A

Selected Answers to Exercises

Answers to Exercises from Chapter 2

Answer to Exercise 2.5 We give an informal argument using a formal proof concept: that of induction over n. The case $n = 0$ is straightforward ($E^0_B\varphi$ is φ and this is false only if $\neg\varphi$ holds, the latter having an empty sequence of $\hat{K}$-operators.) Let us also consider the case $n = 1$ separately, since we will use it in the induction step. $E^1_B\varphi$ is false, if and only if not everybody from B knows that φ, or, in other words, there is an agent a^1 in B that considers $\neg\varphi$ possible: $\hat{K}_{a^1}\neg\varphi$.

Let us now assume that up to a specific n, we know that $E^n_B\varphi$ is false if and only if there is a sequence of agents names $a^1, a^2, \dots a^n$ ($a^i \in B, i \leq n$) such that $\hat{K}_{a^1}\hat{K}_{a^2}\dots\hat{K}_{a^n}\neg\varphi$ holds. Since the a^i's are only variables over names, we might phrase the induction hypothesis alternatively as:

$$E^n_B\varphi \text{ is false iff} \quad \text{(A.1)}$$
$$\text{there are } a^2, a^3, \dots a^{n+1} \text{ such that } \hat{K}_{a^2}\hat{K}_{a^3}\dots\hat{K}_{a^{n+1}}\neg\varphi \text{ holds}$$

Now consider the case $n+1$. $E^{n+1}_B\varphi$ is false if and only if $E_B E^n_B\varphi$ is false, or, more precisely, if $E^1_B E^n_B\varphi$ is false. Using our established result for 1 iteration, we have that $E^1_B E^n_B\varphi$ is false iff for some agent $a^1 \in B$, we have $\hat{K}^1_a\neg E^n_B\varphi$. We now can apply the induction hypothesis (A.1) to $E^n_B\varphi$ to conclude that $E^1_B E^n_B\varphi$ is false iff for some sequence of agent names $a^1, a^2 \dots a^{n+1}$ we have $\hat{K}_{a^1}\hat{K}_{a^2}\dots\hat{K}_{a^{n+1}}\neg\varphi$. □

Answer to Exercise 2.8

1. The formalisation of each item under 1 is the corresponding item under 2 (so 2 (a) formalises 1 (a), etc.).
2. Hint: have first a look at our proof of Equation (2.2) on page 20, or try Exercise 2.9 first. □

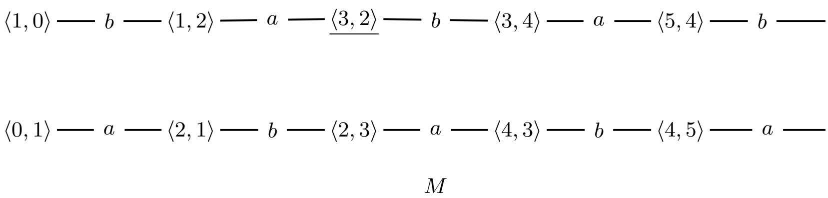

Figure A.1. Modelling Consecutive Numbers.

Answer to Exercise 2.10 Since the accessibility relations are all equivalences, we write $\sim_a$ and $\sim_b$, respectively.

1. See Figure A.1. Note that the model M consists of two disjunctive parts, depending on who wears an odd, and who wears an even number. Once Anne and Bill see the numbers on the other's forehead, they know in 'which part of the model they are'.
2. We show that $M, \langle 1,0\rangle \models K_a a_1 \wedge \mathsf{win_a}$. First of all, note that $\langle 1,0\rangle \sim_a t$ iff $t = \langle 1,0\rangle$: if the state would be $\langle 1,0\rangle$, Anne would see a 0, and exactly know her own number: 1! Hence, we have $M, \langle 1,0\rangle \models K_a a_1$. Since $\mathsf{win_a}$ is defined as a knowing the number on her head, we have the desired property.
3. The previous item demonstrates that $\langle 1,0\rangle$ qualifies for this. By symmetry, we also obtain $\langle 0,1\rangle$, in which Bill can win the game. These are also the only two states in which a player can win the game, since in every other state s, each player considers a state t possible in which his or her own number is different that that in s. When given that $\langle 3,2\rangle$ is the actual state, the two states $\langle 1,0\rangle$ and $\langle 0,1\rangle$ in which an agent can win, differ, in the following respect: $\langle 1,0\rangle$ can be reached, using the agents' access, from the state $\langle 3,2\rangle$, since we have $\langle 3,2\rangle \sim_a \langle 1,2\rangle$ and $\langle 1,2\rangle \sim_b \langle 1,0\rangle$. We can express this in the modal language by saying that in $\langle 3,2\rangle$ it holds that $\hat{K}_a\hat{K}_b(a_1 \wedge b_0)$, and hence $M, \langle 3,2\rangle \models \hat{K}_a\hat{K}_b\mathsf{win_a}$. On the other hand, from $\langle 3,2\rangle$, the state $\langle 0,1\rangle$ is not reachable: $M, \langle 3,2\rangle \models \neg\hat{K}_a\mathsf{win_b} \wedge \neg\hat{K}_b\mathsf{win_b} \wedge \neg\hat{K}_a\hat{K}_b\mathsf{win_b} \wedge \neg\hat{K}_b\hat{K}_a\mathsf{win_b}$. In fact, we have for any n that $M, \langle 3,2\rangle \models \neg\hat{K}_{x_1}\hat{K}_{x_2}\ldots\hat{K}_{x_n}\mathsf{win_b}$, $x_i \in \{a,b\}$.
4. We show items 4 and 7 of Example 2.4.
   To start with 4, note that $M, \langle 3,2\rangle \models K_a\varphi$ iff φ is true in both $\langle 3,2\rangle$ and $\langle 1,2\rangle$. In this case, $\varphi = K_b\psi$, and this is true in $\langle 3,2\rangle$ if ψ is true in $\langle 3,2\rangle$ and $\langle 3,4\rangle$, and $K_b\psi$ is true in $\langle 1,2\rangle$ if ψ is true in both $\langle 1,2\rangle$ and $\langle 1,0\rangle$. Hence we have to check whether ψ holds in $\langle 3,2\rangle, \langle 3,4\rangle, \langle 1,2\rangle$ and $\langle 1,0\rangle$. In our case, $\psi = K_a\chi$, so we have to check whether χ holds in all states accessible for a from the four states just mentioned, hence in $\langle 3,2\rangle, \langle 1,2\rangle, \langle 3,4\rangle, \langle 5,4\rangle$ and $\langle 1,0\rangle$ (remember that every state is accessible to itself, for every agent). Now, in our case, $\chi = (b_0 \vee b_2 \vee b_4)$, which is indeed true in those 5 states, so that we have verified $M, \langle 3,2\rangle \models K_aK_bK_a(b_0 \vee b_2 \vee b_4)$.

Regarding item 7, we have to demonstrate both $M, \langle 3,2\rangle \models E_{\{a,b\}}\neg a_5$ and $M, \langle 3,2\rangle \models \neg E_{\{a,b\}}E_{\{a,b\}}\neg a_5$. For the first, note that all the states that a considers possible in $\langle 3,2\rangle$ are $\langle 3,2\rangle$ itself and $\langle 1,2\rangle$. In both of them, $\neg a_5$ holds, hence $M, \langle 3,2\rangle \models K_a\neg a_5$. Similarly, since $\neg a_5$ holds in $\langle 3,2\rangle$ and $\langle 3,4\rangle$, which are all the states t for which $R_b\langle 3,2\rangle t$, we also have $M, \langle 3,2\rangle \models K_b\neg a_5$. The definition of everybody knows ($E_{\{a,b\}}\varphi$ means $K_a\varphi \wedge K_b\varphi$) then yields the desired result. To finally show that $M, \langle 3,2\rangle \models \neg E_{\{a,b\}}E_{\{a,b\}}\neg a_5$, note that $\neg E_{\{a,b\}}E_{\{a,b\}}\neg a_5$ is equivalent to $\neg E_{\{a,b\}}\neg\neg E_{\{a,b\}}\neg a_5$, which, by using the definition of $\hat{E}_{\{a,b\}}$, is equivalent to $\hat{E}_{\{a,b\}}\hat{E}_{\{a,b\}}a_5$. And $M, \langle 3,2\rangle \models \hat{E}_{\{a,b\}}\hat{E}_{\{a,b\}}a_5$ holds since we have all of $R_{E_{\{a,b\}}}\langle 3,2\rangle\langle 3,4\rangle$ and $R_{E_{\{a,b\}}}\langle 3,4\rangle\langle 5,4\rangle$ and $M, \langle 5,4\rangle \models a_5$ (for the definition of $R_{E_{\{a,b\}}}$, go to Definition 2.30). □

Answer to Exercise 2.18

1. Note that, by the axiom of propositional tautologies, we have the following:

$$(\varphi \to \chi) \to ((\chi \to \psi) \to (\varphi \to \psi))$$

   and then apply Modus Ponens to that and the two antecedents of HS.
2. Suppose that $\vdash \varphi \to \psi$. By necessitation, we derive $\vdash K_a(\varphi \to \psi)$. Distribution says that $\vdash K_a(\varphi \to \psi) \to (K_a\varphi \to K_a\psi)$. Applying modus ponens to what we have so far, gives $\vdash K_a\varphi \to K_a\psi$.
3. The equivalence between K and K' follows from the following instance of *Prop*:
   $(\alpha \to (\beta \to \gamma)) \leftrightarrow ((\alpha \wedge \beta) \to \gamma)$.
   For K'', from $\vdash (\varphi \wedge \psi) \to \varphi$ and the the second item of this exercise, we infer $\vdash K_a(\varphi \wedge \psi) \to K_a\varphi$. Idem for $\vdash K_a(\varphi \wedge \psi) \to K_a\psi$. The required conclusion now follows by applying modus ponens twice to the following instance of *Prop*:

$$\begin{array}{l}(K_a(\varphi \wedge \psi) \to K_a\varphi) \\ \to ((K_a(\varphi \wedge \psi) \to K_a\psi) \to (K_a(\varphi \wedge \psi) \to (K_a\varphi \wedge K_a\psi)))\end{array}$$ □

   For the converse, note that K' and hence K' follows from K'' by this instance of *Prop*:
   $(K_a(\varphi \wedge \psi) \to (K_a\varphi \wedge K_a\psi)) \to (K_a(\varphi \wedge \psi) \to (K_a\varphi \to K_a\psi))$
4. Here is a formal proof:

1	$\varphi \to (\psi \to (\varphi \wedge \psi))$	*Prop*
2	$K_a\varphi \to K_a(\psi \to (\varphi \wedge \psi))$	1, item 2.18.2
3	$(K_a\varphi \wedge K_a\psi) \to K_a\varphi$	*Prop*
4	$(K_a\varphi \wedge K_a\psi) \to K_a(\psi \to (\varphi \wedge \psi))$	$HS, 3, 2$
5	$K_a(\psi \to (\varphi \wedge \psi)) \to (K_a\psi \to K_a(\varphi \wedge \psi))$	K, item 2.18.3
6	$(K_a\varphi \wedge K_a\psi) \to (K_a\psi \to K_a(\varphi \wedge \psi))$	$HS, 4, 5$

7 $((K_a\varphi \wedge K_a\psi) \to (K_a\psi \to K_a(\varphi \wedge \psi))) \to$
$((K_a\varphi \wedge K_a\psi) \to K_a(\varphi \wedge \psi))$ — *Prop*
8 $(K_a\varphi \wedge K_a\psi) \to K_a(\varphi \wedge \psi)$ — $MP, 6, 7$

Answer to Exercise 2.37 For the first four items, see Exercise 2.1.2.1 in [148], and for the fifth item, consult Exercise 2.1.2.3 in the same reference. Finally, for the last item, for the 'only if' direction, use Theorem 2.38. Here we give a proof of the 'if' part. Assume $B' \subseteq B$. Note that we then immediately have $E_B\varphi \to E_{B'}\varphi(*)$, by definition of everybody knows, and propositional steps.

