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List of Symbols and Abbreviations

	Definition/Description	Page
ACICARA	Reference [22]: A Course in Calculus and Real Analysis	VI
$\mathbb{R}$	set of all real numbers	1
$\mathbb{R}^n$	n-dimensional Euclidean space	1
$\mathbb{N}$	set of all positive integers	1
$\mathbf{x} = (x_1, \ldots, x_n)$	vector in $\mathbb{R}^n$ with coordinates $x_1, \ldots, x_n$	1
$\mathbf{0}$	zero vector	1
$\mathbf{x} \cdot \mathbf{y}$	dot product of $\mathbf{x}$ and $\mathbf{y}$	3
$\lvert\mathbf{x}\rvert$	norm $\sqrt{x_1^2 + \cdots + x_n^2}$ of the vector $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n$	3
$\mathbf{x} \leq \mathbf{y}$	$\mathbf{x}$ is less than or equal to $\mathbf{y}$ in the product order on $\mathbb{R}^n$	5
$I_{a,b}$	closed interval between real numbers a and b	6
$I_{\mathbf{a},\mathbf{b}}$	$I_{a_1,b_1} \times \cdots \times I_{a_n,b_n}$, where $\mathbf{a} = (a_1, \ldots, a_n)$ and $\mathbf{b} = (b_1, \ldots, b_n)$	6
$I \times J$	$\big\{(x, y) \in \mathbb{R}^2 : x \in I \text{ and } y \in J\big\}$	7
$\mathbb{S}_r(\mathbf{c})$	$\{\mathbf{x} \in \mathbb{R}^n : \lvert x_i - c_i\rvert < r \text{ for } i = 1, \ldots, n\}$	7
$\mathbb{B}_r(\mathbf{c})$	$\{\mathbf{x} \in \mathbb{R}^n : \lvert\mathbf{x} - \mathbf{c}\rvert < r\}$	7
$\operatorname{diam}(D)$	diameter of a nonempty bounded subset D of $\mathbb{R}^n$	8
$\mathbb{Q}$	set of all rational numbers	9
$I + J$	$\{x + y : x \in I \text{ and } y \in J\}$, where I, J are intervals in $\mathbb{R}$	15
$V(f)$	total variation of a function f of bounded variation	17
v_f	total variation function corresponding to f	18
$W(f)$	total bivariation of a function f of bounded bivariation	20
w_f	total bivariation function corresponding to f	22
$\triangle_{\mathbf{a}}^{\mathbf{b}} f$	alternating difference of $f(c_1, \ldots, c_n)$, where $c_i \in \{a_i, b_i\}$	25
$\triangle_{(a_1,a_2)}^{(b_1,b_2)} f$	$f(b_1, b_2) + f(a_1, a_2) - f(b_1, a_2) - f(a_1, b_2)$	25
$\triangle_{(a_1,a_2,a_3)}^{(b_1,b_2,b_3)} f$	$f(b_1, b_2, b_3) + f(b_1, a_2, a_3) + f(a_1, b_2, a_3) + f(a_1, a_2, b_3)$ $-f(b_1, b_2, a_3) - f(a_1, b_2, b_3) - f(b_1, a_2, b_3) - f(a_1, a_2, a_3)$	25
IVP	Intermediate Value Property	29
$[x]$	integer part of a real number x	29

