PERSPECTIVE

In the course of this book, we have argued for the actuality of two traditions: that of Peircean pragmatism and Husserlian phenomenology. We have even argued that they share a considerable amount of valuable thought - particularly involving the Peircean notion of diagrammatical reasoning and the Husserlian conception of the synthetic a priori of material ontologies.

Roughly, these two ideas connect in two ways. Diagrammatical reasoning is a major road to the understanding of ontological structure – and diagrams, in turn, form a core notion in the material ontology of semiotics.

These ideas are developed in two steps – one purely philosophical, and one intervening in the material regions of biology, pictures, and literature.

The development of this meeting between Peirce and Husserl is thus only to a limited extent seen as an issue of history of ideas. Rather, it is taken to be an intervention in the actual discussions of phenomenology, ontology, and semiotics, of biosemiotics, pictures, and literature – and this is why the discussions of Peirce and Husserl have – all through the book – been embedded in a host of actual issues and discussions with other philosophers, scientists, and critics.

The resulting *diagrammatology* is intended to contributing to the survival of the enlightened idea of the study of the humanities as a rational, interdisciplinary endeavor – during a Dark Age at the humanist faculties strangely wedding constructivist irrationalism to glib marketing and political correctness.

APPENDIX: PEIRCE'S CONCEPTION OF CONTINUITY BETWEEN MATHEMATICS AND PHILOSOPHY

> Mathematical reasoning consists in thnking how things already remarked may be conceived as making a part of a hitherto unremarked system, especially by means of the introduction of the hypothesis of continuity where no continuity hitherto had been thought of . . .
>
> Peirce ('Spinoza's Ethic', NATION III, 85)

When René Thom claims, in the preface to his *Ésquisse d'une sémiophysique* (1988), to be the first philosopher of the continuum since Aristotle, then it is correct only to the extent that there are not indeed very many philosophers who have been interested in that concept – unlike the heavy interest in continuity in mathematics. But Thom's claim, in any case, not only overlooks figures like Leibniz, Kant, Veronese, and Weyl, but most conspicuously Peirce's effort in the field, especially in the mature versions of his thought in the years around 1900. The concept of continuity is central to the whole of Peirce's thought. 'Of all conceptions Continuity is by far the most difficult for Philosophy to handle' (R&LOT 242), writes Peirce in 1898, and consequently continuity is 'the master key which adepts tell us unlocks the arcana of philosophy,' (untitled manuscript, 1897, 1.163), in short it is '. . . the most difficult, the most important, the most worth study of all philosophical ideas.' ('The Categories', undated, NEM IV, 310).

It plays, in particular, a main part in Peirce's metaphysics and consequently – following Peirce's Kantian idea that metaphysics mirrors logic[479] – in Peirce's comprehensive logic. This is why it also becomes of seminal importance for what connects logic and metaphysics in Peirce: his conceptual realism. Peirce's 'extreme realism' is built on the concept of continuity. It is regrettable, thus, that excellent discussions of his realism – like Boler or Haas – only marginally involves continuity; this might be for editorial reasons, though, since some of the decisive texts have only been published since their books. Finally, the concept of continuity provides the basis – often overlooked – for Peirce's more well-known semiotics, in particular in relation to the central concept of the icon.

Continuity as a metaphysical concept, however, is intimately tied to the mathematical presentation of it in Peirce, following his architectonic system for the sciences, according to which mathematics is the first science.[480] Peirce was one of the first philosophers to see the problems in a consistent mathematical treatment of infinity and continuity[481]; according to himself his very first definitions in the field go back to 1867, but it became central from at least 1880 (Peirce's strongly mathematical period, when he held a position at the Johns Hopkins University from

1879–84), that is, before he could have known neither the works of Dedekind nor Cantor (*Was sind und was sollen die Zahlen* appears in 1888[482], and Cantor's papers only gain recognition among international mathematicians during the eighties[483]). Potter and Shields (1977) refer to Peirce's 'pre-Cantorian' period up until around 1884 when he held the Kantian idea that continuity could be defined as infinte divisibility. During the 80s, Peirce became familiar with the writings of Cantor whom he greatly admired. Not without certain aberrations and criticisms, he basically shares Cantor's definition of continuity up until the mid-90s. After around 1895–96, Peirce gradually develops his own version of set theory with the aim of providing a continuity description on that basis, and so his main papers on the issue fall in the period from 1897 to 1908. The continuum definition now revolves around the idea that in continuity, all parts have parts of the same kind. Potter and Shields refers to this period as 'Kantistic', maybe a little wildleading as Peirce's earlier, pre-Cantorian idea of continuity as infinite divisibility was also a Kantian idea. Potter and Shields outline a last period from 1908 to Peirce's death in which he takes the continuity of *time* to be the measure stick of all continuity, thus prolonging an older tendency to describe continuity as no collection at all.[484] Detailed treatments of the problem of continuity thus only appear in the decades around the turn of the century; main points in his reflection on the issue are to be found in 'The Law of Mind' (1892, EPI, 312ff, 6.102ff), 'The Logic of Quantity' (1893, 4.85), Robin-numbers 14 and 28 (christened 'On Quantity' and 'Multitude and Continuity' by Eisele) from 1895 and 1897, 'Fallibilism, Continuity, and Evolution' (c. 1897, 1.141) the lecture series known as 'Reasoning and the Logic of Things' (1898, R&LOT), the 'Lowell Lectures' (1903, partly in CP 4.510, partly in NEM), the papers on existential graphs (1903, 4.418), 'The Bedrock beneath Pragmaticism' (1906, parts in CP (4.553n2; 4.561n1; 6.174–176)),'Some Amazing Mazes' (1908, 4.585), and 'A Sketch of Logical Critics' (1911, EPII, 451ff, partly 6.177) – and recurring reflections on the continuum are to be found in much of his mature work in this period.

THE LAW OF MIND

Let us begin by presenting the whole complex of problems which Peirce connects to the concept of continuity, as it appears in his 'Cantorian' period in his characteristic excursive style, in which he in a few lines moves from mathematics to ontology to psychology to religion. A critical presentation of 'The Law of Mind' (1892) which forms Peirce's first large discussion of his metaphysics of continuity, his so-called 'synechism', will illustrate the range of problems he expects it to solve.

'The Law of Mind' is Peirce's first major discussion of the problem. In the preceding article in the series written for *The Monist*, of which 'The Law of Mind' is the third, Peirce proposed the concept of 'tychism' as referring to a doctrine of ontological absolute chance. He now adds the idea that this concept must give rise to an evolutionary cosmology 'in which all the regularities of nature and of mind are regarded as products of growth, and to a Schelling-fashioned idealism

which holds matter to be mere specialized and partially deadened mind' (6.102).[485] This programmatic monism is now taken as the basis for the validity of the law of mind – briefly, the tendency of ideas to spread – holding for mind in a more narrow sense as well as for matter, corresponding to Peirce's contention that the ideas of Thirdness constitute a 'thought-like' reality without reference to any thinking human subject. Peirce refers to the fact that many years earlier (he thinks of 'Questions Concerning Certain Faculties Claimed for Man' and 'Some Consequenses of Four Incapacities', both from 1868) he had tried to develop this doctrine, albeit by then being too blinded by nominalist prejudices. What Peirce refers to in these early papers is partly the problem of epistemology having to begin from something 'first' which he rejected on the basis of an Achilleus-and-the-Tortoise argument from continuity (EPI, 26; 5.263) claiming that science is always-already in the process of development. And partly the idea that science continually approaches reality due to the ongoing effort of the community of researchers (EPI, 52; 5.311).[486] In these early papers, his reflections referred to continuity in the development of knowledge only, hence the later rejection of the ideas as nominalist. In 'The Law of Mind', on the contrary, the idea is revised so as to refer to ideas understood as fully objective entities: the very Law of that title refers to the fact that 'ideas tend to spread continuously and to affect certain others which stand to them in a peculiar relation of affectibility. In spreading they lose intensity, and especially the power of affecting others, but gain generality and become welded with other ideas.' (EPI, 313; 6.104) Thus the strongly idealist formulation of the Law, and the rest of the paper consists of comments to this. Peirce's introductory argumentation of the objecitivist continuity of ideas rests on an analysis of *memory*.[487] How is it possible at all for past ideas to be present in actuality? If not by some kind of vicar or representative (which we might never remain sure to understand), then only by 'direct perception'. The problem has a certain analogy to the old Husserl's famous discussions in 'Ursprung der Geometrie': how is it possible to secure the stable heritage of knowledge in a represented form? – only by the maintenance of that original evidence which gave rise to its symbolization in the first place.[488] In Peirce, the answer is '... by direct perception.' (EPI, 314; 6.109). This is why an idea, if present now, can not be complete past at all; the present now must be connected with the relevant parts of the past by a series of infinitesimal causal steps – by continuity. The fact that ideas may thus possess a constancy during time becomes an argument in Peirce for a continuous conception of time, an idea which consequently cannot tolerate the idea of a time line dissolved into a powder of separate points. Consciousness must thus – monism here allowing Peirce to pass directly from experienced time to consciousness – cover an extended interval of time which can not, on the other hand, be a finite interval (in that case time would not pass), but precisely an infinitesimal interval. This is a decisive step in Peirce's argumentation which explains his metaphysical sticking to the formulation of calculus in infinitesimal terms in his philosophy of mathematics (as against Weierstrass's version using limits, which had become the canonical version in Peirce's time and well until the 1960s and the appearance of non-standard analysis).

To Peirce, infinitesimals are not only mere conventions for calculating; they possess real existence, and it is they that grant the continuous connectedness of time as well as of consciousness. The fact that the *object* of consciousness must also be continuous, Peirce now tries to argue with a (rather weak) argument of Kantian flavour: '. . . in this infinitesimal interval, not only is consciousness continuous in a subjective sense, that is, considered as a subject or substance having the attribute of duration, but also, because it is immediate consciousness, its object is *ipso facto* continuous.' (EPI, 315; 6.111) This subjectivist – and hence potentially nominalist – argument is probably part of the reason why Peirce later sees 'The Law of Mind' as a mere step in his development of a continuity theory. Time consists of *moments* possessing an extension, not of *instants* being mere points. Instants may appear only relative to moments, not as autonomous entities in themselves. In this claim for an ontologial argument for the existence of infinitesimals, Peirce may be compared to the non-standard analysis of the recent decades (A. Robinson) which reintroduced infinitesimals, even if not for ontological reasons, and without any direct practical implications to the extent that 'non-standard analysis' is consistent with the standard theory using limits (cf. below).[489] Peirce, on the contrary, has a metaphysical background for preferring infinitesimals – his logic of relations: 'The illumination of a subject by a strict notation for the logic of relatives had shown me clearly and evidently that the idea of an infinitesimal involves no contradiction, before I became acquainted with the writings of Dr. Georg Cantor [. . .] in which the same view is defended with extraordinary genius and penetrating logic.' (EPI, 316; 6.113). Peirce here overlooks the fact that Cantor in no way endorsed infinitesimals, quite on the contrary. It allows him, however, to introduce his old distinction (from 'The Logic of Number', 1881) between finite and infinite sets, defined by the fact that only finite sets are subject to de Morgan's 'syllogism of transposed quantities'. This idea is equivalent to Dedekind's and Cantor's definition of infinite sets as sets with subsets having the same size as the sets themselves,[490] and as they it takes as its basis the Bolzano measuring of the relative size of sets the one-to-one pairing of their elements. In Peirce's gay example, reference is made to Balzac's *Physiologie du mariage*, wherein it is claimed that any young Frenchman boasts of having seduced a French woman. As a woman in this use of the word may be seduced only once, and as there are as many French men as women, it follows that no French woman escapes seduction. But this argument only holds provided the set in question is finite. If the French population is growing (and why shouldn't it, considering the amount of seductions), the conclusion is not necessarily true. In this case, the example corresponds to Cantor's well-known proof – which Peirce refers (EPI, 316; 6.115) – that there are as many even numbers as there are whole numbers, because to any whole number, an even may be found, and vice versa. But still they remain a subset of the whole numbers.[491]

Now follows Peirce's first attempt at classifying transfinite sets. He claims that 'of infinite collections there are but two grades of magnitude, the *endless* and the *innumerable*' (EPI, 317; 6.116). This is obviously wrong, as Peirce later acknowledged, but let us follow his argument. The merely endless sets are countable, just like

the finite sets, but even larger sets exist which are not countable, because they are not subject to what Peirce calls the Fermat inference. Peirce gives an unnecessarily complicated algebraic example; the idea is simple and equivalent to mathematical induction (which Peirce for terminological reasons finds 'improperly'). The Fermat inference consists in the possibility of proving a theorem valid for all n, if it is possible to prove it for $n = 1$ and then prove that if it is valid for n, then also for $n + 1$. This conclusion requires, of course, that the set in question – as may the whole numbers – be arranged in a list, in a countable set. And that is not the case for all sets, as proved by Cantor: the irrationals may not be so arranged, because the distinction between them requires an infinite decimal expansion: 'Now if they cannot be exactly expressed and discriminated, clearly they cannot be ranged in a linear series.' (EPI, 319; 6.117). Peirce does not refer Cantor's proof, the famous diagonal proof, later so crucial to computer science, but the very wording of his own argument is interesting: he interprets uncountability as implying that the single elements of such a set may not be clearly distinguished as individuals, and this points forward to his own ontological interpretation of the concept of continuity as a concept of a hyper-set in which the single elements totally lose identity. In contradistinction to the bipartition of infinite sets, Peirce immediately (EPI, 319–20; 6.118) introduces a further distinction of the uncountables in two. First, he claims with Cantor that a line contains exactly as many points as a 2-D plane or a 3-D body.[492] The coordinates of the single points may be compressed into one single decimal expansion, no matter how many dimensions (and correlatively, coordinates) the entity in question may have: place the first coordinate's first decimal on the first place of the number; the second coordinate's first decimal on the second pace; the third coordinate's first decimal on the third place, and so on. This, of course, becomes impossible if the number of dimensions is infinite; a set of infinite sets of numbers which may each of them vary uncountably must transgress the simple uncountable set, and Peirce consequently calls it endlessly infinite: 'The single individuals of such a collection could not, however, be designated, even approximately, so that this is indeed a magnitude concerning which it would be possible to reason only in the most general way, if at all.' (EPI, 319; 6.118). Again, Peirce forwards the idea that the single, distinct individual may become blurred in very large sets, without his argument being particularly clear: it is not evident why it should not be possible to reason about single individuals in such a set, and later Peirce must admit that the elements of even uncountable sets are distinct so that any idea of the anticipation of continuity in transfinite sets must be argued in other ways. The fact that Peirce has not yet grasped the rules of calculation for transfinite numbers is demonstrated by his idea that the product of two uncountable numbers should be larger than each (as e.g. the set of possible pairs composed by one number from each set cf. note 16). It will later occur that a given transfinite entity must appear as exponent in order to let a larger transfinite set appear: $\text{Aleph-n} < 2^{\text{Aleph}-n}$.

These deliberations, however, take Peirce to the decisive question: what *is* continuity? Here he takes his basis in one version of Kant's definition – infinite divisibility – which quickly, and following Cantor, must be dismissed: the rationals

are infinitely divisible but not continuous, because between two given rational numbers uncountably many irrational numbers may be found.[493] Cantor's definition is, instead, that a continuous series is '*concatenated* and *perfect*. By a concatenated series, he means such a one that if any two points are given in it, and any finite distance, however small, it is possible to proceed from the first point to the second through a succession of points of the series each at a distance, from the preceding one, less than a given distance. This is true of the series of rational fractions ranged in order of their magnitude. By a perfect series, he means one which contains every point such that there is no distance so small that this point has not an infinity of points of the series within that distance of it. This is true of the series of numbers between 0 and 1 capable of being expressed by decimals in which only the digits 0 and 1 occur.' (EPI, 320; 6.120).

Peirce here refers Cantor 1879–84, 194 where the corresponding concepts are 'perfekt' and 'zusammenhängende', and the continuum consequently 'perfekt-zusammenhängende'.[494] 'Perfekt' is here defined as being the same as non-reducible, that is, not reducible by 'Ableitungen', which successively isolate the limit points of a set,[495] while 'zusammenhängende' in our days' terminology will be dense (in itself dense).

Even if Peirce extensionally agrees with this definition (in so far as he finds it includes continua and excludes non-continua), he is not satisfied for intensional reasons: the definition is metric (which is only true in a relative sense: 'Es handelt sich also um eine 'metrische' Eigenschaft des Kontinuums' (194); Cantor here refers to the definition of Zusammenhang by distances less than a certain neighborhood distance epsilon) – which is not the case for the distinction between continuous and discontinuous. The fact that a perfect series is defined by containing every point of a certain description is regarded by Peirce as a definition by negation (because points not caught by this description are excluded), while no positive idea is given about 'what all the points are'. This cricitism seems to presuppose Peirce's own idea of the continuum which excludes that it may be described as a set of points of any properties whatsoever, but requires a description on its own level, prerequisite to any division of it into points; without the presupposition of this idea, Peirce's criticism seems ill-placed. Finally, he attacks Cantor for giving no distinct definition of the concept of continuity, because his definition falls in two parts, not yielding any intuition of what the continuum as an entity really is. As mentioned, Cantor believed at this point that the continuum hypothesis could be proved within set theory, so that the continuum was equal to the power of the arithmetical line which was again equal to the first transfinite cardinal after the countables, and he consequently viewed the definition given as holding for the geometrical line. Peirce in a certain sense agreed in the continuum hypothesis, in another sense he did not – and none of them knew, of course, it was later to be proved indecidable in set theory (Cantor's version as it was later formalized by Zermelo and Fraenkel).

Peirce now undertakes to provide an alternative definition based on Kant's admittedly incomplete suggestion. How are the 'holes' in Kant's definition (infinite divisibility corresponding to density) to be 'mended'? 'What is required (…) is to

state in non-metrical terms that if a series of points up to a limit is included in a continuum the limit is included' – from the argument that given a 'hole' in a Kantian series, at most one of the end points may belong to the set (if both belonged to the set, then there would be, according to Kant's definition, at least a further point between them belonging to the set, cf. the Dedekind cut). So if we can require both points to belong to the series, the hole is 'mended'. Peirce notes that this seem to be the property Aristotle saw in defining the continuum as something whose parts has a common limit (Peirce refers to Physica 227a, Metaphysica 1069a[496]). This definition Peirce baptizes 'Aristotelicity', to be added to the 'Kanticity' of infinite divisibility: 'The property of Aristotelicity may be roughly stated thus: a continuum contains the end point belonging to every endless series of points which it contains. An obvious corollary is that every continuum contains its limits. But in using this principle it is necessary to observe that a series may be continuous except in this, that it omits one or both of its limits.' (EPI, 321; 6.123).[497]

Frankly, this definition of continuity seems no less 'double' than Cantor's. Partly, it requires density ('Kanticity'), partly it requires that any delimited part must contain its limit ('Aristotelicity', corresponding to Cantor's 'perfection'[498]), moreover, it corresponds well to the standard definition today. Peirce now connects it to the existence of infinitesimals: the word 'infinitesimal' simply is Latin for the ordinal number the 'infinitieth' and it refers to the fact that incommensurable numbers need an infinitieth number in their decimal expansion in order to be fully described. This is why continuity makes possible infinitesimal entities, Peirce claims in an argument which must presuppose the continuum hypothesis in Cantor's version. His criticism of Cantor here – as Marco Panza puts it (Salanskis (ed.) 27) – seems to amount to the fact that Cantor wants to give a definition in order to constitute the object, while Peirce rather conceives of the task as to satisfactorily describe something already given beforehand, cf. Dipert's (1997) characterization of Peirce's disagreement with Dedekind as pertaining to intensional vs. extensional description of sets. Rather than a mathematical difference, it is a disagreement as to the requirements for the very character of a description.

After this mathematical interlude, Peirce returns to the philosophical use of the concept of the continuum. First a problem of perception: which color should be ascribed to the borderline between to colored surfaces? As the color of a point, seen from a continuist point of view, must be identified as the color in a certain neighborhood around the point, we may only say that the color of the borderline is half of each of the two adjacent colors. The present now must be another case of such a limit surface, consisting half of past, half of future – corresponding to the definition of velocity in the differential calculus as the middle value of speed in an infinitesimal interval. These ideas painlessly are generalized to consciousness: 'Just so my immediate feeling is my feeling through an infinitesimal duration containing the present instant.' (EPI, 322; 6.126)

These quick analyses of time and consciousness now hold decisive implications for the Law of Mind, for the distinction between past and future makes the Law of Mind – unlike physical laws of force – irreversible. The present now is influenced

by the past, not by the future. From this truism[499], Peirce derives radical consequences: the continuity between past and the present implies that '... every state of feeling is affectible by every earlier state'. In this version, the theory seems rather radical, because it does not delimit the influence to a local neighborhood around the single 'state of feeling' – if it is not to be read in a fashion making consciousness itself identical to that local space. That seems not to be the case, though, if we are to believe the following definition of an original continuum of feelings towards which human consciousness only possesses a highly restricted window: 'The development of the human mind has practically extinguished all feelings, except a few sporadic kinds, sounds, colors, smells, warmth, etc., which now appear to be disconnected and disparate.' This idea is connected to Peirce's cosmology, which is for the first time developed in this series of lectures – according to which the beginning of evolution was constituted by an infinite space of possibilities, from which evolution gradually selects some to realize. The realized feelings are in themselves continuous (albeit in less dimensions than the original space of possibilities) so they appear temporally articulated in infinitesimals. Correspondingly, they have a continuous spatial extension, which takes Peirce into his strange doctrine of slime, of protoplasma which is seen as the physiological continuum permitting emotion to enter into motion. This spatial continuum has wide implications in Peirce: 'Since space is continuous, it follows that there must be an immediate community of feeling between parts of mind infinitesimally near together. Without this, I believe it would have been impossible for minds external to one another ever to become coördinated, and equally impossible for any coördination to be established in the action of the nerve-matter of one brain.' The continuity of space and the continuity of feeling are taken to solve all at once both the problem of the existence of other minds and of the mind-matter relation. Both these problems' character of the transgression of boundaries is taken to rely on the lack of insight in a monistic continuity underlying the apparent limits.[500]

Continuity, thanks to monism, immediately becomes the question of the continuity of ideas. Ideas are defined by three aspects, after Peirce's three metaphysical categories: their quality as feelings; the energy with which they affect other ideas[501]; their tendency to imply other ideas. Energy is supposed to diminish, as the idea spreads. It seems like Peirce here argues in some kind of analogy with the decrease of light intensity proportional to the squared distance. Its quality, unlike its intensity, is unchanged (a weak recollection of a strong red colour is not a weak red...). The continuity of feeling of course implies that 'A finite interval of time generally contains an innumerable series of feelings; and when these become welded together in association, the result is a general idea. For we have just seen how by continuous spreading an idea becomes generalized.' Now we approach the very exegesis of the Law of Mind, and there are several interesting points to comment here. If we accept the premiss that any finite time interval contains an uncountable number of feelings, then there is still some distance to the conclusion that these should be able to synthesize into one general idea. It is obviously only possible in some cases, namely when they are 'welded together in association'. Here Peirce's doctrine of

ideas all of a sudden becomes rather brutally psychological, and even in a rather primitive associationism, not unlike the British empiricists: if anything may be associated with anything, then the Mind may entertain any 'general idea' it might wish. Consequently, Peirce must describe this general idea as 'vague' due to its unlimited character, still, on the other hand, it is claimed to have the inner property of a 'living feeling' with strong anti-nominalist implications. Ideas are seemingly essentially related, 'we can directly perceive the one gradually modified and shaping itself into the other. Nor can there any longer be any difficulty about one idea resembling another, when we can pass along the continuous field of quality from one to the other and back again to the point which we had marked.' (EPI, 326; 6.139). Just like the case in the other great phenomenologist Husserl, a refined and continuous version of the psychology of association pays a central role in the description of the synthezicing activity of the mind in Peirce.[502] As the synthesis is here described, its object is not merely ordinary associations of ideas potentially being able to synthesize anything, but specifically *iconic* associations, connections by similarity, objectively existing in the multidimensional quality space. Thus, it is a variety of eidetic variation in this quality space which grants the real connections of ideas in Peirce, making possible, in turn, their association for the mind as well as in the world. After an interlude about the intensity of ideas, Peirce specifies this connection: 'We can now see what the affection of one idea by another consists in. It is that the affected idea is attached as a logical predicate to the affecting idea as subject. So when a feeling emerges into immediate consciousness, it always appears as a modification of a more or less general object already in the mind.' (EPI, 326; 6.142). This points towards Peirce's theories of abduction and his distinction between abduction, induction, abstraction, and prescission in the field which were earlier most often covered by simplicist empiricist-psychological abstraction theories (see Chap. 11). Ideas may not at all be connected without continuity (6.143). It may happen, however, in a universe suffused by chance – cf. Peirce's 'tychism' – that ideas are associated which may not form a general idea. But association does not follow mere coincidence, generally, there is a Darwinian tendency, by selection, presumably, in ideas to gather in more general ideas. The scientific quest for truth in the community of researchers, the development of civilisation in general, and behind them biological evolution, lie, implicitly, behind this argument. This 'mental law' conforms to the forms of logic, however, deduction corresponds to the fact that a general idea releases a reaction (like when the frog's dissected legs react when pinched), in induction, a habit is constructed which is present as a general idea in a series of single feelings, and in abduction (here: hypothesis), the general idea is called for by a single feeling (the logical inference drawn from a single observation to the fact that the entity in question may belong to a general category making its behavior understandable). Only deduction is necessary, while the two other inference forms are probable and possible, respectively (in this context both are called probable, EPI, 329; 6.147), mental action is not necessary and invariable, quite on the contrary it is insecure, plastic, and vital. This argument draws the consequence of monism: the Law of Mind is claimed valid not only

for human thought but also for the evolution of the world even down to details like the claim that the mind's tendency to follow logical forms is also shared by the movement processes in the world: this is what constitutes Peirce's famous and extreme 'logical realism'.

This now permits Peirce to restate the Law and its implications: 1) from a nominalist point of view ideas may not be similar to each other nor influence each other, 2) momentaneous feeling flows together into one continuum of feeling, gaining generality – in this conceptual realism, similarity, connections between feelings and the world cease to constitute problems, 3) because of this, general feelings are no longer mere words, but *even more* than the feelings in which they incarnate (cf. 'extreme realism'), 4) the highest law – which Peirce does not abstain from calling 'heavenly and living harmony' – does not require the single feelings to resign from their individuality, but merely to influence one another. The potentially totalitarian implications of this somewhat Hegelian-Schellingian idealism is sought avoided by this turn: the Law does not require the single ideas to do something *determined*, they are just urged to self-organize without the result of this process being dictated beforehand, 5) we are thus unable, at our present level of knowledge, to say to which extent this evolution is governed.[503] (EPI, 330; 6.150ff).

With this theory, Peirce declares, in yet a surprising turn, we are close to an explanation of personality. Personality is also a general idea which may not be grasped in one glance: 'It has to be lived in time: nor can any finite time embrace it in all its fullness.' Our finitude as human beings here dampens realism: we are not able to grasp sufficiently complex general ideas in finite time: personalities and other such generalities belonging to the realm of continuity.[504]

To sum up, Peirce's metaphysical doctrine of continuity, this 'synechism', entails logical realism, objective idealism and tychism as well as a thoroughgoing evolutionism.

As is evident from this presentation, Peirce's mathematical speculations on continuity is intimately related to his phenomenology (Firstness and Thirdness, characterized by potential and actual continuity, respectively), his philosophy of mind (the doctrine of motivated associationism), epistemology (the tight analogy between evolutionary processes in the world and logical inference processes in the mind), ontology and metaphysics (the continuous character of being, the continuous reality of universals), not to speak about religious speculations (a continuous and therefore (!?) personal God). Even if the motivation for Peirce's strong interest in the continuum is obviously extra-mathematical and related to the need for an ontological 'glue' to make the different parts of his architectonic stay together, Peirce maintains that the possible mathematical definition of the term is necessary for its use elsewhere. This is implied by his conception of mathematics (inherited from his father, the mathematician Benjamin Peirce) as the science which draws necessary conclusions from hypotheses. Even if mathematics is in itself hypothetical through and through (and thus not in itself true or false, it is true only as a corpus of if-then propositions which may have mutually contradictory implications, so as e.g. the different non-Euclidean geometries), it is the reasonings of mathematics to

which the more empirical sciences have to look in order to find formal structures. This is why the mathematical determination of the continuum is necessary for Peirce in order to develop the concept in its ontological and empirical applications. The 'Law of Mind' is often referred to as insufficient (and rightly so) in later Peirce texts, among other things exactly with reference to its conception of continuity. Its chaotic mixture of subjects, though, form a good introduction to grasp the wide span of issues continuity is supposed to explain in the mature Peirce's thought in the years around the turn of the century.

CONTINUITY BEYOND THE TRANSFINITES

The presentation of the purely mathematical aspects of the concept of continuity is – as so often the case in Peirce – spread out in a long series of more or less finished works from the period around the turn of the century. One of the most thorough versions is to be found in Robin nr. 28, published in NEM III with the title 'Multitude and Continuity' (dated by Robin as 1897?). The title is well chosen, for the goal of this cautious presentation is to situate the concept of continuity in relation to the doctrine, inspired by Cantor, of transfinite sets and their size, 'multitude' (Peirce's translation of Cantor's 'Mächtigkeit', our days' 'power'). It might surprise that Peirce choses to treat Cantor's development of the transfinites so detailed as is the case: his main point of view remains, as already in 'The Law of Mind' that the continuum is a primitive phenomenon which may not be derived from simpler phenomena – and yet he devotes great energy to the reconstruction of his own version of an analytical, Cantor-style theory whose aim is to build continuity 'from below' by means of sets of points. We shall return to this seeming paradox.

Peirce had developed his own terminology since the beginning of the eighties[505] which calls for a presentation. 'Multitude' is, as mentioned, his term for Cantor's 'Mächtigkeit', measuring the size of sets from one-to-one mappings; Peirce often uses the term as synonymous with a set equipped with such a measure. 'Collection' is his translation of Cantor's 'Menge' (today: 'set') even if the two terms do not coincide in all respects. 'Enumerable' is his term for the power of sets with a finite number of elements. 'Innumerable' refers to the opposite property and thus corresponds to Cantor's concept of 'transfinite'. This field may, of course, be subdivided into several categories, of which the smallest is called 'denumerable', corresponding to Cantor's 'abzählbar' (today 'countable'), referring to the power of the natural numbers and related sets. Sets transgressing this size are 'abnumeral' (today 'uncountable'), – referring e.g. to the power of the reals. This power constitutes – Peirce here more or less tacitly following the continuum hypothesis (CH) – the first abnumeral number. Cantor believed to have given a proof of the CH, and general opinion of the period held it to be true and possible to prove; as mentioned, it was only in 1964 finally proved indecidable with respect to ZFC set theory. Peirce's overall stance here thus does not differ from that of the period.[506] With respect to the use of the concept of 'abnumeral', Peirce sometimes lets it include the 'denumerable' so that countable sets are seen as the zeroth abnumeral number. Later texts often substitute

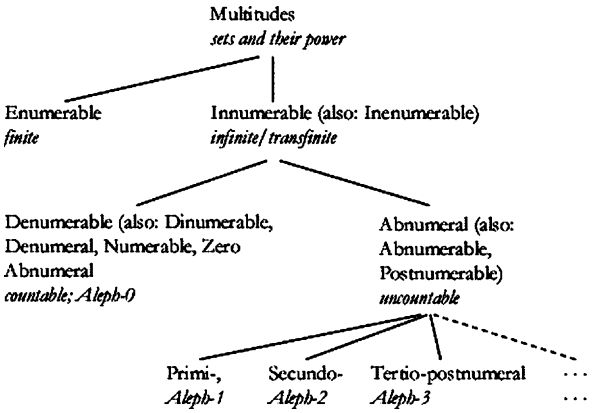

Figure 55.

'-postnumeral' for 'abnumeral', the former with the addition of a prefix addressing which uncountable set is referred to ('primi-', 'secundo-', 'tertiopostnumeral', etc.), just like 'denumerable' is later changed into 'denumeral' (because only one class of countables exist), and 'abnumeral' conversely into 'abnumerable' (because several such classes exist). We may sum up Peirce's concepts in the diagram above.