1	$C_B\varphi \to E_BC_B\varphi$	$mix, Prop, MP$
2	$E_BC_B\varphi \to E_{B'}C_B\varphi$	$(*)$
3	$C_B\varphi \to E_{B'}C_B\varphi$	$(1, 2, HS)$
4	$C_{B'}(C_B\varphi \to E_{B'}C_B\varphi)$	$3, necessitation$
5	$C_{B'}(C_B\varphi \to E_{B'}C_B\varphi) \to (C_B\varphi \to C_{B'}C_B\varphi)$	$induction for\ C_{B'}$
6	$C_B\varphi \to C_{B'}C_B\varphi$	$MP, 4, 5$
7	$C_B\varphi \to \varphi$	$mix, Prop, MP$
8	$C_{B'}(C_B\varphi \to \varphi)$	$7, necessitation$
9	$C_{B'}C_B\varphi \to C_{B'}\varphi$	$8, mix, distribution, MP$
10	$C_B\varphi \to C_{B'}\varphi$	$6, 9, HS$

Answer to Exercise 2.41 Note that **KD45** is an extension of **K**, so we can use the properties of Exercise 2.18.

1	$B(\varphi \wedge \neg B\varphi) \to (B\varphi \wedge B\neg B\varphi)$	K'', Exercise 2.18
2	$B\varphi \to BB\varphi$	*Axiom* 4
3	$(B\varphi \to BB\varphi) \to ((B\varphi \wedge B\neg B\varphi) \to (BB\varphi \wedge B\neg B\varphi))$	*Prop*
4	$(B\varphi \wedge B\neg B\varphi) \to (BB\varphi \wedge B\neg B\varphi))$	$2, 3, MP$
5	$B(\varphi \wedge \neg B\varphi) \to (BB\varphi \wedge B\neg B\varphi)$	$HS, 1, 4$
6	$(BB\varphi \wedge B\neg B\varphi) \to B(B\varphi \wedge \neg B\varphi)$	Exercise 2.18.4
7	$B(\varphi \wedge \neg B\varphi) \to B(B\varphi \wedge \neg B\varphi)$	$HS, 5, 6$
8	$(B\varphi \wedge \neg B\varphi) \to \bot$	*Prop*
9	$B(B\varphi \wedge \neg B\varphi) \to B\bot$	8, Exercise 2.18.2
10	$\neg B\bot \to \neg B(B\varphi \wedge \neg B\varphi)$	$9, Prop$
11	$\neg B(B\varphi \wedge \neg B\varphi)$	$MP, 10, D$

Answers to Exercises from Chapter 3

Answer to Exercise 3.8

1. Suppose $\oplus$ satisfies $(\mathcal{K} \oplus 1), (\mathcal{K} \oplus 2)$ and $(\mathcal{K} \oplus 3)$, and also $(\mathcal{K} \oplus min)^{\{1,2,3\}}$. We show that is also satisfies $(\mathcal{K} \oplus 4)$. Suppose $\mathcal{K}$ is a belief set and $\varphi \in \mathcal{K}$. By $(\mathcal{K} \oplus 3)$ we know that $\mathcal{K} \subseteq \mathcal{K} \oplus \varphi$. Now, if the

other direction does not hold, we can show that we are be able to find a smaller candidate for the expansion of $\mathcal{K}$ with φ. To be more precise, suppose $\mathcal{K} \oplus \varphi \not\subseteq \mathcal{K}$. Let $\mathcal{K} \oplus' \varphi = \mathcal{K}$, and for all other φ and $\mathcal{K}$, we let $\oplus'$ be equal to $\oplus$. It is easy to check that $\oplus'$ verifies $(\mathcal{K} \oplus 1)$ (we assumed that $\mathcal{K}$ is a belief set), $(\mathcal{K} \oplus 2)$ (we assume the antecedent of $(\mathcal{K} \oplus 4)$, i.e. $\varphi \in \mathcal{K}$) and $(\mathcal{K} \oplus 3)$ (we even have put $\mathcal{K}$ *equal* to $\mathcal{K} \oplus' \varphi$). We have $(\mathcal{K} \oplus' \varphi) = \mathcal{K} \subset \mathcal{K} \oplus \varphi$: contradicting the fact that $\oplus$ give the smallest set satisfying the first three postulates!.

2. Suppose $\mathcal{K} \subseteq \mathcal{H}$ but at the same time $\mathcal{K} \oplus \varphi \not\subseteq \mathcal{H} \oplus \varphi$, for a given $\mathcal{K}, \mathcal{H}$ and φ. Then for some ψ we have $\psi \in \mathcal{K} \oplus \varphi$, but $\psi \notin \mathcal{H} \oplus \varphi$. Now define $\oplus'$ by $\mathcal{K} \oplus' \varphi = (\mathcal{K} \oplus \varphi) \cap (\mathcal{H} \oplus \varphi)$ (and, for all 'other' $\mathcal{K}, \mathcal{H}$ and φ, we can define $\oplus'$ to equal $\oplus$). Since the intersection of two belief sets is a belief set, $\oplus'$ satisfies postulate $(\mathcal{K} \oplus 1)$. Postulate $(\mathcal{K} \oplus 2)$ also holds for it, since φ is an element of both $\mathcal{K} \oplus \varphi$ and $\mathcal{H} \oplus \varphi$. On top of this, $\oplus'$ satisfies $(\mathcal{K} \oplus 3)$: $\mathcal{K} \subseteq (\mathcal{K} \oplus \varphi) \cap (\mathcal{H} \oplus \varphi) = \mathcal{K} \oplus' \varphi$. Hence, $\oplus'$ satisfies the postulates $(\mathcal{K} \oplus 1), (\mathcal{K} \oplus 2)$ and $(\mathcal{K} \oplus 3)$. But since $(\mathcal{K} \oplus' \varphi) \subseteq (\mathcal{K} \oplus \varphi)$, and moreover $\mathcal{K} \oplus \varphi$ is not the smallest set satisfying $(\mathcal{K} \oplus 1), (\mathcal{K} \oplus 2)$ and $(\mathcal{K} \oplus 3)$. □

Answer to Exercise 3.9 This is done by an easy case-distinction:

$\varphi \in \mathcal{K}$ In this case, the property to be proven follows directly from $(\mathcal{K} \ominus 5)$.

$\varphi \notin \mathcal{K}$ Now, by $(\mathcal{K} \ominus 3)$, $\mathcal{K} \ominus \varphi = \mathcal{K}$, and, by $(\mathcal{K} \circledast 3)$, we also have $\mathcal{K} \subseteq \mathcal{K} \oplus \varphi$. Together this gives the desired $\mathcal{K} \subseteq (\mathcal{K} \ominus \varphi) \oplus \varphi$. □

Answer to Exercise 3.10 The $\subseteq$-direction is immediate from $(\mathcal{K} \ominus 2)$. For the other direction, we use $(\mathcal{K} \ominus 5)$: Since $\mathcal{K}$ is a belief base, we know that $\top \in \mathcal{K}$. So, by the postulate just referred to, we have $\mathcal{K} \subseteq (\mathcal{K} \ominus \top) \oplus \top$. Since, by $(\mathcal{K} \ominus 1)$, the set $\mathcal{K} \ominus \top$ is also a belief set, we have $\top \in (\mathcal{K} \ominus \top)$ and hence, by the postulate $(\mathcal{K} \oplus 4)$, we derive $(\mathcal{K} \ominus \top) \oplus \top = \mathcal{K} \ominus \top$, giving us $\mathcal{K} \subseteq (\mathcal{K} \ominus \top) \oplus \top = \mathcal{K} \oplus \top$.

Note that only the postulates $(\mathcal{K} \ominus 1)(\mathcal{K} \ominus 2)$ and $(\mathcal{K} \ominus 5)$ for contraction are needed, with the rather weak postulate $(\mathcal{K} \oplus 4)$ for expansion. □

Answer to Exercise 3.26 We first show the hint. Suppose that $\mathcal{K}' \in (\mathcal{K} \perp (\varphi \wedge \psi))$. This implies that:

1. $\mathcal{K}' \subseteq \mathcal{K}$
2. $\mathcal{K}' \nvdash \varphi \wedge \psi$
3. for any $\gamma \in \mathcal{K} \setminus \mathcal{K}'$, we have $\gamma \to (\varphi \wedge \psi) \in \mathcal{K}'$.

In order to show that $\mathcal{K}' \in (\mathcal{K} \perp \varphi) \cup (\mathcal{K} \perp \psi)$, notice, by item 2 above, that either $\mathcal{K}' \nvdash \varphi$, or $\mathcal{K}' \nvdash \psi$. Suppose, without lack of generality, that the first holds (*). Then we have:

1. $\mathcal{K}' \subseteq \mathcal{K}$ (immediately from item 1 above)
2. $\mathcal{K}' \nvdash \varphi$ (from (*))

3. for any $\gamma \in \mathcal{K} \setminus \mathcal{K}'$, we have $\gamma \to \varphi \in \mathcal{K}'$ (immediately from item 3 above and the fact that $\mathcal{K}'$ is a belief set).

