

# INDEX

- $\alpha$ -cut Theorem, 51, 99, 122
- $I$ -indexed product, 3
- $l$ -crisp, 81
- $s$ -crisp, 81
- $t$ -conorm, 40, 52, 155
- $t$ -norm, 40, 52, 154, 171
- 0–1 crisp, 48
- Adjoint, 13, 47, 65, 167
  - lower, 13, 15, 47, 65
  - upper, 13
  - upper left, 15, 47
  - upper right, 15, 47, 65
- Allegory, 57
  - distributive, 63
  - division, 65
  - representable, 58, 95
- Antimorphism, 11, 25, 75, 111–112, 126–127, 131, 134, 140
- Antisymmetric, 5
- Antitone, 6, 16, 25, 63, 68
- Approximate equality, 171
- Arrow category, 94
  - representable, 95
  - with cuts, 122
- Associative, 2, 9, 40, 163
- Atom, 22
- Bijection, 3, 60
- Bijjective, 2, 60
- Boolean algebra, 20, 85, 93, 99
- Boolean values, 3
- Brouwerian lattice, 18, 43, 68
  - complete, 43, 68
  - proper, 147
- Cartesian product, 2
- Category, 55
  - arrow, 94
  - representable, 95
  - with cuts, 122
- Dedekind, 68, 94
- Goguen, 128
  - representable, 139
  - locally small, 56, 68
- Class, 55
- Cloos, 40, 160
- Closed set, 23
- Clog, 40
- Closure operation, 11, 14
- Coclosure operation, 11
- Commutative, 9, 40
- Complement, 1, 20
- Complete, 10
- Completely irreducible, 22
- Composition, 2, 52, 57, 164
- Consistent, 9
- Continuous, 10, 24, 32, 40
- Contractive, 11

- Control rule, 176
- Converse, 58
- Converse operation, 58
- Conversion, 2
- Crisp, 48, 95
  - l*-crisp, 81
  - s*-crisp, 81
  - 0–1 crisp, 48
- Crispness, 48
- Cut, 48
- Decision module, 170, 179
- Dedekind category, 68, 94
  - simple, 71, 106
  - uniform, 70, 102
- Defuzzification, 170, 179
- Difference, 1
- Distributive, 17, 63
- Domain, 2
- Endofunction, 11, 24
- Endorelation, 2
- Environment, 87, 144
- Equation, 87, 144
- Extensive, 11
- Faithful, 56
- Field, 56
- Filter, 32
  - maximal, 33
  - prime, 33
  - complete, 35
  - principal, 32
- Fixed point, 23
  - induction, 25, 27, 29
  - least, 23, 27
  - theorem, 24
- Formula, 86
- Full, 56
- Function, 2
  - antitone, 6, 16, 25, 63, 68
  - contractive, 11
  - extensive, 11
  - idempotent, 11
  - linear, 56
  - monotone, 6, 11, 16
- Functor, 56
  - faithful, 56
  - full, 56
- Fuzzification, 142, 170, 176
- Fuzzy controller, 142, 169
- Galois correspondence, 13, 49
- Goguen category, 128
  - representable, 139
- Greatest element, 7
- Greatest lower bound, 7
- Heyting algebra, 18
- Heyting algebras, 18
- Homogenous relation, 2
- Homomorphism, 3, 9, 11, 19, 56, 58, 65, 68, 95, 130
  - Brouwerian lattice, 19
  - continuous, 10
  - lattice, 11
    - complete, 11
  - lower co-semilattice, 9
    - complete, 10
  - lower semilattice, 9
    - complete, 10
  - upper co-semilattice, 10
    - complete, 10
  - upper semilattice, 9
    - complete, 10
- Ideal relation, 48, 79
- Idempotent, 9, 11
- Identity, 2
- Image, 2
- Injection, 74, 173
  - crisp, 173
- Injective, 2, 60
- Intensifying modifier, 170
- Intersection, 1
- Interval, 185
- Irreducible, 22
  - completely, 22
- Isomorphic, 3
- Isomorphism, 3, 56
- Join, 8
- Kernel, 48, 94
- Kernel operation, 11, 14, 108
- Lattice-ordered operator set, 40
  - commutative, 40
  - complete, 40
- Lattice-ordered semigroup, 40, 52
  - commutative, 40
  - complete, 40
- Lattice, 10
  - atomic, 23
  - complete, 10
  - completely distributive, 17
  - completely downwards-distributive, 17
  - completely upwards-distributive, 17
  - distributive, 17, 63
  - proper, 35, 147
  - sublattice, 10
    - complete, 10
- Least element, 7
- Least upper bound, 7
- Left residual, 65
- Linear, 5
- Linear element, 8, 79
- Linear function, 56
- Linguistic entity, 170
- Linguistic modifier, 170
- Linguistic variable, 170
- Locally small, 56, 68