We said above that Peirce in this text – as in general – supports the continuum hypothesis (CH) as it is usually presented: Aleph-1 $= 2^{\text{Aleph}-0}$. This is, however, a truth with some modifications. Peirce (most often) agrees fully in the equation as given here: that the first cardinal number after the countables is equal to two elevated to the power of the countables. In this he agrees for several reasons. First, he agrees with Cantor in Cantor's theorem proving that the set of subsets (today: the power set) of a set with a members is given by 2^a, and that this number is always larger than a. This proof Peirce repeats at several occasions – and he even thinks he preceded Cantor in proving it. Furthermore, he agrees in the non-trivial content of CH that no further powers exist between the power of the natural numbers and the power of the reals: 'Let us now ask, what is the smallest multitude which exceeds a denumerable multitude? That there is a multitude which exceeds a denumerable multitude has already been shown. Namely, the possible combinations of whole numbers is such a multitude. The possible combinations of whole numbers each of which consists of an enumerable collection of whole numbers is merely denumerable. But the possible combinations of denumerable whole numbers form a collection which exceeds a denumerable collection. I call this multitude the *first [abnumeral multitude].*' (NEM III, 83–84).[507] The opposite idea would be that even if the reals can be proved (Cantor's diagonal proof) to have greater power than the natural numbers, it is not thereby proved that a certain specified subset of the reals might not be greater than the natural numbers and smaller than the whole of the reals so that $2^{Aleph-0}$ would equal Aleph-2 or even a higher Aleph; this is what Cohen's forcing technique in the 1960s proved indecidable. Peirce even agrees with

Cantor in what was later called the generalized continuum hypothesis (GCH): the idea that this relation not only holds between the Alephs 0 and 1, but between each successive pair of Alephs: Aleph-n + 1 = $2^{Aleph-n}$, so that the transfinite numbers form a well-ordered set of Alephs where the next Aleph in line has the same power as the set of subsets of the former Aleph which again equals the set of ordinal numbers having the former Aleph as its power. When Peirce does not, in another sense, agree with CH at all, it refers to a prerequisite so taken for granted that it does not at all appear in the canonical formulation of it. It is the idea that Aleph-1, the power of the reals, is supposed to be identical to the power of the arithmetical line, in turn identical to the power of the geometrical continuum (which is apparent only in the name of the hypothesis). This intuition which dates back at least to Descartes' analytical geometry identifying arithmetical numbers and geometrical line, so that for any point on the line there is a number, and vice versa, seems so evident that it need not be mentioned. In a Peircean light, however, CH as we usually understand it, should rather be stated as Aleph-1 = $2^{Aleph-0} = c$, where c refers to the power of the geometrical line. It is the last part of this equation which makes Peirce strongly disagree, and it is his alternative idea of c which the text in question sets out to prove. This idea is already inherent in the special way Peirce conceives the relation between infinite entities. Let us take a look at his 'proof' (which is of course not valid) of CH, after he has just articulated it in the quote above:

> That there is no multitude greater than the denumerable multitude and less than the first abnumeral, I argue as follows: the demonstration does not seem to be perfect [sic]; but I think it is only because I do not succeed in stating it quite right. The addition, or aggregation, of denumerable collections is entirely without effect upon the multitude, even if the multitude of collections aggregated is denumerable. The multiplication of denumerable collections, that is the formation of all possible compounds each being a set composed of an individual out of each collection, is also without effect, if the multitude of denumerable factors is enumerable. But if the multitude of factors is denumerable, the result is the first abnumeral. Now since there is no multitude greater than every enumerable multitude and less than the denumerable multitude, it follows that there is no multitude greater than the product of every enumerable collection of denumerable factors and less than the products of a denumerable collection of denumerable factors which is the first abnumeral multitude. (84)

This sounds immediately seductive: Peirce lists the types of calculation – addition and multiplication – which are without effect on the size of a countable set in so far as they are used a finite number of times. Is multiplication used, on the other hand, a countably infinite number of times (corresponding to $2^{Aleph-0}$) you will get a larger set consisting of possible combinations, that is, possible subsets of the countable set. This implies, Peirce infers, that when you pass from a finite to a countably infinite set of operations, then you go from a countable set to an uncountable set as a result. But if this should count as a proof, then the burden is passed a step down in the Aleph hierarchy, the premiss being that there is no transfinite entity less than Aleph-0, that is, no infinite large numbers exist which are smaller than the set of the natural numbers. But it is, in fact, a possible implication of Peirce's point of view that such numbers exist.[508] But even apart from such Peircean peculiarities in the proof, it does not hold. It might be possible to define a set with power intermediate

between the naturals and the reals with *other* means than multiplication. This is not excluded by the argumentation.

Peirce now goes on to consider uncountable sets and he observes, with Cantor, that just like it was the case with Aleph-0, the product of a finite set of uncountable factors is still only uncountable (here, Peirce has learnt things since 'Law of Mind'): if every factor of n factors is given in decimal expansion, then we may make one combined decimal expansion so that the first factor of the first number is given the first place the first factor of the second number the second place . . . the second factor of the first number in place $n+1$, the second factor of the second number in place $n+2$, etc. This implies that the product of a finite set of uncountable numbers may be described in one single decimal expansion and thus be contained in Aleph-1. But again the picture changes – according to Peirce – by an infinite (but countable) amount of factors, taking us into Aleph-2. Peirce's argument, thus: $\text{Aleph-1}^{\text{Aleph}-0} = \text{Aleph-2}$. But this is not the case, given his premisses; the result remains caught up in Aleph-1. The exponent must be uncountable in order for a shift to take place: Peirce here argues against GCH – or he has rather misunderstood it. About Aleph-2 Peirce goes on:'Mathematics offers no example of such a multitude. Mathematics has no occasion to consider multitudes as great as this' (85).[509] In the same way, Peirce constructs Aleph-3 and then he continues:

> And so the multitudes succeed one another indefinitely; and the constituent individuals of collections of those multitudes are distinct from one another.
>
> Thus, the whole series of multitude, so far as yet made out, begins with the multitude of a non-existent collection, or zero, and then comes the multitude of a single object, and then the multitude of 2, and so on increasing by one without end. After these multitudes comes the denumerable multitude which may be called the zero abnumeral multitude, then the first abnumeral multitude, then the second abnumeral, and so on increasing in order by one without end. All these multitudes thus form two denumerable series, and consequently there is only a denumerable multitude of different possible multitudes, so far as yet made out. (85–86)

All this might appear to be a (partially unsuccessful) attempt of an relatively orthodox presentation of Cantor's set theory; yet there is a strange emphasis on the role played by the countable sets in Peirce's account. In all cases it is the transition from a finite to a countably infinite set of operations which is supposed to give power change as its result. This is of course not allowed by GCH, according to which Aleph-$x+1$ is achieved by elevating two to Aleph-xth power (and not only to the Aleph-0th power, as Peirce seems to assume). Peirce furthermore seems to have the idea that the whole series af Aleph powers is itself countable – opposed to Cantorian set theory in which Alephs may get ever larger transfinite ordinal numbers as their indices. Despite the generativity in the set of transfinite sets, it appears to Peirce to be somehow 'tamed' by the countable infinity, by the natural numbers.

After these preparations we now reach Peirce's own continuum hypothesis, presenting his idea of the relation of the geometrical line to transfinite sets (PCH, we might call it). It is given first in the form of a hypothesis:

Let us now suppose that there is a collection of distinct objects of each of those multitudes. Then, taking any one of those collections, no matter what, there is, among the whole collection of those collections, a denumerable multitude of collections each of which is greater than the collection chosen. Let us then throw all the distinct individuals of those collections so as to form an aggregate collection. This aggregate collection is greater than any of the single collections; for it has, as we have just seen, a denumerable collection of parts greater than any one of those collections. I shall call it a *supermultitudinous* collection. (86)

Peirce imagines the construction of a set constituted by the sum of the elements in a sum of sets composed by one set from each power class. In doing so, he of course unknowingly uses what was later identified as the Axiom of Choice (AC) and proved indecidable – it allows us to chose one element from each of an infinte set of sets. This union of sets of all powers U(Aleph-n), $n \to \infty$, Peirce now claims constitutes a 'supermultitudinous' collection *S*, larger than any cardinal. It is of course true, as Peirce claims, that such a set will be larger than any single Aleph-n, because it contains the elements of a set with the size Aleph-$n + 1$, and he even adds an equivalent to CH on this higher level: 'It seems to be sufficiently evident that there is no collection at once greater than every abnumeral collection and less than such a supermultitudinous collection.' (86), a claim which may be proved if (in addition to Peirce's other, fallacious premisses), the simple CH is assumed to be valid.

But why is it not the case that this set is merely identical to Aleph-ω_0, as it would be the case in orthodox set theory? First of all we must admit that Peirce differs from the later standard ZFC set theory by not assuming the so-called 'Axiom of Replacement' saying that for every set it holds that if the elements are replaced by something else (e.g. other sets), the result will still be a set. By this axiom, ZFC goes beyond the countable set of cardinal numbers because we may substitute for every number of the natural numbers the cardinal having the number in question as index and then take the union of the sets in this set – this set will have a cardinality exceeding any cardinal with a countable index. Peirce does in fact construct such a set, but he now argues that the decisive interest in such a set lies in the fact that it may be proved that this set *S* has properties differing from all other transfinite sets, making it the equivalent of the very power of the continuum. The kernel of this argumentation is the following 'proof' which just as little as his 'proof' for CH is valid. Still, it expresses nicely which metaphysical requirements a Peircean continuum definition must meet:

The collection of possible ways of distributing the individuals of a supermultitudinous collection, *S*, into two abodes is no greater than that supermultitudinous collection, *S*, itself. For denoting by γ the denumerable multitude, the abnumeral multitudes are [...] Exp γ, $(\text{Exp})^2\gamma$, $(\text{Exp})^3\gamma$, etc.[510]; and the magnitude of the supermultitudinous collection is the limit of this series. It is, in short, the result of a denumerable succession of exponential operations upon the denumerable multitude. But the magnitude of the collection of possible ways of distributing the individuals of a collection into two abodes is simply the result of an exponential operation upon the magnitude of the collection itself. *Hence the magnitude of the ways of distributing the individuals of a supermultitudinous collection into two abodes is obtained by adding one more to the collection of exponential operations successively performed upon the denumerable multitude.* But this collection of operations being denumerable, the addition of one

operation to it does not increase the magnitude. Hence, the collection of possible ways of distributing the individuals of a supermultitudinous collection into two abodes equals that collection itself.

But we have already seen that the multitude of ways of distributing the individuals of a collection of distinct individuals into two abodes exceeds the collection itself. *Hence, it follows that a supermultitudinous collection is so great that its individuals are no longer distinct from one another.* (86–87, our italics)

Peirce here argues that the set S is larger than any transfinite number and consequently makes its single elements lose individuality. In the above quote we have emphasized the two decisive steps in the argumentation. In the period immediately before the first step we find the correct claim that the power of a set of subsets of a set is an exponential function of the power of the set. But after this claim, the argumentation fails when Peirce claims that the power of S should be obtained by adding 'one more' to the series of exponential operations quoted. What is that supposed to mean? In the series 'Exp γ, $(\text{Exp})^2\gamma$, $(\text{Exp})^3\gamma$, etc.' it is already presupposed that it goes towards infinity; hence S will not be obtained by the 'one more'. It would rather require a countably infinite ordinal ω_0 as index for the set, as it would be the case in standard Cantorian set theory where the series of Aleph-n's is followed by a series of Aleph-ω_n's, and so on, with no largest transfinite set showing up. By this fallacious argumentation Peirce 'proves' that S has the same power as the set of its own subsets: $S = 2^S$, at odds with Cantor's theorem $x < 2^x$. This contradiction Peirce attempts to solve – in the second step – by claiming that this inconsistency implies that S can not at all be a set (for which $x < 2^x$ must hold). Which kind of an entity, then, is S? S ceases to have distinct elements, because $x < 2^x$ describes the power relation between a set and the set of possibilities for combining its distinct elements into two sets (corresponding to its power set). 'It follows', Pmirce bravely claims, but why it should be exactly the distinctness of this definition that must be given up is not clear. It would be equally possible to conclude that S does not exist; that S is inconsistent; that S is neither a set nor a collection or class; that it is impossible to take out subsets from $S \ldots$ etc.[511]

No matter which construction Peirce makes of S, though, it is evident that it suffers from exactly the same inconsistency as does Cantor's 'set of all numbers' Ω which cannot include its own ordinal number, because the inclusion of that will make its own power larger. Cantor saw this already in the 90s, and if Peirce realized this he might have a sounder version of his idea of S: Cantor's inconsistent set of all numbers might be interpreted as referring to the fact that the continuum as constructed by points is necessarily inconsistent.

Thus, even if Peirce's 'proof' of the existence of a superpower set is thus far from convincing, it demonstrates how he imagines the continuum conceived of as something that transgresses any possible transfinite number referring to sets of distinct elements. An obvious question is why Peirce is so concerned to grasp the continuum in relation to the series of cardinal numbers when, at the same time, he conceives of it as a 'superpower' beyond all of them. The ensuing presentation answers that question. He is forced to ask the question whether S really 'stops' the transfinite series, that is, whether all supermultitudinous classes will now have the same size ('magnitude'; Peirce must of course give up the notion of 'multitude'

here). He begins by considering a subset of *S*, namely finite decimal expansions, of which he may claim that 'all individuals which are marked each by an enumerable collection of characters form a denumerable collection of individuals.' (87). The crucial feature of this claim is that the set of finite combinations is not in itself finite, but countably infinite; this simple implication of the calculation of a power set has an interesting implication to Peirce: 'Any one of them or any collection of them may be taken away without affecting the remainder: but yet there are already symptoms of incipient cohesiveness in them, a premonition of continuity. Remember that the multitude of grades of enumerable multitude is denumerable; and the multitude of all the grades of multitude is no greater.' Time and time again, Peirce seeks an anticipation at lower levels of the transfinite hierarchy of the cohesion of the continuum and the vanishing of the distinctness of its elements; here his focus upon the countable infinity recurs. Peirce introduces the term 'arithm' to signify the set of size levels *below* a certain set, so that the arithm for 0 is 0, for 1 is 1, for 2 is 2, and generally is identical with the number itself for all finite numbers. But as there are only – to Peirce – countably many transfinite numbers, then it must be the case that, '... the arithm of all higher multitudes is the same. It is the denumerable multitude, which may be called *infinity*, ∞' (88) (which would not be the case in ordinary set theory where PCH is not assumed). Peirce interprets this as follows: '... this unity of the arithms of all the higher multitudes is the first embryon of continuity' (88) It is difficult to appreciate what exactly are the implications of this claim, but Peirce makes the following explicitation:

> The entire collection of those individuals that are marked by enumerable series of characters, is *denumerable*. But as soon as the denumerable multitude of interpolations has been made, the collections of characters attached to the last inserted individuals are denumerable, and the collection of individuals here is of the first abnumeral multitude. Stop it at any point you please, however early, if all the individuals marked by enumerable series of characters are there, there is also along with them a first abnumeral multitude of individuals marked by denumerable series of characters. *They stick together.* (88)

The quote refers to the imagination of a procedure where you are on the way to fill the set of finite numbers to the top, an infinite operation because of the infinity of the set. Peirce then imagines – contrary to intuitionism – the termination of this infinite series of procedures which is only marked by the fact that one all of a sudden must use infinitely many figures in order to give the next numbers, the first irrationals, so to speak, so that you are already in the process of construing the next, uncountable, set, the reals. You can not fulfill the finishing of the countable set without already implying its uncountable successor, this is Peirce's decisive argument making the bottom-up construction of transfinite sets *presuppose* the existence of higher-level sets. It is in this way, allegedly, that the continuum is anticipated in these sets' 'sticking together': we may not terminate the construction of Aleph-0 without already beginning on construing Aleph-1. By introducing the operational point of view, Peirce in one sense anticipates constructivism, even if with a totally different aim: constructivists will claim that a process involving an infinity of operations may not be ascribed any reality; Peirce will quite on the

contrary claim that when you not merely define your way ahead formalistically, but imagine how the operations are to take place, then it is indecidable beforehand 'when' the construction of a transfinite set has terminated. You will first 'discover' this when elements of a higher set 'come along'. You cannot have Aleph-0 without being already fully involved with Aleph-1. The quotation marks above indicate that the apparent psychology of this presentation should only be taken for heuristics: the anti-psychologist Peirce will not in his interest in the character of the construction operation refer to psychology, but only raise the question as to the criterion for the conceivable, succesful termination of the procedure. The fact that there are no other conceivable criteria than the appearance of higher elements from a set of the next power then becomes the crucial argument: '. . . and before there can be any past after the denumerable collection is all there, a first abnumeral collection must already be there.' (92) Peirce repeats this point in different guises over several pages without any substantially new arguments occurring. It seems problematic to assume that the *whole* of Aleph-1 is supposed to appear in the completion of Aleph-0; it must be sufficient if one single real 'showed up'; in which sense would this one number presuppose the existence of whole sets of Aleph-1 power?

The argumentation might seem made with a guided hand. Elsewhere, Peirce may suggest that it is of no importance where in the Aleph series the strange transition to S takes place: 'It is, however, of secondary importance at what particular place in the series of multitudes this phenomenon occurs. It must occur somewhere . . .' (NEM III, 121 n1). While continuity thus seems anticipated in arithmetics, it is the case, conversely, that numbers as distinct entities may only express discrete order, never size ('magnitude'): numbers 'cannot possibly express continuity' (93). On the contrary it will be the case that between a limit (e.g. π) and a series converging onto it, any magnitude whatsoever may be interpolated. This leads back to S, for may this 'set' be so interpolated? Here, the existence of S is again defended with a fragile argument. Peirce talks about the possible interpolation of the different 'postnumeral quantities', that is, uncountable sets: 'But if we suppose that *all* such orders of systems of quantities have been inserted, there is no longer any room for inserting any more. For to do so we must select some quantity to be thus isolated in our representation. Now whatever one we take, there will always be quantities of higher orders filling up the spaces on the two sides.' (95) Here, the argument for the claim that $c = S$ comes from the infinitesimal side, form the possibility of 'filling up' the geometrical line. Here we hear the surprising claim that if we have filled up the line with all Aleph-n's, then there is room for nothing more, and thus the whole line will be complete. But there is no precise definition of this 'filling up', and the argument is not valid. It would be possible to fabricate a completely parallel argument (satisfying constructivists!) against the existence of the reals: take a real number, it must lie between two fractions, but no matter which divisor is chosen for the fractions, there will be fractions with higher divisors 'filling up'. But what is the proof that there is no more space left?

It seems like Peirce himself realizes the fragility of his argument: the text concludes symptomatically with a whole bunch of further arguments. The first is

a *Gedankenexperiment* with knifes cutting the line into parts, a surprising picture in so far as it runs counter to the whole Cantor-like tendency in the argument up till now, having attempted to synthesize the line from a combination of elements: 'Across a line a collection of blades may come down simultaneously, and so long as the collection of blades is not as great that they merge into one another, owing to their supermultitude, they will cut the line up into as great a collection of pieces each of which will be a line, – just as completely a line as was the whole. This I say is the intuitional idea of a line with which *the synthetic geometer* really works, his virtual hypothesis whether he recognizes it or not; (…) cast aside all analytical theories about lines, and looking at the matter from a synthetical point of view to make the mental experiment and say whether it is not true that the line refuses to be cut up into points by any discrete multitude of knifes, however great.' (96, our italics). This would imply that if you – the synthetic geometer – should throw away all the analytical arguments, then this should also include the vacillating proofs of the preceding pages trying to construct the line from the 'merging' and 'welding' of distinct entities. This you might do without if you accept the continuum as a geometrically intuitive primitive object, on which points are mere different classes of intersections. The number of points into which the line presumably may be cut, is of course also taken to be *S*, of which Peirce gives no less than two blitz proofs:

> The possible lines into which any line may be cut at one chop exceed any discrete multitude. Now the points on a line form a collection at least as great as the collection of the possible lines into which it can be chopped. Hence, the points of a line are supermultitudinous.
>
> The other proof is this:
>
> A line consists wholly of points; but in order to chop a line into points, the two ends of each piece must unite; and to do that without shrinkage they must merge into one another. Hence the collection of blades of the chopper must be so great that its constituent blades are no longer distinct. (97)

It is not difficult to complete this 'proof' which just like the first one rests on the intuition of the line as composed of a set of small lines any of which is larger than points. The premiss is thus that the synthetic line contains pieces of line in the number *S*, not so far from what should be proved, and you need only the further premiss that points are smaller than lines in order for the proof to be finished. The first premiss, however, is little less problematic than is the conclusion.

It is interesting that Peirce – after all these proof attempts – return to his definition of mathematics as the science which draws necessary conclusions from hypotheses. It must be this idea which lies behind his claim that: 'This I declare to be the synthetic geometer's hypothesis of the relation of a line to its points. *But it does not affect my argument if it be not so. It is sufficient for my main purpose that it is a perfectly consistent hypothesis*. For all I am trying to do is to elucidate the conception of a supermultitudinous collection and show that it involves no contradiction.' (97, our italics) The whole of the argument is so to speak fictitious, what Peirce wants to prove is only that a certain concept – *S* – is consistent – not that it exists in any more ontological sense. Mathematical existence in this claim seems formalistically to be reduced to consistency. After having regarded and rejected the possibility that *S*'s 'cohesiveness' could be due to a certain tricky arrangement

of its elements only, Peirce admits, before his final conclusion, that 'There is, I confess, a paradoxical aspect in the proposition that a collection may be so great that its individuals lose their separate identities. But the key of that paradox will probably ultimately be discovered to lie in *some unnoticed condition* in the general hypothesis of a collection which requires the mergency of individuals.' (100, our italics)[512]

But the proof that PCH is consistent in relation to set theory (if we assumed it could in fact be proved) is not the same as the claim that it is implied by set theory. 'Half' of Cohen's proof of CH's indecidability in relation to ZFC set theory consists of Gödel's proof of its consistency in relation to ZFC set theory. Peirce consequently only believes he has proved that PCH is not inconsistent with set theory, and consequently he must await the appearance of a new axiom, 'some unnoticed condition' in the definition of the concept of set may finally connect the two theories – not unlike Gödel when he, 50 years later, hoped for a solution to the CH problem by the assumption of new, abstract, and complicated axioms concerning the concept of set which might be obtained by phenomenological, non-formal means.

If we look at Peirce's argumentation for *S* from a distance, there are several paradoxes to be solved. We already mentioned one: Peirce's strange interest in the construction of the transfinite hierarchy of sets, given his explicit insistence on the intuition of the synthetic line before any divisions – a line which it should not, then, be necessary to create by the 'merging' of discrete points.

Another strange issue is the fact that given Peirce's idea of mathematics as the pure investigation of the implications of hypotheses, it seems peculiar he tries to found the decisive concept in his metaphysics on a mathematical argument. The answer to this question can only be that Peirce, in metaphysics and logic, regards mathematics as a sort of aide, a supply of deductively valid diagrams whose application in less hypothetical cases mathematics assumes no responsibility for. But in that case, Peirce's argument is metaphysical and not mathematical, and all the mathematical argumentation only goes to show that the idea of a superpower continuum is not inconsistent: the mathematical argument, were it valid, does not *prove S*'s existence, but only its *possible* existence – the further verification of the hypothesis is no mathematical issue but a case for phenomenology, logic, semiotics, and metaphysics.

A third reason for puzzlement is the combination of similarities and differences in relation to prominent positions in the philosophy of mathematics of the twentieth century The idea of mathematics as the investigation of hypotheses approaches Peirce to Hilbertian formalism; just like his unproblematic acceptance of objects requiring infinities of operations to be constructed (which is no problem for the formalist as long as the calculus in question may be stated in a finitist metalanguage) – but on the other hand, Peirce displays certain affinities with intuitionism, partly in the synthetic aspects of his arguments for *S*, partly in his idea that the generativity of transfinite cardinals must somewhere be stopped. Peirce will not be

hindered in undertaking innumerably many operations, but he can not subscribe to the idea of Aleph-ω many operations.

A fourth reason, finally, is the tension between his insistence that the continuum exceeds any attempt at grasping it analytically with discrete means – and on the other hand his claim that sufficienty large discrete sets seem to be inhabited by germlike tendencies towards continuity, the idea that one cardinal in some sense implies the next one – in general the idea that the transition between discontinuity and continuity must, in itself, be continuous. We shall return to these points.

In a lecture series from 1898 which has recently been published in its entirety under the title *Reasoning and the Logic of Things* (RLOT) Peirce again takes up the definition of the continuum, now connected to his claim for the reality of infinitesimals and the most elaborated version of his continuum metaphysics. Let us discuss Peirce's metaphysics of infinitesimals and the publishers' Ken Ketner and Hilary Putnam's reconstruction of Peirce's position.

INFINITESIMALS AND THE STATUS OF MATHEMATICS

In the third of the 1898 lectures, Peirce introduces his logic of relations, to which we return below.[513] We shall not here discuss Peirce's diagrammatical notation techniques for logic, but only take note of those ideas implying continuity and infinitesimals. Peirce implicitly takes his point of departure in the traditional predicate judgment 'S is P' which is generalized to the extent that more than one subject may appear in it. This is well known from everyday language, as for instance in constructions like 'X sells Y to Z for the price of Æ'. Peirce then determines the predicate as that structure in the construction which is left when all subjects are replaced with empty slots. In this way the predicate outlines a relation between a plurality of subjects, and predicates may take potentially infinitely many empty slots to be filled with subjects. By his famous theorem of reduction, Peirce claims that all predicates with a valency larger than three may be reduced to a combination of predicates each of which are no more than trivalent. Trivalent predicates exist, on the other hand, which may not be further decomposed into bivalent predicates. The predicate in the logic of relations thus implies a more extensive generality (and, consequently, possibility for comparison) than does the classical predicate judgment. Now subjects may be related not only in terms of satisfying (or not) the same predicate, but also in terms of satisfying one predicate in different slots; appear in analogous slots in different predicates, etc. Similarity, in this respect, is just a simple relation among many, and the concept of a class of objects defined by a predicate is substituted with the concept of a 'system' defined with the interrelations between its objects (156).

Second, it is crucial to note that to Peirce this relational logic makes possible a tripartion of the types of signs necessary to describe these relations. First *verbs* describing predicates (to which he counts adjectives, common nouns, etc.), second *demonstrative pronouns* (including proper names), referring to the subjects involved

in the expression, and finally logical operators (negation and conjunctions, etc., see Chap. 1). These three classes of signs now provide the meaning, reference, and inference, respectively, of the proposition.[514] (155).

The crucial issue here with respect to the continuum is that the classes (Peirce uses the term even if he has just argued it ought to be replaced with 'system') of objects referred to by propositions of the logic of relations very often possess generality (predicates are general and subjects may be). That this generality can only be defined by continuity forms a cornerstone of his semantic realism.

Now he runs through, in high speed, his 'proof' for the supermultitudinous set *S*, and then he divulges in new deliberations on the nature of the continuity of a line. Take, he says, a circle, it is a set of points, because a particle which in every moment contains a point, moves along the periphery and draws a line which consists only of points in which the particle has been. This appears to be a traditional description of the matter, but Peirce, importantly, adds that no point on this line has any distinct identity; it is only when we single out a point on the line that it becomes distinct from the others – because a discontinuity is then created. If we now cut the line in this point, *then the marked point has become two points*. This claim, of course, is contrary to the concept of the Dedekind cut which defines a real number (and hence, in the classical interpretation of the arithmetical line, a point on this line) by dividing the line in a closed and an open section (that is, one part containing its limit point and another not so):

> It has become two points. And if those two ends were joined together so as to show the place, – they would become one single point. But if the junction ceased to have any distinguishing character, that is any discontinuity, there would not be any distinct point there. If *we* could not distinguish the junction it would not appear distinct. But the line is a mere conception. It is nothing but that which we can show; and therefore it follows that if there were no discontinuity there would *be* no distinct point there . . .(160).

The two points are two as soon as we can distinguish them. What is at stake here is of course not mere psychological distinction skill, the crucial thing is what can be demonstrated with logical means. One should not be fooled, however, by the apparent simplicity of the criterion; even if the line is only our 'conception', the evidence it presents to us is not complete – cf. also Peirce's Gödelian idea that new axioms might settle the continuum hypothesis.

Even more counterintuitive to the present conception of the line is the following claim: one can remove one point from the end of the line segment, but still an end point will remain – as against standard arithmetic, according to which we are left with an open interval with no limit point included in the line if we remove the limit point from a closed interval. As is evident, Peirce's conception of the geometrical line is very different from the arithmetical line:

> 'The end of a line might burst into any discrete multitude of points whatever, and they would all have been one point before the explosion. Points might fly off, in multitude and order like all the real irrational quantities from 0 to 1; and they might all have had that order of succession in the line and yet all have been at one point.'(160)

Points in an amount corresponding to the real numbers may be removed in an ordered series from one single point on the line. As is evident, this conception of the line also implies a completely different conception of the point which is close to the Aristotelian idea of the line being primitive and the point being the addition of a discontinuity to the line. It goes even further than Aristotle's idea which does not contain any idea of a point being able to contain other points, let alone an ordered series of points.

The editors of *Reasoning and the Logic of Things*, Ketner and Putnam, have added to their preface an attempt to construe a consistent hypothesis from Peirce's doctrine of point and line. They observe that none less than Gödel remarked that it seems intuitively correct (as against the Dedekind cut) to assume that if the geometrical line is cut into two parts, the result will be two symmetrical pieces (personal communication to Putnam, 38n). The division of the line should not be conceived of as the mapping of sets of points (because the line is no set of points), but quite on the contrary as an irreducibly geometrical transformation, and they propose that Peirce's point of view may be expressed in analogy with non-standard analysis which, like him, reintroduces infinitesimals in analysis. Thus, when Peirce claims that the infinite series of points 'are' one single point, then this is no identity claim, but rather a *mereological* use of 'to be' so that it should be read as saying that the point 'consists of' of many points, just like in the proposition 'India and Pakistan were one country in 1940'. Hence, Peirce's conception of the point implies that the points may have parts – and transfinite reasoning, of course, parts may have the same power as the whole they partake in.

Formalized in non-standard analysis, they continue:

> . . . we say that two points P and Q whose distance is infinitesimal are 'identical modulo the infinitesimals', and we symbolize this by using a wiggly equals sign: $P \approx Q$. If P is a point, the collection of all points Q such that $P \approx Q$ is called the *monad* of P.(44)

It can now be shown that every point must lie within the monad of a standard point, and they propose that Peirce's idea can be expressed in the following way: his concept of 'point' corresponds to the concept of 'monad', and parts of points corresponds to normal points. Then it becomes meaningful to say that every point (monad) contains a set of parts (points) with the power of the reals. Given Peirce's identification of the power of the geometrical line with the supermultitudinous set S, one may infer that the set of non-standard points on the line (exceeding Aleph-1) must be vast; in Peirce's construction of this phenomenon it does not, however, give meaning to distinguish standard and non-standard points, respectively, because Peirce does not share the ambition of non-standard analysis to correspond to standard analysis. So is it, from this point of view, possible to 'remove' a monad from the end of the line? Ketner and Putnam plausibly argue that this point of view must be constructivist: by removing the monad we create a new monad of possible points; what is removed is a 'notion of a possibility', and it would give no meaning to say that we removed the whole set of points which could ever be construed in the monad of the original point (53). We may add to Ketner and Putnam's discussion that Peirce has already elsewhere, in his analysis of the continuity of time (which

we have already touched upon), proposed a distinction analogous to that between monad and point, namely *instant/moment*. The latter has an infinitesimal extension and hence actual reality as opposed to the former which are mere, punctual aspects of the latter and which may be actualized or not.

Construed in this way, Peirce's position with respect to infinitesimals is consistent, and Ketner and Putnam may propose that Peirce's use of the concepts of potentiality and actuality in relation to the possible and actualized points takes him close to Putnam's own 'modal logic view of set theory'[515] (50): sets are not mathematical objects in the sense that we, e.g., have the set of all real numbers[516] before us, the sets are constructions which implies no more and no less than given a certain set of operations we *may* construct them. In this respect – they continue – Peirce comes close to e.g. Brouwer's intuitionism with the crucial difference that Brouwer would let the finitude of the human mind prohibit infinite series of operations (which implies, in turn, the sacrificing of well-known mathematical objects as for instance the real numbers), while Peirce, given the free domain of potentiality on the one hand and the doctrine of 'real possibilities' on the other, did not erect prohibition signs of that kind. Given the potentiality of concepts, Peirce does not refrain from assuming a set transgressing the power of any transfinite set, his S – but this has the epistemological reason that intuition and perception always already contains this set. The faculty of intuition thus radically exceeds what any potential, transfinite series of operations would ever be able to accomplish.

These vast sets of points *are* not, actually, on the line which is not, in turn, composed of them. As an Aristotelian, Peirce may indeed say that the points on the line *potentially* are there but that they are only actualized when we perform some operation making them distinct. It is this Aristotelian train of thought which forms the base for Peirce's constant link between the continuum and the concept of potentiality – and it is *this* metaphysical intution which provides the motor in Peirce's various attempts to chart the domain of the transfinites: multitudes of ever so great a power are distinct and hence potentially actualized while the realm of the merely possible always remain greater than that of the actual. This is why Peirce seeks a mathematical foundation for the point of view that the continuum exceeds any transfinite power.[517] In this way, Peirce founds his doctrine on a modal triad: the potential, the actual, and the real. Already the whole numbers constitute a set which we can only potentially construe completely but which in principle would be actualisable as infinite procedures can be completed. Beyond the actual lies, consequently, a realm which is not only potentially actualizable but, as Peirce will put it, realizable: evolution does in fact realize continua in the shape of 'real possibilities', as universals with real existence. Beyond that, on the other side, remains the continuum in itself, which may never be actualized in toto. Ketner and Putnam express this idea in terms of possible-world semantics: '. . . one can never construct *all possible* points on the line, because there is no possible world in which there are actually Ω [their Cantorian name for the size of S, fs] 'distinct individuals.': '(. . .) *What answers to our conception of a continuum is a possibility of repeated division which can never be exhausted in any possible world, not even*

in a possible world in which one can complete abnumerably infinite processes.' Also here, the two of them continue the ideas of Putnam 1967: '. . . it is not possible for all *possible* sets to exist in any one world!' (311)

THE RETURN OF NON-COMPOSITIONALITY – A PEIRCEAN CONTINUUM HYPOTHESIS AS METAPHYSICAL POSSIBILITY?