In other words, we have $\mathcal{K}' \in (\mathcal{K} \bot \varphi)$, as required.

To prove the main claim, we have to show that, for any φ and ψ,

$$(\mathcal{K} \ominus \varphi \cap \mathcal{K} \ominus \psi) \subseteq \mathcal{K} \ominus (\varphi \wedge \psi) \tag{A.2}$$

In case we have one of the three situations in which $\vdash \varphi$ or $\vdash \psi$ or $\vdash \varphi \wedge \psi$, (A.2) follows immediately. This is seen as follows: if $\vdash \varphi \wedge \psi$, all three sets $\mathcal{K} \ominus \cdot$ in equation (A.2) are equal to $\mathcal{K}$, due to Exercise 3.20, and hence the equation is trivially true. If only $\vdash \varphi$, we have to show $\mathcal{K} \ominus \psi \subseteq \mathcal{K} \ominus (\varphi \wedge \psi)$. But this is obvious, since $\vdash \psi \leftrightarrow (\varphi \wedge \psi)$, so that we can apply Exercise 3.20 once more.

So, now we may assume $\not\vdash \varphi$ and $\not\vdash \psi$. Using the definition of contractions, showing that (A.2) holds is then equivalent to showing

$$\bigcap S(\mathcal{K} \bot \varphi) \cap \bigcap S(\mathcal{K} \bot \psi) \subseteq \bigcap S(\mathcal{K} \bot (\varphi \wedge \psi)) \tag{A.3}$$

Now, let $\mathcal{K}'$ be a belief set occurring in the left hand side of (A.3). This means that $\mathcal{K}'$ fails to imply φ, it fails to imply ψ, it is $\leq$-best among $\mathcal{K} \bot \varphi$ and also $\leq$-best among $\mathcal{K} \bot \psi$. Then, obviously, $\mathcal{K}'$ fails to imply $\varphi \wedge \psi$. Is $\mathcal{K}'$ also $\leq$-best among $\mathcal{K} \bot (\varphi \wedge \psi)$? Suppose not. This would mean there is a $\mathcal{K}'' \in (\mathcal{K} \bot (\varphi \wedge \psi))$, such that $\mathcal{K}'' \not\leq \mathcal{K}'$. But then, by the hint just proven, also $\mathcal{K}'' \in (\mathcal{K} \bot \varphi)$ or $\mathcal{K}'' \in (\mathcal{K} \bot \psi)$. But then, $\mathcal{K}'$ cannot be $\leq$-best in both $\mathcal{K} \bot \varphi$ and $\mathcal{K} \bot \psi$. □

Answer to Exercise 3.8

1. Suppose $\oplus$ satisfies $(\mathcal{K} \oplus 1), (\mathcal{K} \oplus 2)$ and $(\mathcal{K} \oplus 3)$, and also $(\mathcal{K} \oplus min)^{\{1,2,3\}}$. We show that is also satisfies $(\mathcal{K} \oplus 4)$. Suppose $\mathcal{K}$ is a belief set and $\varphi \in \mathcal{K}$. By $(\mathcal{K} \oplus 3)$ we know that $\mathcal{K} \subseteq \mathcal{K} \oplus \varphi$. Now, if the other direction does not hold, we can show that we are be able to find a smaller candidate for the expansion of $\mathcal{K}$ with φ. To be more precise, suppose $\mathcal{K} \oplus \varphi \not\subseteq \mathcal{K}$. Let $\mathcal{K} \oplus' \varphi = \mathcal{K}$, and for all other φ and $\mathcal{K}$, we let $\oplus'$ be equal to $\oplus$. It is easy to check that $\oplus'$ verifies $(\mathcal{K} \oplus 1)$ (we assumed that $\mathcal{K}$ is a belief set), $(\mathcal{K} \oplus 2)$ (we assume the antecedent of $(\mathcal{K} \oplus 4)$, i.e. $\varphi \in \mathcal{K}$) and $(\mathcal{K} \oplus 3)$ (we even have put $\mathcal{K}$ *equal* to $\mathcal{K} \oplus' \varphi$). We have $(\mathcal{K} \oplus' \varphi) = \mathcal{K} \subset \mathcal{K} \oplus \varphi$: contradicting the fact that $\oplus$ give the smallest set satisfying the first three postulates!.
2. Suppose $\mathcal{K} \subseteq \mathcal{H}$ but at the same time $\mathcal{K} \oplus \varphi \not\subseteq \mathcal{H} \oplus \varphi$, for a given $\mathcal{K}, \mathcal{H}$ and φ. Then for some ψ we have $\psi \in \mathcal{K} \oplus \varphi$, but $\psi \notin \mathcal{H} \oplus \varphi$. Now define $\oplus'$ by $\mathcal{K} \oplus' \varphi = (\mathcal{K} \oplus \varphi) \cap (\mathcal{H} \oplus \varphi)$ (and, for all 'other' $\mathcal{K}, \mathcal{H}$ and φ, we can define $\oplus'$ to equal $\oplus$). Since the intersection of two belief sets is a belief set, $\oplus'$ satisfies postulate $(\mathcal{K} \oplus 1)$. Postulate $(\mathcal{K} \oplus 2)$ also holds for it, since φ is an element of both $\mathcal{K} \oplus \varphi$ and $\mathcal{H} \oplus \varphi$. On top of this, $\oplus'$ satisfies $(\mathcal{K} \oplus 3)$: $\mathcal{K} \subseteq (\mathcal{K} \oplus \varphi) \cap (\mathcal{H} \oplus \varphi) = \mathcal{K} \oplus' \varphi$. Hence, $\oplus'$ satisfies the

postulates $(\mathcal{K} \oplus 1), (\mathcal{K} \oplus 2)$ and $(\mathcal{K} \oplus 3)$. But since $(\mathcal{K} \oplus' \varphi) \subseteq (\mathcal{K} \oplus \varphi)$, and moreover $\mathcal{K} \oplus \varphi$ is not the smallest set satisfying $(\mathcal{K} \oplus 1), (\mathcal{K} \oplus 2)$ and $(\mathcal{K} \oplus 3)$. □

Answers to Exercises from Chapter 4

Answer to Exercise 4.9 We show that $Hexa, 012 \models [\neg 1_a]\hat{K}_a K_b 0_a$:

We have that $Hexa, 012 \models [\neg 1_a]\hat{K}_a K_b 0_a$ iff ($Hexa, 012 \models \neg 1_a$ implies $Hexa|\neg 1_a, 012 \models \hat{K}_a K_b 0_a$). The premiss is satisfied as before. For the conclusion, $Hexa|\neg 1_a, 012 \models \hat{K}_a K_b 0_a$ iff there is a state s such that $012 \sim_a s$ and $Hexa|\neg 1_a, s \models K_b 0_a$. State $021 = s$ satisfies that: $012 \sim_a 021$ and $Hexa|\neg 1_a, 021 \models K_b 0_a$. The last is, because for all states, if $021 \sim_b s$, then $Hexa|\neg 1_a, s \models 0_a$. The only $\sim_b$-accessible state from 021 in $Hexa|\neg 1_a$ is 021 itself, and $021 \in V_{0_a} = \{012, 021\}$.

The other parts of this exercise are left to the reader. □

Answer to Exercise 4.15 For example, $Hexa, 012 \models [\neg 0_a]1_a$ but $Hexa, 012 \not\models \langle \neg 0_a \rangle 1_a$. For the first, we have that by definition

$$Hexa, 012 \models [\neg 0_a]1_a \quad \text{iff} \quad (Hexa, 012 \models \neg 0_a \text{ implies } Hexa|\neg 0_a, 012 \models 1_a)$$

This is true because $Hexa, 012 \not\models \neg 0_a$ (i.e., 'box'-type modal operators are satisfied if there are no accessible worlds at all). For the second, we have that by definition

$$Hexa, 012 \models \langle \neg 0_a \rangle 1_a \quad \text{iff} \quad (Hexa, 012 \models \neg 0_a \text{ and } Hexa|\neg 0_a, 012 \models 1_a)$$

This is false, because $Hexa, 012 \not\models \neg 0_a$ and therefore the conjunction is false (i.e., for 'diamond'-type modal operators to be satisfied there must at least be an accessible world — the announcement must be executable). □

Answer to Exercise 4.16 We do Proposition 4.13. We only need to prove one more equivalence:

$M, s \models \varphi \rightarrow \langle \varphi \rangle \psi$
$\Leftrightarrow$
$M, s \models \varphi$ implies $M, s \models \langle \varphi \rangle \psi$
$\Leftrightarrow$
$M, s \models \varphi$ implies ($M, s \models \varphi$ and $M|\varphi, s \models \psi$)
$\Leftrightarrow$ propositional
$M, s \models \varphi$ implies $M|\varphi, s \models \psi$
$\Leftrightarrow$
$M, s \models [\varphi]\psi$
□

Answer to Exercise 4.23 We prove that $[\varphi](\psi \rightarrow \chi) \leftrightarrow ([\varphi]\psi \rightarrow [\varphi]\chi)$. The direction $[\varphi](\psi \rightarrow \chi) \rightarrow ([\varphi]\psi \rightarrow [\varphi]\chi)$ will be obvious. For the other direction, let M and s be arbitrary and assume $M, s \models [\varphi]\psi \rightarrow [\varphi]\chi$. To prove that $M, s \models [\varphi](\psi \rightarrow \chi)$, we assume that $M, s \models \varphi$ and have to prove that $M|\varphi, s \models \psi \rightarrow \chi$. Therefore, suppose that $M|\varphi, s \models \psi$. If $M|\varphi, s \models \psi$ then $M, s \models [\varphi]\psi$. Using our assumption $M, s \models [\varphi]\psi \rightarrow [\varphi]\chi$, it follows that $M, s \models [\varphi]\chi$. From that and $M, s \models \varphi$ follows $M|\varphi, s \models \chi$. Therefore $M|\varphi, s \models \psi \rightarrow \chi$. Done. □

Answer to Exercise 4.25 Let M and s be arbitrary. Then we have:

$M, s \models \langle\varphi\rangle\neg\psi$
$\Leftrightarrow$
$M, s \models \varphi$ and $M|\varphi, s \models \neg\psi$
$\Leftrightarrow$ propositional
$M, s \models \varphi$ and ($M, s \models \varphi$ implies $M|\varphi, s \models \neg\psi$)
$\Leftrightarrow$
$M, s \models \varphi$ and $M, s \models [\varphi]\neg\psi$
$\Leftrightarrow$
$M, s \models \varphi \wedge [\varphi]\neg\psi$
$\Leftrightarrow$ duality
$M, s \models \varphi \wedge \neg\langle\varphi\rangle\psi$
□

Answer to Exercise 4.35 φ may be successful but not $\neg\varphi$:

For example $\neg(p \wedge \neg K_a p)$ is successful, but its negation $p \wedge \neg K_a p$ is—as we already know—unsuccessful. That $\neg(p \wedge \neg K_a p)$ is successful can be shown directly, but it also suffices to observe that $\neg(p \wedge \neg K_a p)$ is equivalent to $\neg p \vee K_a p$, and that that formula is in the language fragment that is preserved under taking arbitrary submodels, and therefore a fortiori in the unique submodel resulting from its announcement. See Proposition 4.37, later.