- Loos, 40, 52, 150
- Losg, 40, 163
- Lower adjoint, 65
- Lower bound, 7, 170
  - greatest, 7
- Lukasiewicz, 171
- Map, 60
- Meet, 7
- Modifier, 170
  - intensifying, 170
  - ordering-based, 170
  - weakening, 170
- Modular law, 58
- Monotone, 6, 9, 16
- Morphism, 55
- Natural numbers, 3
- Object, 55
- Ordering-based modifier, 170
- Ordering, 5, 170
- Partial identity, 61, 72
- Poset, 5
  - linear, 5
- Power set, 2
- Pre-functor, 57, 69, 130
- Predicate, 1, 24
  - admissible, 24
  - continuous, 24
- Projection, 75, 142
- Pseudo-complement, 20
- Pseudo-Representation Theorem, 136
- Range, 2
- Real numbers, 3
- Reflexive, 5
- Relation, 2, 43, 57
  - L*-fuzzy, 43
  - antisymmetric, 5
  - approximate equality, 171
  - bijjective, 2, 60
  - composition, 2
  - converse, 2
  - crisp, 48, 95
  - domain, 2
  - endorelation, 2
  - homogeneous, 2
  - ideal, 48, 79
  - identity, 2
  - image, 2
  - injective, 2, 60
  - ordering, 5
    - reversed, 6
  - partial identity, 61, 72
  - range, 2
  - reflexive, 5
  - scalar, 48, 79
    - linear, 79
  - sliceable, 110
  - source, 2, 43
    - surjective, 2, 60
    - symmetric idempotent, 75
    - target, 2, 43
    - total, 2, 60
    - transitive, 5
      - univalent, 2, 60
- Relational product, 75, 142
- Relational sum, 74, 142
- Relative pseudo-complement, 18
- Representable, 58, 95, 139
- Residual
  - left, 65
  - right, 65
- Residuated operations, 15, 18, 47, 167
- Reversed ordering, 6
- Right residual, 65
- Rule base, 170, 176
- Scalar, 48, 79
  - linear, 79
- Semilattice, 7
  - complete, 7
  - lower, 7
  - upper, 8
- Set-theoretic injection, 74
- Set-theoretic projection, 75
- Set, 1
  - cartesian product, 2
  - closed, 23
  - complement, 1
  - comprehension, 1
  - difference, 1
  - intersection, 1
  - power set, 2
  - union, 1
- Simple, 71, 106
- Sliceable, 110
- Source, 2, 43
- Splitting, 75, 142
- Sublattice, 10
- Subsemilattice, 8
  - lower, 8
  - upper, 8
- Support, 48, 94
- Surjective, 2, 60
- Symmetric idempotent relation, 75
- Symmetric quotient, 65
- Target, 2, 43
- Tarski-rule, 70
- Term, 86
- Total, 2, 60
- Transitive, 5
- Triangular fuzzy set, 171
- Ultrafilter, 33
- Uniform, 70, 102
- Union, 1
- Unit, 75
- Unit interval, 3