Putnam and Ketner's Peirce interpretation forms part of the tendency to question the Cantor-Dedekind interpretation of the reals which has been state of the art for most of the twentieth century set theory and arithmetics. The force of that position (**R** as an uncountable, connex set (which happens, that is true, to make discontinuous real functions much more probable than continuous functions!)) has been to unite arithmetics and spatiality, so that different geometrical and topological spaces can be conceived as having $\mathbf{R}^n$ as their base. Thus cardinality and topology are unified (because uncountability and connexity are identified), logic and mathematics are unified (Salanskis 194). The continuum hypothesis is undecidable, that is true, but this inconvenience has been neglegted and has had few consequences in practice. The duality within formalism of pragmatism (we assume CH when needed) on the one hand, and scepticism (in reality we know nothing about continuity to the extent that it exceeds formalisations) on the other now gives way to a new obsession with what Salanskis calls 'the return of non-compositionality'. This has been evident, inter alia, in the volume *Le labyrinthe du continu* which he edited (1992) with the participation of a series of mathematicians and philosophers of mathematics. Non-standard analysis with the return of infinitesimals, non-Cantorian set theory, renewed interest in Gödelian Platonism, the search for new axioms in set theory – all these tendencies might point in the direction of Peirce's metaphysics of the continuum, and in this section we shal briefly consider a bundle of these actual deliberations on the status of the continuum.

The possible consistency of Peirce's construction – despite all the blatant errors in his attempts at appropriation of Cantorian set theory – lies first and foremost in Cohen's proof of the autonomy of the continuum hypothesis. By the so-called 'forcing' technique this proof shows that within standard ZFC set theory, the continuum may be ascribed any cardinality you wish without this leading to inconsistency.[518] It could be added that Peirce's idea of *S* has an interesting equivalent in Cantor's concept of the set of all ordinals Ω which he as mentioned realized was an inconsistent concept, an 'absolutely infinite set'. The proof of the inconsistency of this set seems to be vaguely foreboded by Peirce in his 'proof' for the existence of *S* – the fact that the sum of the numbers preceding a given number equals that number gives the consequence that 'the set of all numbers' is inconsistent. Peirce commits the error of assuming the countable series of transfinite cardinals to constitute a superior limit to the series of transfinites, but apart from that his argumentation takes the same course as the argument that 'the set of all numbers' is inconsistent. We can add that Peirce certainly would not oppose the idea that *S* is inconsistent. In his trivalent logic this could be interpreted as

saying that claims involving elements of *S* have the third, undecided truth value between truth and falsity – anticipating intuitionist logic. Precisely the infinitesimal parts of the line are ascribed such a status by Peirce: he claims that the law of excluded middle (implied, of course, by a two-value logic) does not hold for the domain of the possible and particularly not for these points which are consequently 'vague'. On the other hand, if falsity is taken to be the only source of inconsistency in logic, *S* would yet be consistent in this (admittedly) more broad use of the word.

All the time, certain voices from the past kept protesting against the scepticist triumph of formalism in the twentieth century. On the one hand, the hyper-scepticism of intuitionism connected to (psychological, it must be admitted) references to intuition (Weyl, Brouwer), on the other hand, Gödel and his updating of Platonism. Gödel kept insisting that the continuum had existence as an autonomous object to be investigated. It just needed a more complicated and competent version of set theory, obtained by the addition of new axioms, in order to be decidable. Gödel claimed – and rightfully so – that formalism tends to overlook the fact that there must always be extra-formal motivations for the assumption of new axioms – and he claimed, moreover, that we have no reason to expect such motivation to be particularly simple in every case. Against the formalist main current Gödel thus interpreted his own famous 1931 proof, not as an indication that you had to stay within formalisms (because it proved that every sufficiently complex formalism makes possible true propositions which cannot be proved), but quite on the contrary as an anti-formalist insight that formalisms are motivated from their outside. Thus, a compuational theory of consciousness was impossible and the main task ought to be the search for new and more ambitious axioms by an intuitive, phenomenological procedure, not a formal one. The older Gödel for the same reason took a strong interest in Husserl and assumed that a 'clarification of meaning which does not proceed by logical definitions' could take things further, for instance by a phenomenological analysis of the concept of 'set'[519]. Gödel shifted between believing and disbelieving CH, but his Platonism remains a corrective to the extreme formalism which will renounce any intuition at all (or better: *Anschauung*) access to mathematics except the tiny subset of intuition needed to read finitist calculi of symbols, a subset which is assumed to be fully controllable.[520] Apart from this, it remains undecidable (if we are to take formalism at face value) what a formal calculus is in fact 'about'. Opposed to this, Gödel's point of view is an ongoing questioning of formalism, to the extent that intuition – in this case the primitive intuition of the continuum – is taken to be irreducible and the formalisms are seen as vehicles for knowledge about intuitions, not for the extermination of them.

In the last decades, Platonism and its claim that CH must be decidable because mathematics is taken to have real, autonomous entities as its objects, have had a renaissance, e.g. in Penelope Maddy's philosophy of mathematics (Maddy 1990 and 1997) claiming a so-called physicalist Platonism. It takes its point of departure in physicalism in much of recent American epistemology, as it is expressed e.g. in the

classical Quine-Putnam position where the applicability of mathematics in physics is the main argument for a kind of Platonism. This Platonism claims, at least, that mathematics is not only a mental construct of ours nor empty logical formalisms (even if Quine still clings to conventionalism). As Maddy rightly observes, this position does not account for the validity of non-applied mathematics, neither for the classical issue of the apparent evidence in simple mathematical intuitions. Yet, she agrees in the demand for naturalization of mathematics, taken in the sense that mathematics must be seen as a science on a par with other empirical sciences[521], so that a physical explication is needed to account for the mathematician's knowledge about his objects. This idea gained authority due to a classical paper by Benacerraf ('Mathematical Truth', 1973, reprinted in Benacerraf and Putnam 1982) which takes as its main argument against Platonism that it does not account for how it should be possible to achieve knowledge about timeless, abstract entities without any 'causal power'. This idea is influenced by the 'causal reference'-semantics of its time claiming that reference in general would be explainable by causal influence of the object on the utterance. This somewhat simplistic idea is taken up by Maddy in her idea that sets may be perceived directly. In a certain sense, this is of course correct as a Gestalt Theory claim, and in this sense it is important because it can be argued that many sets have an objective, empirical existence. But its naïvety relies in the fact that this claim is identified with the requirement that the objects in question have a physical existence only, that is, a purely material existence. Another problem is that this idea holds only for finite sets – nobody ever saw a transfinite set, so taken at face value, such an idea might lead to strongly finitist ideas which Maddy does not embrace. A more cautions expression might be to say that the schematic aspect of perception which allows us 'directly' to see shape, contour, number, sets, etc. of everyday perceptual objects is the same ability which is active in the pure intuition of mathematics.[522]

But let us leave the physicalist part of Maddy's Platonism in order to discuss her insistence that realism demands of CH that it be decided.[523] Her point of departure is Cohen's 1966 proof that CH is undecidable in ZFC. This proof basically assumes a tension between the two methods for obtaining higher cardinals (by the power of ever larger sets of ordinals; by the power sets of the preceding cardinals, respectively). Cohen finds an incurable opposition between these two methods, because exponentiality in the power set construction is always 'stronger' than the generation through ordinals. Thus, Aleph-1 must be less than $2^{\text{Aleph-0}}$ – and CH consequently false:

> A point of view which the author feels may eventually come to be accepted is that CH is *obviously* false … Aleph-1 is the set of countable ordinals and this is merely a special and the simplest way of generating a higher cardinal. The set [of subsets of the natural numbers] is, in contrast, generated by a totally new and more powerful principle, namely the Power Set Axiom. It is unreasonable to expect that any description of a larger cardinal which attempts to build up that cardinal from ideas deriving from the Replacement Axiom can even reach [a set of size $2^{\text{Aleph-0}}$]. Thus, [$2^{\text{Aleph-0}}$] is greater than Aleph-n, Aleph-ω, Aleph-α, where $\alpha = \text{Aleph-}\omega$, etc. This point of view regards [the power set of the set of natural numbers] as an incredibly rich set given to us by one bold new axiom, which can never be approached by any piecemeal process of construction. (Cohen (1966), 151; quoted from Maddy 131)

– a theory which just like many in our days considers the continuum large as compared to Aleph-1 (unlike Gödel who most of his life did not believe in CH neither, but tended to identify the continuum with Aleph-2). In this light, the assumption of CH would be an unrealistic delimitation of the continuum, but the matter may, as Maddy says, just as well be seen the other way around: that CH ascribes to Aleph-1 an unrealistically large size. Cohen's proof, however, paved the ways for the attempts which have since then been made to construe a new and better version of CH by reparing Cantor's original version or by stating a version making the continuum stronger than Aleph-1.

The former road takes its departure in Gödel's first half of the indecidability proof (the proof for CHs consistency in ZFC) which is built on a version of the classical, iterative set theory with its axioms – but delimited in so far as only those subsets are allowed which may be explicitly defined by predicative formulae, that is, expressions which refer to sets already constructed. This so-called constructible universe is called L, and Gödel's proof thus adds a constructibility axiom to ZFC which identifies the whole universe of set theory with the set of merely constructible sets L, so that $V = L$. Gödel's proof now demonstrates that no contraditions may be derived from $ZFC + V = L$ which were not already in ZFC. But as it may be proved that CH follows from $ZFC + V = L$, CH can not be disproved by ZFC. $ZFC + V = L$ is thus a straw man built with this one intention – but could it not be converted into a positive idea so that $V = L$ is taken as a new axiom which makes not only CH and GCH but also AC (so all sets may be well-ordered) true? This solution has tempted many which find it an attractive and even 'natural' solution, because constructibility may be seen as a consequence of the very concept of 'set' (Keith Devlin). In this train of thought, the set concept is interpreted in a way appealing to Peirce: intensionally (as an explicitation of certain predicates referring to properties which, in turn, determine the elements) instead of extensionally as defined by its elements. This solution, however, is untenable, also for Maddy, because it goes against the fundamental, combinatorial assumption of set theory by simply prohibiting the existence of certain subsets.[524]

The other main road leading from Cohen 1966 receives Maddy's support: to see the continuum as stronger than Aleph-1. This strategy must thus seek new axioms supporting such an idea, partly the different large cardinal axioms and so-called 'dependency axioms'. Already Gödel rested his head on such axioms, and the first of them, the axiom of inaccessible cardinals (going back to Zermelo as well as Sierpinski and Tarski around 1930), is built on the simple intuition that all sets which may be built from the other axioms in themselves constitute a set, on which those very same procedures may in turn be used (Maddy 135) – by which procedure otherwise inaccessible cardinals are reached (because of the infinity axiom is used on already infinite sets, so to speak). This operation may now be repeated, which, in turn, gives as a result the axioms of 'measurable cardinals' and 'supercompact cardinals'. The former assumes that there are inaccessibly many cardinals between the inaccessible cardinals and the first measurable cardinal, and was proposed

already by Ulam in 1930 and gaining support because of its promising implications elsewhere in mathematics.

Unfortunately, it was quickly proved in 1967 (Solovay and Levy) that none of these attempts decides CH. Axioms of 'very large large cardinals' led to the introduction of so-called supercompact cardinals (there are measurably many measurable cardinals below the first supercompact (SC) cardinal) which proves projective determination, and ZFC+SC were thus seen by many in the 1990s as an attractive maximalist alternative to the minimalist ZFC+V = L solution. But the status of CH has not yet been settled by this development:

> If [CH] is true in L, a minimal environment, perhaps it is false in a maximized world of large cardinals. There is some evidence in that direction [...] But in Cantor's favoured form - Aleph-1 = $2^{\text{Aleph-0}}$ - it remains false; the continuum isn't of size Aleph-alpha for any alpha because it can't be well-ordered.(142–43)

No unanimous consequence for CH follows, and different positions among the adherents of ZFC + SC seem to exist – this, in turn, because they do not agree whether CH or its negation is the most restrictive claim (143). Even if no solution thus is at hand, Maddy calls on a realist basis for research to continue, and she does not hide that her sympathy lies with ZFC + SC. The strange thing is that this preference in no clear way is connected to her physicalist Platonism.

Maddy is concerned with the question whether arguments for the different axioms have inner or outer character, respectively, referring to whether the arguments are related to the plausibility of axiom itself or whether they relate to intertheoretical effects in relation to other parts of mathematics. She concludes that the arguments for very grand cardinals do not fit easily into one of those groups, but rather follows certain 'rules of thumb' which are – just like Gödel imagined, in fact – is connected to basic intuitions about what the concept of set entails. These are intuitions like the idea that the universe of sets continues to as large cardinals as possible or the idea that the hierarchy of cardinals must contain subsets which looks alike – a sort of generalisation of the axiom of replacement. The reason why Maddy refuses to give these rules of thumb an 'inner' status goes as follows: '... because I think they extend beyond anything that could plausibly be traced to an underlying perceptual, neurological foundation.' (141)

But this already goes for **N** taken as an actual infinity?! Maddy admits that the idea that sets are formed from already existing elements and that they are formed by combination without necessitating any explicit rules, does have an intuitive basis, while the continuation of the hierarchy to infinity (cf. already the axiom of infinity) and the continuation of it as far as possible, is based on the 'methodology of set theory' rather than any simple intuition (ibid.). But is this methodology without intuitive basis? It seems as if Maddy's concept of intuition is constrained by her physicalism to a small, strongly finitist inventory. It is indeed correct that no sets larger than finite or, at most countably infinte sets, seem to have any probable existence in the physical world. This gives rise to further questions: a really stubborn physicalist would probably not find questions far upwards in higher cardinals interesting (cf. 'materialist' Soviet mathematics refusing the 'idealism' of

set theory), because they really only pertain to non-applied math? And, as Maddy herself touches upon: the number of neurons and their interconnections in the brain is vast but still finite: how could they (presupposing a computational theory of consciousness which Maddy does not discuss further) give rise to representations of the infinite? But there is no need to go so far as to the strong (even if yet undecidable) arguments for the existence of non-computable processes in the physics of the brain (like Roger Penrose) to find Maddy's version of basic mathematical intuition unsatisfactory. For it *is* indeed the case that a simple intuition of the continuum exists which does not, it must be admitted, imply the assumption of actual infinity but only refers to the continuum as a primitive phenomenon characterized by homogeneity between its parts, lack of distinction between parts, etc. – to a large extent corresponding to the Aristotelian notion of the continuum as potentiality. The two Aristotelians Peirce and Thom refer, both of them, to the idea that the givenness of continuity for intuition in some sense grants its reality. Thom: the idea of the continuum can not be an illusion – it is intersubjectively verifiable and it is even subject to commercial exploitation, as in cinema where we without any problems interpret the presentation of 24 discontinuous pictures per second as the continuity of time: 'thus, it is an "objective" scientific fact and must, as such, be "objectively" explained. If you do not have an immanentist theory of the continuum in our neurophysiology, how will you explain to me the origin of the illusion!'[525] Essentially the same argument may be found in Peirce who argues that continuity – together with thirdness and argumentation – is not the privilege of the mind, because it can be found already in nature: 'Whatever unanalyzable element *sui generis seems* to be in nature, although it be not really where it seems to be, yet must really be [in] nature somewhere, since nothing else could have produced even the false appearance of such an element *sui generis*. (. . .) . . . the very fact that there seems to be Thirdness in the world, even though it be not where it seems to be, proves that real Thirdness must somewhere be. If the continuity of our inward and outward sense be not real, still it proves that continuity there really is, for how else should sense have the power of creating it?' ('Detached ideas continued', 1898, NEM IV, 344) The Peirce-Thom argument has a strong thrust against physicalists whether they be nominalists, or, like Maddy, realists, because the core of the argument is naturalist: basic existing ideas which can not be reduced to other ideas (such as may unicorns, etc.) can not be naturalistically dismissed as illusions. They must possess birthright in nature itself, for otherwise the naturalisation of epistemology can not be pulled through . . . One can add, a propos set theory, that it is the primitive idea of the continuum which gives rise to the rarely discussed premiss in CH that $c = \mathbf{R}$ – that the real numbers (whichever vast cardinality they may possess) do in fact exhaust the geometrical line – and hence, in turn, gives rise to the whole search operation for grand grand grand cardinals, or, the other way around, a constructivist constraint on the real numbers. From a Peircean point of view, the search for very large cardinals could indirectly appear as a proof for continuity as a primitive intuition, because the larger cardinals are being invoked in attempts at making CH decidable, the more probable the inexhaustibility of the continuum seems to be.

But Peirce's rejection of identifying the continuum with the real numbers makes him, on the other hand, immune to the possibility of CH being decided by some new grand cardinal axiom: even the eventual identification of **R** with some mighty cardinal will, from his point of view, never exhaust the intuitive continuum.

LACK OF DISTINCTION

A further mathematical development which might support a consistent interpretation of Peirce's concept of the single elements' lack of distinctness in uncountable sets is Gregory Chaitin's algorithmic information theory and its 'random numbers' which follow from an extension of Gödel's theorem. Chaitin defines a random number as a number which can not be expressed in any shorter way than its own decimal expansion. Many numbers may be written shorter than that, the integers, the rational numbers (being expressible as fractions), but also many irrational reals, π, e – in short, all real numbers which may be expressed by some finite algorithm. The interesting issue in Chaitin's theory is that it can be proved that the random numbers (unlike the non-random numbers) have the same power as the reals – so that 'almost all' numbers on the arithmetic line are random. On the other hand, it can be proved that it cannot be proved about one particular number that it is in fact random. It is impossible, given a candidate random number, to exclude the possibility that some subtle order in its decimal expansion might once be found, thereby making the decimal expansion reducible to some shorter expression. This implies that random numbers are uncomputable, they may not appear as the output of a computer (not only because of the infinte amount of time printing would require) – they can not be determined distinctly. In this interpretation, the random numbers as a vast subset of the reals seems to satisfy Peirce's demand for non-distinctness, vagueness: it is impossible to pinpoint any single random number even if they are overwhelmingly there.

Chaitin does not involve the continuum question, but a related argument is presented by Harthong (cf. Salanskis, 207) who makes a distinction between simple, programmable integers on the one hand and complicated, aleatoric integers on the other hand. For sufficiently large natural numbers, the simple numbers are surrounded by complicated numbers – just like standard reals are surrounded by non-standard reals – and Harthong consequently proposes a metaphorical interpretation of the continuum: it is like the integers viewed at a far distance: '**Z** vu de loin'! Thus the question of the continuum is taken by him to be an irreducible *scalar problem*: the same phenomenon may, in different blow-ups, be continuous and discrete, respectively. In a philosophical bent, Harthong proposes that the duality continuous/discontinuous (just like René Thoms idea of 'apories fondatrices' for different sciences) constitutes a veritable antinomy for pure reason in the tradition of Kant's well-known four antinomies from *Kritik der reinen Vernunft*[526], and that it for that reason constitutes a question which remains necessarily unsolvable.

Salanskis sums up in his synthesizing paper which touches upon many of the positions outlined here[527] when he depicts the actual understanding of the continuum

as a necessary surplus exceeding any possible description of it: '... what thus appears in this set of works of non standardists continuing the Borelian tradition, is that the continuum is some sort of 'effect' which results from an *excess*. If the approach from the point of view of 'de-setting' the destiny of the Cantor-Dedekind continuum emphasizes *incommensurability*, what is won on the other hand is the attention towards the constructive and its place in the world of mathematical entities and it makes us confront the philosophical theme of *excess*. The link between those two terms seems clair: the excess engenders incommensurability, and the distribution of incommensurability on the line adds to it the constitutive *quality* of the continuum ...'[528] This excess appears, according to Salanskis, rather as an *indetermination within* the finite (rather than the Gödelian idea, as a *determination of* the finite calculus). This difference points towards Petitot's reformulation of Gödel's position.

René Thom repeats, in the Salanskis volume, his conviction about the irreducibility of the continuum, from a simple argument. The continuum is ontologically prior to discontinuities, measured by a simple criterion: X is ontologically prior to Y, if X naturally takes Y as a predicate, but not vice versa. (This hypothesis is interesting, by the way, to compare to Peirce's ontological observations as to the subject-predicate distinction.) By the application of this criterion he reaches the observation that the continuum may easily be modfied discontinually – which in some sense corresponds to taking it as a predicate ('a broken continuum') while the other way around is impossible: a discontinuous entity or set of entities can not take a continuous predicate without becoming in some way continuous itself (138). We may imagine something like 'merging points' as an example.The sets of set theory are like flour sacks (without sacks, because they are the result of the mind) in which every flour particle is endowed with a label, Thom sarcastically says. Instead he refers to an Aristotelian space characterised by total qualitative homogeneity where 'rien n'y est identifiable' (141) as a paradigm for the continuum. The points, then, could be derived only secondarily by taking Aristotelian *chora*, places, and letting them shrink to zero size. Generally, discontinuities are introduced in the continuum by *cuts*. Even non-standard analysis is judged too rigid in its insistence to stay within the confines of set theory where the continuum is supposed to be the result of an arithmetical derivation process. Thom hints at the possibility that richer classes of irrationals may be defined and ends up with a promising Gödelian-style hint: maybe the concept of point has not yet revealed all its secrecies for us. We are tempted to believe that Thom has up his sleeve a more or less vague idea that the point in some yet unknown way will appear able to demonstrate its continuous affinities. We shall return to Thom's deliberations on the continuum and the subject-predicate calculus.

The most detailed analysis of the continuum in Salanskis' volume is undertaken by Thom's disciple Jean Petitot[529] who, at the same time, makes a plaidoyer for a Kantian-transcendental rehabilitation of Gödel's Platonism. Petitot's point of departure is the observation of a strange duplicity, a so-called 'bimodality' regarding the status of the continuum in mathematics and physics, respectively, as critically

construed objectivities. While the continuum in mathematics has the status of 'meaning', as a thing-in-itself and consequently is endowed with the inaccessibility of the *noumenon* as a mere regulative idea, the continuum in physics has the status of an unexplained empirical phenomenon appearing in the sensuous intuition – logically anterior to the constitution of a physical objectivity by the transcendental-aesthetical determination of the status of form and the development of physical concepts therein by means of schematisms. Here, the *Ding an sich* of physics on the one hand becomes the 'inner substance' of matter, and, on the other, it refers to forms more complicated than those which mechanical physical objectivity governs: phenomena of self-organisation and related morphological phenomena. Petitot's idea now is that this constitutive duality between mathematical inaccessibility and physical givenness is what gives rise to the constantly diverging interpretations of the continuum. He thus separates 4 interpretations:

1. (Neo-)Aristotelian ideas of the continuum as primitive phenomenological intuition, characterized by the merging of parts and by the merely potential infinite point sets which may, in turn, be realised herein. This position evidently encompasses Thom and Peirce, and, in addition to them, Brentano, Stumpf, Husserl, Weyl, Poincaré, Gestalt Theory – they all chose different ways of interpreting this basic status of the continuum. Brentano, Poincaré, and the gestaltists have a tendendy to psychologize it (and thus make of it a sort of necessary illusion), while Peirce, Husserl, Weyl accordingly should phenomenologize it. Naturally it is correct that the latter will refuse a psychologisation of the continuum, and Peirce and Thom can, as we saw above, directly use basically the same argument in this refusal[530], but the question is whether it is correct to make Peirce a phenomenologist as opposed to Thom. As we have seen, the continuum has a basic ontological status in Peirce, and Thom's and his positions rather seem very close. Husserl of course takes the intuitive plurality af continuous morphologies – 'inexakte Essensen' – to be irreducible, but also as a fact distinct from scientific, purely formal symbolization so that there is no direct road from the former to the latter.
2. A second position is the continuum as a case of actual infinity given in intuition; this includes Kant, Neo-Kantians, Veronese and Salanskis.
3. A third interpretation is the positivist-eliminativist in Carnap and nowadays Hartry Field who conceive of the continuum as a purely physical phenomenon.
4. A fourth interpretation is Gödel's who as already mentioned regards it as possible to grasp the continuum – including its inexhaustibility and potentiality – by set theoretical tools, if they are enlarged with suitable stronger axioms. To this, it must be added, despite Petitot's placing of Gödel in 'the opposite end of the spectrum' (246) that he, as already mentioned, stick to a phenomenologically primitive intuition as a prerequisite to the formalizations of set theory. Gödel's position has, of course, appeared like a constant thorn in the flesh on Hilbertians taking further Hilbert's program (e.g. Feferman). Feferman has approached the problem via his idea of a 'reverse mathematics' (250) which, given a theorem, seeks which axioms will be sufficient to prove

that theorem – an abductive method which Petitot praises as 'deflationist' and highly anti-Platonist. Thus is indeed the intention behind Feferman's argument, but the question remains whether this procedure is necessarily anti-Platonist. Gödel's own idea (based on CH as a theorem from which to search which axioms may prove or disprove it) has exactly the same reverse methodology, even if, admittedly, other expectations. We can not here go into Feferman's technical fight with Gödel: the decisive thing is that he sticks to the idea that the power set construction ($P(x) = 2^x$) is well-defined for all sets – whereas Feferman claims that this Platonism is medieval and untenable, because CH is essentially undecidable and thus will remain unsolved forever.

Petitot's solution is here – as so often – to seek a middle way between the two extremes: he wants to keep Gödel's Platonism equipped with a criticist, transcendental basis which will, on the other hand, agree with Feferman in the principal indecidability of CH (not only in relation to ZFC). Petitot here argues on the basis of a discussion (235) of the status of the concept of mathematical existence. Either this concept is formalistically interpreted as referring solely to true expressions involving existential quantifiers – or it is interpreted as referring to something extra-mathematical (that is, something outside of the symbolic calculus). But the former case is unsatisfactory because it is possible to reduce existential quantifiers by means of alternative logics and the introduction of sufficiently many ideal elements (and also, supposedly, also because this idea suffers from the constant Hilbertian problem that a consequent 'formale Redeweise' exterminates any appeal to intuition). The latter possibility is also unsatisfactory, however, because it gives the continuum the status of an object where instead it must be maintained as a Kantian 'pure intuition', that is, determined by mathematical idealities. This interpretation contributes a classically Kantian duplicity: the continuum may be determined as objective reality – at the same time as keeping its character of being a 'horizon-object' escaping any complete determination. This Kantian have-your-cake-and-eat-it-too is made possible in Petitot by the interpretation of mathematical existence as derivative from a transcendental constitution of objectivity: 'The transcendental Platonism is 'negative' and not 'positive' like the ontologically vulgar Platonism. It permits to *invert* the philosophical claims of anti-Platonism *and to transform the phenomena of incompleteness and indecidability into arguments supporting Platonism*.'[531] These apparently depressing results should, quite on the contrary, be taken as the very signs that objectivity is no ontology; they are so to speak in themselves Platonist properties, not of the object, but of the determination of the object.

Petitot runs through the series of different proposals for additional axioms which we have already mentioned, e.g. the axiom of measurable cardinals (building on the intuition – a sort of set theory analogue to the cosmological principle in astronomy – that the continuum must be 'equally dense' on all scales. Just like the domain of the transfinites is inaccessible from finite side and requires the axiom of infinity, there must be greater numbers inaccessible from the transfinites, etc.). An extension of these axioms lead to the axiom of determinacy which prohibits that certain complicated sets may be 'forced' into being interpreted as projective[532]. The toughest

version is the axiom of complete determination (AD) which claims that any subset of **R** is determined which makes CH false because **R** may now be ascribed very large cardinalities (even if there still will be no transfinite number between Aleph-0 and $2^{\text{Aleph}-0}$), because **R** can no longer be well-ordered (AD excludes AC). Other, weaker versions of AD only applies this demand on projective sets and allows the well-known 'pathological' sets associated with AC to unfold in the non-determined parts of the set universe. This construction, according to Petitot, contains a sufficient realisation of our intuitive idea of the continuum (258). This is what made Gödel insist that ZFC does not contain a satisfying description of the objects of mathematics which transgress the abilities of formal systems – and to insist on the objectivity of mathematics, our intuitive access to it prior to formalisms, a non-computational theory of consciousness, and a philosophy critical against pure conventionalism, scepticism, syntax without semantics, antiplatonism, etc. These Gödelian standards are now reinterpreted by Petitot in a criticist way as referring to intuition, transcendental aesthetics and the synthetic a priori judgments as a charting of it – that is, as referring to the fact that the intuition of the continuum might be ineradicable but is, on the other hand, inexhaustible by formal descriptions.

At this point, Petitot's new foundation for Platonism seems to support a renaissance for Peirce's intuitions of the continuum as being at the same time primitive and inexhaustible. Yet, Petitot's position involves some open issues. In his presentation of the 'bimodality' of the continuum it seems strange – also given his emphasis on intuition in relation to Gödel – that the mathematical intuition of the continuum is relegated to thing-in-itself while the form of manifestation of mathematical phenomena is a pure, Hilbertian 'littéralité symbolique', and the form of objectivity of mathematics correlatively purely symbolic. It seems as if Petitot in a Hilbertian craze relegates intuition out of mathematics as such, which symptomatically gives rise to the fact that no schematisms to production of synthetic a priori judgments are needed here: mathematics is through and through analytic. Even if it might indeed be plausible to relegate the continuum as object to the *Ding-an-sich* position, it hardly seems reasonable that its 'density', 'homogeneity', 'mergedness', 'potentiality' and so on are not represented as primitive phenomena for pure intuition. Maybe this development in Petitot's thought is connected to the conclusion of the paper where he – and plausibly so – supports Roger Penrose's claim that the non-computationality of consciousness (Gödel) for a naturalist viewpoint must necessarily lead to the claim that parts of physics are non-computational. This deep idea – and the research program connected to it regarding a non-computational physics of the mind, maybe even implying a Penrosean revolution in quantum mechanics – leads Petitot to the strange claim that 'Obviously, the riddle of pure intuition remains unsolved, that is, of how we are affected by the outside. This problem is not first of all mathematical, but *physical and cognitive*.'[533] Penrose's argument certainly leads to the assumption of a physical and empirical foundation of consciousness – but this refers to *empirical* consciousness, to which pure intuition could hardly be taken to belong. Does Petitot here commit an advanced version of the classical mistake of identifying the empirical and the transcendental subject to the disfavor of the latter?

In the transcendental realm seems to remain only Hilbertian finitism, while all the questions regarding intuition may be thrown to the lions of empirical cognitive research. Even if it may be correct, as Petitot adds, that Kant vaguely anticipated such points of view in his *Opus postumum*, when his idea of a 'phenomenon of the phenomenon' in that work refers to a non-mechanical and non-atomistic physics, then this as yet remains a speculative idea and no empirical physical result. Should one really take the consequence of this unhappy partition between a Hilbertian finitist metamathematics referring only to computational symbol strings on the one hand and an vague ungraspable 'meaning' open to empirical cognitive science only, then it might amount to something like a resurrection of parts of that 'transcendental Jalta' which Petitot has himself bravely spent much of his carreer demolishing.[534] Petitot's hasty naturalisation of pure intuition probably must be seen as an insistence that it is, naturally, natural and indeed highly interesting to investigate the way any single one of the Kantian faculties are implemented in empirical subjects – without thereby forgetting that such an ambitious piece of psychology will still remain a special science referring to a regional objectivity, in turn to be grasped by transcendental tools (among them the ineradicable pure intuition itself with its inventarium of Hilbertian finitist tools as well as primitive, pre-combinatorial and pre-constructive intuitions about the continuum.)

Still, Petitot's delicate distinction between the continuum as thing-in-itself on the one hand and mathematical formalizations on the other (whether they involve Hilbertian symbolisms only or richer representations) forms a strong parallel to Peirce's version of an Aristotelian continuum. Peirce's interpretation of the continuum in terms of possibilities (firstness) and 'real possibilities' (thirdness) indeed places it at a distance from any finite actualization whatsoever.

To sum up, we may say that various actual mathematical and philosophical discussions of the continuum support the reactualization of a version of Peirce's continuum doctrine. The set theoretical definitions of supermultitudinous set *S* and the search for its exact place in the hierarchy of cardinals must given up, while the very idea of the continuum as a basic intuition equipped with a potentiality exceeding any formal attempt at actualization is preserved. Moreover, the actual initiatives seem to take further Peirce's intution that even if set theory may not exhaust the continuum, then it may indirectly depict it, to the extent that different formalisation attempts in some way or other display the 'indistinctness' which is a crucial property of that intuition. This goes for e.g. the indecidability of generic subsets of reals even in **R** (Chaitin, Harthong), it goes for the 'halo' of points in non-standard analysis (Robinson, Harthong), it goes for the different categories of 'inaccessibility' regarding the axioms of 'large cardinals', it also goes for the 'ungovernable' sets which the axioms of partial determination allow for (and which also Chaitin refers to when he points to the fact that there is no winning strategy in the even-uneven game), it goes for the determination of the continuum as 'excess', referring to the fundamental status of incommensurability in all reasoning about the continuum (Salanskis), it goes for the arguments for the ontological anteriority of continuity in relation to discontinuities, connected to an ontological interpretation

of any predicate calculus (Thom), and it finally goes for the criticist relativisation of Gödel's Platonism which makes CH's apparent indecidability to an argument *for* a Platonist idea of the continuum rather than the opposite (Petitot).