φ and ψ may be successful but not $[\varphi]\psi$:

Consider a model M with $\{s, t\}$ as the set of possible worlds. There is only one accessibility relation $\sim_a = \{(s, s), (s, t,), (t, s), (t, t)\}$ and only one propositional variable p, which is only true in t, i.e. $V_p = \{t\}$. We take the epistemic state (M, s). Now consider the formula $[\neg p \rightarrow K_a \neg p]\bot$. The subformulas $\neg p \rightarrow K_a \neg p$ and $\bot$ are both successful. However: $M, s \models \langle[\neg p \rightarrow K_a \neg p]\bot\rangle\neg[\neg p \rightarrow K_a \neg p]\bot$. This can be seen as follows. The formula $\neg p \rightarrow K_a \neg p$ is true in t, but false in s. Therefore $[p \rightarrow K_a p]\bot$ is trivially true in s. It is obviously false in t. So M restricted to this formula consists of s only. In this model $\neg p \rightarrow K_a \neg p$ is true. Therefore $\langle\neg p \rightarrow K_a \neg p\rangle\top$, which is equivalent to $[\neg p \rightarrow K_a \neg p]\bot$, is true there as well.

An example where φ and ψ are successful but not $\varphi \rightarrow \psi$ is left to the reader. □

Answer to Exercise 4.48 We can assume $\vdash \psi \leftrightarrow \chi$ throughout. We show that $\vdash \varphi(p/\psi) \leftrightarrow \varphi(p/\chi)$ by induction on φ.

If $\varphi = p$, then $\vdash p(p/\psi) \leftrightarrow p(p/\chi)$ equals $\vdash \psi \leftrightarrow \chi$ which was assumed.

If $\varphi = q \neq p$, then the substitution results in $\vdash q \leftrightarrow q$ which is a tautology.

If $\varphi = \neg\varphi$, then $\vdash (\neg\varphi)(p/\psi) \leftrightarrow (\neg\varphi)(p/\chi)$ becomes $\vdash \neg\varphi(p/\psi) \leftrightarrow \neg\varphi(p/\chi)$ and, for one direction of the equivalence, we use an instance of the tautology 'contraposition', the induction hypothesis, and modus ponens:

1	$(\varphi(p/\chi) \rightarrow \varphi(p/\psi)) \rightarrow (\neg\varphi(p/\psi) \rightarrow \neg\varphi(p/\chi))$	tautology
2	$\varphi(p/\chi) \rightarrow \varphi(p/\psi)$	induction hypothesis
3	$\neg\varphi(p/\psi) \rightarrow \neg\varphi(p/\chi)$	1,2, modus ponens

If $\varphi = \varphi_1 \wedge \varphi_2$, then we have to show $\vdash (\varphi_1 \wedge \varphi_2)(p/\chi) \leftrightarrow (\varphi_1 \wedge \varphi_2)(p/\psi)$, which is by definition $\vdash \varphi_1(p/\chi) \wedge \varphi_2(p/\chi) \leftrightarrow \varphi_1(p/\psi) \wedge \varphi_2(p/\psi)$, and our induction hypotheses are $\vdash \varphi_1(p/\chi) \leftrightarrow \varphi_1(p/\psi)$ and $\vdash \varphi_2(p/\chi) \leftrightarrow \varphi_2(p/\psi)$. This can be achieved by simple propositional reasoning.

If $\varphi = K_a\varphi$ and if $\varphi = [\varphi_1]\varphi_2$, we use necessitation. Details are left to the reader. □

Answer to Exercise 4.49 We show that $\vdash (\varphi \rightarrow [\varphi]\psi) \leftrightarrow [\varphi]\psi$ by induction on ψ. Simple steps have not been justified in the derivations, e.g., we often use Proposition 4.46 without reference. In (only) the case p we explicitly repeat the right side of the equivalence. Alternatively, one can make derivations with assumptions, but such are avoided in our minimal treatment of Hilbert-style axiomatics.

Case p:		
1	$[\varphi]p \leftrightarrow [\varphi]p$	
2	$(\varphi \rightarrow p) \leftrightarrow [\varphi]p$	
3	$(\varphi \rightarrow (\varphi \rightarrow p)) \leftrightarrow [\varphi]p$	
4	$(\varphi \rightarrow [\varphi]p) \leftrightarrow [\varphi]p$	
Case $\psi \wedge \chi$:		
1	$[\varphi](\psi \wedge \chi) \leftrightarrow [\varphi](\psi \wedge \chi)$	
2	$[\varphi]\psi \wedge [\varphi]\chi \leftrightarrow \dots$	announcement and conjunction
3	$(\varphi \rightarrow [\varphi]\psi) \wedge (\varphi \rightarrow [\varphi]\chi)$	induction
4	$\varphi \rightarrow ([\varphi]\psi \wedge [\varphi]\chi)$	propositional
5	$\varphi \rightarrow [\varphi](\psi \wedge \chi)$	announcement and conjunction
Case $\neg\psi$ (induction hypothesis is not used):		
1	$[\varphi]\neg\psi$	
2	$\varphi \rightarrow \neg[\varphi]\psi$	announcement and negation
3	$\varphi \rightarrow (\varphi \rightarrow \neg[\varphi]\psi)$	propositional
4	$\varphi \rightarrow [\varphi]\neg\psi$	
Case $K_a\psi$ (induction hypothesis not used):		
1	$[\varphi]K_a\psi$	

2 $\varphi \rightarrow K_a[\varphi]\psi$
3 $\varphi \rightarrow \varphi \rightarrow K_a[\varphi]\psi$
4 $\varphi \rightarrow [\varphi]K_a\psi$
Case $[\psi]\chi$:
1 $[\varphi][\psi]\chi$
2 $[\varphi \wedge [\varphi]\psi]\chi$ announcement composition
3 $(\varphi \wedge [\varphi]\psi) \rightarrow [\varphi \wedge [\varphi]\psi]\chi$ induction
4 $\varphi \rightarrow [\varphi]\psi \rightarrow [\varphi \wedge [\varphi]\psi]\chi$ propositional
5 $\varphi \rightarrow \varphi \rightarrow [\varphi]\psi \rightarrow [\varphi \wedge [\varphi]\psi]\chi$ propositional
6 $\varphi \rightarrow [\varphi][\psi]\chi$ similar to 1–4

Answer to Exercise 4.50 We use the equivalence of $\varphi \rightarrow \psi$ and $\neg(\varphi \wedge \neg\psi)$. As for other items of the proof of Proposition 4.46, the proof consists of a sequence of equivalences, and to improve readability we delete the right hand side. Line numbering and justifications are omitted — most steps are propositional.

$[\varphi](\psi \rightarrow \chi) \leftrightarrow [\varphi](\psi \rightarrow \chi)$
$[\varphi]\neg(\psi \wedge \neg\chi) \leftrightarrow \dots$
$[\varphi]\neg(\psi \wedge \neg\chi)$
$\varphi \rightarrow \neg[\varphi](\psi \wedge \neg\chi)$
$\varphi \rightarrow \neg([\varphi]\psi \wedge [\varphi]\neg\chi)$
$\varphi \rightarrow \neg([\varphi]\psi \wedge (\varphi \rightarrow \neg[\varphi]\chi))$
$\varphi \rightarrow (\neg[\varphi]\psi \vee (\varphi \wedge [\varphi]\chi))$
$\varphi \rightarrow ((\neg[\varphi]\psi \vee [\varphi]\chi) \wedge (\neg[\varphi]\psi \vee \varphi))$
$\varphi \rightarrow (([\varphi]\psi \rightarrow [\varphi]\chi) \wedge ([\varphi]\psi \rightarrow \varphi))$
$(\varphi \rightarrow [\varphi]\psi \rightarrow [\varphi]\chi) \wedge (\varphi \rightarrow [\varphi]\psi \rightarrow \varphi)$
$\varphi \rightarrow [\varphi]\psi \rightarrow [\varphi]\chi$
$[\varphi]\psi \rightarrow \varphi \rightarrow [\varphi]\chi$
$[\varphi]\psi \rightarrow [\varphi]\chi$
□

Answer to Exercise 4.55 Proof of $\vdash C_B(\varphi \rightarrow E_B\varphi) \rightarrow \varphi \rightarrow C_B\varphi$. We outline the derivation as follows:

First, note that a propositionally equivalent form of the induction axiom is $C_B(\varphi \rightarrow E_B\varphi) \wedge \varphi \rightarrow C_B\varphi$. Next, for $\varphi = \top$ the rule for announcement and common knowledge becomes: from $\chi \rightarrow \psi$ and $\chi \rightarrow E_B\chi$ follows $\chi \rightarrow C_B\psi$. This we will now apply in the following form: from $C_B(\varphi \rightarrow E_B\varphi) \wedge \varphi \rightarrow \varphi$ and $C_B(\varphi \rightarrow E_B\varphi) \wedge \varphi \rightarrow E_B(C_B(\varphi \rightarrow E_B\varphi) \wedge \varphi)$ follows $C_B(\varphi \rightarrow E_B\varphi) \wedge \varphi \rightarrow C_B\varphi$. From the two premises of this conclusion, the first one, $C_B(\varphi \rightarrow E_B\varphi) \wedge \varphi \rightarrow \varphi$, is a tautology. The second can be derived from $C_B(\varphi \rightarrow E_B\varphi) \wedge \varphi \rightarrow E_BC_B(\varphi \rightarrow E_B\varphi)$ (i) and $C_B(\varphi \rightarrow E_B\varphi) \wedge \varphi \rightarrow E_B\varphi$ (ii). Formula scheme (i) can also be written as $C_B(\varphi \rightarrow E_B\varphi) \rightarrow \varphi \rightarrow E_BC_B(\varphi \rightarrow E_B\varphi)$ which is a weakening of $C_B(\varphi \rightarrow E_B\varphi) \rightarrow E_BC_B(\varphi \rightarrow E_B\varphi)$ which makes it clear that this is an instance of 'use of common knowledge'. Whereas (ii) can also be written as $C_B(\varphi \to E_B\varphi) \to \varphi \to E_B\varphi$ which makes it another instance of 'use of common knowledge'. □

Answer to Exercise 4.57 We prove the generalisation of 'partial functionality' ($\vdash (\varphi \to [\varphi]\psi) \leftrightarrow [\varphi]\psi$). 'Substitution of equals' is left to the reader. 'Announcement and implication' did not require induction so remains valid as a matter of course.