Univalent, 2, 60

Upper bound, 7, 170

  least, 7

Upper right adjoint, 65

Validity, 88

Value, 88

Vector space, 56

Weakening modifier, 170

Zermelo-Fraenkel, 1, 56

ZF, 1, 56

ZFC, 1

# SYMBOLS

$x \in A$	1
$A \subseteq B$	1
$\mathfrak{P}$	1
$\{x \in A \mid \mathfrak{P}(x)\}$	1
$A \cup B$	1, 43
$A \cap B$	1, 43
$A \setminus B$	1
$\overline{A}, \bar{x}$	1, 20
$A \times B$	2, 74
$\mathcal{P}(A)$	2
$R : A \rightarrow B$	2
$R^T$	2, 43
$R \circ S$	2, 43
$R(x, y)$	2
$\mathbb{I}_A$	2, 44, 55
$\text{ran}(R)$	2
$\text{dom}(R)$	2
$f(x), f(A)$	2
$A \rightarrow B$	2
$f^{-1}$	3
$\prod_{i \in I} A_i$	3

$\mathbb{B}$	3
$\mathbb{N}$	3
$\mathbb{N}^\infty$	3, 23
$\mathbb{R}$	3
$[0, 1]$	3
$\leq$	5
$\preceq$	6
$P_1 \preceq P_2$	6
$P_1 \succeq P_2$	6
$P_1 \xrightarrow{*} P_2$	6
$\mathcal{L}$	7
$x \wedge y$	7
$x \vee y$	8
$0, 1$	8
$x \nabla y$	12, 27
$x:y$	18
$\neg x$	20
$\text{At}(\mathcal{L})$	23
$\mu_f(a)$	23
$X_{f,a}$	24
$\mathcal{L}_1 \xrightarrow{\text{anti}} \mathcal{L}_2$	25

$\tau(f)$	26	$\mathcal{R}^{\mathcal{L}}$	78
$\varphi(f)$	27	$\alpha_A$	79
$\mathcal{F}_{\mathcal{L}}$	35	$\text{Sc}_{\mathcal{R}}(A)$	79
$\psi$	36	$\neg:\alpha_A$	79
$\vartheta(f)$	37	$\phi, \phi^{-1}$	80
$(\mathcal{L}, *, e, z)$	40	$R^s$	81
$\perp_{AB}$	44, 63	$R^l$	81
$\prod_{AB}$	44, 68	$\Phi_s$	81
$S \cdot R$	47	$\Phi_l$	81
$Q \cdot S$	47	$\sigma[A/a]$	87
$\alpha_A^u$	48	$\sigma[R/r : a \rightarrow b]$	87
$R_u$	48	$\mathcal{V}_{\mathcal{R}}(t)(\sigma)$	88
$R^{\downarrow}$	48, 94	$\mathcal{R} \models_{\sigma} \Theta$	88
$R^{\uparrow}$	48, 94	$\mathcal{A}$	94
$Q \cap_* R$	52	$\mathcal{A}^{\uparrow}$	95
$Q \circ_* S$	52	$\text{Sc}[\mathcal{A}]$	104
$\mathcal{C}[A, B]$	55	$\varrho$	107
$f; g$	55	$\hat{F}$	119
<b>Set</b>	56	$\mathcal{G}$	128
<b>Rel</b>	56	$P_{\mathcal{G}}, P_{\mathcal{G}}^{-1}$	136
<b>[0, 1]-Rel</b>	56	$Q \otimes_* R$	151
<b><math>\mathcal{L}</math>-Rel</b>	56	$\alpha_A \tilde{\otimes}_* \beta_A$	153
<b>PO</b>	56	$Q \triangleleft_*^{\otimes} R$	168
<b>Vct<math>_{\mathbb{F}}</math></b>	56	$R \triangleright_*^{\otimes} S$	168
<b>ZF</b>	56	greater than $_*(E, M)$	171
$F, F_{Obj}, F_{Mor}$	56	less than $_*(E, M)$	171
$\mathcal{R}$	57	$t_L$	171
$Q \sqcap R$	57	extremely $_*(M)$	172
$\sqsubseteq$	57	very $_*(M)$	172
$Q \sim$	58	more or less $_*(M)$	172
$\mathcal{R}_{\geq}^P$	63	roughly $_*(M)$	172
$Q \sqcup R$	63	very $_*(M)$	172
$S/R$	65	roughly $_*(M)$	172
$Q \setminus S$	65	$\mathcal{P}_{\emptyset}(A)$	181
$\text{syQ}(Q, R)$	65	$\varepsilon_A^{\emptyset}$	181
$\sum_{i \in I} A_i, A + B$	74	$[x.y]$	185
<b>I</b>	75		

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