PEIRCE AS PHILOSOPHER OF MATHEMATICS

Given that Peirce's position as reconstructed by Putnam and interpreted within the framework of his own (partly) intuitionist logic is consistent, we may ask the anachronistic question: to which position in the philosophy of mathematics would it belong? We have already mentioned Kurt Gödel whose controversial philosophy of mathematics is apt to put Peirce's position into profile. The implication of Gödel's 1931 proof – proving the incompleteness of any logical system which contains finite arithmetic – has very often been taken to constitute the definitive farewell to Platonism – the idea that the objects of mathematics have some sort of independent existence which our mathematical operations serve to reveal – with the further formalist implication that mathematics must now rely upon the tentative experiment with different axiom sets which remain our constructions, without any claim to referring to some ideal world.[535] Against this consequence, Gödel took his incompleteness proof to show that *formal* methods might never make mathematics complete, and that this fact must force mathematics to seek refuge in non-formal, phenomenological methods able to clarify the abstract content of our intuitive assumptions and thereby lead to new axioms which might make some of the indecidable axioms (AC, CH, etc.) decidable. Thus he tended to regard AC as true and CH false (so that $2^{\text{Aleph-0th}} = \text{Aleph-2}$ – even if he changed to the opposite viewpoint late in life.)

Gödel is thus in opposition to formalism and Hilbertian finitism, claiming that proofs may be obtained by symbolic calculi alone and let intuition about mathematical objects remain on the metamathematical level where axioms are selected and proofs interpreted – and he is also, on the other hand, in opposition to intuitionism with its very tough interpretation of human finitude tending towards psychologism and, in many cases, towards constructivism prohibiting infinite operations and the principle of excluded middle.

Many aspects of Peirce's philosophy of mathematics seem to make him quite the opposite of Gödel: Peirce's constructivism implies that he does not regard the objects given in mathematical intuition (the line, for instance) as being exhaustible by operations, so that the actually construed always exists within a modal horizon of possibilities. His insistence, moreover, close to formalism, that we are free to choose widely different procedures, each of them consistent, but not mutually consistent, within this field of possibility (cf. his obsession with non-Euclidean geometries) which might seem to give him a MacLanean affinity. Still, we find other respects in which Peirce may express himself as a through-and-through mathematical Platonist, just like he, in analogy to Gödel, may expect CH to be settled by future axioms – all in all the idea that he sees the issue of the reality of mathematics as a question which may be settled with metaphysical means. How is this strange knot to be

untied? Let us consider one of the cases where Peirce declares himself Platonist. He observes an ellipsis drawn on a blackboard as a continuous figure, 'inheriting' its continuity form the plane on which it is drawn. Its continuity is actualised due to its potentially infinite set of actual tangents governed by that habit which constitutes the ellipsis. Then he claims:

> Many such reacting systems may spring up in the original continuum; and each of *these* may itself act as first line from which a larger system may be built in which it in turn will merge its individuality.
>
> At the same time all this, be it remembered, is not of the order of the existing universe, but is merely a Platonic world, of which we are, therefore, to conceive that there are many, both coördinated and subordinated to one another; until finally out of one of these Platonic worlds is differentiated the particular actual universe of existence in which we happen to be. (R&LOT 1898, 263)

Here, we are close to Peirce's cosmology where the undifferentiated continuum appears as a sort of ever-creative Ur-cosmos, the beginning of everything. Here we find a surprising chain of cosmic development:

Continuum → many Platonic worlds → one Platonic world → the particular actual universe

We will not here discuss this extravagant onto-cosmological claim as a proposition referring to an real genetic developing process, but restrict us to discussing its implication in philosophy of mathematics, as a series of presupposition relations. Mathematics, to Peirce, is no science of actuality, its domain is the iconic field, formally apprehended, its point of departure hypotheses, and its task is to draw necessary conclusions from them. No formalist would disagree. Peirce and the formalist split, however, when Peirce's idea of 'many Platonic worlds' is taken to imply that *all competing consistent axiom system have a Platonic real character*, they are not only symbolic nor human constructs from which we can demand no further reality. Quite on the contrary, different axiom systems so to speak map a series of different, possible Platonic worlds, and the role of mathematics is to cover this field of possibility. 'The particular actual universe' as an object must of course be covered by different empirical special scientists – what will happen, then, with 'the one Platonic world'? It is investigated by applied mathematics, not in the special sciences, however, but in metaphysics. The import of mathematics in various special sciences guarantees the validity of their deductive schemata and diagrams and thus their generality beyond the always finite field of particular empirical observations. It is those very empirical observations, on the other hand, which investigates which one combination of the possible Platonic worlds we actually live in. This is probably why Peirce may seek to let metaphysical considerations decide CH: mathematics may supply many different consistent systems, they are merely hypothetical – what is decisive is what metaphysics, in the Kantian way built on logic, as a real science will chose in the mathematical candy store.[536] As against Gödel, Peirce would (probably) support formalists and MacLaneists: there are several different axiom systems for set theory, and we can not from consistency alone decide which of them is mathematically correct – being consistent they are equally correct. But against formalists, on the other hand, Peirce would say: *all* these different systems have equal objective Platonic reality – they are true

investigations in the domain of the possible and thus not only (MacLane) 'correct'. As regards the role of intuition, Peirce would agree with Gödel and intutionists, each of which will claim intuition's role to be irreducible, and against the formalist tendency to strong reduction or even elimination of intuition. Peirce would also agree in the intuitionists' skepticism towards the law of the excluded middle, and he even counts as an important forerunner to their doctrine in his characterization of firstness and thirdness, as not obeying the law of contradiction and the law of excluded middle, respectively. But against (at least) the constructivist aspect of intuitionism, Peirce will argue against the idea that human finitude must constrain us to prohibit infinite operations[537] – here he would rather agree with Gödel that intuition is nothing simply given but something which must be investigated further with phenomenological means – diagrammatical reasoning – and potentially with new formalist results as its consequence. Even if we are finite beings, our finitude is indefinite, because its exact limits are not given and will only be clearer from ongoing research.

This will also further clarify Peirce's idea of the continuum. Its basic motivation is metaphysical; it comes from the generality of universals in the logic of relations. It comes from the basic fact that predicates do not possess any directives for which existing objects they hold for. Thus, they are potentially valid for an inexhaustible continuum of possible subjects. This is why any extensional semantics trying to define the property expressed by the predicate by the existing subjects possessing the property, is impossible, because these subjects may not be listed, not even in any possible world. This Aristotelian universalism whose special Scotist background in Peirce we covered in Chap. 2 becomes the motivation for his metaphysical continuism, and mathematics becomes the formal apparatus to investigate it. Peirce's realism with respect to universals is thus the decisive basis for his continuity doctrine: universals are would-be's, and such phenomena may be described only as general continua of possible events. The competing Platonic worlds which are mapped by mathematics grow out of an original continuity which eo ipso *is given before even the mathematics supermarket of competing sets of hypotheses*. Mathematics thus *must* be continuist; even if it possesses its full rights, even duty, to investigate discontinuous hypotheses, then it is a metaphysical premiss that they may only be understood on the background of a more encompassing continuity. This is due to metaphysics – which given its systematic place in Peirce's encyclopaedia is restricted to deal with general properties of *this* world (and thus much more constrained than mathematics). But it is also due to that much more encompassing metaphysics which is not explicitly called by that name in Peirce, but which appears in the very shaping of Peirce's encyclopaedic architectonics, and in which mathematics is first science, dealing with a continuum of pure possibilities.[538]

NOTES

[1] References to Peirce and Husserl are in much-used cases abbreviated (like EPII or LU) – key to abbreviations is in the bibliography. As to Peirce's work, references are to the *Writings* and *Essential Peirce* when possible. If the same texts appear also in the older *Collected Papers*, references to that edition are given immediately afterwards. In many cases, though, reference is possible only to *Collected Papers*. In both cases, references to that work follow normal procedure: volume number followed by paragraph number (like 3.445).

[2] Lakoff and Nuñez (2003) has a similar point in investigating the metaphors and blendings used to obtain central mathematical results – their book considerably adds to our understanding of how embodied beings may get in touch with mathematical truths by means of the abilities charted in the cognitive semantics tradition. I do not agree, though, in their jump from this epistemological investigation and to reductionist ontological claims that mathematical objects simply *are* nothing but the result of such mapping processes (rather than independent objectivities *investigated* by such processes). This seems like a confusion of *Genesis* and *Geltung*, origin and validity, and thus reawakens that psychologism which both Peirce and Husserl were out to bury.

[3] The art historian W.J.T. Mitchell has, on a couple of occasions, used the word 'diagrammatology' in a sense not wholly unlike my own, based on the plausible idea that our access to literary (and other) form takes place by means of ' "sensible" and "spatial" constructs'. Mitchell first used it in a small debate article (W.J.T. Mitchell 1981): 'Diagrammatology', in *Critical Inquiry*, Spring 1981, Vol. 7, nr. 3, 622–33, responding to Leon Surette's critique of his own paper 'Spatial Form in Literature') – here, he poses the question: 'If we cannot get at form except through the mediation of things like diagrams, do we not then need something like a diagrammatology, a systematic study of the way that relationships among elements are represented and interpreted by graphic constructions' (623). More recently, he has taken up the notion again in an internet interview (O.N.C.Wang 1997) 'The Last Formalist, or W.J.T. Mitchell as Romantic Dinosaur', Interview with Mitchell by O.N.C. Wang, in *Romantic Circles Praxis Series,* Series Editors: Orrin N. C. Wang and John Morillo (http://www.rc.umd.edu/praxis/mitchell/mitch-about.html) which does little more than refer back to the earlier, 'strange, little article'.

[4] With diagrammatology in general as its subject, this book does not contain analyses of particular diagrammatic representation systems – however important this subject indeed is. Peirce's general doctrine of diagrams developed in the years after the turn of the century is motivated, to a large extent, in his parallel development of existential graphs. The results of these developments, Alpha-, Beta-, and Gamma-graphs for logic representation are thus discussed here only in the context of constituting an important part of the specific material from which Peirce's general diagram doctrine is developed.

[5] Quote from the presentation of a planned work with the title *The Principles of Philosophy: or, Logic, Physics, and Psychics, considered as a unity, in the Light of the Nineteenth Century*, c. 1893; the CP bibliography Vol. 8, 283.

[6] I originally planned to open this book with a thorough chapter connecting Peirce's mathematical attempts at understanding the continuum with his scattered outlines of a continuity metaphysics. It was, however, judged too harsh as a starting point which might dissuade too many otherwise interested readers from continuing – and to some extent also wildleading in relation to the overall diagrammatological aim of the book. A more detailed discussion of these issues may now be found in this book's Appendix – the curious reader is encouraged to continue there. Further discussion of Peirce's continuity concept may be found in Parker 1998.

[7] For an actual elaboration on Peircean knowledge in the limit, see Hendricks 2001.

[8] Yet, one time Peirce might refer to Poncelet when talking about his continuity principle – namely in a prospect for a book never written about nineteenth century ideas (in the bibliography of CP, Vol. 8, 283).

These ideas embody the principle of continuity, and among them is listed 'modern mathematics and its imaginaries'. This might refer to Poncelet whose generalizing interpretation of geometrical figurae allowed them to refer not only to possibilities present but also possibilities absent (thus, the figure of two crossing lines might refer to all such different crossings, but also to the case of the two lines being parallel, that is, crossing at a point in infinity, cf. Greaves 2002, 46–47). This is, in fact, Poncelet's continuity principle – that what holds for one figure, also holds for continuous deformations of it – and it seems likely that Peirce's principle is influenced by him, the more so because of the evident relevance of such general figure interpretation for Peirce's diagram doctrine.

[9] Peirce's conception of the system of sciences is in constant development. Here, we stick to the relatively elaborated versions which he crafts after the turn of the century, especially the 1902 Carnegie application (MS CSP L75, i NEM IV and LCS, Joseph Ransdell's synthesis of L75), 'A Detailed Classification of the Sciences' from 'Minute Logic', 1902 (EPII, 115ff; 1.203ff) and 'Syllabus' from 1903 (EPII, 258ff; 1. 180ff).

[10] Peirce takes the term 'Grammatica speculativa' after a treatize ascribed to Duns Scotus (properly written by Thomas of Erfurt which Peirce was then in no position to know), meaning 'mirroring grammar', that is, a grammar mirroring metaphysics. Hence its connection to epistemology: 'Kant, who first raised these questions to prominence, called this doctrine *transcendentale Elementarlehre*, and made it a large part of his *Critic of the Pure Reason.* But the *Grammatica Speculativa* of Scotus is an earlier and interesting attempt. The common German word is *Erkenntnistheorie*, sometimes translated Epistemology.' ('Minute Logic', 1902, 2.206) Thus, in Peirce semiotics and epistemology are generally two sides of the same coin.

[11] As Thirdnesses, teleologies are naturally general, vague, variable and bundle-creating ('Minute Logic', 1902, EPII, 118ff; 1.206–7), and we must not expect sharp, but only provisional, fallible (but gradually improving) demarcations between the sciences.

[12] Cf. Parker 1992, 67.

[13] Short (2004) argues that the sign theory of the early Peirce had the consequence of making of the concrete object a bundle of qualities only (much like the trope-bundles in present discussions), necessitating him in 1885 to add the Scotist idea of 'haecceitas', 'thisness' and the corresponding sign type of indices, in order to grasp actual existence without giving up the idea of the particular as not fully determined.

[14] In Peirce's theory of perception, perceptual judgments form a borderline case of judgments because they, as a rule, and unlike other judgments, are beyond conscious control. They judge on preceding sensory 'percepts' to which we have no direct access but which must be postulated as the matter of perceptual judgments – but the generality revealed in perceptual judgments must inhere already in these percepts even if they are not propositional in structure.

[15] Even if the phenomenological categories are logically anterior to logic, they have, in Peirce, been discovered through logic, by generalizing abstraction from logical relations (in the three categories, it is easy to recognize the properties of predicate (1), subject (2), and copula (3)). But here Peirce separates what the theory of science of our days would call context of discovery from context of justification.

[16] 'The acquiring [of] a habit is nothing but an objective generalization taking place in time. It is the fundamental logical law in course of realization. When I call it objective, I do not mean to say that there really is any difference between the objective and the subjective, except that the subjective is less developed and as yet less generalized. It is only a false word which I insert because after all we cannot make ourselves understood if we merely say what we mean.' ('Abstract of 8 lectures', undated, NEM IV, 140)

[17] A tautology or a petitio principii, it might be critically argued?

[18] The quotes are from 'Abstracts of 8 lectures', Robin-nr. 942 which is undated. Clearly, it is a rather late text, in any case later than 'Law of Mind'(1892), and probably also later than the 'Reasoning and the Logic of Things' series (1898).

[19] This is, in fact, an interpretation of the Kantian doctrine of human finitude: we are finite beings because we do not possess intellectual intuition. Being bound to sensory experience, we must organize this in different ways. This finitude is here seen as a result of the fact that sensory experience is limited to

certain contingently constrained senses which defines what we may imagine in intuition. As a contrast, we might imagine a god who has the ability of sensing in the whole continuum of qualities.

[20] This idea, due to the attempted definition of Firstness as that whose constituents are independent of anything else, evidently runs counter to Peirce's insistence on non-distinctiveness as basic property in continua. It seems like Peirce's determination of Firstness qualities as *vague* is a compromise between these two ideas.

[21] In Peirces evolutionism, he even ventures the hypothesis that Thirdness is growing as a tendency towards the appearance of still more correlations between still more Secondness events. Thus the relation between the three categories as presented here may be given two different interpretations. One is structural, picturing the atemporal relation between the three. Another is temporal, supposing that Firstness is also first in a temporal sense, so that the growth of Secondness and Thirdness are cosmological processes empirically taking place. Here, we stick to the former interpretation; it seems uncertain whether evolutionism prospers from the attempt at rendering it in these categories. Cf. the discussion in the biosemiotics section.

[22] The difference between ordinary modern logical use of PC and PEM, referring to propositional logic, and Peirce's use, referring to contradictory predicates within propositions, has only recently been clearly outlined in Lane (1997).

[23] The rain examples pertaining to both PC and PEM are from MS CSP 678, quoted from Lane (1999), 289.

[24] Modern modal logic regarding modal propositions also has necessity not obeying standard PEM and possibility not obeying standard PC. If N is the necessity operator, then PC holds: *Not both Np and N non-p* – but not PEM: *Either Np or N non-p*. And, conversely, if M is the possibility operator, then PEM holds: *Either Mp or M non-p* but not PC: *Not both(Mp and M non-p*. (Øhrstrøm and Hasle 1995, 144-145). Modal propositional logic thus give parallel results to Peirce where modality is expressed inside the S-P proposition in first order predicate logic.

[25] The semiotic description is adapted from "Pragmaticism, Prag. [4]", c. 1905; 5.505.

[26] Peirce's introduction of a three-value logic in 1909 has no direct connection to the analysis of generals by means of non-appliance of PEM; it rather points to discontinuous limit cases in continua (like the status of the borderline between two coloured areas, etc.), calling for a third, limit truth-value where PEM does apply but is false. These issues have often been confused, but is admirably sorted out by Lane (1999).

[27] Noble (1989) argues against Murphey's (1961) idea that it was Peirce's new conception of the continuum which gave rise to the doctrine of real possibilities, claiming that it was rather real possibilities which – after the abandonment of Cantorian continuity – gave rise to Peirce's mature idea of the continuum as inexhaustible (170). Our contention that Peirce's continuum concept is metaphysically rather than mathematically motivated seems to support Noble's account. We return to 'real possibilities' in Chap. 2.

[28] Or, more properly, the inside of the cut appears in the mode of 'If'. Only if the cut contains a further cut, thus constituting a 'Scroll', the contents of the cut are denied.

[29] We shall not go further into Peirce's three systems of logic graphs here, just notice that Peirce's work on them undoubtedly formed part of his motivation for a general diagrammatology. For the time being, a fertile research into the possibilities of the Graphs is developing, cf. Barwise and Etchemendy 1995; Øhrstrøm 2001; Pietarinen 2006; Shin 1994, 2000; Sowa 1997, 1999, etc. In Stjernfelt 2006, I argue that the Graphs urged Peirce to add a stronger criterion of iconicity to his operational iconicity discussed in the present book, here called 'optimal iconicity' – with the motivation of shaping the graphs to give as fitting a description of logical reality as possible.

[30] Cf. Stjernfelt 2006.

[31] In the letters to Lady Welby, Peirce explains that he often uses the human mind as an illustration when introducing his sign concept, but that this is only for the sake of giving a clear example - this reference being a 'sop to Cerberus', as he puts it (Dec 23, 1908, EP II, 478). This implies a theory of mind in which consciousness is not a necessary ingredient, because the mere existence of purposive action is sufficient to grant the presence of mind in this broad interpretation: 'I hold that purpose, or rather, final causation, of which purpose is the conscious modification, is the essential subject of

psychologists' own studies; and that consciousness is a special, and not a universal, accompaniment of mind.' (From 'Minute Logic', 1902, 7.366 – for a discussion, see Skagestad 2004, 247f). It must be added that Peirce is often very unclear as to the extension of this mind concept. Does it cover all of Thirdness, so that all regularity in the world, every abstract object which is not a mere *ens rationis*, has a mind-like character – or do purposive, mental phenomena constitute special subclasses of Thirdness? In the more extreme versions of his cosmology, Peirce without further notice presupposes the former; we shall stay content with assuming the latter.

[32] Of course, it is an open issue whether this general definition is not too general. Does it add anything to our knowledge of a physical process to describe it in sign terminology? I tend to say there is not, cf. below. Even if a fact shares the same structure as a proposition, as Peirce says, it *is* no proposition but rather the *fundamentum in re* of a proposition.

[33] We might imagine, as examples, that an animal species can be conceived of as the general conclusion to an evolutionary argument, or, to take a less teleological example: the solar system will be the conclusion to the premises constituted by mechanical laws and a set of initial conditions. In his later years, Peirce is rather definitive on the existence of natural arguments, even to the extent of claims like '. . . the universe is a vast representamen, a great symbol of God's purpose, working out its conclusions in living realities. Now every symbol must have, organically attached to it, its Indices of Reactions and its Icons of Qualities; and such part as these reactions and these qualities play in an argument that, they of course, play in the universe – that Universe being precisely an argument.' (Lectures on Pragmatism, 1903, EP II, 193–4, 5.119)

[34] Peirce often characterizes icons and indices as 'degenerate' symbols. He does not use that term about simpler symbols (propositions, rhemes) as compared to arguments, but still he has an analogous idea: 'If we erase from the argument every trace of monstration of its special purpose, it becomes a proposition; usually a copulate proposition, composed of several members whose mode of conjunction is of the kind expressed by 'and'; which the grammarians call a 'copulative conjunction'. If from a propositional symbol we erase one or more of the parts which separately denote its objects, the remainder is what is called a *rhema*. . .' ('New Elements', 1904, NEM IV, 244). Here, both proposition and rheme are seen as less complete arguments.

[35] The three basic trichotomies pertaining to sign, object, and interpretant, respectively, may not be combined freely. A subtype of a higher triad may not be combined with lower subtypes of lower tirads (as all symbols are legisigns, e.g. symbols do not combine with quail- or sinsigns). Combination of the nine sign aspects recorded in the trichotomies thus only yield ten combined sign types, presented by Peirce in this triangle ('Syllabus', EPII. 296; 2.264) with the simplest sign upper left and the most complex upper right:

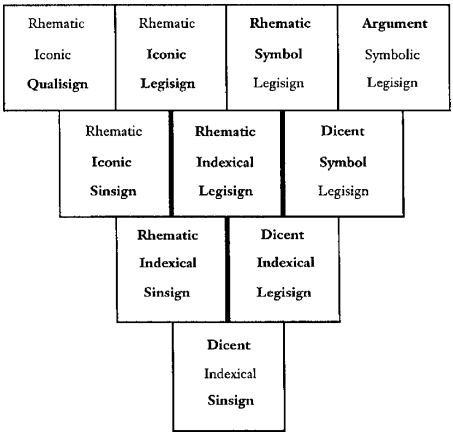

Figure 7.

It is rather rare, however, that Peirce (nor we) need to refer to the precise type of single signs – primarily because signs rarely occur as isolated specimens but most often appear as parts or aspects of an ongoing reasoning process where it is sufficient to highlight the sign aspect actually scrutinized.

[36] These trichotomies are the basic three among the ten trichotomies which Peirce outlined in the Lady Welby letters during the last decade of his life. Most of the others are only sketchily outlined, but some of them merit more study, thus the trichotomy pertaining to the nature of the dynamic objects of signs, distinguishing 'Abstractives, Concretives, and Collectives', that is, roughly, universals, particulars, and sets.

[37] Some authors maintain that there is a decisive difference between the triads of qualisign-sinsign-legisign and tone-token-type, respectively. This is not the case. Both are defined, as the only triad, with respect to subtypes of the sign itself. 'As it is in itself, a sign is either of the nature of an appearance, when I call it a qualisign; or secondly, it is an individual object or event, when I call it a sinsign (the syllable sin being the first syllable of semel, simul, singular, etc.); or thirdly, it is of the nature of a general type, when I call it a legisign.' (Letter to Lady Welby Oct 12, 1904, 8.334 – the same distinction as used in the 'Syllabus' quote given here). Two years later in the 'Prolegomenta to an Apology for Pragmaticism': 'Only one division is concerned with the nature of the Sign itself, and this I now proceed to state.' – and then follows the tone-token-type distinction (1906, 4.536–7). Also the CP editors in a note claim the identity of this triad with the former. Two more years pass, and in another letter to Lady Welby, Peirce takes up a terminological discussion. Here, he prefers the triad potisign-actisign-famisign for the same distinction, even if toying with maintaining tone-token-type with 'mark' substituted for 'tone' (EPII, 488; 8.363–64). In all cases, though, the distinction pertains to the merely potential quality of a sign as opposed to a singular, active sign as opposed to a lawlike sign able to exist in identical tokens. These tokens of a type or sinsigns of a legisign are tokens/sinsigns of a 'special type' because governed by a legisign (not all tokens/sinsigns are so governed), and the term proposed for them is 'replicas' or 'instances'.

[38] Here, Peirce's special conception of 'set' as intensionally defined as a unity is important, cf. his argument with Dedekind on this issue. If sets are defined extensionally, then the unity of a set of sinsigns becomes a fragile notion, and types, important to grant the identity of the same sign from one occurrence to the next, become problematic. As Randall Dipert remarks, Peirce's refusal to accept Dedekind's notion of 'echter Theil' (proper subset) can be seen as an insistence that sets be defined intensionally and in a certain sense constructively (Dipert 1997, 62ff). If we say *P* is a proper subset of *W*, the crucial issue is 'How do we know a set *is* a proper subset of an infinite set? [. . .] some property or relation must hold on some of the members of the collection, *W*, but not others (i.e., those in *P*), and we must know this. In other words, we are back, if we choose Dedekind's method, of requiring certain kinds of knowledge about the internal structure of the original infinite set *W*.' (62–63). In order for an infinite set to be treated as an individual, it must be somehow intensionally defined, for otherwise we should be able to list all its elements. Thus, Peirce's realism and intensionalism make him insist on a constructivist idea of sets based on the reality of rules defining such sets rather than on the actuality of each of the elements.

[39] René Thom has exactly the same argument: to say that a sign is a mere 'flatus vocis' begs the question, for how could a mere blow of air be absolutely identical from one occasion to the next without assuming realism of that sign expression and thus realism about universals?

[40] Douglas Hofstadter has made a strong argument for the complicated inner structures of apparently very simple types, such as for instance letterforms. The possible token instantiations of the letter type A is infinite and even subject to constant innovation in designer typefaces, even if the A 'tone' for a superficial glance seems extremely simple. His conclusion is that attempts to make explicit all possible A's by an algorithm are subject to incompleteness constraints formally analogous to Gödel's 1931 proof (in Gödel 1986). This thus forms a strong argument against semantic extensionalisms without resting on psychological arguments (Hofstadter 1986, see also Stjernfelt. 1992c).

[41] The somewhat strange notion of 'dynamical' object in Peirce refers to a distinction between the 'immediate' object – the object as the sign presents it – on the one hand, and the object as it is in itself, regardless of the sign, on the other. The latter is the 'dynamical' object.

[42] At a conference in May 2005, the British philosopher Galen Strawson claimed that intentionality must always imply consciousness, even for analytical philosophers. Thus, he argued against the idea,

also in analytical philosophy, that there is a broader type of intentionality which does not presuppose consciousness, for instance in living organisms. Just like the Peircean idea that mind is a broader concept than consciousness, and that only certain sophisticated types of mind are equipped with consciousness.

Such an idea Strawson rejected. His reason was one basic argument, the so-called 'stopping problem'. The idea is that if we take the chain of causes connecting an object with the intention intending it, then we lack criteria for where to 'stop' in the causal chain. To him, only the conscious experience can yield such a criterion. In Peircean notions, it is the question of where to 'stop' in the causal chain connecting index and object.

Strawson's idea is that it is only consciousness which makes it possible to 'stop' at the right place in that causal chain. If consciousness is not there to stop it, then any effect in the universe may be taken as a sign of any one of its many causes – and intentionality would be everywhere. This is evidently not an attractive consequence – even if Peirce sometimes spoke as if he was willing to accept something like it. But if we should not accept this consequence, how should we solve the 'stopping problem' for indices without ending in the opposite extreme and embracing Strawson and consciousness as the only source of proper intentionality?

The problem is evidently a major issue for the development of biosemiotics, because the idea here precisely is that biological processes down to biochemical detail can be understood as signs, that is, can be understood as intentional. But how do we see what biological signs below the organism level are 'signs for'? – in some sense they could be said to be signs for any segment in the homoeostatic process they are part of. When von Uexküll's famous tick waiting in its tree smells butylic acid, this chemical releaser may be said to function as a sign for 'mammal', as a sign for 'jump', as a sign for 'blood', as a sign for 'food' or 'reproduction' – because all these phenomena enter into the action chain involving the tick smelling the chemical and acting correspondingly.

But what is the right place to stop? Strawson implies that we cannot determine any stopping place from merely behavioral criteria. How do we frame the stopping problem in order to grant that a sign process is going on, no matter whether the tick has any inner feeling of what's going on? We shall return to the index stopping problem in the biosemiotics section below as well as in the discussion of Eco's idea of indexicality as subject to negotiation.

[43] Sometimes also known as seme, pheme, and delome; or sumisign, dicisign, and suadisign.

[44] It goes without saying that this reference may point to any kind of fictituous reality. Another discussion which we shall not go into here is the dicisign's character of assertion which we will here take as the default case. Of course, it is possible to consider the satiated predicate with its assertive force placed in brackets so that the expression in itself has no reference, just like the addition of assertive force to the expression is not the only speech act possibility. The same satiated predicate can be used in other speech acts than assertion; questions, imperatives, wishes, etc. In his Gamma graphs, Peirce outlines the basics of theory to cover such cases, and so qualifies as an early father of speech act theory.

[45] As part of his elegant definition of the whole third trichotomy: rhemes are signs making explicit their information, dicisigns are signs making explicit their object, and arguments are signs making explicit their interpretant.

[46] Human beings are mortal, Socrates is a human being, Socrates is mortal - this old example implicitly refers to a law due to which a continuum of other human beings than Socrates may be inserted in the latter part of the premiss and the conclusion - hence its law-like generality.

[47] The three distinctions recur in Peirce's abstraction theory where the three types of distinctions dissociation, prescission, and discrimination correspond to Scotus' real, formal, and logical distinctions, respectively. They distinguish between *res* and *res, res* and *realitas, res* and *ens rationis*, respectively. We shall return to Peirce's abstraction theory below, Chap. 11.

[48] '... grâce à laquelle *homme* est suffisament indéterminé pour que, par une intellection unique, il soit conçu quidditativement en tout homme' (Scotus, quoted by de Libera, 345).

[49] Diodorus Cronus and Philo the Logician or Philo of Megara formed part of the so-called 'Dialectical school' at the turn of the fourth and third centuries BC – a school which was among the initiators of propositional logic. The latter was a pupil of the former, and we only know their work from secondary sources. Diodorus outlined the famous 'Master argument' to prove the existence of actuality only

(briefly: every proposition about the past is necessary; the impossible does not follow from the possible; it seems like possibilities exist which are not nor will ever become true. From this follows, Diodorus argues, that nothing possible exists which is not or will not become true. Much about the details of the argument is still unknown.) Thus, when Peirce in his later years toys with Diodoran implication, the realism as to possibilities which he tries to articulate does not have any support from the Master argument (As to Peirce and the Master argument, see Øhrstrøm 2001). Thus, it is rather Diodoran implication which interests Peirce. It seems like Diodoran implication involved time: a conditional is Diodoran true if it is never the case that P is true and Q is false – it must be Philonian true not only now but at all times. But still it does not seem to require any connection between P and Q such as Peirce seems to indicate (Bobzien 2004).

[50] Two years later, he says similarly 'For my part, I am a Philonian; but I do not think that justice has ever been done to the Diodoran side of the question. The Diodoran vaguely feels that there is something wrong about the statement that the proposition "If it is lightening, it will thunder," can be made true merely by its not lightening.' ('Types of Reasoning' (1898), Peirce 1976, 169).

[51] This would not be satisfied by what we know of Diodorus' view of implication either, however, cf. above.

[52] Strict implication avoids the problems quoted, but involves other problems: $[]\, p \rightarrow q$ (it is necessary that p implies q) gives rise to problems like the following. 'If there are lions in South America, then $5 + 2 = 7$'. As $5 + 2 = 7$ in any possible world, the statement read as strict implication is true. In Peirce's later years, he makes some attempts to extend the Beta and Gamma logic graph systems to cover real possibilities, cf. Stjernfelt 2006.

[53] To substantiate the existence of such ontological extravaganza in Peirce, see the discussion of 'The Law of Mind' (which is the earliest locus in Peirce where this tendency is given free rein) in the Appendix. The general problem is that it is not clear how many of the phenomena subsumed under Thirdness are taken to characterize the whole of that category. Take for instance the symbol concept. It is easily taken for a pure sign concept only appearing at some point of biological evolution, and in Chap. 11, we shall indeed discuss it in that way. Peirce's cosmology, however, gives him the interesting idea that nature itself may take habits, and from here to assuming that such habits possess the full character of symbols is a short step which Peirce often does not hesitate to take. See, e.g. the conclusion to his magnificent discussion of his sign theory in 'New Elements' (1904): 'A symbol is an embryonic reality endowed with power of growth into the very truth, the very entelechy of reality. This appears mystical and mysterious simply because we insist on remaining blind to what is plain, that there can be no reality which has not the life of a symbol.' (NEM IV, 262). There are indeed interesting ideas in Peirce's cosmology: 'If we are to explain the universe, we must assume that there was in the beginning a state of things in which there was nothing, no reaction and no quality, no matter, no consciousness, no space and no time, just nothing at all. Not determinately nothing. For that which is determinately not *A* supposes the being of *A* in some mode. Utter indetermination. But a symbol alone is indeterminate. Therefore, Nothing, the indeterminate of the absolute beginning, is a symbol.' ('New Elements', 1904, EPII, 322). But I do not think we can take them as proofs for the omnipresence of symbols in the universe. Such ideas seem to be related to Peirce's revaluation of Hegel – whom he almost despised during the 70s and 80s – in the 90s. Hegel's speculative dialectics and its jumps between Nothing and Everything inspires such parts of Peirce's cosmology: 'With the emergence of Time, the first book, as it might be called, of an Historical *Encyclopädie*, what Hegel terms "Logic" is brought to a close and we pass to what he calls the "Naturphilosophie" ' ('Abstracts of 8 lectures', undated, NEM IV, 138) Such claims are by no means rare in the mature Peirce, take for instance this from 1898 (RLOT, NEM IV 345): '. . . I myself entertain that every true universal, every continuum, is a living and conscious being . . .' Such claims imply there are not only Thirdness, habits, and signs, but also symbols, life, and consciousness already in prebiological nature and we must assume gravitation, prime example of Thirdness in Peirce, suits this description of a conscious, personalized symbol. Still, I hesitate to venture into these waters where all distinctions between real possibilities, Thirdness as such, continuity, symbols, entelechy, life, evolution, etc. vanish. I prefer to remain seeing such claims as mystical and mysterious until further notice, which is why I only treat Peirce's cosmology tangentially in this book. A related issue to which we return in the biosemiotics section is the question whether there are indeed signs and life in prebiological nature, and if so, then in what sense.