We (only) need to add a case $C_A\psi$ to the proof. For the case $\psi = C_A\psi$ from the inductive proof of $\vdash (\varphi \to [\varphi]\psi) \leftrightarrow [\varphi]\psi$, we have to derive $\vdash (\varphi \to [\varphi]C_A\psi) \leftrightarrow [\varphi]C_A\psi$. The direction $\vdash [\varphi]C_A\psi \to (\varphi \to [\varphi]C_A\psi)$ is trivial.

We derive $\vdash (\varphi \to [\varphi]C_A\psi) \to [\varphi]C_A\psi$ by an application of the rule for announcement and common knowledge, namely, from $\vdash (\varphi \to [\varphi]C_A\psi) \to [\varphi]\psi$ and $\vdash (\varphi \to [\varphi]C_A\psi) \land \varphi \to E_A(\varphi \to [\varphi]C_A\psi)$.

$\vdash (\varphi \to [\varphi]C_A\psi) \to [\varphi]\psi$:

Use that by inductive hypothesis, $\vdash [\varphi]\psi \leftrightarrow (\varphi \to [\varphi]\psi)$, and observe that $\vdash (\varphi \to [\varphi]C_A\psi) \to (\varphi \to [\varphi]\psi)$ follows propositionally from $\vdash [\varphi]C_A\psi \to [\varphi]\psi$, which follows from 'use of common knowledge', 'necessitation for announcement' ($[\varphi]$), 'distribution of $[\varphi]$ over $\to$', and modus ponens.

$\vdash (\varphi \to [\varphi]C_A\psi) \land \varphi \to E_A(\varphi \to [\varphi]C_A\psi)$:

1	$C_A\psi \leftrightarrow E_AC_A\psi$	
2	$[\varphi]C_A\psi \leftrightarrow [\varphi]E_AC_A\psi$	necessitation and distribution, and propositional
3	$[\varphi]E_AC_A\psi \leftrightarrow (\varphi \to E_A[\varphi]C_A\psi)$	announcement and knowledge
4	$(\varphi \to E_A[\varphi]C_A\psi) \to (\varphi \to E_A(\varphi \to [\varphi]C_A\psi)$	propositional
5	$[\varphi]C_A\psi \to (\varphi \to E_A(\varphi \to [\varphi]C_A\psi)$	2–4, propositional
6	$([\varphi]C_A\psi \land \varphi) \to E_A(\varphi \to [\varphi]C_A\psi)$	propositional
7	$(\varphi \to [\varphi]C_A\psi) \land \varphi \to E_A(\varphi \to [\varphi]C_A\psi)$	

Answer to Exercise 4.72 We explain in detail why (Anne announces: "I have one of $\{012, 034, 056, 135, 246\}$," and Bill announces "Cath has card 6") is a solution. Let π ($= 012_a \lor 034_a \lor 056_a \lor 135_a \lor 246_a$) be Anne's announcement. We have to show that all of the following hold. Note that the common knowledge requirements are translated into *model* requirements of the commonly known formula:

$Russian, 012.345.6 \models K_a\pi$ *i*
$Russian|K_a\pi \models \mathsf{cignorant}$ *ii*
$Russian|K_a\pi, 012.345.6 \models K_b6_c$ *iii*
$Russian|K_a\pi|K_b6_c \models \mathsf{cignorant} \land \mathsf{bknowsas} \land \mathsf{aknowsbs}$ *iv*

We can now prove these requirements by checking their combinatorial equivalents in the model:

- Hand 012 is in $\{012, 034, 056, 135, 246\}$. Therefore i holds.
- If c holds 0, the remaining hands are $\{135, 246\}$. Each of $1, 2, ..., 6$ both occurs in at least one of $\{135, 246\}$ and is absent in at least one of those (1 occurs in 135 and is absent in 246, 2 occurs in 246 and is absent in 135, etc.). If c holds 1, the remaining hands are $\{034, 056, 246\}$. Each of $0, 2, ..., 6$ both occurs in at least one of $\{034, 056, 246\}$ and is absent in at least one of those (0 occurs in 034 and is absent in 246, ...), etc. for c holding 2, ..., 6. Therefore ii holds.
- From $\{012, 034, 056, 135, 246\}$, Bill can remove any hand that contains either 3, 4, or 5. This leaves only hand 012. In deal 012.345.6 Cath actually holds 6. Therefore iii holds.
- After both communications, the following deals are still possible:

$$\{012.345.6, 034.125.6, 135.024.6\}.$$

They are all different for Anne and for Bill, therefore $\mathsf{bknowsas}$ and $\mathsf{aknowsbs}$ hold. They are all the same for Cath. Each of $0, 1, ..., 5$ both occurs in at least one of $\{012, 034, 135\}$ and is absent in at least one of those. Therefore iv holds. □

Answers to Exercises from Chapter 5

Answer to Exercise 5.16 A side effect of the first action is that b learns that a learns p. But the knowledge b obtained after $L_{ab}?q$ is exactly the same as what he obtained after $L_{ab}L_{ab}?q$, so there should not be that side effect. □

Answer to Exercise 5.26 A simple counterexample is the non-deterministic action $(L_a?p \cup L_b?p)$. We have that $gr(L_a?p \cup L_b?p) = gr(L_a?p) \cap gr(L_b?p) = \emptyset$, but of course, for given (M, s) and (M', s') such that $(M, s)[\![L_a?p \cup L_b?p]\!](M', s')$, it is not the case that $gr(M', s') = \emptyset$. □

Answer to Exercise 5.29 By induction on α. We only show one direction of the crucial case 'learning': Suppose $(M, s) \models \langle L_B\alpha\rangle\top$. Let (M', s') be such that $(M, s)[\![L_B\alpha]\!](M', s')$. From the definition of action interpretation follows $(M, s)[\![\alpha]\!]s'$. Therefore $(M, s) \models \langle\alpha\rangle\top$. By induction, $(M, s) \models \mathsf{pre}(\alpha)$, By definition $\mathsf{pre}(L_B\alpha) = \mathsf{pre}(\alpha)$. Therefore $(M, s) \models \mathsf{pre}(L_B\alpha)$. □

Answer to Exercise 5.39 We now compute the interpretation of the show action in detail, to give one more example of the semantics of epistemic actions. We will refer repeatedly to Definition 5.13 on page 116. We first apply the clause for 'learning' in that definition. To interpret $\mathsf{show} = L_{abc}(!L_{ab}?0_a \cup L_{ab}?1_a \cup L_{ab}?2_a)$ in $(Hexa, 012)$, we interpret the type $L_{ab}?0_a \cup L_{ab}?1_a \cup L_{ab}?2_a$ of the action bound by L_{abc} in any state of the domain of $Hexa$ that is $\{a, b, c\}$-accessible from state 012. Specifically: we interpret $L_{ab}?0_a \cup L_{ab}?1_a \cup$

$L_{ab}?2_a$ in all states of *Hexa*. The resulting states will make up the domain of $(Hexa, 012)[\![\mathsf{show}]\!]$. We then compute access on that domain.

Action $L_{ab}?0_a \cup L_{ab}?1_a \cup L_{ab}?2_a$ has a nonempty interpretation in *any* state of *Hexa*. By applying clause $\cup$ in Definition 5.13, $L_{ab}?0_a \cup L_{ab}?1_a \cup L_{ab}?2_a$ can be interpreted in $(Hexa, 012)$ because $L_{ab}?0_a$ can be interpreted in that state. Similarly, $L_{ab}?0_a \cup L_{ab}?1_a \cup L_{ab}?2_a$ can be interpreted in $(Hexa, 201)$ because $L_{ab}?2_a$ can be interpreted in that state. We compute the first.

Again, we apply clause 'learning' in the definition. To interpret $L_{ab}?0_a$ in $(Hexa, 012)$, we interpret $?0_a$ in any state of *Hexa* that is $\{a, b\}$-accessible from 012, i.e., in all states of *Hexa*. The interpretation is not empty when Anne holds 0, i.e., in $(Hexa, 012)$ and in $(Hexa, 021)$. We compute the first.

We now apply clause 'test' ('$?\varphi$') of the definition. State $(Hexa, 012)[\![?0_a]\!]$ is the restriction of *Hexa* to states where 0_a holds, i.e., 012 and 021, with empty access, and with point 012. Figure A.2 pictures the result.

Having unravelled the interpretation of show to that of its action constituents, we can now compute access for the so far incomplete other stages of the interpretation. The epistemic state $(Hexa, 012)[\![?0_a]\!]$ is one of the factual states of the domain of $(Hexa, 012)[\![L_{ab}?0_a]\!]$. This is visualised in Figure A.2 by linking the large box containing $(Hexa, 012)[\![?0_a]\!]$ with the small box

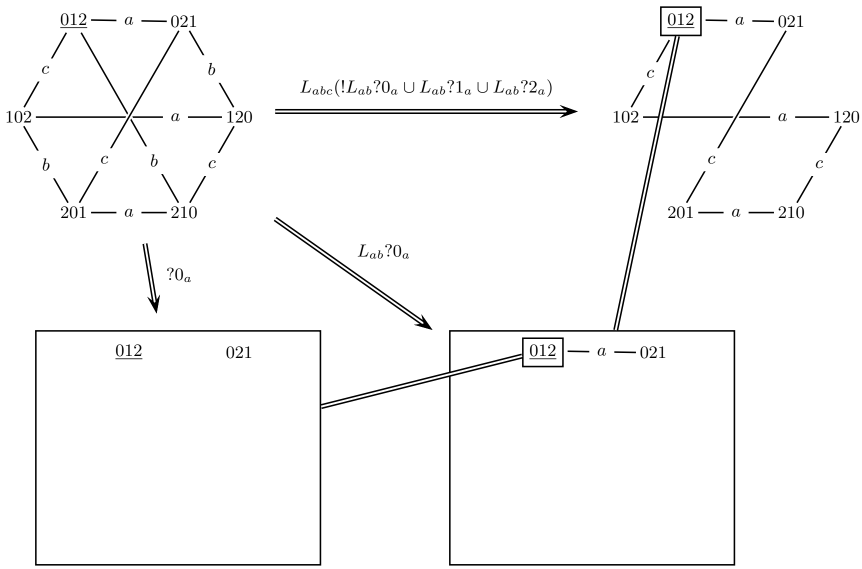

Figure A.2. Stages in the computation of $(Hexa, 012)[\![\mathsf{show}]\!]$. The linked frames visually emphasise identical objects: large frames enclose states that reappear as small framed worlds in the next stage of the computation.