Definition/Description		Page
$\mathbf{x} \preceq \mathbf{y}$	$\mathbf{x}$ is less than or equal to $\mathbf{y}$ in the lexicographic order on $\mathbb{R}^n$	35
$\mathbb{M}_r(\mathbf{c})$	$\{\mathbf{x} \in \mathbb{R}^n : \lvert x_1 - c_1\rvert + \cdots + \lvert x_n - c_n\rvert < r\}$	35
$\lVert\mathbf{x}\rVert_p$	p-norm $\left(\lvert x_1\rvert^p + \cdots + \lvert x_n\rvert^p\right)^{1/p}$ of $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n$	38
$\lVert\mathbf{x}\rVert_\infty$	∞-norm $\max\{\lvert x_1\rvert, \ldots, \lvert x_n\rvert\}$ of $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{R}^n$	38
SIVP	Strong Intermediate Value Property	42
$\big((x_n, y_n)\big)$	sequence in $\mathbb{R}^2$ whose nth term is (x_n, y_n)	43
$(x_n, y_n) \to (x_0, y_0)$	sequence $\big((x_n, y_n)\big)$ converges to (x_0, y_0)	44
$\big((x_{n_k}, y_{n_k})\big)$	subsequence of $\big((x_n, y_n)\big)$	45
$\overline{D}$	closure of a subset D of $\mathbb{R}^n$	46, 48
∂D	boundary of a subset D of $\mathbb{R}^n$	46, 48
$\lim_{(x,y)\to(x_0,y_0)} f(x,y)$	limit of f as (x, y) tends to (x_0, y_0)	67
$\lim_{(x,y)\to(x_0^+,y_0^+)} f(x,y)$	limit of f from first quadrant as (x, y) tends to (x_0, y_0)	71
$\lim_{(x,y)\to(\infty,\infty)} f(x,y)$	limit of f as (x, y) tends to (∞, ∞)	73
f_x, $\dfrac{\partial f}{\partial x}$	partial derivative of f w.r.t. x	84 138
∇f	gradient of f	85, 139
$(f_x)_-$	left(-hand) partial derivative of f w.r.t. x	85
$(f_x)_+$	right(-hand) partial derivative of f w.r.t. x	85
MVT	Mean Value Theorem	87
$\mathbf{D_u} f$	directional derivative of f along $\mathbf{u}$	88, 138
f_{xx}, $\dfrac{\partial^2 f}{\partial x^2}$	partial derivative of f_x w.r.t. x	91, 138
f_{xy}, $\dfrac{\partial^2 f}{\partial y \partial x}$	partial derivative of f_x w.r.t. y	91, 138
$\dfrac{\partial^n f}{\partial x^{n-m} \partial y^m}$	nth-order partial derivative of f $(m = 0, 1, \ldots, n)$	96
$\mathcal{D}_{h,k}$	partial differential operator $h\dfrac{\partial}{\partial x} + k\dfrac{\partial}{\partial y}$	97
$\mathcal{D}^n_{h,k}$	$\left(h\dfrac{\partial}{\partial x} + k\dfrac{\partial}{\partial y}\right)^n = \displaystyle\sum_{m=0}^{n} \binom{n}{m} h^{n-m} k^m \frac{\partial^n}{\partial x^{n-m} \partial y^m}$	98
$\mathbf{D}^n_{\mathbf{u}} f$	nth-order directional derivative of f along $\mathbf{u}$	99
$D^2_{\mathbf{uv}} f$	directional derivative of $\mathbf{D_u} f$ along $\mathbf{v}$	99
$J(\Phi)$	Jacobian of the transformation Φ	123
$\dfrac{\partial(x,y)}{\partial(u,v)}$	Jacobian of the functions x and y w.r.t. u and v	123
Δf	discriminant of f	136
$\mu(P)$	mesh of partition P	187
$\mathsf{P}_{n,k}$	partition into $n \times k$ equal parts	187