It should immediately be said that this absolute idealism tendency is not alone in Peirce - there is also a counter-tendency, so as for instance: 'My reply is that while I hold all logical or intellectual, interpretants to be habits, I by no means say that all habits are such interpretants. It is only *self-controlled* habits that are so, and not all of them, either' ('Pragmatism' 1907, EPII, 431). Logical and final interpretants are the same, so here Peirce delimits final interpretants to self-controlled – that is, consciously controlled, primarily human – habits. See Chap. 11 for our development of this idea.

[54] An attempt at construing a Peircean notion of causation is Hulswit (2002) with the sound aim of elaborate '... an anatomy of (Peircean) events, according to which it belongs to the nature of an event, not only *that* it creates a new event (Secondness) but also that it creates a new event *of a certain type* (Thirdness)' (215). Unluckily, Hulswit choses the same path as the absolute idealist tendency in Peirce, claiming that all causations are inherently teleological: '... Peirce forcefully defends the position that all natural processes are to some extent teleological, and can only be explained on the basis of final causes, which are general principles. The empirical fact that, as far as I know all (physical, biological, psychological, and sociological) processes show a definite tendency toward a general end state – whether it be described as chaotic or as involving a higher level complex order – cannot be explained without referring to certain general principles that somehow direct the process.' (213). The error here is to assume 'end states' of all processes. This begs the question, for only teleological processes could be said to have an 'end state'. To impose an end in a physical process is artificially splitting up its continuous development. Thus, Hulswit's identification of general principles (such as laws) with final causes does not hold (even if teleological processes form a subset of general principles). We would restrict Peircean teleology and mind to be biological properties primarily, thus occurring only locally in certain specifiable processes, cf. the biosemiotics section below.

[55] They do not imply, however, that all concepts of relations etc. have real counterparts. So the relations between relations between relations ... need not be a real would-be and basic relational realism needs not fall victim to Ockham's razor.

[56] Thus, there is also an important link from real possibilities and to Peirce's own method, highlighted by Risto Hilpinen (1995, 274) who calls it 'the method of ideal examples' as opposed to 'attempts to characterize concepts by necessary and sufficient conditions: the latter method has often prompted philosophers to be excessively concerned about 'difficult' and 'borderline' cases which has sometimes hampered fruitful philosophical theorizing.' Being obsessed with category borderlines, we may add, is precisely an extensionalist vice, while the continuist will be rather obsessed with 'ideal examples' (prototypes, if you wish, or 'ideale Spezies' (Husserl)) and their bundles of interconnected real possibilities.

[57] Given Peirce's 'extreme realism', is it not the case that it severely conflicts with his idea of truth as that which the scientific community asymptotically approaches in the long run – just like Hendricks (2001, 101) says, interpreting the latter claim as anti-realist. It is true that this claim has often been interpreted as opting for a consensus or coherence theory of truth, but I think it is equally compatible with a correspondence theory of truth. The idea of ultimate agreement among investigators is, in fact, an *idea*, it is what they would agree upon if they behaved as impeccable scientists and reasoners. So truth is not mixed up with what actual communities of scholars may agree upon or not. On the other hand, the idea is fallibilist: the vast majority of our current scientific views *are* probably true, but a few of them may lack credibility. So overall, science as a whole corresponds to reality, apart from some points. But we have no other means of discerning those points than further research.

[58] It must be noted, however, that it is well-known that similarity within categories is psychologically perceived in a not completely symmetrical way. Rosch's and Lakoff's studies show that there less typical category members are perceived as more similar to the more typical members than vice versa. i.e., penguins and sparrows display a range of similarities, but the penguin is seen as more like a sparrow than a sparrow is like a penguin.

[59] One could add that the opposite seems to be the case regarding transitivity. Similarity is not transitive: if A is similar to B and B is similar to C, then A is similar to C, is generally not the case, not even if we restrict similarity to deal with one single type of quality. One red nuance may be similar to another which in turn is similar to a third nuance – without it being the case that the first and third are similar. Take on the other hand transitivity in signification: if A signifies B, and B signifies C, then A signifies C. This is, if not generally realized in discourse, then in general a possible, that is, valid construction

in the system in question, at least given certain contexts. Peirce seems to admit this principle under the Scholastic notion 'nota notae': 'The logical principle Nota notae est nota rei ipsius, that is, the predicate of the predicate is the predicate of the subject, which is laid down in several places by Aristotle as the general principle of syllogism.' (a planned continuation of 'Syllabus', 1903, 3.590)

[60] 'Das Zeichen hat mit dem Bezeichneten inhaltlich zumeist nichts gemein, es kann ihm Heterogenes ebesowohl bezeichnen, als ihm Homogenes. Das Bild hingegen bezieht sich auf die Sache d u r c h Ä h n l i c h k e i t, und fehlt sie, so ist auch von einem Bilde nicht mehr die Rede. (…) Es wäre eine deskriptiv unrichtige Auffassung der Sachlage, wenn man denken würde, der ganze Unterschied bestehe darin, dass dieselbe Intention, die einmal an die Erscheinung eines dem gemeinten Objekt ä h n l i c h e n Objektes genknüpft ist, ein andermal an die Erscheinung eines ihm u n ä h n l i c h e n Objekts geknüpft sei. Denn auch das Zeichen kann dem Bezeichneten ähnlich sein, ja vollkommen ähnlich. Die Zeichenvorstellung wird dadurch aber nicht zur Bildvorstellung. Die Photographie des Zeichens *A* fassen wir ohne weiteres als Bild dieses Zeichens auf. Gebrauchen wir aber das Zeichen *A* als Zeichen für das Zeichen *A*, wie wenn wir schreiben: *A ist ein römisches Schriftzeichen*, so fassen wir *A* trotz bildmässiger Ähnlichkeit nicht das Bild, sondern eben als Zeichen.
Also die objektive Tatsache der Ähnlichkeit zwischen Erscheinendem und Gemeintem bestimmt keinen Unterschied. Gleichwohl ist sie für den Fall der Bildvorstellung nicht belanglos. Dies zeigt sich in der möglichen Erfüllung; und es ist ja nur die Erinnerung an diese Möglichkeit, welche uns die 'objektive' Ähnlichkeit hier heranziehen ließ. Die Bildvorstellung hat offenbar die Eigentümlichkeit, daß, wo immer ihr Erfüllung zuteil wird, ihr als 'Bild' erscheinender Gegenstand sich mit dem in erfüllenden Akte g e g e b e n e n Gegenstand durch Ähnlichkeit identifiziert. Indem wir dies als Eigentümlichkeit der Bildvorstellung bezeichnet haben, ist schon gesagt, daß hier die E r f ü l l u n g d e s Ä h n l i c h e n d u r c h Ä h n l i c h e s d e n C h a r a k t e r d e r E r f ü l l u n g s s y n t h e s i s a l s e i n e r i m a g i n a t i v e n i n n e r l i c h b e s t i m m t.' (LU II, 54–55)

[61] 'An icon is a representamen which fulfills the function of a representamen by virtue of a character which it possesses in itself, and would possess just the same though its object did not exist.' (5.73, Lectures on Pragmatism, 1903). In general, the relation of Peirce's sign descriptions to his pragmatism has been undervalued cf. e.g. 'It appears to me that the essential function of a sign is to render inefficient relations efficient, – not to set them into action, but to establish a habit or general rule whereby they will act on occasion.' (8.332, Letter to Lady Welby, Oct 12, 1904). Peirce's sign distinctions are made with reference to a basically functionalist (in a pragmatic sense of the word) sign conception.

[62] The animal known as a walking stick evidently functions as a sign fooling hungry birds, but there's no conscious, let alone human intention behind this functioning.

[63] Thus, it could be argued, Hilbert the father of formalism was no formalist (Hempel, in Benacerraf and Putnam (1983) makes this point). He insisted not on formalism but on finitism, that is, the formal means of proof should be finite, even if the mathematical content of the theorems proved might be infinite, and thus it was his attempt at solving the same infinity problem as the intuitionists, not to make formalisms the only goal of science.

[64] Of course, one should not overrate the 'turn' fact. Many foolish ideas have presented themselves as 'turns'.

[65] A related quarrel loomed large in early Cognitive Science under the label of the 'imagery debate'. Do mental pictures form real parts of thought processes, and may the pictural properties of such figures play effective roles in reasoning? Or are such pictures merely secondary to a basically symbolical, propositional thought process having pictures as epiphenomena or taking them as objects? Stephen Kosslyn's and Zenon Pylyshyn's old debate is even resurfacing these years. For a Peircean point of view, however, iconicity and propositionality is not in any way mutually exclusive, cf. Chap. 4 on diagrams.

[66] Actually, this similarity problem involves Gödel's well-known incompleteness theorem and entails that the set of possible A's may not be delimited beforehand because of being not computable. Of course, such insights do not interest the crusader of difference.

[67] Peirce makes exactly the same observation, in fact, when stating his pre-Popperian argument against the necessity of inductive reference.

[68] The reason why Goodman does not think this is the case is probably because he sees the idea of a 'common property' as something which may easily be formalized in a discrete symbolic algebra. If phenomenon 1 has the property *a* and phenomenon 2 has the property *a,* then it takes no larger amount of insight to state that $a = a$. But if quality spaces are continuous, this proposition forms a mere surface referring to continuous transformations which need not be simple.

[69] Thus, there must be an internal contradiction in the theory; the idea of a 'sémiotique naturelle' being impossible without the possibility to identify similar phenomena as similar. I hasten to say that this theory has in other respects, in particular dealing with narrative issues, proved fertile cf. Chap. 17–18.

[70] One can even find cooperations between the Nietzschean version and the extreme formalist version of anti-iconism, as for instance in the Italian Hjelmslevo-Nietzscheanism of Alessandro Zinna, a direct heir to the Ecoists' Hjelmslevo-Marxism of the seventies. When first you get the strange idea it is easy to see how this coupling works: Hjelmslev's sign-function destabilizes the sign to the extent that it is not a part of the language system, and his definition of form of expression and form of content, respectively, as the giving form to amorphous substance yields a place for Nietzschean active nihilists to undertake these creative acts of producing not only signs, but also expression and content forms.

[71] Even if Eco does not know the Erlanger program and thinks that geometrical similarity is only invariance to scalar transformations (and not, for instance, projections, rotations, mirrorings, topological 'stretchings' etc.).

[72] Already Peirce, by the way, raged against the mysticist claims that the Gizeh pyramids documented ancient Egyptian knowledge of the expansion of π, of e, the precise position of the North Star etc. – claims so well parodied in *Foucault's Pendulum*.

[73] The argument here is structurally analogous to Kant's argument for the indispensability of an *Urteilsraft*, an ability to judge. How can a rule be applied to a particular case? You may not propose another rule to take care of the application, for then you run into an infinite regress: how can this second rule be applied, then, and so on. The same goes for the attempt to get rid of similarity by means of conventions: how do you apply the conventions to similar cases – by means of yet other conventions . . .?

[74] We shall not go into Eco's arguments here except to notice that his treatment of the token-type problem is at best superficial. He obviously thinks that expression types are 'ruled by a *ratio facilis*', that is, they are easy to produce and reproduce, even if he admits that tokens of content types may be more difficult to handle. But even expression tokens constitute a very complicated problem, the bundle of transformations securing the various kind of similarities between various subtypes of a type has been investigated by Douglas Hofstadter (1986) and myself (1992).

[75] In short, one could sum up, it conceives of the thing as if it was an animal. René Thom's idea of the semantics of the substantive as a generalized animal here receives an indirect support. It is a pity that the consequences of this theory are not more thoroughly discussed – does this ascription of generality and organic teleology precede categorization *as such*? If so, is it not a necessary corollary that animal species categories are prototypical categories, so that other empirical concepts receive their (deceptive?) natural-kind character from this implicit biology of meaning? (No wonder, if this be the case, that philosophers always chose animals when talking about natural kinds, from Kant's dog over 'the cat is on the mat' and to Eco's own platypus . . .).

[76] Nolan 1994, 230.

[77] In a variety of sources, we find indications of such a distinction, for instance Groupe μ's insistence on the autonomous existence of visual types as distinct from their virtual linguistic denomination (1992), Jakobson's distinction between privative and qualitative oppositions, and Medin and Barsalou's (1987) distinction between prototype and boundary classifications, respectively, in categorical perception – see Chap. 10.

[78] Eco's promising negotiation theory of semantics ought to be compared to Hintikka's game-theoretical semantics.

[79] Rather late in the development of his theory, Peirce saw the icon as being not general, even if the icon can be used as an icon of a continuity of objects which has the same quality: '. . . neither icon nor index possesses generality' ('A Guess at the Riddle', c. 1890, 1.372). Peirce's development of the notion of hypoicons, including diagrams, introduces an important change to this idea, making general icons not only possible, but central to his epistemology (cf. Chap. 4). The existence of general

pictures has often been ridiculed in philosophy (Locke's famous triangle), but an important undercurrent continues a position cognate to Peirce's, see e.g. H.H. Price's overlooked *Thinking and Experience* (written in 1953 in the midst of Wittgenstein language game ecstasy in Britain) where the importance of 'images as general symbols' is emphasized. Price makes a strong case for the central role of general images in the psychology of thought. Regarding diagrams as external representations, James Robert Brown's *Philosophy of Mathematics* (1999) makes, disguised as a phil-of-math introduction, a strong case for the possibility of pictures to 'provide solid evidence, too, evidence which is as rigorous as any traditional verbal/symbolical proof' (192). Brown's involvement with computer aided mathematical imagery suggests that information technology may function as a partisan in the renaissance of the acknowledgment of general images.

[80] Peirce does not identify the copula with the verb to be, rather it is the very assertion of the predicate+subject structure. Here, he anticipates speech act theory by admitting questions, imperatives and the like as proposition types on a par with assertions.

[81] This problem of the reducibility of relations entails an enormous and still unfinished discussion. Löwenheim and Quine have, each in their way, tried to show that all logical propositions may be reduced to dyads, but the general validity of their results are still subject to doubt, cf. Burch 1991. As to actants in linguistics, René Thom (1980) has argued there is a formal, topological reason for the apparent four-actant limit in sentences.

[82] Cf. Tom Short's (1983) important observation regarding 'How to Make our Ideas Clear': less clear ideas are not prevented from participating in making more general ideas clear. It suffices that the role played by the less clear ideas in the clearer picture is, in fact, clear. Clarity is not necessarily built up from below using crystal clear basic elements but may reside on certain levels of organization, cf. the 'bottomlessness' of mereology in contrast to set theory.

[83] Maybe this would also provide an answer to the old Husserl's famous problem in *Ursprung der Geometrie* (1939) about how original evidence might be transferred through history by means of symbols not containing this evidence . . .

[84] The whole problem of the synthesis of the various profiles of the object is another, even if it possesses interesting analogues to the diagrammatic conception of the object. One profile of the object already contains hidden similarities in a Peircean conception.

[85] See Roberts 1973, for a groundbreaking treatment of Peirce's logic graphs. Among recent diagrammatic scholars investigating Peirce's logic graphs could be mentioned John Sowa, Allwein/Barwise, Sun-Joo Shin and Ahti-Veikko Pietarinen. Shin claims the heuristic virtues of existential graphs and aims at rearticulating Peirce's basic graph operations for practical purposes, while Pietarinen argues for a philosophically deeper relevance of the 'endoporeutic' (outside-in) interpretation direction of Peirce's graphs, the idea that the outermost layer of a diagram must be interpreted before going on to its interior. To Pietarinen, Shin's reformulation of Peirce's conventions attempting to render the graphs compositional goes against the outside-in reading of symbolic calculi and he points to Hintikkan game-theoretical, dialogical intepretations of logic in order to further develop Peirce's graphs. The outside-in reading, to Pietarinen, is important because it makes the interpretation depend on the context implicitly provided by the phemic sheet on which the graphs are drawn – the phemic sheet corresponding not to the universe as such, but to the universe of discouse, depending on tacit understandings between the dialogue partners (Pietarinen 2006, 120–22). Maybe Pietarinen's observation can be generalized to Peirce's diagram doctrine as such: the outside-in reading is preferred because taking the diagram as a gestalt informed by a context. In specific diagram interpretation, the outside-in reading will, of course, mix with inside-out readings in an ongoing trial-and-error process, but it is right for Pietarinen to insist that diagrams generally may not be presupposed to be compositional; compositionality rather forming a restricted subclass of special diagrams. Everyday examples of diagrams like a map of a country or a school timetable are evidently interpreted outside-in rather than inside-out.

[86] Pure icons only exist as a limit category in Peirce – concrete signs being, as a rule, composite. 'Hypoicon' is Peirce's notion, then, for signs whose mode of object reference is primarily iconic.

[87] The force of this idea in metaphor analysis is obvious – and it is recognized, albeit in non-Peircean clothing, by the cognitive semantics tradition mentioned above.

[88] In the development of Peirce's thought, the idea of a general diagrammatology thus precedes his construction of existential graphs rather then the opposite. It is his diagrammatology and his category phenomenology which permits him putting them to use in graphical logic representations as a special case.

[89] This fact is elaborated ingeniously in Hintikka 1997.

[90] In addition to this basic, operational icon criterion, however, Peirce also has a continuous idea of measuring different degrees of iconicity of representations. Thus, he sees his own logic graphs as more iconic than symbolic representations because a variable is here presented by one continuous line of identity as against the repeated occurrence of a number of *x*s with the same reference in a symbolic representation. While the former preserves the unity of the variable, the latter represents it in a shattered way untrue to the unity of the reference of the variable. Thus, in his Beta graphs, he has two different ways of expressing identity between variables – one identifying them by means of a continuous 'identity-line', another identifying them by attaching the same letter ('selective') to them. The former Peirce sees as *more iconical* than the latter (even if the latter may be hueristically superior). This points to another, *optimal* iconicity concept in Peirce in addition to the basic operational iconicity. See Stjernfelt 2006.

[91] It is, for instance, not sufficient to rebaptize objects *a, b, c* . . . in order to perform a formalization if a rational transformation syntax is missing. By this criterion, hence, the infertility of some classical formalization attempts in semiotics becomes understandable; e.g. Hjelmslev's ambitious algebra of glossematics (1975) which did not permit transformational possibilities of any larger interest.

[92] I prefer to count such sign use as diagrammatic, notwithstanding some Peirce's more strict definitions demanding the presence of explicit intentional diagrams. This definition conflicts with other descriptions of diagram use, e.g. his characterization of mental imagery experiments as diagrammatic or his ideas of grammar as a type of diagrams, and is closer to his pragmatic *in actu*-requirement for sign use. I follow the latter tendency in calling icon experimentation involving rule-bound manipulation of icon parts diagrammatic.

[93] This points to the fact that the organization of perception includes highly elaborated diagrammatic capacities without explicit conscious representation.

[94] Lakoff and Johnson's metaphor theory thus involves that structure is mapped from one domain onto another; Fauconnier and Turner's generalized 'blending' theory (comprising also non-oriented mappings) involve a schematic, so-called 'generic' space granting the coherence of the blending output.

[95] For a thorough investigation of Kant's schematism, see Frovin Jørgensen 2005.

[96] Diagrams as 'the main if not the only way we acquire new knowledge of relationships' have been acknowledged as a Peircean doctrine by Johansen 1993, 99.

[97] It must be added, though, that Peirce's attitude towards Kant's famous distinction is not unanimous. In his early and middle period, Peirce simply sees analyticity as identical to deductive necessity, while syntheticity covers ab- and inductive probability (cf. for instance Peirce's discussion with Dr. Carus in CP 6.595 (1893), see Otte 1997 353f), thus pertaining to ideal and real realms, respectively. Consequently, mathematics is taken to be analytic – in contrast, of course, to Kant. Later, the issue becomes more muddled, not less because Peirce now only rarely refers to the analytic/synthetic concepts explicitly. Here, theorematic reasoning – corresponding to the experiment attitude in diagram manipulation – is generally seen as synthetic. We shall return to the issue in more detail in Chap. 8.

[98] Correction in the quote made from Robin (293), 59; NEM IV has 'represent*ed* existential or experiential peculiarities'.

[99] The distinction between pure and applied diagrams roughly corresponds to Kant's distinction between a priori and a posteriori schemata.

[100] Barwise and Etchemendy highlight this important feature in diagrammatic modeling: '5. Every possibility (involving represented objects, properties, and relations) is representable. That is, there is no possible situations that are represented as impossible. 6. Every representation indicates a genuine possibility.' (1995, 215).

[101] But doesn't this example run counter to Peirce's observation that the grammar of natural language is diagrammatic? No, because the contents of the words 'round' and 'square' are not defined by grammar. The diagrammaticality of (parts of) natural language syntax rather lies in its instantiation of some basic logic and ontological categories (argument structure, subject/predicate structure, etc.) It is important to

remember – cf. our painting example above – that concrete signs may possess both diagrammatic and non-diagrammatic aspects, just like they may be composed of differently defined diagrams, the relation between which need not in itself be diagrammatic. Some of the strength of natural language probably lies in precisely this: it freely unites diagrams on different levels (expression, grammar, lexical semantics of the different word classes, narratology), the relative independence of which constitute language's plasticity and its ability to talk about many things, including impossible objects.

[102] Of course, this requires that the diagram is consistent. But the very syntax of a diagram forces it to be consistent: it is impossible to draw a square circle. This does not imply, however, that it may not be in many cases rather or extremely difficult to determine whether a given diagram is in fact consistent. For instance, an equation – a subspecies of algebraic diagrams – may hide an inconsistency very difficult to ascertain at first glance but which requires lot of work to determine: if you can derive a contradiction from it (the reductio ad absurdum method), then it is false (if we do not admit intuitionist logic etc.). The seminal difference is that you cannot derive from the mere grammar of the symbolic expression 'a square circle' an analogous contradiction; in order to do so, you have to attempt to make a diagram of its content.

[103] Yet, this distinction is in many cases impossible to draw beforehand, so to speak – cf. for instance the fact that a certain amount of empirical data shows up to yield a Gaussian distribution: on a first glance, this result may be conceived of as an empirical law, but it might hide a deeper law, yet uncovered, which would rather make the distribution a logical result of general mathematical principles.

[104] This icon-index distinction in Peirce of course refers back to Kant's contention that existence (haecceity, referred to by an index) is no predicate (quality, referred to by an icon), just like it refers forward to Kripkean reference theory's rigid designators (as a certain class of indices).

[105] Many basic proofs in mathematics may be represented in more or less immediately accessible visual diagrams, see e.g. Nelsen 1993.

[106] See Misfeldt 2006 for an empirical investigation emphasizing the change between different representations of the same object in mathematical thought.

[107] Our description of the workings of such experiment processes might give the idea that they are psychological and thus dependent upon a person's psychological grasping of the diagrams. This is not, however, the case. It is a crucial part of Peirce's pragmatism that it shares a basic anti-psychologism with Husserlian phenomenology (cf. Chap. 6). Pragmatism insists that it is possible actions on diagrams which count – but such actions need not be performed by psychological means (cf. Peirce's notion of mind being much broader than that of psyche). Diagram operations are, by their very nature, purely formal and do not owe their validity to the psychology of those performing them. If the description of such processes may in some cases sound as if informed by psychology, this is only for the sake of understanding. When talking about the 'imaginary moment' as a phase in diagram manipulation, this of course refers to the psychology of the manipulator, but the decisive thing is that this moment is made possible by structural iconicity between diagram and object – not by the psychology of he or she who contemplates that iconicity. The case is parallel to when Peirce himself refers to human minds as bearer of signs, but immediately adds that this is only a pedagogical 'sob to Cerberus' to make his own conception easier understandable (Letter to Lady Welby 23 December 1908, EPII, 478). The validity of diagrammatical representations in general depends just as little on psychology as does the special case of logical formalisms.

[108] Peirce makes a distinction making this understandable – between corollaries and theorems. The former are propositions directly read from a diagram; the latter propositions only to be found after some more or less 'ingenious' experiment. The distinction is valid, but can not be sharp: there is a continuum between, say, measuring a distance on a map; measuring the same distance with corrections according to the map projection used; constructing that projection; proving that the geometry of the surface of a sphere is isomorphous to a non-Euclidean geometry . . . We return to this distinction below.

[109] It must be added that the truth of Kekulé's discovery story is a matter of ongoing debate in the history of science and has not yet been definitively settled. The story is only recorded by Kekulé himself in 1890, in a celebration speech 25 years after the discovery was published. The case is even more complicated from the fact that the German Chemists' Society at a conference in 1886 published a mock-periodical in which they appear not as the deutsche but as the 'durstige chemische Gesellschaft'. In this

joke, the Benzene ring is depicted with the Carbon atoms as six apes grapping each others' arms and legs (playing on the similarity between 'Affe' and 'Affinität' (ape and affinity) in German). Hence, it is argued, Kekulé's 1890 memory might have been influenced by this recent joke. Thus, the story was an object of doubt rather early, and already in 1927, Kekulé's son stated in an article that according to his childhood recollections, his father had often told the story many years before it appeared in print without ever referring to any apes – thus adding to its probability (Sponsel and Rathsmann-Sponsel 2000). Also the source of the 'Uroboros' motive – the snake biting its own tail – is discussed, and maybe traced to a sign on a pharmacy door which Kekulé remembered (ibid.). On the other hand, it has been pointed out that the German chemist Josef Loschmidt was a forerunner of Kekulé because he had, already in 1861, described a long series of organic molecules as involving ring-shaped Carbon structures (albeit not the simplest one, Benzene). Kekulé knew Loschmidt's work as is evident from his dismissive references to it already in the same year where he refuses the shapes given by Loschmidt to have any connection to real molecule shapes (Bader and Parker 2001). This thus forms a strong argument that Kekulé's 1863–65 discovery may have been influenced by his reading of Loschmidt some years earlier – but, on the other hand, it does not prove this influence may not have appeared in the guise of the half-dreamt snake ring of the original anecdote.

[110] Ernst Cassirer's concept of 'symbolic pregnance' may be interpreted as referring to such cases of 'spontaneous proto-diagrams'.

[111] Another example would be maps with high direction sensitivity but no metric, e.g. maps of the starry sky as seen from the earth; distances on this map measured in minutes and seconds of arc do not refer to real distances between stars in the universe, while directions do refer to real orientations in space.

[112] Hintikka's distinction is most thoroughly presented in the papers collected in the volume *Lingua Universalis vs. Calculus Ratiocinator* (1997).

[113] The drawbacks of this tradition have large overlaps with those ridiculed by Barry Smith's concept of 'fantology' (2005).

[114] In the logic-as-universal-language tradition, Peirce's distinction will be invisible, because any chain of reasoning will here be represented as a valid, finished symbol string *post hoc*, so that the theorematic, constructive part may merely be added as further premisses among others to the inference at issue. Peirce's distinction, however, becomes crucial because the viewpoint of the logic-as-calculus stance rather envisages the issues *ante hoc* – logic deals with the solution of problems and thus displays a continuity with heuristics and theory of science which is absent in the language tradition. When seen from the problem rather than from the solution aspect, Peirce's distinction suddenly becomes pertinent: given a set of premisses, it is of huge importance how and what to construct on their basis in order to reach a desired result.

[115] It is not so easy, however, that the universal-language and the calculus tradition are simply anti-iconic and iconic, respectively. Hintikka, e.g. places Hilbert and Goodman firmly in the calculus tradition. Hilbert is, according to Hintikka, no formalist, rather a fore-runner of model theory and, as discussed later, much less anti-iconist than often assumed, while Goodman the staunch anti-iconist is a calculus supporter because of his plurality of languages. On the other hand, the position of being universal-language and iconist at the same time is also possible – cf. the younger Wittgenstein with his picture theory of language maintained at the same time as the ineffability of semantics, famously making it impossible actually to point out any particular examples of the logical atoms claimed to found the theory of *Tractatus*. So even if Hintikka's calculus-language distinction is indeed orthogonal to the iconist-anti-iconist distinction, the combination of universal-language and anti-iconism is strong (and stronger than the combination of universal-language and iconism) in both analytic and continental traditions.

[116] See some preliminary remarks in May and Stjernfelt 1996.

[117] The German mathematician Moritz Pasch explicitly noticed this geometrical error and proposed a pure geometry in terms of purely formal manipulation of symbols with no regard to their intuitive signification, an idea that was fully developed by his famous pupil David Hilbert's formalism.

[118] The concept of 'symbol' has a history so confused that it almost ought to be completely discarded; in any case, any use of it should be explicit about the precise signification intended, cf. Sørensen 1963.

In formalism, symbols are arbitrary, simple signs to be manipulated syntactically; in Peirce they are not necessarily simple, and dependent on iconic meaning and indexical reference. On the symbol concept in the Kantian tradition, see my 'Die Vermittlung zwischen Anschauung und Denken' (2000).

[119] Greaves, in turn, quotes from a paper by Michael Hallett who provided translation and italics.

[120] Greaves has even found an amazing quote by Hilbert's close collaborator Paul Bernays in an unpublished lecture from 1921, where the stroke counting ability is directly expressed in terms of basic iconicity suggesting, as Greaves says, 'a distinctly Peircean explanation': 'The philosopher is inclined to speak of this representation [between sign and number] as a relation of meaning. However, one should note that, in contrast to the usual relation between word and meaning, there is [in this example] the essential difference that *the object doing the representing contains the essential properties of the object to be represented.* Thus the relations which are to be investigated between the objects represented *are to be found in* the objects doing the representing, and thus can be established through consideration of these.' (190–191; translated by Michael Hallett, emphases by Greaves). The intuition necessary for the metamathematical finitism is hence (strongly restricted, it must be admitted) iconicity. Still remains, of course, the Hilbertian distinction between these finite calculi and the potentially infinite objects they may be taken to refer to.

[121] But even if we grant the basic iconicity of any 'symbolic' calculus, a Peircean approach will still be faced with the problem of evidence in cases where the 'imaginary moment' is precluded or where it simply refuses to appear, cf. for instance the discussion of the computer proof of the four-color map theorem of topology which – because of its enormous size – is hard to understand as an ordinary proof which a skilled reader may adorn with interpretations from beginning to end. In proofs of this type, the trust is put in the infallibility of the computer: each step in the proof is logically valid; ergo the whole proof is valid, even if nobody has ever *observed* its truth in Peircean evidence or in Husserlian 'kategoriale Anschauung'.

[122] 3.467, from 'Grand Logic,' 1893.

[123] Life as such seems formally to involve simple diagrams known as 'categorical perception,' see Chaps. 9–12.

[124] '. . . il n'y a science qu'à partir du moment où on peut plonger le réel dans le virtuel' (Thom 1988, 69)

[125] Of course, this counterfactuality is easier to hide in experimental sciences where diagram experiments may, in many cases, be verified by similar experiments on the object itself. When this possibility falls away, counterfactual speculation prevails, cf. for instance cosmology or issues like the origin of life and origin of language. It is interesting to note, however, that the insight in the connection between counterfactual constructions and scientificity is taken up in non-experimental sciences in recent years. For instance historiography, so long trapped in a positivist determination to record only what actually happened, now seems (through inspiration from, among others, chaos theory and the formal concept of phase space in general qualitative dynamics) to realize that the actual event is only made intelligible through its juxtaposition with a rational idea of what would have happened if some central factors in the initial conditions of the situation were changed (Ferguson 1997).

[126] Nöth uses the term when describing Piaget's structuralism, Todorov's narratology, Jakobson's classification of sign systems as well as in the context of more general issues such as the semiotic concept of 'code' and the description of the transferral of semiotic structures into media such as theatre or gesture.

[127] By 'semiotics' we shall here take those disciplines which in various ways investigate signs and their meaning; we should not delimit ourselves to currents only which explicitly makes use of the term 'semiotics' or related expressions.

[128] We shall here remain so general in order not too quickly to tie ourselves to one specific rendering of these concepts so as for instance function as modelled by set theory as a 1-1 relation between the elements of one set to those of another. We shall return to different versions of the function concept below.

[129] The following presentations are not, of course, thorough introductions to the relevant theories' assumptions and methods, but are centered around the transformation concepts in play. Some of the theories I have discussed in more detail elsewhere.

[130] Cf. the detailed criticism in Scubla 1998 where the different attempts at formalization of the 'canonical formula of myth' is presented: the equivalence between dual opposition pairs and the mediations between them; Boolean logic, group theory, the Klein bottle as well as Jean Petitot's catastrophy theory

interpretation of it. I cannot here run through these vast discussions, a discussion of the last example can be found in Stjernfelt 1992a.