containing an underlined 012. The epistemic state $(Hexa, 012)[\![?0_a]\!]$ is also the point of the epistemic state $(Hexa, 012)[\![L_{ab}?0_a]\!]$. The other factual state of that epistemic state is $(Hexa, 021)[\![?0_a]\!]$. As Anne does not occur in (the group of) either of these, and as their origins under the interpretation of $?0_a$ are the same to Anne—$012 \sim_a 021$ in *Hexa*—we may conclude that $(Hexa, 012)[\![?0_a]\!] \sim_a (Hexa, 021)[\![?0_a]\!]$ in $(Hexa, 012)[\![L_{ab}?0_a]\!]$. For the same reason, both states are reflexive for both Anne and Bill in $(Hexa, 012)[\![L_{ab}?0_a]\!]$. Access for Bill is different: $(Hexa, 012)[\![?0_a]\!] \not\sim_b (Hexa, 021)[\![?0_a]\!]$, because in $012 \not\sim_b 021$ in *Hexa*. In other words: even though Bill does not occur either in the groups of epistemic states $(Hexa, 012)[\![?0_a]\!]$ and $(Hexa, 021)[\![?0_a]\!]$, he can tell them apart in $(Hexa, 012)[\![L_{ab}?0_a]\!]$, because he already could tell deal 012 apart from deal 021 (in 012 he holds card 1, whereas in 021 he holds card 2). The valuation of atoms does not change as a result of action execution. Therefore state $(Hexa, 012)[\![?0_a]\!]$ is named 012 again, and state $(Hexa, 021)[\![?0_a]\!]$ is named 021 again, in Figure A.2 that pictures the result. Indeed, these names were convenient mnemonic devices 'showing' what facts are true in a state.

Similarly to the computation of $(Hexa, 012)[\![L_{ab}?0_a]\!]$, compute the five other epistemic states where Anne and Bill learn Anne's card. These six together form the domain of $(Hexa, 012)[\![\mathsf{show}]\!]$. We compute access for the players on this domain in some typical cases. Again, reflexivity trivially follows for all states. Concerning Anne, we have that $(Hexa, 012)[\![L_{ab}?0_a]\!] \sim_a (Hexa, 021)[\![L_{ab}?0_a]\!]$ as factual states, because $012 \sim_a 021$ and also, as epistemic states, $(Hexa, 012)[\![L_{ab}?0_a]\!] \sim_a (Hexa, 021)[\![L_{ab}?0_a]\!]$. The two epistemic states in the latter are the same for Anne, because the points of these states are the same for Anne (here we have applied Definition 5.12 in the domain of the model underlying epistemic state $(Hexa, 012)[\![L_{ab}?0_a]\!]$). Note that from "$(Hexa, 012)[\![L_{ab}?0_a]\!] \sim_a (Hexa, 021)[\![L_{ab}?0_a]\!]$ as epistemic states" and the fact that a is in the group of both epistemic states, already follows that $012 \sim_a 021$. The first condition in the clause for computing access is superfluous when the agent occurs in the epistemic states that are being compared.

Concerning Cath, we have $(Hexa, 012)[\![L_{ab}?0_a]\!] \sim_c (Hexa, 102)[\![L_{ab}?1_a]\!]$, because $c \notin \{a, b\}$ (so that they are, as epistemic states, the same for Cath) and $012 \sim_c 102$ in *Hexa*. In this case, unlike the access we just computed for Anne, the condition $012 \sim_c 102$ is essential.

Concerning Bill, we have that $(Hexa, 012)[\![L_{ab}?0_a]\!] \not\sim_b (Hexa, 210)[\![L_{ab}?2_a]\!]$ as factual states, because Bill occurs in both and $(Hexa, 012)[\![L_{ab}?0_a]\!] \not\sim_b (Hexa, 210)[\![L_{ab}?2_a]\!]$ as epistemic states. The last is, because we cannot by $(\sim_b)$ shifting the point of the latter find an epistemic state that is bisimilar to the former. This is obvious, as the underlying models of $(Hexa, 012)[\![L_{ab}?0_a]\!]$ and $(Hexa, 210)[\![L_{ab}?2_a]\!]$ have a different structure.

Again, the valuation of atoms in the factual states of the resulting epistemic state $(Hexa, 012)[\![\mathsf{show}]\!]$ does not change. Therefore, factual state $(Hexa, 012)[\![L_{ab}?0_a]\!]$ is named 012 again in Figure A.2, etc. The point of $(Hexa, 012)[\![\mathsf{show}]\!]$ is $(Hexa, 012)[\![L_{ab}?0_a]\!]$, because the unique epistemic state such that $(Hexa, 012)[\![L_{ab}?0_a \cup L_{ab}?1_a \cup L_{ab}?2_a]\!]$ is $(Hexa, 012)[\![L_{ab}?0_a]\!]$ (see

Definition 5.13 again: the point of an epistemic state that is the result of executing $L_B\alpha$ is an epistemic state that is the result of executing α). We have now completed the interpretation. Figure A.2 pictures the result.

In any world of the resulting model, Bill knows the deal of cards. Anne does not know the cards of Bill and Cath, although Anne knows that Bill knows it. Cath also knows that Bill knows the deal of cards. □

Answer to Exercise 5.42 A counterexample to the first is the first minimal sequence, where agent 1 makes the same call twice to agents over 4. A counterexample to the second is, e.g., call 14 in the sequence starting with $12, 34, 13, 24, 14, \ldots$. □

Answers to Exercises from Chapter 6

Answer to Exercise 6.4 Let $\boldsymbol{\varphi}_n$ and $\boldsymbol{\alpha}_n$ be the set of formulas and actions, respectively, constructed at step n of the inductive construction in Definition 6.3. The first steps of the inductive construction deliver the following—in $\boldsymbol{\alpha}_2$ the expression (F, s) stands for an arbitrary finite pointed frame.

$$\boldsymbol{\varphi}_0 = \emptyset$$
$$\boldsymbol{\alpha}_0 = \emptyset$$
$$\boldsymbol{\varphi}_1 = \{p\}$$
$$\boldsymbol{\alpha}_1 = \emptyset$$
$$\boldsymbol{\varphi}_2 = \{p, \neg p, p \wedge p, K_a p, K_b p, C_a p, C_b p, C_{ab} p\}$$
$$\boldsymbol{\alpha}_2 = \{(\langle F, \mathsf{pre}\rangle, \mathsf{s}) \mid \mathsf{pre}(\mathsf{s}') = p \text{ for all } \mathsf{s}' \in \mathcal{D}(F)\}$$

The set of action models constructed in $\boldsymbol{\alpha}_2$ is already infinite, as there are an infinite number of action frames. Note that at this stage as preconditions are only formulas allowed that have already been constructed in $\boldsymbol{\varphi}_1$, in other words, only p. After stages $\boldsymbol{\varphi}_2$ and $\boldsymbol{\alpha}_2$, matters get out of hand quickly. For example, $\boldsymbol{\varphi}_3$ contains an infinite number of expressions $[\langle F, \mathsf{pre}\rangle, \mathsf{s}]\varphi$ with φ one of the eight formulas in $\boldsymbol{\varphi}_2$, and $(\langle F, \mathsf{pre}\rangle, \mathsf{s})$ one of the infinitely many action models constructed in step $\boldsymbol{\alpha}_2$. Set $\boldsymbol{\alpha}_3$ is the first where preconditions $\neg p$ are allowed, because $\neg p \in \boldsymbol{\varphi}_2$. This is indeed the level where we find the required epistemic action $(\mathsf{Read}, \mathsf{p})$, as the pointed frame np—b—$\underline{\mathsf{p}}$ is one of the (F, s) used in the inductive construction. □

Answer to Exercise 6.14 We show that $[\mathsf{pub}(\varphi)]\psi$ is equivalent to $[\varphi]\psi$. The remaining two items are left to the reader.

The action model $\mathsf{pub}(\varphi)$ is formally $(\langle\{\mathsf{pub}\}, \sim, \mathsf{pre}\rangle, \mathsf{pub})$ such that $\sim_a = \{(\mathsf{pub}, \mathsf{pub})\}$ for all $a \in A$, and $\mathsf{pre}(\mathsf{pub}) = \varphi$: it consists of a single state pub, that is publicly accessible, and with precondition φ. For convenience in this proof, we name the model $\langle\{\mathsf{pub}\}, \sim, \mathsf{pre}\rangle$: Pub. Let (M, s) be an arbitrary epistemic model. Either $M, s \not\models \varphi$, in which case both $M, s \models [\mathsf{pub}(\varphi)]\psi$ and

$M, s \models [\varphi]\psi$ by definition, or $M, s \models \varphi$. The model $M \otimes \mathsf{Pub}$ is constructed as follows:

Domain: Its domain consists of all pairs (t, pub) such that $M, t \models \varphi$. Note that, modulo naming of states, this is the set $[\![\varphi]\!]_M$—the set of all φ-states in M.

Access: For an arbitrary agent a, access $\sim_a$ is defined as $(s, \mathsf{pub}) \sim_a (s', \mathsf{pub})$ iff $s \sim_a s'$ and $\mathsf{pub} \sim_a \mathsf{pub}$. As $\mathsf{pub} \sim_a \mathsf{pub}$ is true, we have that $(s, \mathsf{pub}) \sim_a (s', \mathsf{pub})$ iff $s \sim_a s'$. In other words: access between states in the model $M \otimes \mathsf{Pub}$ is simply the restriction of access in M to the domain $[\![\varphi]\!]_M$.

Valuation: The valuation in $M \otimes \mathsf{Pub}$ remains unchanged, i.e., $s \in V_p$ iff $(s, \mathsf{pub}) \in V_p$.