	Definition/Description	Page
$m(f)$	infimum of f on a rectangle	187
$M(f)$	supremum of f on a rectangle	187
$m_{i,j}(f)$	infimum of f on (i,j)th subrectangle	187
$M_{i,j}(f)$	supremum of f on (i,j)th subrectangle	187
$L(P,f)$	lower double sum of f w.r.t. partition P	188, 267
$U(P,f)$	upper double sum of f w.r.t. partition P	188, 267
$L(f)$	lower double integral of f	188, 267
$U(f)$	upper double integral of f	188, 267
$\iint_{[a,b]\times[c,d]} f$	double integral of f on the rectangle $[a,b]\times[c,d]$; also denoted by $\iint_{[a,b]\times[c,d]} f(x,y)d(x,y)$	193
$\mathrm{Vol}(E_f)$	volume of the solid under the surface $z=f(x,y)$	193
FTC	Fundamental Theorem of Calculus	208
$S(P,f)$	Riemann double sum for f w.r.t. partition P	223
f^*	extension of $f: D\to\mathbb{R}$ to a rectangle R containing D obtained by setting $f^*=0$ on $R\setminus D$	226
$\iint_D f$	double integral of f over a subset D of $\mathbb{R}^2$	226, 437, 444
f^+	positive part of a function f	230
f^-	negative part of a function f	230
1_D	constant function on D having value 1 at each point	241
$\mathrm{Area}(D)$	area of a bounded subset D of $\mathbb{R}^2$	241
$\Phi:=(\phi_1,\phi_2)$	transformation Φ with component functions ϕ_1 and ϕ_2	247
$\iiint_K f$	triple integral of f on a cuboid K in $\mathbb{R}^3$; also denoted by $\iiint_K f(x,y,z)d(x,y,z)$	268
$\iiint_D f$	triple integral of f over a subset D of $\mathbb{R}^3$	270
$\mathrm{Vol}(D)$	volume of a bounded subset D of $\mathbb{R}^3$	274, 297
$\mathrm{Area}(S)$	surface area of a (piecewise) smooth surface S	314, 317
$\mathrm{Av}(f)$	average of a function f	323
$\mathrm{Av}(f;w)$	weighted average of a function f w.r.t. a function w	324
$(\overline{x},\,\overline{y})$	centroid of a planar region	324
$(\overline{x},\,\overline{y},\,\overline{z})$	centroid of a surface or of a solid in 3-space	326, 329
$(Q\times R)(f)$	product cubature rule for f on a rectangle	339
$(Q\times \widetilde{R})(f)$	product cubature rule for f over an elementary region	345
$(\widetilde{Q}\times R)(f)$	product cubature rule for f over an elementary region	345
$\mathsf{C}(f)$	cubature rule for f analogous to the Midpoint Rule	351
$\mathsf{T}(f)$	cubature rule for f analogous to the Trapezoidal Rule	351
$\mathsf{S}(f)$	cubature rule for f analogous to Simpson's Rule	352

	Definition/Description	**Page**
$\mathsf{C}_n(f)$	compound cubature rule corresponding to $\mathsf{C}(f)$	353
$\mathsf{T}_n(f)$	compound cubature rule corresponding to $\mathsf{T}(f)$	353
$\mathsf{S}_n(f)$	compound cubature rule corresponding to $\mathsf{S}(f)$	354
$(a_{m,n})$	double sequence whose (m,n)th term is $a_{m,n}$	370
$a_{m,n} \to a$	double sequence $(a_{m,n})$ converges to a real number a	370
$\lim_{(m,n)\to(\infty,\infty)} a_{m,n}$	limit of double sequence $(a_{m,n})$	370
$a_{m,n} \to \infty$	double sequence $(a_{m,n})$ diverges to ∞	370
$a_{m,n} \to -\infty$	double sequence $(a_{m,n})$ diverges to $-\infty$	370
$\sum\sum_{(k,\ell)} a_{k,\ell}$	double series whose double sequence of terms is $(a_{k,\ell})$	376
$A_{m,n}$	(m,n)th partial double sum of $\sum\sum_{(k,\ell)} a_{k,\ell}$	376
$\sum_{\ell} a_{k,\ell}$	row-series corresponding to $\sum\sum_{(k,\ell)} a_{k,\ell}$	381
$\sum_{k} a_{k,\ell}$	column-series corresponding to $\sum\sum_{(k,\ell)} a_{k,\ell}$	381
$\sum_{j} c_j$	diagonal series corresponding to $\sum\sum_{(k,\ell)} a_{k,\ell}$	385
$\sum\sum_{(k,\ell)} c_{k,\ell} x^k y^\ell$	double power series around $(0,0)$	403
$\iint_{[a,\infty)\times[c,\infty)} f$	improper double integral of f on $[a,\infty)\times[c,\infty)$; also denoted by $\iint_{[a,\infty)\times[c,\infty)} f(s,t)d(s,t)$	416
$A(D)$	area of an unbounded subset D of $\mathbb{R}^2$	443

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