[131] 'Or, la notion de transformation est inhérente à l'analyse structurale. Je dirais même que toute les erreurs, tous les abus commis sur ou avec la notion de structure proviennent du fait que leurs auteurs n'ont pas compris qu'il est impossible de la concevoir séparée de la notion de transformation. La structure ne se réduit pas au système: ensemble composé d'éléments et des relations qui les unissent. Pour qu'on puisse parler de structure, il faut qu'entre les éléments et les relations de plusieurs ensembles apparaissent des rapports invariants, tels qu'on puisse passer d'un ensemble à l'autre au moyen d'une transformation' (Lévi-Strauss 1988, 159).

[132] 'Quant à la méthode comparative, elle ne consiste pas, je l'ai souvent dit, à comparer d'abort et à généraliser ensuite. Contrairement à ce qu'on croit souvent, c'est la généralisation qui fonde et rend possible la comparaison' (ibid. 179).

[133] D'Arcy Thompson thus is the heir to a sort of structuralism widely disregarded in the French structuralism of the 60s, namely that of Goethe (see Cassirer 1945). Goethe's theory of the 'Urpflanze', the arch plant, and its metamorphoses of course transgresses – as Jean Petitot remarks – Kant's prohibition against intellectual intuition, as when he intuitively seeks to grasp the schema for vegetative morphogenesis from an inner power which modifies the arche-part of the plant, the leaf, into assuming different functions. Yet his idea that such a schema might be possible constitutes a source for later and more rational attempts at formalizing such schemata (cf. the unproblematic talk about 'Baupläne' for each of the large groups of animals). The concept of metamorphosis in Goethe thus forms a first outline of a biological concept of transformation.

[134] If we look back on Lévi-Strauss' inspiration from d'Arcy Thompson, it is general: he does not, generally, use continuous transformations, but discontinuous ones only (as in the equivalence between semantic opposition pairs), even if some of the models in which he tries to formalize these transformations are indeed continuous (topology).

[135] The discussion in this section is based on Kline 1972.

[136] Many more could be mentioned, thus the concept of 'categorical perception' in phonetics, generalized in Stevan Harnad (where a discontinuous partition of the continuum of perception allows that a particular, concrete perception is transformed to the category of a typical one); Eleanor Rosch's prototype semantics (where a similar argument is made on the semantical level); Gestalt Theory (where stable structures in perception may be reached by transformation from concretely perceived instantiations, often even with apparently insufficient input in cases where a pregnant form is completed).

[137] I have done that in Stjernfelt 1992a.

[138] Thus, the central unfolding of a function germ in Catastrophe Theory constitutes a transformation having the character of an eidetic variation making clear which possibilities lie in the function germ in question.

[139] I have discussed the principal aspects of this idea in Stjernfelt 1992c.

[140] It may be added that the programmatic binarism of this part of the theory remains problematic.

[141] As Jean Petitot argued in his catastrophe theory modeling of the basic structures of that theory, cf. Petitot 1985.

[142] See Bundgaard et al. 2006.

[139] All these ideas in Husserl have a mathematical background, namely his never published dissertation on 'Beiträge zur Variationsrechnung' from 1882 with references to Felix Klein. We return to eidetic variation in Chap. 8.

[140] This is a central argument in Jean Petitot's philosophy which is strongly presented in the introduction to *Naturalizing Phenomenology* (Petitot et al. 1999).

[141] See Chaps. 6 and 12.

[142] Jaakko Hintikka (1983) has highlighted the importance of this distinction: 'theorematic' reasoning requires the introduction of a new set of quantified objects and hence, in the diagrammatic representation of symbolic logic, a further quantifier.

[143] State changes must be understood as temporal transformation invested with material ontological categories pertinent for the field investigated. In physics, invariance refers to the constance of matter and energy, in other sciences other invariants are picked out: political power is redistributed in political

science; intersubjectivity in sociology; meaning in semiotics; gene pool and life in biology (cf. the centrality of 'survival'). These transformations in real time only become understandable when they are embedded in a more comprehensive virtual space accessible by eidetic variation, cf. René Thom's idea: understanding is the nesting of the real within the virtual.

[144] Cf. the Goodman-Eco discussion above.

[145] Thus, transformation as procedure is crucial to the enlightenment project. A decisive component here is the giving up of any idea of metaphysically unprenetrable distinctions between domains – which may be transgressed by the transformation of structure between them (cf. Newton's merging together of supra- and sublunar worlds or the search in cognitive science for neural consciousness correlates (NCCs)). Such transformations, of course, have trial-and-error character (to avoid dogmatic assumptions of wrong transformations), and one might assume that transformation is what realizes the otherwise incredible step from rule to case (the necessity of judgment in Kant following from the fact that no rule is possible to guide the use of the rule, for this should be governed by a further rule, etc.) – by a trial-and-error transformation of rule-bound structure onto the particular case.

[146] The references to English versions of *Logische Untersuchungen* quotes are to Findlay's translation, Husserl 1970.

[147] Herbert Spiegelberg's pioneering 1956 article has not given rise to much further work. Here, Spiegelberg finds the following four basic agreements between the phenomenology of the two: the intuitive approach to the immediately given without preconceived theories; the disregard of questions of reality or unreality in that investigation; the radical difference between phenomenology and psychology; and the foundational role of phenomenology for philosophy and logic (182). This indeed forms a basic agreement, and as Spiegelberg says, makes it legitimate to talk both of them being phenomenologists. Spiegelberg also lists a series of points in which Peirce differs from Husserl, most notably his emphasis of the discovery of categories and the absence of intentionality as a main concern – and he concludes a brief review of the two's mutual knowledge of the other that the similarities primarily are based in their common situation: two mathematicians turned philosophers who attempt to found philosophy as a rigorous science on phenomena given in experience – in short, as two historical parallels, not unlike Newton and Leibniz. Still we will argue that considerably more similarities may be studied than the four mentioned in Spiegelberg's paper – while his basic conclusion as to mutual influence remains correct, even in the light of the further information which has appeared since his paper (NEM, *Hua* XXII, etc.).

A few further papers have commented upon the relation between the two. The fine article by Dougherty 1983 on Peirce's phenomenological defence of deduction concludes that Peirce's phenomenological approach is what allows him to articulate the 'applicability of the ideal triad to the real world' due to his different notions of abstraction (cf. Chap. 11), and thus reconcile formal and empirical justifications of deduction. By doing so, he has reached a result '. . . remarkably similar to that of Husserl' (ibid.), namely founding phenomenology as a method to attain the ideal realm of meanings.

Leila Haaparanta 2001 continues Dougherty's observation and emphasizes the commonality between the two in bracketing existence presuppositions and utilizing related abstraction methods in their research. Inspired by Hintikka, she points to a commonality in Peirce and Husserl in the analogy to geometrical method in both of them; in Peirce explicitly, of course, in Husserl, in the idea of working 'backwards' from experience – cf. Pietarinen 2006 on the importance of Peirce's backwards, 'endoporeutic' interpretation rule for his logic diagrams.

[152] '. . . durch ihre besondere Originalität, Einfachkeit und Eleganz imponiert. Sie erscheint, zumal nach der Vereinfachung, die *Schröder* ihr zuteil werden läßt, als eminent brauchbar, und es wäre demgemäß ein größerer Reichtum an sie anschließender und ausgerechneter Aufgaben erwünscht gewesen' (43).

[153] Peirce thus explicitly refers to Hegel for the notion, and this takes place in a period where Perice highly admires Hegel and *Phänomenologie des Geistes* especially – as against his earlier anti-Hegelianism of the 70s and 80s. Still, the temporal coincidence with Peirce's acquaintance with LU is striking – just as the similarities in their way of using the concept, whereas Peirce's 'phenomenology' has little to do with the Hegelian emergence of the Geist through history.

[154] Spiegelberg 1956 points to the fact that Peirce probably gave up 'phenomenology' for ethics-of-terminology reasons between Oct 3, 1904 when he writes to James about his need to find a new term and

Oct 12 the same year when he writes to Lady Welby, now using the term 'ideoscopy'. Presumably, he felt that the Hegelian use of the word were too far from his own and that Hegel must be given primacy.

[155] Rollinger 1999 (58) relates an amusing anecdote of categorial intuition: in Husserl's copy of Brentano's *Vom Ursprung sittlicher Erkenntnis*, 'in the margin next to the passage where it is claimed that perceivability cannot be the same as existence since the non-real exists and yet cannot be perceived (p. 62), it is written "categorial perception!" ' – Kategoriale Wahrnehmung!

[156] Measured against this central role in Husserlian epistemology, categorial intuition has hardly received the interest it deserves. Among the most important contributions to its clarification count the following: Bernet 1988, 1988a; Cobb-Stevens 1990; Lohmar 1987, 1990, 1998; Seebohm 1990; Sokolowski 1964, 1974, 1982; Ströker 1978, to which I shall refer in the following. Most of these accounts aim at a reconstruction of Husserl's views (and are very useful, even fertile in so doing) and do not attempt to go into comparisons with other accounts of the problem. I suspect, however, that such comparisons might bring us closer to the problem itself.

[157] Husserl's own examples of the higher-order objects grasped in categorial intution in the 6.*LI* include the identity of an object, the relation of part to whole, relations, collections, the 'ideierende Abstraktion' and its intiution of essences, the determinate and indeterminate grasping of single objects ('das A', 'ein A'). Sokolowski 1982 presents a thorough analysis of the steps from an unanalyzed experience to its categorial articulation in subject and predicate. Lohmar 1998 articulates a general 3-step structure for categorial intuition: 'Gesamtwahrnehmung, Sonderwahrnehmungen, kategoriale Synthesis'.

[158] Even if 'Wesensschau' is a later term, it can be seen as a successor concept to categorial intuition, cf. Hintikka 2003, 187.

[159] This is, of course, what makes transcendental phenomenology possible as a project: the idea of taking the essences of consciousness as an object of eidetic study. It seems possible, however, to retain the idea of the possibility of fulfilment of (certain) intentions aimed at eidetic phenomena as well as phenomenology as an anti-psychologist science of consciousness, without assuming the constitutive nature of the latter. This would preclude (or, at least, bracket) the possibility of a transcendental phenomenology, but preserve eidetic phenomenology.

[160] Later, in his drafts of a new version of the sixth *LI*, Husserl took the position that they *always* involve such meaning, cf. Melle 2002.

[161] 'Le paradoxe est qu'une forme catégoriale signitive ne peut être remplie que par une intuition déjà elle-même catégorisée.'

[162] We find in the three degenerate act types (imaginative, indexical, and signitive, respectively) a not coincidential parallel to Peirce's three different ways of signifying an object (icon, index, symbol). This has been noted by Münch 1993, 218.

[163] As is evident, this variation procedure is modeled upon function analysis in mathematics, even in Husserl's terminology.

[164] As to the history of the concept of ontology, see Øhrstrøm et al. 2005.

[165] This point and its relation to the transcendence issue is not always clearly emphasized; a strong exception is Willard 1982, 397.

[166] In fact, if categoriality were identified with formal logic only, then some version of logical positivism might be the outcome. But it is not necessary to identify categoriality nor the propositional stance with language. Rather, language is one (prominent, to be sure) instrument developed on the basis of the cognitive potentials of abstraction and categoriality. A very broad definition of categoriality – comprising all higher-level acts founded on perception – is proposed in Smith (2000). Similarly, Bernet (1988) points to the fact that categoriality in general 'is about the intuitive givenness of ideal objects' (33).

[167] The intimate connection between categorial intuition and this 'theory of theories' is highlighted in Cobb-Stevens 1990.

[168] In Peirce, this problem, of course, is solved by taking general meaning to have a continuum of merely possible (but vague) referents as its extension and his idea that generality corresponds to the possibility of choosing any one of these – close to Husserl's idea of 'no matter which one'. When Husserl talks about 'representation' here, it must not be read symbolically as in normal Anglo-Saxon usage – in that case, intuitive representations would lose their direct object contact (cf. Roy 1999, 111 – 147).

[169] Here, Husserl is on a par with Peirce for whom the diagram is not the particular drawing on the page nor the reader's perception of it. Peirce thus distinguishes between the diagram token – the particular drawing on the page, corresponding to Husserl's 'Anhalt' – and the diagram type which we are able to grasp through a reading of that token, governed by a symbolic sign (which, in Peirce's terminology, implies generality).

[170] See Chaps. 8 and 12.

[171] Lohmar (1987, 1990, 1998) highlights this example in order to correct an error which Husserl himself later detects (1939). Husserl's idea in the LU was that the categorial act of collection by means of the 'and' operator could only reach fulfilment by a 'reflection upon the act itself', because it is the very act that constitutes the collection of entities envisaged (all possible entities whatsoever may be so collected). As an alternative to this strange idea where the performance of an act becomes the intuition fulfilling that same act, Lohmar points to fulfilment as *Deckungssynthesis* – covering synthesis – between partial intentions. Thus, collection would be so to speak a zero-*Deckungseinheit* and is probably involved in all more complicated acts because it simply co-localizes its entities in one and the same categorial place.

[172] This corresponds to Peirce's semiotics where the classification of objects in types does not require symbols, while the opposite is the case.

[173] We have already remarked that a strong case can be made that the set of possible typefaces or writing styles is so large that it cannot be exhausted by computational algorithms (cf. Hofstadter 1986; Stjernfelt 1992c). Thus, the very concept of writing style cannot be the result of a variation procedure limited to computational strength. Thus it points to the fact that the variation implied in grasping essences does not always – if ever – proceed to completion through all possible variants. Rather we intuit the fact that such a variation may go on indefinitely.

[174] We can ask, then, what is implied in the *mere* signification of the word 'steam engine', before the analogical fulfilment by the general imagination of the model? 'Steam engine' is a composite noun, that is, it means an engine somehow concerned with steam. Depending on the underlying schema chosen, such an engine could work by steam, produce stem, fight against steam, etc. The syntax of noun composition only tell us it is an engine in some way concerned with steam (thus founded upon simpler signitive acts aimed at 'steam', and 'engine', respectively; but these are both acts concerned with general objects and thus already presupposing categoriality in the form of generalizing abstraction). The same structure characterizes Descartes' famous 'chiliogon'-example which Husserl uses to argue against the representative use of diagrams in geometry. True, we can not imagine such a figure in its complete shape, and thus the understanding of the P-S structure of the word, literally 'thousand-edge', rests on our categorial understanding of the syntactical structure as well as our categorial understanding of each of the composite elements: a figure with a thousand edges. A further step in the *Erfüllungsreihe* may now prompt us to try and construct the figure in imagination. We still cannot imagine it as a figure, but we may imagine the procedure to construct it (take a rectangle and subdivide the sides until you get a number close to thousand, then add or subtract sides until you get thousand) – just like Husserl himself, when describing eidetic variation, emphasizes that the imagination can not cover all cases, but we can imagine the imagination act to go on . . . An contrast case is the well-known 'round square' with its impossibility of intuively construing such an object. All such composite expressions prompt an *Erfüllungsreihe* prescribed both by their syntactical structure, by their founding acts. The compound noun problem forms a center of the discussion of grammatical 'blending' in Turner and Fauconnier's cognitive semantics (Bundgaard et al. 2006; Fauconnier and Turner 2002).

[175] However, a problem remains concerning the purely empirical species concepts. Husserl distinguishes three cases: sensuous abstraction giving sensuous concepts, categorial abstraction yielding pure categorial concepts, and a large group of *mixed* concepts (with the examples 'color', 'house', 'judgment', 'wish' for the first ones, 'unity', 'plurality', 'connection', 'concept' for the second ones, and 'coloredness', 'virtue', 'parallel axiom' as examples for the mixed group.) The latter two can of course be seen as direct parallels to Peirce's pure and applied diagrams, respectively. Peirce also admits the existence of concepts not (yet?) analyzable in diagrams and mentions 'murder' as an example. Still, even concepts as these are not without diagrammatic content. Both Husserl's 'house' and Peirce's 'murder' are founded concepts; both presupposes a schema of wilful, human action oriented towards a goal and towards eliminating certain factors opposing that goal (life of some person in the former case; cold, rain, theft,

etc. in the latter. The reason why Peirce will not take 'murder' as a diagram is that its semantics contains no rational relations. But this only implies that it is a 'stiffened' diagram: it is not possible to perform any information-yielding experiments on it. But it is still a diagram in so far it is a schematic relation able to subsume instantiations by variation. Thus, I believe that the field of pure sensuous concepts is probably narrower than both Husserl and Peirce suppose; they seem to be restricted to primitive sense qualities and even then, the categorial apparatus of variation is necessary for isolating them from phenomenological experience.

[176] Thus, I perfectly agree in his insistence that it is a mistake for Husserl to claim that the 'vague morphologies' principally resist mathematization. It is interesting to note that this was not unanimously Husserl's contention in the first version of the LU where he still claims the ideal that 'Die vagen Gebilde der Anschauung mittels exakter Begriffe möglichst deutlich zu charakterisieren, ist überhaupt eine phänomenologische Aufgabe . . .', even if he immediately admits that this task '. . . lange nicht genug angegriffen und auch in Beziehung auf die vorliegende Untersuchungen nicht gelöst ist' (3. LU, Hua XIX, 249). The second version replaces this contention with a longer argumentation to emphasize that 'Offenbar sind die Wesensgestaltungen aller anschauliche Gegebenheiten als solcher prinzipiell nicht unter 'exakte' oder 'Ideal-Begriffe', wie es die mathematischen sind, zu bringen' (ibid.). (Eng. 'The descriptive concepts of all pure description, i.e. of description adapted to intuition immediately and with truth and so of all phenomenological description, differ in principle from those which dominate objective science. To clear up these matters is a phenomenological task never yet seriously undertaken and not carried out in relation to our present distinction'; 3rd LU, Vol. 2, 451).

[177] Of course, discontinuous operation procedures are possible, so as e.g. the stepwise construal of $(5^3)^4$. But discontinuity is dependent on continuity, Peirce would argue: the only way we are able to synthesize the single components of a discontinuous procedure into one state-of-affairs is by embedding them in a continuous space. Thus, discontinuous phenomena are always embedded into continuous ones, and discontinuous objects or calculi presuppose (explicitly or not), continuity.

[178] Holenstein 1972, 288. Husserl's argument is built on §22 in LU 3, where he notes that not every whole requires a specific moment of unity, only the 'zerstückbare' require that. All really uniting consists of relations of foundation, and unity is a categorial predicate, on the one hand, at the same time as this unity is given directly, sensuously in Gestalts.

[179] This fact is, of course, what gives rise to the attempts at finding a vocabulary of simple schemata, so as e.g. the 'kinaesthetic image schemas' of Lakoff 1987 or the schemata in Peruzzi 1999. The precise amount of such schemata which the human mind is able to process is no doubt governed by empirical psychological regularities – but an a priori, phenomenological corollary is that *any* possible mind will have to do with some finite vocabulary or other of such simple diagram atoms. This will, in fact, be the phenomenological equivalent of the impossibility of 'intellectual intuition': if we possessed such a faculty, infinitely complicated diagrams would be possible to process in one glance.

[180] 'Mereology', from Greek *meros*, part. As a matter of fact, it might have been called 'merology', but after the Polish logician Lesniewski the form quoted has become ubiquitous. Lesniewski, in his works from the 20s and 30s, considered mereology to be one out of three basic branches of philosophy, 'protothetic', 'ontology', and 'mereology', respectively. Protothetic is a doctrine of propositions and their interrelations and it forms the logical basis of his theory; ontology is based on a distributive rendering of class membership, so that distributive class expressions are identical with general nouns applicable to individuals. Mereology, then, considers collective class expressions understood as being composed of parts. By the distinguishing of the two latter branches of philosophy, Russell's paradox is avoided, and mereology is weaker than set theory because it only admits one relation of inclusion (part of), opposed to the two in set theory (membership relation and subset relation, which in Lesniewski's thought are separated as belonging to ontology and mereology, respectively). Mereology in this sense of the word thus has the advantage of being 'bottomless', the compositional foundation of a class upon the existence of ultimate *Ur-elemente* being avoided. This implies that mereology is 'phenomenological' in so far as it may describe a given level of phenomenal existence without recourse to a bottom level of atomistic ontology, a crucial aspect of the semiotic perspectives in mereology. This implies the possible affinity of mereology to strongly nominalist positions claiming mereology to be without any ontological presuppositions whatsoever (as in Lesniewski's case, and after him, Goodman) – even if this is no

necessity, and mereology may as well be connected to neutral (the younger Husserl) or strong realist positions (Smith). Lesniewski was influenced by the part-whole reflections of the Brentanian tradition: Husserl's third and fourth *Logische Untersuchungen*, in turn influenced by Brentano's *Deskriptive Psychologie* (1890), and Carl Stumpf's *Über den psychologischen Ursprung der Raumvorstellung* (1873). For further accounts for these developments, see Smith 1982, 1994. In this context, we shall stick to mereological aspects of decidedly semiotic currents of thought.

Even if none of the authors discussed here use the word, I have chosen it as shorthand for 'doctrine of whole and parts' and similar complicated expressions.

[181] Of course, these two alternatives display, each of them, a host of subvariants of more and less sophisticated types, ranging from a completely compositional logicist theory in the one end and to almost mystical insistances on the autonomy of wholes in the other. Moreover, there is not necessarily a contradiction between the two; it is perfectly possible to imagine compromises, so as e.g. an emphasis on the primacy of the holist level giving rise to motivated, iconic syntaxes governing its parts – combined with a recognition of the possibility for these syntaxes of assuming, once established, an autonomous status involving local compositionality.

[182] Cf. Hua XII which contains among other writings 'Zur Logik der Zeichen (Semiotik)'.

[183] Husserl's disciple from the 1910s, Roman Ingarden, has probably developed the most ambitious further detailing of dependency theory, distinguishing no less than four different ontological species of dependency: autonomy/heteronomy, originality/derivation, self-sufficiency/connectivity, independence/dependency which may further combine into eight types of signs (Ingarden 1965–74, 123). It seems evident that dependency is in need of further subdistinctions (a head is an independent part of the body, as an autonomous part, but it is a special part because the body ceases to function as a body if it is in fact cut off). Ingarden's suggestions have not, however, yet given rise to a fertile further development.

[184] Fassen wir irgendein Paar von Teilen eines Ganzen ins Auge, so bestehen folgenden Möglichkeiten:

1. Zwischen beiden Teilen besteht ein Verhältnis der Fundierung.
2. es besteht diese Verhältnis nicht. Im ersteren Falle kann die Fundierung
   a) eine **gegenseitige**,
   b) eine **einseitige** sein . . . (LU II, 264–65)

[185] After Holenstein 1976, 58.

[186] Jakobson 1985, 281.

[187] Jakobson 1971b, 713.

[188] Jakobson 1985, 203.

[189] Ibid. 189. Here, he claims that the third Logical Investigation is '. . . one of the milestones for the initial advance of structural linguistics . . .'.

[190] Like in 'Parts and Wholes in Language' where he begins: 'In the second part of Edmund Husserl's *Logische Untersuchungen* – still one of the most inspiring contributions to the phenomenology of language – two studies devoted to 'Wholes and Parts' introduce the philosopher's meditations on 'the Idea of Pure Grammar'. In spite of manifold aspects of interdependence between wholes and parts in language, linguists have been prone to disregard this mutual relationship.' (Jakobson 1971b, 280), and further 'The structure of the verbal code is perhaps the most striking and intricate example of whole-part relations that are built hierarchically' (282–83).

[191] Jakobson 1971a, 314.

[192] It is a strange fact that Jakobson does not explicitly refer to the *Logische Untersuchungen* in his definitions of the zero-sign or the markedness/unmarkedness distinction. Holenstein 1975, 1976 does not mention any such direct influence either.

[193] 'Eine der wesentlichen Eigenschaften der phonologischen Korrelation besteht darin, dass die beiden Glieder eines Korrelationspaares nicht gleichberechtigt sind: das eine Glied besitzt das betreffende Merkmal, das andere besitzt es nicht; das erste wird als *merkmalhaltig* bezeichnet, das zweite – als *merkmallos* . . .' Jakobson 1971b, 3, with reference to Prince Troubetzkoy.

[194] Jakobson 1971b, 213.

[195] Even politically correct language politics, eager to avoid the asymmetry inherent in the markedness-unmarkedness distinction, can not avoid this, cf. long marked forms like 'African-American' vs. short unmarked forms like 'African' or 'American'.

[196] For a comparative discussion of the bases of Hjelmslev's theory in relation to Brøndal's, see Jørgensen and Stjernfelt 1987.
[197] The only reference to Husserl found in Hjelmslev is in the early *Principes*, but even if it refers to the fourth investigation, it is pejorative: '... la théorie étrange du philosophe HUSSERL' (40). None of two recent comprehensive investigations on Hjelmslev mentions any possible relation to Husserl (Gregersen 1992; Rasmussen 1992).
[198] Diderichsen returns over and over again to the fact that linguistics in general tends to focus upon 'three main types of grammatical connexion', and at several occasions he mentions in that context Husserl's mereological analyses from *Logische Untersuchungen* as strikingly similar to structural linguistics (Diderichsen 1966, 107 [1947]; 137 [1948]; 207 [1952]) but he yields no indication as to the possible relationship between Husserl and Hjelmslev.
[199] Marty, whose 1908 *Untersuchungen zur Grundlegung der allgemeinen Grammatik und Sprachphilosophie* refers to the *Logische Untersuchungen.* Hjelmslev refers to Marty at several occasions, but not directly in connection to the dependency algebra.
[200] So, it seems like mid-twentieth century mereology has left behind some interesting ruins – or drafts for impressive new constructions? – in Ingarden's and Hjelmslev's complicated mereologies, hopefully to be developed by future research.
[201] Here, Diderichsen is admirably clear in his early insistence that formal glossematic description is impossible without a prior phenomenological sensibility for identities and differences (Diderichsen 1966, 123 (1948)).
[202] This consequence is still visible in Greimasian semiotics, having inherited the whole of its metatheoretical apparatus from the *Prolegomena*. In the semiotics of the Paris school, the orthodoxy teaches that denominations of theoretical as well as metatheoretical terms are completely arbitrary; still they are invariably chosen so they are relatively easily understandable with reference to ordinary language or linguistic tradition.
[203] For instance Ronald Langacker at the 'Wholes and their Parts' conference in Bolzano, where a first draft of this chapter was given as a paper.
[204] The shortest presentation is probably: 'For mathematical reasoning consists in constructing a diagram according to a general precept, in observing certain relations between parts of that diagram not explicitly required by the precept, showing that these relations will hold for all such diagrams, and in formulating this conclusion in general terms. All valid necessary reasoning is in fact thus diagrammatic' ('Lessons from the History of Science', 1896, 1.54). The object of mathematics will be pure diagrams of any kind, while ordinary reasoning as well as the empirical sciences will use diagrams applied in being constrained by existing relations as well.
[205] Maybe this very general notion of diagram – parts with unspecified interrelations being open to abstraction to classes of interrelations – makes Peirce's schema an early forerunner of category theory. Finally, I imagine Peirce's very wide concept of diagram may not only generalize Kant's idea of a schema, but also serve as a unifying concept for the various schema-like ideas that prevail in present-day cognitive semantics (image schemas, force dynamics, landmark-trajectory, frames, scripts etc.). If so, then the semantic processes charted in this tradition – metaphorical mapping, blending, conceptual extension etc. – would be understandable as specific variants of diagram manipulation.
[206] Austria avoided the wave of German idealism (partly due to politically motivated prohibitions) and thus maintained a realist Aristotelian curriculum in philosophy all the way to 1918. This contributed to the Austrian commitment to ontological issues when such issues were long since surrounded by epistemological (on the Continent) or logical (the Anglo-Saxon world) scepticism, cf. Smith 1994.
[207] Smith 1992, 1996b.
[208] Peer Bundgaard (2004a, 2004b) has pointed to the fact that Husserl actually outlines *two* different a priori theories in that investigation, namely of grammar and semantics respectively.
[209] Cf. Barry Smith 1992
[210] It is interesting to note that parts of semiotics have undertaken similar investigations, even if in less ontological clothing, cf. Greimas' fertile analyses of 'challenge' and 'anger' in Greimas 1982. In Chap. 18, we shall attempt a related analysis of 'espionage'.
[211] See Caldwell 2004 and Grassl and Smith (eds.) 1986.

[212] In the original, the first question was: '*Do the empirical theories with the help of which we seek to approximate a good or true picture of reality rest on any non-empirical presuppositions?*' We must assume this version is a typo since it 'turns the wrong way'. Extreme empiricists would hardly embrace non-empirical presuppositions. In order to give meaning, we have 'inverted' the question to fit the answer.

[213] Cf. Cassirer's interest in mediate concepts in, for instance, biology, cf. next chapter. It must be added that Carnap was not unaware of the coordination problem between these two tiers – the discussion of this problem was central in early positivism. Still, he thought to have solved it with the distinction between analytical and descriptive parts of the logical-linguistic framework and thus lost sight of it as a special problem (Friedman, 82n).

[214] Friedman's ideas have an independent correlate in the French Neo-Kantian Jean Petitot and his idea of an epistemological 'golden rule' requiring an intermediate level of objectivity-constituting concepts between mathematical models used in science on the one hand and empirical data on the other (see Petitot 1992; Stjernfelt 1992a). Petitot is even, from time to time, collaborating with Barry Smith.

[215] A considerable wave of interest is also palpable in analytical philosophy, cf. the Boghossian and Peacocke volume *New Essays on the A Priori*. I think it is fair to say, though, that these purely philosophical discussions are less interested in history of science – and, maybe for the same reason, less interested in the synthetical a priori.

[216] What does one do when trying to find a – conceivable or imaginable – counterexample to a synap sentence? One *varies* the states of affairs mentioned, performs a diagram experiment, in short.

[217] This and the following are my translations. 'Das Eidos, das *reine Wesen*, kann sich intuitiv in Erfahrungsgegebenheiten, in solchen der Wahrnehmung, Erinnerung usw., exemplifizieren, ebensogut aber *auch in bloßen Phantasiegegebennheiten*. Demgemäß können wir, ein Wesen selbst und *originär* zu erfassen, von entsprechenden Anschauungen ausgehen, *ebensowohl aber auch von nicht erfahrenden, nicht-daseinserfassenden, vielmehr 'bloß einbildenden' Anschauungen.*

Erzeugen wir in der freien Phantasie irgendwelche Raumgestaltungen, Melodien, soziale Vorgänge, u. dgl., oder fingieren wir Akte des Erfahrens, des Gefallens oder Mißfallens, des Wollens u.dgl., so können wir daran durch 'Ideation' mannigfache reine Wesen originär erschauen und ev. sogar adäquat: sei es die Wesen von räumlicher Gestalt, von Melodie, sozialem Vorgang, usw. *überhaupt*, sei es von Gestalt, Melodie, usw. des betreffenden besonderen *Typus*.' (16–17)

[218] '1. erzeugences Durchlaufen der Mannigfaltigkeit der Variationen;

2. einheitliche Verknüpfung in fortwährender Deckung;

3. herausschauende aktive Identifizierung des Kongruierenden gegenüber den Differenzen.' (EU 419)

[219] Husserl's theory is that empirical generals – in an analogy to the pure generals – form successive layers, beginning with 'concrete generals' based on repetition of the same experience ('gleiche' – as it can not, of course, be numerically identical experiences, it must be 'same' experiences, give and take some margin), while generals based on similarities only form species comprising several such concrete generals. The same two steps are repeated when it comes to partial sameness or similarity: here generals comprising abstract aspects, and, in turn, species of such aspects, are constituted. Common to all these empirical generals on different levels, however, is that they refer merely to 'real possibilities' as distinct from the 'pure possibilities' covered by the variation manifold referred to by pure concepts. It is crucial to keep in mind, here, that the distinction between empirical and pure does not – as in Kant – coincide with the distinction between material and formal. This forms the very basis of the synthetical a priori in Husserl: the pure concepts may refer to a priori structures, both in formal and material domains.

[220] 'Sie beruht auf der Abwandlung einer erfahrenen oder phantasierten Gegenständllichkeit zum beliebigen Exempel, das zugleich den Charakter des leitenden 'Vorbildes' erhält, des Ausgangsgliedes für die Erzeugung einer offen endlosen Mannigfaltigkeit von Varianten, also auf einer *Variation*. M.a.W. wir lassen uns vom Faktum als Vorbild für seine Umgestaltung in reiner Phantasie leiten' (EU, 410–11).

[221] '…das besagt nicht, daß ein *wirkliches* Fortgehen in die Unendlichkeit erforderlich wäre, eine wirkliche Erzeugung aller Varianten…' (EU, 412).

[222] 'Es kommt vielmehr darauf an, daß die Variation als Prozeß der Variantenbildung selbst eine *Beliebigkeitsgestalt* habe, daß der Prozeß im Bewußtsein beliebiger Fortbildung von Varianten vollzogen sei.' (EU, 413).

[223] 'Zu jeder Variationsmannigfaltigkeit gehört wesentlich dieses merkwürdige und so überaus wichtige Bewußtsein des 'und so weiter nach Belieben'. Dadurch allein ist gegeben, was wir eine 'offen unendliche' Mannigfaltigkeit nennen; evidenterweise ist sie dieselbe, ob wir langehin erzeugend oder beliebig Passendes heranziehend fortschreiten . . .' (EU, 413).