Together this describes the model $M|\varphi$ as in public announcement logic. The truth definition

$$M, s \models [\mathsf{pub}(\varphi)]\psi \text{ iff for all } M', s' \colon (M, s)[\![\mathsf{pub}(\varphi)]\!](M', s') \text{ implies } M', s' \models \psi$$

is in this case

$$M, s \models [\mathsf{pub}(\varphi)]\psi \text{ iff } M, s \models \varphi \text{ implies } M \otimes \mathsf{Pub}, (s, \mathsf{pub}) \models \psi$$

We have just shown that this computes to

$$M, s \models \varphi \text{ implies } M|\varphi, (s, \mathsf{pub}) \models \psi$$

In other words, as the names of states are irrelevant

$$M, s \models \varphi \text{ implies } M|\varphi, s \models \psi$$

The last defines

$$M, s \models [\varphi]\psi \ .$$ □

Answer to Exercise 6.18 Let the agents be Anne, Bill, and Cath, and the cards 0, 1, and 2, and the actual card deal 012 (Anne holds 0, Bill holds 1, Cath holds 2).

- Anne says that she holds card 0: singleton action model with universal access for a, b, c and precondition 0_a.
- Anne shows card 0 to Bill: three-point action model with universal access for c and identity for a and b, preconditions 0_a, 1_a, and 2_a (for Anne holding 0, 1, and 2, respectively), and point 0_a.
- Following a request from Bill to tell him a card she does not hold, Anne whispers into Bill's ear that she does not hold card 2: as the previous description, but now with preconditions $\neg 0_a$, $\neg 1_a$, and $\neg 2_a$ (for Anne *not* holding 0, 1, and 2, respectively), and point $\neg 2_a$. □

Answer to Exercise 6.39 We provide a derivation of the second item.

1	$[\mathsf{Read},\mathsf{np}]\neg p \leftrightarrow (\neg p \rightarrow \neg[\mathsf{Read},\mathsf{np}]p)$	action and negation
2	$[\mathsf{Read},\mathsf{np}]p \leftrightarrow (\neg p \rightarrow p)$	$\mathsf{pre}(\mathsf{np}) = \neg p$, atomic permanence
3	$(\neg p \rightarrow p) \leftrightarrow p$	tautology
4	$[\mathsf{Read},\mathsf{np}]p \leftrightarrow p$	$2,3$, propositional
5	$\neg p \rightarrow \neg[\mathsf{Read},\mathsf{np}]p$	4, propositional
6	$[\mathsf{Read},\mathsf{np}]\neg p$	$1,5$, propositional
7	$K_a[\mathsf{Read},\mathsf{np}]\neg p$	6, necessitation for K_a
8	$\neg p \rightarrow K_a[\mathsf{Read},\mathsf{np}]\neg p$	7, weakening
9	$[\mathsf{Read},\mathsf{np}]K_a\neg p \leftrightarrow (\neg p \rightarrow K_a[\mathsf{Read},\mathsf{np}]\neg p)$	action and knowledge
10	$[\mathsf{Read},\mathsf{np}]K_a\neg p$	$8,9$, propositional
11	$[\mathsf{Read},\mathsf{np}](K_a p \vee K_a\neg p)$	10, taut. $\varphi \rightarrow (\psi \vee \varphi)$, nec., prop. steps
12	$[\mathsf{Read},\mathsf{p}]K_a p$	Example 6.38
13	$[\mathsf{Read},\mathsf{p}](K_a p \vee K_a\neg p)$	12, taut. $\varphi \rightarrow (\varphi \vee \psi)$, nec., prop. steps
14	$[\mathsf{Read},\mathsf{p}](K_a p \vee K_a\neg p) \wedge [\mathsf{Read},\mathsf{np}](K_a p \vee K_a\neg p)$	$11,13$, propositional
15	$[\mathsf{Read}](K_a p \vee K_a\neg p)$	14, non-determinism, propositional

In the last step, note that action model Read is defined as non-deterministic choice $(\mathsf{Read},\mathsf{p}) \cup (\mathsf{Read},\mathsf{np})$. □

Answer to Exercise 6.41 We give the descriptions in $\mathcal{L}_!^{\mathrm{act}}$ of the four actions given the restriction to 0...5. In all cases, the description for the general case cannot be made because it would be of infinite length. Or, in other words, because the domain of the corresponding action state is infinite. Action descriptions are constrained to have finite domains. (Allowing only a finite subset of a possibly infinite domain to be named, does not help either in this case.) Below, let i_a stand for 'Agent a is being told the natural number i; below, we give the types of action, the points will be obvious.

$$L_{ab}(\textstyle\bigcup_{i=0..5}?(i_a \wedge i_b))$$
$$L_{ab}(\textstyle\bigcup_{i=0..4}?(i_a \wedge (i+1)_b) \cup \bigcup_{i=1..5}?(i_a \wedge (i-1)_b))$$
$$L_{ab}(!L_a?4_a \cup \textstyle\bigcup_{i=0,1,2,3,5} L_a?i_a)$$
$$L_{ab}(!L_b?3_b \cup \textstyle\bigcup_{i=0,1,2,4,5} L_b?i_b)$$

The corresponding action models consist of: 25, 10, 6, and 6 points, respectively. Obviously, all these models are of infinite size in the general case. □

Answers to Exercises from Chapter 7

Answer to Exercise 7.15

1. Suppose $\varphi \in \Phi$. Suppose $\Gamma \vdash \varphi$. Since Γ is consistent, so is $\Gamma \cup \{\varphi\}$. Since Γ is also maximal in Φ, it must be the case that $\varphi \in \Gamma$.
2. Suppose $\neg\varphi \in \Phi$. Therefore $\varphi \in \Phi$. From left to right. Suppose that $\varphi \in \Gamma$. By consistency $\neg\varphi \notin \Gamma$.

From right to left. Suppose that $\neg\varphi \notin \Gamma$. By maximality it must then be the case that $\Gamma \cup \{\neg\varphi\}$ is inconsistent. Therefore $\Gamma \vdash \varphi$. By 1 of this lemma $\varphi \in \Gamma$.

3. Suppose $(\varphi \wedge \psi) \in \Phi$. From left to right. Suppose $(\varphi \wedge \psi) \in \Gamma$. Then $\Gamma \vdash \varphi$ and $\Gamma \vdash \psi$. Since Φ is closed under subformulas, also $\varphi \in \Phi$ and $\psi \in \Phi$. Therefore, by 1 of this lemma $\varphi \in \Gamma$ and $\psi \in \Gamma$.
   From right to left. Suppose $\varphi \in \Gamma$ and $\psi \in \Gamma$. Therefore $\Gamma \vdash (\varphi \wedge \psi)$. Therefore by 1 of this lemma $(\varphi \wedge \psi) \in \Gamma$.
4. Suppose that $\underline{\Gamma} \wedge \hat{K}_a\underline{\Delta}$ is consistent. Suppose that it is not the case that $\Gamma \sim_a^c \Delta$. Therefore, there is a formula φ such that
   a) $K_a\varphi \in \Gamma$ but $K_a\varphi \notin \Delta$, or
   b) $K_a\varphi \notin \Gamma$ but $K_a\varphi \in \Delta$.

   We proceed by these cases
   a) By 2 of this lemma and the fact that Φ is closed under single negations, $\neg K_a\varphi \in \Delta$. However, by positive introspection $\Gamma \vdash K_aK_a\varphi$. Note that $K_aK_a\varphi \wedge \hat{K}_a\neg K_a\varphi$ is inconsistent. However $\vdash \hat{K}_a\underline{\Delta} \to \hat{K}_a\neg K_a\varphi$. Therefore $\underline{\Gamma} \wedge \hat{K}_a\underline{\Delta}$ is inconsistent, contradicting our initial assumption.
   b) By 2 of this lemma and the fact that Φ is closed under single negations, $\neg K_a\varphi \in \Gamma$. However, $\vdash \hat{K}_a\underline{\Delta} \to \hat{K}_aK_a\varphi$ and $\vdash \hat{K}_aK_a\varphi \to K_a\varphi$. Therefore $\underline{\Gamma} \wedge \hat{K}_a\underline{\Delta}$ is inconsistent, contradicting our initial assumption. □

   In both cases we are led to a contradiction. Therefore $\Gamma \sim_a^c \Delta$.
5. From right to left is trivial by the truth axiom.
   From left to right. Suppose that $\{K_a\varphi \mid K_a\varphi \in \Gamma\} \vdash \psi$. Therefore $\vdash \bigwedge\{K_a\varphi \mid K_a\varphi \in \Gamma\} \to \psi$. By necessitation and distribution $\vdash \bigwedge\{K_aK_a\varphi \mid K_a\varphi \in \Gamma\} \to K_a\psi$. By positive introspection $\{K_a\varphi \mid K_a\varphi \in \Gamma\} \vdash \bigwedge\{K_aK_a\varphi \mid K_a\varphi \in \Gamma\}$. Therefore $\{K_a\varphi \mid K_a\varphi \in \Gamma\} \vdash K_a\varphi$.

Answer to Exercise 7.16 Suppose that $\bigvee\{\underline{\Gamma} \mid \Gamma$ is maximal consistent in $cl(\varphi)\}$ is not a tautology, i.e., that $\neg\bigvee\{\underline{\Gamma} \mid \Gamma$ is maximal consistent in $cl(\varphi)\}$ is consistent. Therefore $\bigwedge\{\neg\underline{\Gamma} \mid \Gamma$ is maximal consistent in $cl(\varphi)\}$ is consistent. Therefore $\bigwedge\{\bigvee\{\neg\varphi \mid \varphi \in \Gamma\} \mid \Gamma$ is maximal consistent in $cl(\varphi)\}$ is consistent. Therefore, for every maximal consistent Γ in $cl(\varphi)$ there is a formula $\varphi_\Gamma \in \Gamma$ such that $\{\neg\varphi_\Gamma \mid \Gamma$ is maximal consistent in $cl(\varphi)\}$ is consistent. Therefore, by the Lindenbaum Lemma (and the law of double negation) there is a maximal consistent set in $cl(\varphi)$ that is inconsistent with every maximal consistent set in $cl(\varphi)$. That is a contradiction. Therefore $\vdash \bigvee\{\underline{\Gamma} \mid \Gamma$ is maximal consistent in $cl(\varphi)\}$. □

Answer to Exercise 7.23

1. By induction on ψ.
   Base case If ψ is a propositional variable, its complexity is 1 and it is its only subformula.

Induction hypothesis $c(\psi) \geq c(\varphi)$ if $\varphi \in Sub(\psi)$ and $c(\chi) \geq c(\varphi)$ if $\varphi \in Sub(\chi)$.

