
.001	3.08
.0001	3.72

	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

II.3.2 Poisson Table

The table below tabulates $P(X \leq x)$ for a given x when $X \sim Poi(\lambda)$. For example, if $\lambda = 2.5$, then $P(X \leq 4) = .8912$. The individual probabilities are found by subtraction. Thus, if $\lambda = 2.5$, then $P(X = 4) = P(X \leq 4) - P(X \leq 3) = .8912 - .7576 = .1336$.

	λ									
x	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	1.0000	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	1.0000	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8	1.0000	1.0000	1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9972	0.9933	0.9863
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9976	0.9945
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Author Index

Subject Index