[224] A 'composite photograph' was – and is – the name of a technique where different variants of a type are superposed by subjecting the film to the light from each of the variants for exactly the same amount of time – then differences are levelled out, and a type appears. The technique is nowadays used to display the 'favourite woman' of a decade by the composite photograph of a series of actresses and models. See Hookway 2002 for Peirce's use of the metaphor.

[225] '. . . die logische Forderung der Individualität als eines Gegenstandes, als eines identischen Substrates für Prädikate, bzw. für objektive Wahrheiten (die unter dem Satz vom Widerspruch stehen), ist nicht erfüllt durch die Vereinzelung eines Begriffsumfanges, sondern steht unter Bedingungen der Zeit. Das sagt, daß wir für die individuelle Vereinzelung unter der Forderung einer Möglichkeit einstimmiger Ausweisung in einem *kontinuierlichen* Zusammenhang wirklicher und möglicher (an die wirklichen anschließbarer) Anschauungen stehen' (431, our italics).

[226] von Hayek's (1978) idea that the whole of our perception apparatus is geared to the extraction of abstract features of the environment and that abstractions (Peircean prescissions) are thus phenomenologically primary as compared to particulars might support Peirce's controversial idea of the direct perception of generalities.

[227] See Chap. 4.

[228] Earlier, an exception to this rule made it possible to let a broken coastline signify that nothing beyond that point had yet been explored, a sign pertaining not to geographical ontology, but rather to that of discovery.

[229] He even proposes a third interpretation: that what is synthetic is only what is theorematical in the sharpened sense of deriving from the introduction of some new axiom (and not theorematical in the sense of deriving form some well-known theorem which has not been mentioned in the problem at issue). According to Levy, these two 'degrees' of theorematicity must generally be distinguished – between, e.g. the introduction of an auxiliary line in a geometrical proof where the existence of such a line is granted beforehand by the axioms of geometry even if the line has not in any way been mentioned in context of the present problem, on the one hand, and e.g. the introduction of the Power Set Axiom in Peirce's proof for the claim that there is no greatest multitude, on the other. Even if this distinction is indeed valuable, to let the analytic/synthetic distinction follow it would be to historicize it to unrecognizability.

[230] In both special sciences and everyday reasoning, diagrams and mathematics may be *implicit*, though – just like is the case in syllogisms, in logical arguments more general, or in diagrams underlying grammar according to Peirce. There is no need that we should in every single case be able actually to draw out the diagram in question (even if it generally makes explicit what was only implicit and thus may open up a new set of possibilities of diagram experiments). Just as being the case in many symbols (covering both terms, propositions, and arguments in Peirce) – in most cases the content of those symbols may rest implicit and not made the explicit object of a diagram. In this respect, diagrams may play a role parallel to the variation procedure in the eidetic variation which, in Husserl's terminology might remain a result of 'passive synthesis' before yielding the resulting concept as an object of active intention; Husserl's intentionality distinction between passive and active may here be covered by Peirce's semiotic distinction between implicit and explicit. Such implications, however, will not be part of reasoning proper which, in Peirce, comprises only inferences which are explicitly controllable and subject to an ongoing scrutiny (cf. Chap. 11).

[231] Another such problem is: given fallibilistic apriorism, it is possible for individuals or cultures to be at fault regarding parts of a given synthetic a priori conceptual cluster – how does this idea fit with the clain that synthetic a priori laws are generally tacitly known and govern common sense actions in different domains? One answer is that the common sense world has a basic pragmatic quality which to some extent sets it apart from scientific accounts for the same phenomena, cf. for instance the mainly Aristotelian 'naive physics' sufficient for most everyday tasks but at odds with real physics. See Petitot and Smith 1990.

[232] In Stepanov 1971, cf. Sebeok 1991, 7. Around the same time, C.H. Waddington concluded his *Towards a Theoretical Biology* (Vol. III, 1972, 289) with the idea that language should become 'a paradigm for the theory of General Biology', and René Thom founded his biolinguistics in Thom 1972.
[233] The latter half of the *Kritik der Urteilskraft*, 1790.
[234] For instance René Thom (1972), or Emmeche and Hoffmeyer (1991).
[235] Hoffmeyer has recently summed up his own work in the field in *Biosemiotik* (2005).
[236] Translations from *Kritik der Urteilskraft* are my own. 'Ich behaupte aber, dass in jeder besonderen Naturlehre nur so viel *eigentliche* Wissenschaft angetroffen werden könne, als darin *Mathematik* ist . . .' (Kant 1786, A IX).
'. . . eine reine Naturlehre über bestimmte Naturdinge (Körperlehre und Seelenlehre) ist nur vermittelst der Mathematik möglich . . .' (ibid. A X)
[237] 'Dass seine Form nicht nach blossen Naturgesetzen möglich sei, d.i. solchen, welche von uns durch den Verstand allein, auf Gegenstände der Sinne angewandt, erkannt werden können; sondern dass selbst ihr empirischens Erkenntnis, ihrer Ursache und Wirkung nach, Begriffe der Vernunft voraussetze' (Kant 1974 [1790], 232).
[238] '. . . ein Ding existiert als Naturzweck, wenn es von sich selbst [. . .] Ursache und Wirkung ist' (Kant 1974 [1790], 233).
[239] That is to say, it could not be mathematized with the mathematics available to Kant. But there is no reason to accept Kant's identification of mechanical explanation with mathematical formalization of a given domain; the latter is far richer than the former. Whether more advanced mathematics might do (parts of) the job is a possibility open for an historicized version of Kant's apriorism, permitting the synthetic a priori field to develop. As we have already mentioned, Petitot has a strong argument for the possibility of qualitative dynamics to mathematically describe phenomenological aspects of non-mechanical reality aspects – 'pheno-physics'.
[240] 'dieser Begriff fürht nun notwendig auf die Idee der gesamten Natur als einer Systems nach der Regel der Zwecke, welcher Idee nun aller Mechanism der Natur nach Principien der Vernunft (. . .) untergeordnet werden muss' (op. cit. 242).
[241] 'Auch Schönheit der Natur, d.i. ihre Zusammensstimmung mit dem freien Spiele unsere Erkenntnisvermögen in der Auffassung und Beurteilung ihrer Erscheinung kann auf die Art als objektive Zweckmässigkeit der Natur in ihrem Ganzen als System, worin der Mensch ein Glied ist, betrachtet werden' (op. cit. 243).
[242] One could say that Kant admits the anthropomorphic character of the concept of organism and related notions – but at the same time he offers a naturalistic (!) explanation of it: it because we ourselves are animals and thus teleological beings that we are able to discern organic properties in other objects. Thus, the tragedy of teleology is not that it is wrong or misleading, it is, rather, that we are unable to understand its 'inside'.
[243] In this interpretation of Kant, it must be emphasized, we do not subscribe to subjectivism, nor to the idea of a prioric regularities as a privilege of some transcendental subject. They are, rather, objective conditions valid for, but not necessarily known by, any possible subject whatever, cf. Chap. 8.
[244] On Cassirer's Uexküll inspiration, see Stjernfelt (in press b).
[245] My translation. 'Sie setze an die Stelle des Zweckbegriffs den Ordnungs- und Systembegriff, und sie charakterisiert das Leben dadurch, dass sie ihm die Eigenschaft des Systems zuschreibt' (Cassirer 1957/1991, 221).
[246] Of course, the problem here deals with the idea that semiotic concepts might contribute to the very basis of biology, not, for instance, to its import into more restricted subarea like zoo-semiotics where their use is much more unproblematic, at least for higher animals with elaborate mental representations of the surroundings. Already phyto- and myco-semiotics investigating plants and fungi, take the problem much deeper into the metaphysics of biology.
[247] The semiotic interpretation of the behaviour of E. Coli presented here has been developed through discussions with the biologist Claus Emmeche.
[248] The illustrations are from Alberts et al. 1994.
[249] Such as the fine introduction in Alberts et al. 1994, 773–85.
[250] The fact that man and bacterium alike may be fooled by exactly the same chemical supports such a hypothesis. The qualia of sweetness, somehow actualized by our gustatory system, is no doubt lacking

in the bacterium and bears witness to a much more sophisticated process of tasting in higher animals – but still exploiting the same rather simple mechanisms. Categorial perception by means of 'active sites' probably plays a central role in the important transition from uni- to multicellular animals studied by Lynn Margulis. According to her endosymbiont hypothesis, already the eucaryote cell forms a symbiosis between simpler species: mitochondria, chloroplast, cell kern, and maybe the undilipodia facilitating cell movement like cilia or the flagella as in E. Coli. Further symbiosis at a higher level requires the coordinated cooperation between bacterial colonies able to communicate, and the interesting intermediate forms behaving like single cells in some phases and environments only to change to more coordinated behaviour in other environments are possible only through elaborate intercell communication. During unfavorable environment constraints, e.g. the outer bacteria of the colony are able to secrete chemical signs in order to communicate the danger to the more inner layers, causing them to take defensive action and change into a more intert, spore-like form. Simultaneously, the whole colony changes shape forming a sort of fruit stalk able to let these spores spread to faraway, hopefully more favorable locations. To that extent, bacterial colonies share certain of the coordinated behaviours in much more sophisticated multicellular organisms where intercell sign use may be harder to observe directly.

[251] Even if biologists like de Loof and Broeck (1995) maintain a short definition not far from what follows here: life as the total sum of all acts of communication performed by a compartment (an organized, bounded whole) from the lowest to the highest level at a given moment *t*.

[252] Rosen (1989) argues from Gödel's incompleteness theorem for the introduction of semantic notions in biology. Gödel's theorem may be expressed as stating that for a given, sufficiently rich system, syntactic truth (that is, theorems provable in the system) is but a subset of semantic truth (true theorems not provable in the system). Rosen calls 'complex systems' material systems which embody this property, in opposition to mechanic systems, and sees living beings as prototypical examples of such complex systems. As is evident, this argument is structurally analogous to the anti-functionalist Gödel argument in AI (from Lucas to Penrose).

[253] A priori analyses of various domains' conceptual structures still often have an ill reputation due to empiricist prejudices or to the widespread myth that a priori analyses necessarily implies subjectivism; see Chap. 8.

[254] We also find this crucial idea in Kauffman as a property in 'autonomous agents', e.g. his (2000).

[255] In Stjernfelt 1992b, I proposed this property as a prerequisite to all formation of signs. The notion of 'categorical perception' is taken from experimental phonetics (Liberman, Studdert-Kennedy etc.) where it refers to the ability of language speakers to immediately identify and categorize different variants of spoken phonemes, cf. Petitot 1985a.The fact that the set of macromolecules possessing the same active site can probably not be mapped by any effective procedure supports the Robert Rosen-like idea that the semantics inherent in this primitive categorization is subjected to undecidability limitations. The set of macromolecules triggering a specific reaction in the organism is (just like the whole of organic chemistry) potentially open-ended and thus can not be defined extensionally. I did not know, in 1992, that Giorgio Prodi had some years before put forward almost exactly the same proposal (Prodi 1988).

[256] This crucial difference is noted already by Pattee 1977 ('our *conceptual* distinction between a molecule and its message or interpretation depends on the *physical* distinction between strong and weak forces', 265) and more recently by Igambardiev's (1992) distinction between information and energetic levels (128).

[257] This crucial size differences between the sign and its signification probably entails that semiotic systems must constitute a part of what Bak (1997) calls self-organized critical systems. The idea here is that a long range of different systems from physics and geology over biology and well into the human and social sciences (sandpiles, earthquakes, traffic jams etc.) display a characteristical relation between the size of events (avalanches in a sand pile, for instance) and their frequency, so that a power law holds between event size and event frequency. Bak argues that brains must be at the self-organized critical limit by a negative argument: if brains were subcritical, too few events would influence their behavior, rendering them insensible; if they were supercritical, too many events would influence them and make them chaotic (Bak 1997, 176–77). This brain argument may be generalized to all semiotic systems: they must react with insignificant changes to most small impulses but with significant changes to few small impulses; the latter class now constitute signs of various 'sizes' dependent on the size of events they trigger.

[258] I borrow this use of the term from Merrell (1997).
[259] These ideas resume deliberations in Thom 1975, 128f.
[260] And even science – according to René Thom's fertile definition of science as the nesting of actual events into a space of virtuality – finds its roots here.
[261] As discussed in Chap. 4, the concept of *schema* goes back to Kant's first Critique (as a notion for the possible meeting point for concepts and intuitions) and plays a central role for realist semiotics from Cassirer to cognitive semantics. Santa Fe complexity theory school has recently realized the necessity of the presence of *schemata* in complex, adaptive systems (Gell-Mann 1995; Martin 1994) and found coarse graining and perception of regularities among their chief properties.
[262] As to consciousness involving qualia, experienced selfhood, etc., we have no behavioral criteria for determining its occurrence during evolution, but a first hypothesis might be that it appears as an integrating factor in organisms with central nervous systems, facilitating more efficient perception-action cycles.
[263] Peirce's pan-psychism of course implies that even inanimate matter would possess some kind of dull 'feeling'. A recent argument for the existence of proto-consciousness in living beings has been put forward by Penrose (1990, 1994): the remarkable fact that most life reacts similarly to the same basic types of anesthetic chemicals. The question of when consciousness shows up in the course of evolution, however interesting it is, is not decisive to the discussion of the relevance of semiotics for biology, so we shall leave it in this context.
[264] Thus, the code duality (analogue/digital, cell architecture/genome) discussed by my colleagues in the Copenhagen biosemiotic school is probably no logical necessity for life as such, even if it seems to be for the reproduction of it.
[265] Elsewhere, Peirce restricts himself to claiming that the universe is representable (because of appearing in the three categories) – this would be a more modest, Scotean claim that symbols and arguments may chart the universe because they mirror structures in it and thus have a *fundamentum in re* in such proto-semiotic *realitates*, rather than claiming that the universe *consists* of symbols.
[266] A further support for this thesis comes from Santa Fe theoretical biologist Stuart Kauffman and his theory of 'autonomous agents'. The idea here is that they derive from 'autocatalytic sets' of chemical compounds giving rise to chemical cycles. When they become able to stabilize and reproduce, we get 'autonomous agents' as a general notion for (not only biological) organisms. Furthermore, by the introduction of Per Bak's 'self-organized criticality' into this framework, Kauffman makes the guess that the biosphere as a whole – maybe even the whole universe – evolves at the maximum speed possible into what he calls the 'adjacent possible', defined by consisting of yet unexisting chemical compounds at one-reaction-distance from existing compounds. If this radical version of a naturalized Enlightenment optimism is really correct, it would probably imply that biological evolution takes the shortest way possible in the evolution of still more intelligent beings. Kauffman specifies in this context an interesting assumption in our context: even if the biosphere (and maybe cosmos as a whole, he speculates) thus must be supercritical with respect to the autocatalytic power of its chemical networks, the single autonomous agent (organism, cell, or whatever) must necessarily be subcritical inside its skin – if not, autocatalytic bursts of still new compounds (following Bak's power law distribution of events at the critical limit) would rapidly destroy it. This hypothesis – speculative as it is – would imply interesting semiotic consequences; the Kantian notion of the finitude of the subject (here, the agent) in relation to the thing itself (here, the biosphere) would have a naturalistic counterpart, as would the Hegelian idea of a 'ruse of reason' where the individual agent unknowingly contributes to the common best. Semiotically speaking, this asymmetry between the agent and its environment would arithmetically necessitate categorization (by a finite mind of an infinity of possibilities) and hence schematic signs as general and underdetermined means of coping with an overdetermined environment, cf. our remarks on the economy of sign use above.
[267] See, for instance, Emmeche et al. 1997 and 2000.
[268] Thom 1990, 43: 'Efficient causality and final causality may be subsumed under the heading of formal causality. It should be observed that in Aristotelian theory final causality is in fact always conditional. An 'act,' described as a process by an analytical model, may be stopped short if some impediment intervenes during its course. So the act may fail because of interaction with an accidental factor. When so considered, Aristotelian finality is perfectly compatible with our modern view of (local) determinism.'

[269] The fact that biosemiotic vocabulary is indispensable also for higher ontological levels does not in any way imply that higher domains may be *reduced* to biology. von Uexküll's dark *Staatsbiologie* – in fact no less than a fascist biologist doctrine of state – should be sufficient as a warning against such shortcircuits.

[270] It might be argued that this privilege of biology might be a coincidence due to our biological being. Kauffman (2000), for instance, toys with the possibility that his equivalent concept to biosemiotic organisms, 'coevolving autonomous agents' may be applied on a cosmological scale (such as the organization of galaxies) as well as on a microphysical scale (the organization of elementary particles and below). Of course, there may be several levels of organization between the fermion-boson level and Planck scale; we do not know and might never come to know.

[271] A lineage involving scholars like Goethe, Geoffroy Saint-Hilaire, von Baer, d'Arcy Thompson, Spemann, Driesch, Waddington, Brian Goodwin, René Thom, Robert Rosen, Stuart Kauffman, inter alia.

[272] René Thom has attempted to rescue the term vitalism, claiming that it is reductionism which is a dogmatist view, trying to repudiate the reality of the organism, evident for all to see. His insistence on the term, however, does not include the usual assumptions of the existence of extra-physical forces or the like, so his idea of saving the term, however sympathetical, should probably be left behind. Nevertheless, in this connection he makes a surprising attack on Neo-Darwinism, hinted at above: it is Neo-Darwinism which is the metaphysical theory, not at all so empiricist as it often sees itself, and what is more, in contrast to the epigenetic tradition's emphasis on ontogenesis, the Neo-Darwinist emphasis on phylogenesis gives it a *holism* on the species level, contrary to its own reductionist claims, in so far as it is the genome of a species which is considered the main object of research. (Thom 1990, 600)

[273] 'Umwelt' is often quoted as von Uexküll's own terminological neologism; in fact, it was already used by Goethe and originates from the Danish-German Enlightenment-Early Romanticism poet Jens Baggesen who coined it in an ode to Napoleon in the year 1800.

[274] cf. von Uexküll 1973, 220.

[275] The inclusion of von Uexküll in the current repertoire of semiotics is thus largely due to Sebeok. But despite his exile status in relation to biology departments, there is a series of other local 'rediscoveries'. Most notably, probably, his status as ancestor to present-day ethology via Lorenz and Tinbergen. But also his role in the biological parts of Cassirer's theory of knowledge should be mentioned (Cassirer (1991) quotes with approval the 'nicht-stoffliche Ordnung' of the *Theoretische Biologie* as a central concept for an anti-vitalist biology – see Stjernfelt (in press a), in René Thom's catastrophe theory (1975, 1990) with his insistence of the centripetal vs. centrifugal construction principles of machines and organisms, respectively (despite the non-mechanist teleology inherent in both), and, most elaborated, in Merleau-Ponty of around 1957–60: in his lecture notes for that period (1995), dealing with the concept of nature, we find a large chapter on the phenomenological importance of the *Umwelt* concept: its introduction of 'plans naturels' making possible the 'commencement de culture' and its gradual evolution, being 'moins en moins orienté vers un but et, de plus en plus, interprétation de symboles.' (231) – see Chap. 10. Finally, the 'embodiment' wave of recent cognitive studies also displays a growing von Uexküll interest.

[276] The illustration is from von Uexküll 1973, 105. Such circles are most often, in von Uexküll, connected to individual purposes of the organism: drinking, preying, eating, mating, fighting, etc. Collective functional circles of more complex kind, relevant to a flock or herd Umwelt must be presupposed in the semiotically interesting cases of social animals; we shall not go into this issue here.

[277] A Kantian idea, to be sure, prefigured in Kant's idea of causal circles in 'Kritik der teleologischen Urteilskraft' – see the previous chapter.

[278] This goes for animals, even down to unicellular animals, but not for plants in so far they are generally seen as possessing a truncated *Umwelt* all right (von Uexküll calls it a *Wohnhülle*, a dwelling place), but no functional circle because of their stationary life and lack of nervous system, making them unable to move and hence unable to perform *Wirken*. This distinction seems somewhat problematic (heliotropes or carnivorous plants e.g. seem to form spectacular exceptions and not the only ones) but we shall leave further discussion of it here.

[279] An example given by several occasions, e.g. von Uexküll 1973, 1982, 56–57.

[280] Prominent figures in a Neo-Kantian semiotic family tree would include Peirce, Cassirer, and in our days Jean Petitot and the American cognitive semantics traditions; the differentia specifica will be the assignment of a central role to schemas or schema-like concepts in the respective semiotic theory. For a discussion of Neo-Kantian semiotics and Peirce, Cassirer, and Heidegger, see Stjernfelt 2000. Taken in a more wide sense, of course, Neo-Kantianism as meaning merely semiotic reflections inspired by Kant, it would include the doctrines of Nietzsche, Heidegger, brands of *Lebensphilosophie* as well as evolutionary epistemology.

[281] The German quote can be found in von Uexküll 1973, 9.

[282] This attempt at the organism's auto-constitution of space, however, is mysterious: a single segment of the (in itself spatially defined) functional circle is taken to be constitutive of space . . . So the same problem remains here: if the organism is taken to be a subject in the constitutive sense of the word, then it is posited anterior to time and space, and its body remains part of the empirical world just like any other object – if the organism, on the other hand, is taken to be the empirical animal, then its bodily sensations are seminal to the construction of its surrounding space-time – but not of space-time as such.

[283] The same is the case in von Uexküll's famous tick example. The tick *Umwelt* as it is described here, has little to do with the simplicity we must suppose pertains to the real tick *Umwelt* but is ripe with concepts from the scientist's *Umwelt*: the branch, the fall, the mammal etc.; and the example lives from our intuitive grasping of a mapping from the scientist's *Umwelt* to the tick *Umwelt* and back. But this mapping presupposes a space in which both these *Umwelten* partially coincide, namely an objective space from which scientist and tick *Umwelt* select each their sets of relevant signs. If no such space is supposed, the mapping between scientist and tick *Umwelt* can not be presupposed, and the *Umwelt* hypothesis will fall (consequently, the idea of an ontological 'pluriverse' of *Umwelten* is contradictory).

[284] Merleau-Ponty finds a development in von Uexküll from an early Kantianism in which the *Umgebung* behind the various *Umwelten* plays the role as Ding-an-sich and to a later Schellingianism, in which Nature becomes a name for the integration of all *Umwelten*. (Merleau-Ponty 1995, 232)

[285] Translations from *Theoretische Biologie* are mine. 'Das Material, aus dem sich eine fremde Umwelt aufbaut, besteht unter allen Umständen aus unserer objektivierten Qualität, weil uns andere Qualitäten gar nicht zugänglich sind. Das einzige Unterschied zu unserer Umwelt besteht darin, dass sie geringer an Zahl sind. [. . .] Ein Ort, der für uns mehr nach links liegt als ein anderer, liegt auch in der fremden Umwelt, wenn beide Orte als Merkmale in ihr vorhanden sind, ebenfalls weiter nach links, auch wenn die Anzahl von örtlichen Merkmalen die sie voneinander trennen, geringer ist als in unserer Welt.' (von Uexküll 1973, 104)

[286] An analogous argument as to the possibility of inter-species *Umwelt* mappings due to 'general-purpose sensory and effector mechanisms' can be found in Cariani 1998, 252–53. The possibility of mappings within and between different functional circles in the same species must is in itself count as a core contribution to the semiotic generativity in higher animals and humans especially: they constitute metaphors, blendings, metonymies etc. and thus form part of the biological basis of cognitive semantics' 'literary mind', cf. Turner 1996.

[287] This does not, of course, rule out the issue of the experienced qualia consciousness and its 'inside'. It is perfectly possible, even highly probabale, that higher animals (that is, animals possessing a central nervous system) entertain inner states of this kind, and the question whether lower animals or even plants share related properties remains undecided. Yet, this question must be kept apart from the functional *Umwelt* definition, because the existence of a functional *Umwelt* in a species does not imply the presence of qualia consciousness. As to research into the qualia consciousness issue, however, we have not, at present, any scientifically valid method to ascertain nor investigate such states.

[288] Thus it will no longer be the case, as von Uexküll quotes the radical conservative sociologist Werner Sombart: 'No 'forest' exists as an objectively prescribed environment. There exists only a forester-, hunter-, botanist-, walker, nature-enthusiast-, wood gatherer-, berry-picker- and a fairytale-forest in which Hansel and Gretel lose their way.' (29). The fact that the forest is defined exactly with reference to these and related groups (and not e.g. football players, state secretaries, unemployed, Irishmen, or other collective subjects) is an objective fact which we – so Sombart – are able to registrate and which thus forms part of the characterization of the forest, not its dissolution. The Nietzschean skepticism in this quote is related to other radical conservative and irrationalist vitalist features in von Uexküll, for instance his pessimism toward civilization, not to mention his Nazist leanings.

[289] 'Sobald die Umrisse der Körper als Merkmale auftreten, ändert sich das Bild der Merkwelt von Grund aus, denn nun beginnt das Nebeneinander im Raum eine ausschlaggebende Rolle zu spielen. [. . .] Erst wenn räumliche Unterschiede im Merkmal selbst auftreten, kann man von einer höheren Stufe der Merkwelt sprechen.' (189)

'Voraussetzung für die Existenz von Gegenständen in der Merkwelt eines Tieres bildet die Fähigkeit des Tieres, eigene Funktionsregel für sein Handeln zu bilden.' (191)

[290] 'Bref, c'est la thème de mélodie, beaucoup plus que l'idée d'une nature-sujet ou d'une chose supra-sensible qui exprime le mieux l'intuition de l'animal selon Uexküll. Le sujet animal est sa réalisation, trans-spatiale et trans-temporelle. Le thème de la mélodie animale n'est pas en dehors de sa réalisation manifeste, c'est un thématisme variable que l'animal ne cherche pas à réaliser par la copie d'un modèle, mais qui hante ses réalisations particulières, sans que ces thèmes soient le but de l'organisme.' (Merleau-Ponty 1995, 233)

[291] Probably the ability to distinguish pure from false tones (relative to a given scale) is innate in humans: already 4-months old babies express disgust when faced with false tones (Wilson 1998, 166).

[292] The place of these categorical perception examples in functional circles even anticipates a central issue in the recent decades' category research: Eleanor Rosch's observation that there is a 'basic level' of categorization relevant to corresponding basic motor routines involving those objects. Functional circle categorical perception is defined with respect to the *Wirken* and hence on a motor routine basic level.

[293] The categorical perception tradition in cognitive psychology has its roots in phonetics but has since then spread onto other senses (e.g. color perception) and species (higher animals categorical perception). In Stevan Harnad's central 1987 anthology around the subject, most papers still focus upon perception issues, but one seminal paper (Medin and Barsalou) undertakes a detailed comparison of categorical perception research with semantic categorization and finds much more similarities than expected. Many brands of structuralism tacitly assume the same parallel (e.g. in Hjelmslev's thoroughgoing formal parallelism between expression and content). The same continuity between perceptive and semantic categorization is often tacitly assumed in the cognitive linguistics and cognitive semantics traditions in the recent decades. Categorical perception in the Uexküllian *Umwelt* connection is a strong argument for this continuity to the extent that the perceptual categorizations in primitive animals are directly linked to semantic categorizations (namely of the corresponding *Wirkmale*, specific actions). Thus, we here immediately use the categorical perception concept about animal categorization. Generally, no sharp distinction seems to delimit perceptual and semantical categories which rather form parts of a continuum.

[294] Cf. Günter Ehret and Patricia K. Kuhl in chaps. 10 and 12, respectively, in Harnad 1987.

[295] It is striking how much Harnad's model mirrors features in Peirce's classical account for the generality of signs, where token and type denotes the particularity and generality of the sign itself, while the type due to its typicality in turn may be used as a symbol to signify some other general object.

[296] von Uexküll even, in a *lebensphilosophical* gesture typical for his time, makes the specificity of the human *Umwelt* a tragic problem for our species. It is strange that he can, on the one hand, claim that his theory of meaning 'culminates in explicating this connection' (the relation between a human need and the corresponding utensilia, such as coffee and coffee-cup), at the same time as he on the other hand can state that 'It should suffice to point out that with all our utensils we have built bridges between ourselves and nature. In so doing, we have come no closer to nature; in fact we have removed ourselves from her. [. . .] In the city we are exclusively surrounded by artifacts [. . .] The much-lauded technology has lost all feeling for nature: Indeed, it presumes to solve the most profound questions of life – such as the relationship between man and god-like nature – with totally inadequate mathematics.' (66–67) It is by no means an evident corollary to his theory that man should possess this tragic distance to nature; quite on the contrary. If Uexküll's concepts are removed from his creationism and *lebensphilosophische* pessimism of culture, it may be integrated into evolutionism such that man's especially malleable *Umwelt* becomes a consequence of the ever more complex *Umwelten* of higher animals, cf. Jesper Hoffmeyer's idea of a gradient of growing 'semiotic freedom' during evolution.

[297] Even if the *Bedeutungslehre* is sceptical against neutral objects in animal functional circles, it admits the possibility of distinguishing properties in the object with greater or lesser relevance for the animal. This, Uexküll argues, is what gives rise to the Aristotelian essentia/accidentia distinction, referring to a rank-ordering of properties by importance according to the functional circle in question

(28). This distinction may receive a formal interpretation in the light of categorical perception, implying the crucial strengthening of cross-boundary distinctions (relevant for the essentia) as well as the weakening of the intra-boundary properties (which are hence accidental, according to the categorization in question).

[298] 'Bei diesem Vorgang ist die Melodie der Richtungszeichen das Gestaltende, die uns aber nur in der 'Gestalt' zum Bewusstsein kommt. Die gestaltende Melodie nannte Kant *ein 'Schema'*, und die verborgene Kunst des Gestaltens in unserem Gemüt nannte er den 'Schematismus'.' (118)

[299] Here, we disagree with Chebanov's otherwise exciting paper (1999) and its claims that the concept of *Umwelt* does not entail semiosis. A similar discussion which we mentioned earlier was raised at the conference World and Mind in Copenhagen May 2005 where the British philosopher Galen Strawson argued against the existence of non-conscious intentionality from a so-called 'stopping problem'. If we take the causal chain involved in a perception or other intention, then the intention selects its object in that chain. Not so for non-conscious processes, Strawson argued: here the object intented can not be distinguished from other segments of the chain such as the light falling upon the object or my retina or my visual cortex. Obviously, it would be absurd to say that I see my neurons working in my visual cortex. When consciousness is absent as stopping criterion, so Strawson, there is no criterion for determining the object of intention and hence not for intentionality at all. Strawson's argument begs the question, however, in presupposing intention to be translatable into a causal chain which is, of course, not intentional. If intention, in an Uexküllian manner, is defined by the functional circle, any further segment of that circle is functionally intented. Even if microscopically continuous, the perception and action signs cut up the circle in characteristic schematic segments, due to granularity, and in the circle, a segment may serve as a sign for a later segment. Butylic acid serves as a sign for falling, serving as a sign for warm skin (or for crawling back on a branch), serving as a sign for biting . . . Thus, the stopping problem is posed in a wrong way as pertaining to linear rays of intention. It overlooks that such lines basically constitute segments of the functional meaning circle – what is intended is intended with respect to its virtual inclusion in this circle.

[300] Here, we recognize two different sets of semiotic oppositions: contrary and contradictory relations (in e.g. Greimas) vs. marked/unmarked (Jakobson), the former privileging boundaries over prototypes, the latter prototypes over boundaries.

[301] Niklas Luhmann's autopoiesis theory seems to be an attempt at generalizing this insight in sociology, seeing societies as consisting of systems with each their defined *Umwelten*.

[302] Thus, the centrality of some version of CP in any functional circle definition points to it as being a formal a priori prerequisite of biosemiotics – adding to the arguments connecting biosemiotics to formal ontology in Chap. 9.

[303] It should be noted that this recursive definition of symbols does not correspond to Peirce's original account. In Deacon, indices are made out of icons plus icons holding between icons, and symbols, in turn, out of indices plus indices holding between indices. In Peirce, however, the three are irreducible, and the icon-index structure rather forms the internal anatomy of the symbol without being sufficient for its compositional definition.

[304] So as for instance that the increase in brain size necessitating more protein made early man turn to a more carnivorous behavior. While the *Männerbund* went hunting, the mothers nursing the children were waiting for protein to be brought home. The hunting man's gene pool was threatened, however, by his woman's possible unfaithfulness during hunt, and she and her child was correlatively threatened by protein undernourishment if he did not return. This situation calls for stabilization by marriage which in turn requires stable institutions guaranteed by language – in turn calling for (further) development of symbol use. This hypothesis is interesting indeed, but it includes many specific issues and premisses which are not our concern in this context.