Induction step We proceed by cases

negation Suppose that φ is a subformula of $\neg\psi$. Then φ is either $\neg\psi$ or a subformula of ψ. In the former case, we simply observe that the complexity of every formula is greater than or equal to its own complexity. In the latter case, the complexity of $\neg\psi$ equals $1+c(\psi)$. Therefore, if φ is a subformula of ψ it follows immediately from the induction hypothesis that $c(\psi) \geq c(\varphi)$.

conjunction Suppose that φ is a subformula of $\psi \wedge \chi$. Then φ is either $\psi \wedge \chi$ or it is a subformula of ψ or χ. Again in the former case, the complexity of every formula is greater than or equal to its own complexity. In the latter case the complexity of $\psi \wedge \chi$ equals $1 + \max(c(\varphi), c(\psi))$. Simple arithmetic and the induction hypothesis gives us that $c(\psi \wedge \chi) \geq c(\varphi)$.

individual epistemic operator This is completely analogous to the case for negation.

common knowledge This is also completely analogous to the case for negation.

public announcement Suppose that φ is a subformula of $[\psi]\chi$. Then φ is either $[\psi]\chi$ or it is a subformula of ψ or χ. Again, in the former case, the complexity of every formula is greater than or equal to its own complexity. In the latter case, the complexity of $[\psi]\chi$ equals $(4 + c(\psi)) \cdot c(\chi)$. Simple arithmetic and the induction hypothesis gives us that $c([\psi]\chi) \geq c(\varphi)$.

2. $$\begin{aligned} c([\varphi]p) &= (4 + c(\varphi)) \cdot 1 \\ &= 4 + c(\varphi) \end{aligned}$$
   and
   $$\begin{aligned} c(\varphi \to p) &= c(\neg(\varphi \wedge \neg p)) \\ &= 1 + c(\varphi \wedge \neg p) \\ &= 2 + \max(c(\varphi), 2) \end{aligned}$$
   The latter equals $2 + c(\varphi)$ or 3. Both are less than $4 + c(\varphi)$.
3. $$\begin{aligned} c([\varphi]\neg\psi) &= (4 + c(\varphi)) \cdot (1 + c(\psi)) \\ &= 4 + c(\varphi) + 4 \cdot c(\psi) + c(\varphi) \cdot c(\psi) \end{aligned}$$
   and
   $$\begin{aligned} c(\varphi \to \neg[\varphi]\psi) &= c(\neg(\varphi \wedge \neg\neg[\varphi]\psi)) \\ &= 1 + c(\varphi \wedge \neg\neg[\varphi]\psi) \\ &= 2 + \max(c(\varphi), 2 + ((4 + c(\varphi)) \cdot c(\psi))) \\ &= 2 + \max(c(\varphi), 2 + 4 \cdot c(\psi) + c(\varphi) \cdot c(\psi)) \end{aligned}$$
   The latter equals $2 + c(\varphi)$ or $4 + 4 \cdot c(\psi) + c(\varphi) \cdot c(\psi)$. Both are less than $4 + c(\varphi) + 4 \cdot c(\psi) + c(\varphi) \cdot c(\psi)$.
4. Assume, without loss of generality, that $c(\psi) \geq c(\chi)$. Then
   $$\begin{aligned} c([\varphi](\psi \wedge \chi)) &= (4 + c(\varphi)) \cdot (1 + \max(c(\psi), c(\chi))) \\ &= 4 + c(\varphi) + 4 \cdot \max(c(\psi), c(\chi)) + c(\varphi) \cdot \max(c(\psi), c(\chi)) \\ &= 4 + c(\varphi) + 4 \cdot c(\psi) + c(\varphi) \cdot c(\psi) \end{aligned}$$

and

$$\begin{aligned} c([\varphi]\psi \wedge [\varphi]\chi) &= 1 + \max((4 + c(\varphi)) \cdot c(\psi), (4 + c(\varphi)) \cdot c(\chi)) \\ &= 1 + ((4 + c(\varphi)) \cdot c(\psi)) \\ &= 1 + 4 \cdot c(\psi) + c(\varphi) \cdot c(\psi) \end{aligned}$$

The latter is less than the former

5. This case is completely analogous to the case for negation.
6. $$\begin{aligned} c([\varphi][\psi]\chi) &= (4 + c(\varphi)) \cdot (4 + c(\psi)) \cdot c(\chi) \\ &= (16 + 4 \cdot c(\varphi) + 4 \cdot c(\psi) + c(\varphi) \cdot c(\psi)) \cdot c(\chi) \end{aligned}$$

   and

   $$\begin{aligned} c([\varphi \wedge [\varphi]\psi]\chi) &= (4 + (1 + \max(c(\varphi), (4 + c(\varphi)) \cdot c(\psi)))) \cdot c(\chi) \\ &= (5 + ((4 + c(\varphi)) \cdot c(\psi))) \cdot c(\chi) \\ &= (5 + 4 \cdot c(\psi) + c(\varphi) \cdot c(\psi)) \cdot c(\chi) \end{aligned}$$

   The latter is less than the former. □

Answers to Exercises from Chapter 8

Answer to Exercise 8.23 From left to right. Suppose that $(M, s) \equiv_{\mathcal{L}_K} (M', s')$. Suppose that there is an n such that duplicator does not have a winning strategy for the n-round $\mathcal{L}_K$-game on (M, s) and (M', s'). By Theorem 8.21, there is a formula φ of depth at most n such that $(M, s) \models \varphi$ and $(M', s') \not\models \varphi$. This contradicts the initial assumption.

From right to left. Suppose that for all $n \in \mathbb{N}$ duplicator has a winning strategy for the n-round $\mathcal{L}_K$-game on (M, s) and (M', s'). Suppose that $(M, s) \not\equiv_{\mathcal{L}_K} (M', s')$. Therefore there is some formula φ such that $(M, s) \models \varphi$ and $(M', s') \not\models \varphi$. But according to the assumption duplicator has a winning strategy for the $d(\varphi)$-round $\mathcal{L}_K$-game on (M, s) and (M', s'). By Theorem 8.21, it should be the case that $(M, s) \models \varphi$ iff $(M', s') \models \varphi$, which contradicts our earlier conclusion. □

Answer to Exercise 8.25 Let t and t' be such that duplicator responds with t' if spoiler chooses t or duplicator responds with t' if spoiler chooses t. We have to show these states satisfy the atoms, forth, and back conditions.

The atoms condition follows straightforwardly. Since the strategy is a winning strategy for duplicator, the states must agree on atomic properties.

For the forth condition, suppose that there is a state u such that $t \sim_a u$. Then also $s \sim_a u$, because it is an *S5* relation. Therefore duplicator also has some response when spoiler chooses u. Let the response be u'. Of course $s' \sim'_a u'$, and therefore $t' \sim'_a u'$, and also (u, u') are in the bisimulation relation.

The case for the back condition is analogous to the case for the forth condition. □

Answer to Exercise 8.28 We show that the universal relation $S \times S'$ is a bisimulation. Given that $V = V' = \emptyset$, it follows immediately that any pair of states linked in this relation have the same atomic properties.

For the forth condition, take two states $s \in S$ and $s' \in S'$. Suppose that $s \sim_a t$. Given that the relations are *S5*, there is a t' such that $s' \sim'_a t'$. By definition (t, t') are in the universal relation.

The back case is analogous to the forth case. □

Answer to Exercise 8.42 We have to show that $\mathcal{L}_K(\{a\}) \preceq \mathcal{L}_{KC}(\{a\})$, and that $\mathcal{L}_K(\{a\}) \succeq \mathcal{L}_{KC}(\{a\})$. For the first it suffices to observe that $\mathcal{L}_K(\{a\})$ is a sublanguage of $\mathcal{L}_{KC}(\{a\})$. For the second take a formula $\varphi \in \mathcal{L}_{KC}(\{a\})$. Now replace every occurrence of C_a in φ with K_a. This yields a formula $\varphi' \in \mathcal{L}_K(\{a\})$. It follows that $\varphi \equiv \varphi'$ from the fact that in *S5* it is the case that $K_a\varphi \leftrightarrow C_a\varphi$, because $\sim_a = \sim_{\{a\}}$. □

Answer to Exercise 8.43 The formula $C_{ab}\neg K_a p$ is true in state 0 in all models $Spine(n)$, if n is odd, and false if n is even.

There is no such formula in $\mathcal{L}_K$. Suppose there is such a formula φ. Let $d(\varphi) = n$. It follows from Theorem 8.32 that $(Spine(n), 0) \equiv_{\mathcal{L}_K^n} (Spine(n+1), 0)$, which contradicts the assumption. □

Answer to Exercise 8.55 We have to show that $\mathcal{L}^-_{KC\otimes} \preceq \mathcal{L}_{KC\otimes}$ and that $\mathcal{L}_{KC\otimes} \preceq \mathcal{L}^-_{KC\otimes}$. The former is trivial, because $\mathcal{L}^-_{KC\otimes}$ is a sublanguage of $\mathcal{L}_{KC\otimes}$. For the latter we have to show that for every formula $\varphi \in \mathcal{L}_{KC\otimes}$, there is an equivalent formula $\psi \in \mathcal{L}^-_{KC\otimes}$ We prove this by induction on the number of $\cup$ operators in φ.

Base case If the number of $\cup$ operators is 0, then φ is already a formula in $\mathcal{L}^-_{KC\otimes}$.

Induction hypothesis For every $\varphi \in \mathcal{L}_{KC\otimes}$ it holds that if the number of $\cup$ operators in φ is less than or equal to i, then there is a formula $\psi \in \mathcal{L}^-_{KC\otimes}$ which is equivalent to it.

Induction step Suppose that the number of $\cup$ operators in φ is $i+1$. Take a subformula of φ of the form $[\alpha \cup \alpha']\chi$. This formula is equivalent to $[\alpha]\chi \wedge [\alpha']\chi$. The number of occurrences of $\cup$ in $[\alpha]\chi$ is less than $i+1$. Therefore, by the induction hypothesis, there is a formula $\xi \in \mathcal{L}^-_{KC\otimes}$, which is equivalent to it. The same holds for $[\alpha']\chi$, where we find an $\xi' \in \mathcal{L}^-_{KC\otimes}$. So $[\alpha \cup \alpha']\chi$ is equivalent to $\xi \wedge \xi'$ which is in $\mathcal{L}^-_{KC\otimes}$. If we substitute $\xi \wedge \xi$ for $[\alpha \cup \alpha']\chi$ in φ, we obtain a formula with less than $i+1$ occurrences of $\cup$. Therefore the induction hypothesis applies, and therefore there is some formula $\psi \in \mathcal{L}^-_{KC\otimes}$ which is equivalent to φ. □

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