[305] Baldwin was primarily known as one of the pioneers of child psychology. Incidentally, he was also one of Peirce's acquaintances and invited Peirce to contribute his many entries to Baldwin's influential *Dictionary of Philosophy and Psychology* (1901). Sometimes, Peirce expressed ideas not far from Baldwinian evolution: 'For we must remember that the organism has not made the mind, but is only adapted to it. It has become adapted to it by an evolutionary process so that it is not far from correct to say that it is the mind that has made the organism.' ('Abstract of 8 lectures', undated, NEM IV, 141)

[306] After *The Symbolic Species*, Deacon has refined the Baldwinian hypothesis assuming two phases often interchanging in complex evolution, involving the 'masking' and 'unmasking' of selection pressure on a specific gene (Deacon 2003). A gene may be masked by a learned behaviour which makes the effect of that gene superfluous. Deacon's example is early primates being introduced to fruit in the diet with the result that the gene governing Vitamin C synthesis in early primates (as in most other mammals) was masked against selection with the result that the gene degenerated. Now, primates were forced to stick to a Vitamin C rich diet in order to survive, a fact which unmasked a completely different gene, namely in the set of genes governing the anatomy of the retina. Now, colour vision all of a sudden became an advantage in order to get Vitamin C from red, yellow, and blue fruit on the green foliage background, and via two duplications of a retinal gene, primates became able to see colours. Genetic degeneration facilitated by masking is thus an important factor in evolution because it loosens genetic control over certain phenotypical features and allows self-organizing possibilities in the organism to experiment with new properties. Thus, Deacon conjectures that early symbol use in hominids masked genetically constrained stereotypical communication and thus allowed for the brain to self-organize in new ways giving rise to the growth of interconnectedness of the human brain – in turn making more complicated symbol use possible and thus unmasking other cerebral genes and exposing them to Baldwinian selection pressure from linguistically skilled group members.

[307] It may even cover lower animals as well, cf. E. coli's ability to swim upstream in a saccharine gradient which in Peirce's terms must be classified as symbolic with respect to its *esse in futuro* – see Chap. 9. See Harnad 1987 for a long range of investigations of categorical perception tied to behavior – hence forming symbols – in many different species. The issue of the natural extension of Peirce's symbol concept is complex – as already discussed in Chap. 2. There is no agreement upon it, and Peirce's own idea on this point seems to find symbols even in inorganic nature – an idea nowadays supported by John Deely. Thomas Sebeok tended to make semiotics co-extensive with biology, hence only finding symbols in biological nature. Deacon, in turn, heightens the distinctive limit to make symbols a human prerequisite. On this point, we support Sebeok's middle way.

[308] A further problem is that Deacon's reconstruction of the icon-index-symbol triad as referred above makes it compositional, so that higher sign types are presumed reducible to combinations of the lower. But if pure icons – so Peirce – are mere possibilities, taken by themselves, then the actuality dimension of indices can not be created by the composition of ever so many icons; correlatively, the general, future dimension of symbols can not be the result of a composition of ever so many actual, indexical moments. Peirce's description faces the opposite direction: symbols are wholes, and icons and indices are moments of the symbol's anatomy.

[309] Deacon refers at length to the famous *Kanzi* case where a young bonobo on its mother's back learned the symbolic language which scientists were trying to teach its mother. There is little doubt that *Kanzi* is a symbol user, both in Deacon's and (less surprisingly) in Peirce's sense of the word.

[310] See, e.g. 'Minute Logic' (1902):

'Reasoning, properly speaking, cannot be unconsciously performed. A mental operation may be precisely like reasoning in every other respect except that it is performed unconsciously. But that one circumstance will deprive it of the title of reasoning. For reasoning is deliberate, voluntary, critical, controlled, all of which it can only be if it is done consciously. An unconscious act is involuntary: an involuntary act is not subject to control; an uncontrollable act is not deliberate nor subject to criticism in the sense of approval or blame. A performance which cannot be called good or bad differs most essentially from reasoning.' (2.182)

[311] Both must, furthermore, be distinguished from *induction* dealing with a series of related, empirical phenomena and proposing a probable law uniting them. Induction is often by empiricists confused with abstraction, but like Husserl (2nd LU), Peirce keeps these problems apart, and neither of the abstraction types have anything to do with extracting regularities from a set of examples.

[312] The central quote is the following:

'The terms 'precision' and 'abstraction,' which were formerly applied to every kind of separation, are now limited, not merely to mental separation, but to that which arises from *attention to* one element and *neglect of* the other. Exclusive attention consists in a definite concepton or *supposition* of one part of an object, without any supposition of the other. *Abstraction* or precision ought to be

carefully distinguished from two other modes of mental separation, which may be termed *discrimination* and *dissociation*. Discrimination has to do merely with the senses of the terms, and only draws a distinction in meaning. Dissociation is that separation which, in the absence of a constant association, is permitted by the law of association of images. It is the consciousness of one thing, without the necessary simultaneous consciousness of the other. Abstraction or precision, therefore, supposes a greater separation than discrimination, but a less separation than dissociation. Thus I can discriminate red from blue, space from color, and color from space, but not red from color. I can prescind red from blue, and space from color (as is manifest from the fact that I actually believe there is an uncolored space between my face and the wall); but I cannot prescind color from space, nor red from color. I can dissociate red from blue, but not space from color, color from space, nor red from color.

Precision is not a reciprocal process. It is frequently the case, that, while A cannot be prescinded from B, B can be prescinded from A. (…)'

[313] We may note that in this argumentation, the three separation modes are tied to three different modes of presentation: imagining, supposing, and representating, respectively.

[314] Peirce's definition of the distinction abstraction types connects them intimately to a part-whole dependency calculus. This idea is exactly parallel to the connection between Husserl's 2nd and 3rd LU with the anti-empiricist abstraction theory of the 2nd (abstraction is not inductive generalization, abstraction is a special idealizing focusing act related to an object's properties) and the mereology of the 3rd (the properties thus grasped should be seen as different parts and moments of the object, and a calculus is possible to map these parts' internal relationships). This connection between abstraction and mereology is a highly original idea crucial for the possibility of a realist understanding of the cognition of abstract objects.

[315] 'In general, prescission is always accomplished by imagining ourselves in situations in which certain elements of fact cannot be ascertained.' ('Supplement', 1893, 2.428) Prescission thus is Peirce's version of Duns Scotus' famed 'formal distinction'; it refers to a distinction made by the mind, but with a *fundamentum in re.* As to the color-extension issue, Husserl and Peirce analyze it differently. To Husserl, they are mutually dependent, to Peirce, colour is dependent upon extension (cf. Chap. 6–8); we tend to support the latter.

[316] A simple example is the train of thought as follows: *a white particular thing* - (P) - *white things as such* - (A) - *whiteness*, with P for prescission and A for abstraction. A more complicated example is hinted at in set theory and may be reconstructed as follows: *elements* - (P) - *belonging together* - (A) - *a set* - (P) - *bigger/smaller* - (A) - *multitude* - (P) - *relation to other multitudes* - (A) - *cardinal number*… This example is reconstructed from 'Consequences of Critical Common-Sensism' (1905), 5.534. The unlimited character of this abstraction process does not entail it is infinite.

[317] So as for instance ('Consequences of Critical Common-Sensism' (1905), 5.534). The famous quote stems from the third interlude in Molière's last play, *Le malade imaginaire* which introduces a grotesque ceremony of doctors dancing and singing medical latin. Here, a medicine student answers a doctor's question as follows:

'BACHELIERUS/ Mihi a docto doctere/ Domandatur causam et rationem quare/ Opium facit dormire./ A quoi respondeo,/ Quia est in eo/ Virtus dormitiva,/ Cujus est natura/ Sensus assoupire. CHORUS/ Bene, bene, bene, bene respondere./ Dignus, dignus est entrare/ In nostro docto corpore.' (Molière, 660); in my translation: 'Bachelor/ Me the learned doctor/ asks about the cause and reason why/ Opium puts to sleep./ To this I answer/ That there is in it/ A sleep-inducing power/ Whose nature it is/ To weaken the senses.// CHORUS/ Good, good, good, good answer./ Honorable, honorable is it to enter/ into our learned society.'

[318] Peirce's theory of hypostatic abstraction thus forms a strong argument against the current fad in rhetorics where it is claimed that abstract noun use is just a showoff strategy trying to impress the reader with difficult wording (the opium argument of our time), while texts which express 'the same' in more concrete terms are praised as more honest and easier to read. In a Danish context, the two writing styles are even hastily connected to males and females, respectively, rhetoricians taking the party of the latter. If this claim were true, it would do the feminist cause a questionable service, as abstractions are indeed necessary for thought to occur.

[319] Roberts, 64.

[320] Peirce has a primitive mereological intuition in so far he refuses to distinguish between parthood and elementhood (as in set theory) and sees those two as shadings of one and the same basic relation.

[321] Cf. e.g. '. . . I do not think that we need have any further scruple in admitting that abstractions may be real, - indeed, a good deal less open to suspicion of fiction than are the primary substances.' ('Pragmatism Lectures' (1903), Peirce 1997, 136.) This idea forms a very important phenomenological principle in Peirce: the objects to which we have direct access are neither completely abstract nor concrete; they are at different intermediate levels. Thus, both abstract objects and the concrete object are constructions reached by abstraction, and the ladder of levels is virtually bottomless; we have no guarantee that it terminates 'downwards' in some elementary, atom-like entities. As collections are also abstractions, this consideration also goes for scalar properties. This idea is basically a mereological idea (whole-part relations are pertinent on all levels of observation or reflection) and fits nicely with Peirce's proto-mereological refusal of distinguishing element-of and part-of relations (like it was later formalized in Lesniewski).

[322] T.L. Short (1983) traces roots of this idea in Peirce's thought to the famous 'How to Make our Ideas Clear' paper from 1878 where Short emphasizes the crucial idea that 'we *can* use ideas that are *less* clear to make other ideas *more* clear' (290). Thus, the fact that the higher, more abstract terms may be clearer than their concrete basis is a crucial insight in order to avoid infinite regresses and appreciate the role played by abstraction in Peirce.

[323] Helmut Pape: '. . . the 'abstract in concrete form' brought about by a 'realistic hypostatization of relations' is a deductively valid form of reasoning which he at other places calls 'hypostatic abstraction' and which is now called class abstraction.' (Pape 1997, 171) Pape explains in a note: 'It is obviously deductively valid to conclude that, if there is a red rose, the class of red things has at least one member, namely, this rose.' (182n). In the very same volume we read T.L. Short: '. . . neither is the inference to it logically necessary. Rather, that inference could be deductively valid only with the additional, logically contingent premiss that the regularity in question has an explanation (. . .) Absent that assumption, the inference is not deductive but an extreme case of what Peirce called 'abduction' . . .' (297).

[324] This should, of course, be taken to refer to reflective self-consciousness. Pre-reflective self-consciousness (cf. Zahavi 2000) as a moment of any conscious experience is presupposed by reflective self-consciousness, and may probably, unlike the latter, be found in large parts of the animal kingdom and maybe even in lower organisms.

[325] The quote talks about 'races', but we should not take this as an indication that any idea of 'racism' could be found in Peirce's thought; he is merely using the word as coextensive with 'culture' or 'social or geographical group of people', such as was a commonplace at the time.

[326] The Husserlian idea of a pure grammar – to some extent shared by Peirce – may make the Chomsky-Deacon conflict around grammatical inneism irrelevant. If there are indeed a priori rules for grammar, then we should expect evolution (biological as well as linguistic evolution) to conform to them in a gradual approximation, making the riddle of possible innate chunks of universal grammar easily understandable, because no empirical selection pressure will be needed for their articulation. This would correspond to the fact that we have learned elementary arithmetics and performs that easily without anybody wondering about the specific selection pressures giving rise to an 'arithmetic module' in the brain. Terrence Deacon seems on the same track when toying with an 'extraterrestrial' grammar valid for any speaking subjects whatever . . .

[327] The issue of the 'semiotic missing link' is currently the object of intense investigation. A famous suggestion parallel to Deacon's is Michael Tomasello's idea that what separates man from other higher primates is the ability of *shared attention* during learning. Mother and child, for instance, may direct their attention to one and the same object, initiated by the child understanding the mothers gaze direction or pointing gesture, while young apes would rather look at the pointing finger than at the object indicated. There is not, however, any common scientific language or set of criteria making us able to decide between such proposals – or even to decide whether they are mutually exclusive, reconcilable – or even maybe expressing related or identical ideas in different vocabularies. It would not be strange, for instance, if shared attention necessarily involved hypostatic abstraction – the explicit control necessary for the latter being provided by the common discursive field opened by the former. Here, further investigation, both empirical and conceptual, is necessary.

[328] A problem here, though, is that there is hardly any definite boundary separating metaphorical projection from concept extension (both refer to the application of a concept on hitherto uncovered domains). The difference lies in the domains rather than in the extension process.
[329] The grasp of this problem depends on the understanding of 'a priori reasoning'. If it is to be taken in the Kantian tradition where a priori refers to validity prior to any empirical knowledge as well as to evident givenness for any knowing subject whatsoever, it is clear that the CS tradition is right in refusing to indulge in a priori reasoning, cf. Chap. 8.
[330] See Chap. 10.
[331] Doing so, Merleau-Ponty follows the example set by Ernst Cassirer – philosophizing by sifting the results of the special sciences - rather than that of Heidegger and his irrationalist claim that the 'Wissenschaft denkt nicht'. There are more references to Cassirer than to Heidegger in *Phénoménologie de la perception.*
[332] Despite the idea that the *Umwelt* is a subjective construction on part of the organism, von Uexküll's position is not necessarily subjectivist in the sense that it is a construction taking place in the organism's consciousness. von Uexküll's methodology for *Umweltforschung* was, in fact, a sort of behaviorism: an animal's *Umwelt* should be investigated by observation of its anatomy and its interaction with its surroundings, rather than through any sort of empathy.
[333] Merleau-Ponty thus does not hesitate to solve von Uexküll's problem with the possibility of 'neutral objects': higher animals with central nervous systems do, in fact, access such objects, cf. below.
[334] This idea anticipates René Thom's idea that environment objects are prototypically conceived of as a sort of generalized 'animals' so that nouns in general refer to such animal-objects.
[335] In this idea, Merleau-Ponty's concept of symbol approaches the Peircean symbol, being defined by its *esse in futuro* and its ability to anticipate future action by incarnating habits. At the same time, moreover, Merleau-Ponty anticipates Terrence Deacon's fertile idea of symbolicity as a crucial threshold in evolution, cf. the previous chapter.
[336] Translations from *La nature* are mine. 'Le corps appartient à une dynamique de comportement. Le comportement est enlisé dans la corporéite.' (239)
[337] This importance of animal appearance not only goes for its relation to species mates and to predators, it must be assumed, but also in relation to other species with which it competes, enters into symbioses, etc.
[338] 'La phénoménologie dénonce l'attitude naturelle et, en même temps, fait plus qu'aucune autre philosophie pour la réhabiliter.' (104)
[339] 'La chose m'apparaît comme fonction des mouvements de mon corps.' (106)
[340] Do animals hunting in bands not possess a sort of shared attention? Maybe not at any conscious level as known from human learning, but still several stages during the evolution from inter-animality over inter-corporality and to intersubjectivity seem to await proper conceptual determination.
[341] Without explicitly talking about the a priori of regional ontologies, Kauffman thus makes a parallel argument to our charting of a conceptual cluster of biosemiotics in Chap. 9.
[342] It is an interesting and strange implication of Kauffman's work (which it shares with the organism definitions of much romantic *Naturphilosophie*) that this cluster of concepts defining an autonomous agent may not stay a privilege for biology as we know it (geomorphology in geology, galaxy clusters in astronomy, companies in economics, states in international politics, etc. may be other candidates for autonomous agents in Kauffman's definition) – cf. our related discussion in Chap. 9.
[343] Referring to the putting to use of a biological property for another purpose than that for which it originally evolved.
[344] This idea forms Kauffman's suggestion for a fourth thermodynamical law - the universe invades the 'adjacent possible' with the highest possible speed. It remains unclear, though, against what this 'highest speed' could be measured.
[345] What Kauffman calls 'clusters of concepts', Husserl would probably call a regional ontology of a priori concepts for any possible biology.
[346] When I have presented this idea in lectures, it has been challenged by distinguished semioticians, like Susan Petrilli and Lucia Santaella, on Peircean grounds. They claim that Peirce's metaphysical continuity prohibits such thresholds from having any ontological value. I would argue this is a fallacious

argument. It is perfectly possible to maintain a basic continuist ontology and at the same time search for discontinuities segmenting this continuum. This lies already in one of Peirce's basic arguments for the primacy of continuity: the continuum problemlessly includes discontinuities, vice versa not so (and all attempts at 'building' the continuum from discrete entities fail). Moreover, even when no apparent discontinuity is at stake, clear differences are possible (take e.g. the continuum of color which does not imply that there is no difference between, say, yellow and green, having each their prototype (cf. Berlin and Kay's classic investigation of color names)). A sharp demarcation between any two colors will possess a certain arbitrarity but still it will possess a *fundamentum in re*. It is important not to let continuity assume a role of thought prohibition, a new darkness in which all cows become grey. This would violate Peirce's basic motto: Do not block the way of inquiry.

The biosemiotic *Scala Naturae* may be conceived of on the basis of Gould's idea of 'punctuated equilibrium'. Even if evolution is basically continuous, it may be segregated into long periods with little change, separated by short periods with huge changes. Thus, the development of man's unique semiotic possibilities has taken place over a very short timespan on an evolutionary scale, and it may be possible that the same thing holds for the passing of some of the other biosemiotic thresholds.

[347] Some of the thresholds proposed may cross each other, e.g. some of the candidates for 'missing link' thresholds, or the 'intercellular' threshold and the 'Uexküll threshold'.

[348] Mikkel Bogh (2002) argues that the introduction of poststructuralist theories in art history has had double consequences. On the one hand, it heightened the level of reflection compared to the strongly biographical traditional art history – on the other hand, it tended to leave the interest in the formal and phenomenological properties of the artwork to oblivion in favor of mere context.

[349] For a further discussion of negativism as avant garde kitsch, see Stjernfelt 'The Vulgar Metaphysics of Transgression', in *Text und Kontext* 2001 or Thomsen and Stjernfelt 2005.

[350] For a closer gaze, a certain tension thus prevails between two of the basic ideas of the historical avant garde: anti-figuration and anti-convention. Both of them are dubious as ontological constraints on art (why must an artwork not portray something; why could a conventional work not possess artistic value?), but it is interesting in this context that anti-conventionalism naturally implies that the anti-conventional artwork can not rely on purely anti-figurative mechanisms of meaning only (meaning must be either motivated or conventional, or both). Anti-convention must thus base itself on a similarity (but maybe one not seen before!). This lies already in the notion of 'abstract' art: if a picture is abstract, it does not imply it is not figurative, only that its similarity is so general that many different objects fall under it. The famous white square on white (to which we return below) refers to many possible concrete squares, herein lies its abstraction, and the game of providing the abstraction with the concrete objects it subsumes is left over to the viewer. The fact that things are hardly thus conceived by the main line of the art history of our time has its reason elsewhere, in another reason for the scepticism towards similarity, namely that similarity exists as a possibility before the finished artwork and thus offends the idea of the artist as an inspired deity, creating objects never before seen nor possible. The artwork's basis in similarity makes, from this point of view, the artist into a copist; even if the status of a sharp-sighted observer, seeing what nobody before him has been able to see, should not be so poor as a substitute for radical 'creativity'.

[351] In this, the metaphor definition is in agreement with the actual tradition of cognitive semantics where general metaphors and blendings are widespread in ordinary language, even if in an unspectacular way because understanding them is so easy - here the spectacular metaphors and blendings of poetry owe their effects to a sophisticated but derivative use of ordinary linguistic and conceptual material. This forms the cognitive equivalent to the core idea of 'deviation' poetics highlighted by the Russian formalists such as Shklovski.

[352] At the conference 'Thinking With Diagrams 98', Alan Blackwell and Yuri Engelhardt gave an overview over the many existing attempts at diagram taxonomies which all seem insufficient in some way or another (Blackwell and Engelhardt 'A Taxonomy of Diagram Taxonomies', paper given at 'Thinking With Diagrams 98', University of Wales, Aberystwyth). As mentioned Peirce has one sketchy attempt at a tripartition of diagrams into maps, algebra, and graphs (May and Stjernfelt 1996).

[353] That is, they can be used as diagrams and are regularly so used in ordinary picture observation. It must immediately be admitted that even if visual pictures are thus icons and diagrams among others,

their visual specificity may be pointed out referring to the special features of the visual system, cf. e.g. the already classical *Traité du signe visuel* (Groupe μ 1992) where the visual sign is characterized by its possible utilization of a long series of different optical transformations. These transformations are relevant here, because they are visually specific versions of the general rule-governed manipulability of diagrams. Visual diagrams are thus characterized by certain classes of manipulability, and it may be expected that a rational taxonomy of visual pictures may be constructed from the classification of such transformations.

[354] It is possible to argue for a distinction between two types of pictures, those which make possible the construction of a space for the perceiving body, and those without this possibility, cf. next chapter.

[355] A further investigation of this device follows in Chap. 16.

[356] Erik Fischer *C.W. Eckersberg*, Copenhagen 1993. I reviewed the book in the Copenhagen daily *Information* (Oct. 6, 1993) where some of the reflections in this paper were originally born.

[357] John 17.6–19 (*King James Bible* 1611):

6 'I have manifested thy name unto the men which thou gavest me out of the world: thine they were, and thou gavest them me; and they have kept thy word.
7 Now they have known that all things whatsoever thou hast given me are of thee.
8 For I have given unto them the words which thou gavest me; and they have received them, and have known surely that I came out from thee, and they have believed that thou didst send me.
9 I pray for them: I pray not for the world, but for them which thou hast given me; for they are thine.
10 And all mine are thine, and thine are mine; and I am glorified in them.
11 And now I am no more in the world, but these are in the world, and I come to thee. Holy Father, keep through thine own name those whom thou hast given me, that they may be one, as we are.
12 While I was with them in the world, I kept them in thy name: those that thou gavest me I have kept, and none of them is lost, but the son of perdition; that the scripture might be fulfilled.
13 And now come I to thee; and these things I speak in the world, that they might have my joy fulfilled in themselves.
14 I have given them thy word; and the world hath hated them, because they are not of the world, even as I am not of the world.
15 I pray not that thou shouldest take them out of the world, but that thou shouldest keep them from the evil.
16 They are not of the world, even as I am not of the world.
17 Sanctify them through thy truth: thy word is truth.
18 As thou hast sent me into the world, even so have I also sent them into the world.
19 And for their sakes I sanctify myself, that they also might be sanctified through the truth.'

[358] It is a well-known fact, dating at least from Kant's *Kritik der Urteilskraft* (1790), that artworks and biological organisms share this property, making both adequate objects for 'teleological judgment'. For both kinds of objects, however, we should not expect every single detail or feature to serve the whole teleologically; this would be a misplaced and dogmatic use of the concept of artwork or organism, respectively. Rather, artworks make use of completely ordinary structures of perception, thought, objects, which they take over for their own purposes (as argued in Bundgaard 2004) – much like biological organisms are built from ordinary physical structures.

[359] Cf. Groupe μ's criticism of this tendency, op.cit.

[360] See Esmann 1995.

[361] The color white naturally plays a special role in relation to the 'absolute' character of the painting because of the neutral and 'all-encompassing' character of this color (consisting of all visual wavelengths) which gives it a more absolute character than the individual spectral colors. Another experiment might thus substitute for the two white nuances two red nuances, e.g., with a larger degree of concretion as a contrastive result.

[362] An imperfect similarity relation, of course, because the infinitely thin lines of ideal geometry by their very nature cannot be drawn.

[363] Something like this naturally also occurred to Rudolf Arnheim cf. his *Visual Thinking* (Berkeley 1967). Sensitive phenomenologists might feel an intellectualist offense by this emphasis on diagrams, but on a closer look it is by no means strange to the phenomenological tradition. As discussed in

Chaps. 6–8, the experimentative manipulation on a diagram is directly related to the 'eidetic variations' leading to phenomenological 'Wesenserschauung', cf. Husserl's *Erfahrung und Urteil*, and the emphasis on the body in phenomenology from the later Husserl, Merleau-Ponty and onwards (cf. 'la chair' in *Le visible et l'invisible*, Paris 1964) may easily be reconciliated with diagrammatical reasoning. The diagrammatical experiment forms a central part of Peirce's pragmatism, the experiment involves the imagination of something *one could do*, and this imaginable doing has a natural corporal basis: 'It is not a historical fact that the best thinking has been done by words, or aural images. It has been performed by means of visual images and muscular imaginations.' (Peirce: 'Sketch of a new philosophy', undated, NEM IV, 375). In many pictures, there is a large class of diagram experiments which are immediately bodily; in landscape pictures, the viewer involuntarily imagines himself wandering into it, cf. Anne Fastrup's research into Diderot's 'Salons' (Fastrup, in press).

A remarkable consequence of pictures' diagram character is the refusal of the widespread semiotic idea that a picture is a sign to the extent that it implies a code which must be decoded directly, in an analogy to a denotative proposition. Diagrams leave, thanks to their more or less open manipulability a considerable initiative to the observer, and different schools of picture viewing may probably be described by their favorite diagram manipulation types. The systematic and rule-bound character of diagrams prevents, on the other hand, the complete handing over of initiative to the observer, as known from the different forms of scepticist aesthetics, from reception analysis and to (de)constructionism.

[364] It could be said that the analysis of pictures must give up its dogma of the concreteness of the picture and thus make an epistemological self-criticism analogous to historiography. Here, the Rankean idea of the description of concrete events 'wie es eigentlich gewesen' implied the rejection of all speculations of the type 'what if . . .'. But a semiotic analysis of these ideas demonstrates that it is impossible to describe even the most concrete event without the use of universals, types, generalities, and as soon as reference is made to general objects, a 'what if?' is implied, cf. the pragmatic maxim's 'would-bes'. If you say, for instance, that the rearmament of the enemy released the war, then you claim by the same token, that if the enemy had not rearmed, all other things being equal, then the war would not have come - a claim which would normally be rejected out of hand as counterfactual speculation. If you should refrain from indulging in such things, you should positivistically limit yourself to registrate events without reflecting on their type, nor on relations of cause and effect between them. I do not think any historiography has been written which conforms to such ideas. Younger historians (such as Niall Ferguson (ed.) *Virtual History*, London 1997) are now realizing that counterfactual speculations are inevitable - and if you think you can do without them, then you are fooling yourself. The equivalent in analysis of pictures, I would claim, is that if you think you can see and analyze pictures without the counterfactual variations of diagrams, then you fool yourself.

[365] Translations from *Hua* XXXIII are my own.'Wenn hier zwischen Sache und Bild unterschieden wird, so merken wir bald, dass der Begriff des Bildes hier ein doppelter ist. Der abgebildeten Sache steht nämlich ein Doppeltes gegenüber: 1) Das Bild als physisches Ding, als diese bemalte und eingerahmte Leinwand, als dieses bedruckte Papier usw. In diesem Sinn sagen wir, das *Bild* ist verbogen, zerrissen, oder das Bild hängt an der Wand usw. 2) Das Bild als das durch die bestimmte Farben- und Formgebunt so und so erscheinende *Bildobjekt*. Darunter verstehen wir nicht das abgebildete Objekt, das *Bildsujet* sondern das genaue Analogon des Phantasiebildes, nämlich das erscheinende Objekt, das für das Bild*sujet* Repräsentant ist.' (*Hua* XXIII, 19)

[366] 'Drei Objekte haben wir. 1) Das physische Bild, das Ding aus Leinwand, aus Marmor usw. 2) Das repräsentierende oder abbildende Objekt, und 3) das repräsentierte oder abgebildete Objekt.' (ibid.)

[367] 'Was im Inhalt des Bildobjektes repräsentativ fungiert, das ist in eigentümlicher Weise ausgezeichnet: *Es stellt dar*, es *vergegenwärtigt*, verbildlicht, *veranschaulicht*. Das Sujet blickt uns gleichsam durch *diese* Züge an. Diese Züge treten erst in Einzelbeachtung hervor und scheiden sich erst in ihr von den anderen Zügen des Bildobjektes: von Momenten, Teilen, Bestimmtheiten, die entweder ausgeprägt den gegensätzlichen Charakter, den des Widerstreits mit entsprechenden Bestimmtheiten des gemeinten Sujets, haben, oder denen weder der eine noch der andere Charakter anhaftet. Solche charakterlosen Züge verbildlichen nichts, es bleibt aber unbestimmt, wie sich darin das wirkliche Objekt [dar]stellt.' (30–31)

[368] '. . . das Bild muss sich *klar* von der Wirklichkeit scheiden, d.h. rein intuitiv, ohne alle Beihilfe von indirekten Gedanken.' (41)

[369] 'Which relation pertains between picture and object? Which relation pertains between pictorial object and object?'

[370] As to the issue of similarity in icon definitions in general, see Chaps. 2–3.

[371] See Groupe μ 1992 and the chapter on the esquisse. Here they distinguish between the visual sign, the type it signifies, and the object it refers to – where the type is the invariance kept constant in the transformation between the sign and its object. The type thus corresponds to Husserl's pictorial object, and Husserl's two-step transformation is analyzed as a one-step transformation with invariance – which may be viewed as different ways of representing the same state-of-affairs. The type in Groupe μ may be grasped separately with transformation relations to the two poles at each end so that it subsumes both the sign and the object as instantiations. In this idea it seems to be the case that the pictorial object is more general (or, at least, not less general) than the picture and the object.

[372] '*Vielmehr: "Bewusstsein" besteht durch und durch aus Bewusstsein, und schon Empfindung so wie Phantasma ist "Bewusstsein"*' (265)

[373] Noema of course being the object pole in the central analysis of perception as intentional act in the *Ideen*.

[374] I translate 'setzende' by 'assertive'; it refers to an act's character of existence claim as to its object; 'nicht-setzende' correlatively refers to acts not involving such claim.

[375] The context might be Meinong's doctrine of non-existent objects where 'assumptions' play an important role: they are non-assertive propositions (or, propositions minus existence claims) and form a much wider category than fantasy, comprising e.g. assumptive linguistic utterances deprived of fantasy imagery.

[376] 'Ich folge den Aufführungen eines Theaterstückes, oder ich betrachte ein Gemälde. Hier unterscheiden sich, ähnlich wie bei der reproduktiven Phantasie, die im Bilde dargestellten (bildlich produzierten) Vorstellungen, Wahrnehmungen, Urteile, Gefühle, etc., von den in mir, dem Zuschauer, erregten und aktuellen. Aktuell ist die Darstellung selbst.' (383)

[377] 'Aber nur wo ein Bildbewusstsein sich gründet in einem Wahrnehmungszusammenhang (wo eine Perzeption eben in einem perzeptiven Zusammenhang steht, aber sich ihm nicht in Einstemmigkeit einfügt) oder wo ein Bildbewusstsein sich in einem Erinnerungszusammenhang ebenso einfügt, haben wir Bildbewusstsein mit Widerstreit verknüpft; was alles genauer zu fassen wäre. Ich muss mich hüten vor unangemessener Verallgemerinerung: als ob Bild und Widerstreit notwendig zusammengehörte.' ('Zur Lehre vom Bildbewusstsein und Fiktumbewusstsein', c. 1912, 494)

[378] We may add that Husserl now generally sees *illusion* as a category where the *Widerstreit* is lacking, cf. his overall wax doll – human being example (fx. 39; 490). This is analogous to that phase in picture contemplation where the *Widerstreit* is suspended (in the old theory, however, the *Widerstreit* could be called back). It is the close relation between these two forms, of course, which lies at the root of the mathematical and theory-of-science disbelief in pictures as medium of thought.

[379] 'Auseinanderhalten wird man müssen die *Fälle wirklichen Widerstreits*, wie im Fall schauspielerischer Darstellung, und die Fälle *potentiellen Widerstreits*, der an Umgebungsintentionen hängt, die dem anschaulichen Gegenstand anhaften, die aber erst 'aufgewickelt' werden müssen, um zu wirklichen Widerstreiten zu führen.' ('Zur Lehre von den Anschauungen und ihren Modis', c. 1918,510)

[380] Fantasy (incuding pictures) remains in this period fundamentally 'modification'. This idea may puzzle, because it does not refer to any originary sensation of which the modified is a modification. Rather, modification refers to the fact that neither fantasy nor picture is present in 'anschauliche Fülle'. Husserl takes this concept of 'fullness' from the British psychologist Alexander Bain, and it plays a central role in his transcendental period's definition of perception as adequate act. The analogous problem (which phenomenological character has the givenness of actual objects, different from phantasized, ideal, and other objects?) is solved in a different way in Peirce: as we saw, he introduces the scholastic term 'haecceity', thisness from Duns Scotus as a concept for this special insistence characterizing the actually existing object different from merely potential or phantasized objects. In both cases, Husserl and Peirce, this difference is grounded on a 'purified' psychological concept, but while Husserl hints at the possibility of continuous grading (objects may be given with more or less fullness), Peirce's solution points to an all-or-nothing solution with a certain tension to his overall metaphysical continuism.

[381] Taken to the LU letter, a fairy-tale remains a purely 'signitive' intention, so it is striking that Husserl without further ado may involve this example in the discussion of the species of imagination. This points to the fact that the definition of signitive intentions only with difficulties may free itself completely from imaginative components – just like it was argued in the chapter on categorial intuition above.
[382] Ingarden enrolled as Husserl's student in Göttingen in May 1912, only to follow him (with a few breaks) closely in the years to come. He followed him from Göttingen to Freiburg and from around 1916 he was very close to Husserl as his personal assistant. In the summer of 1918, Ingarden returned to the Polish state then in the process of resurrection, but he remained in close contact with Husserl until his death in 1938, despite their well-known disagreement as to the issue of idealism/realism, cf. Ingarden's memories in his edition of Husserl's letters to him (Husserl 1968).
[383] Peirce has an analogous point when he points to the possible growth of information in diagrammatical reasoning by making explicit what was only implicit.
[384] Beilage VII (appr. Sept. 1906) introduces an interesting distinction between several possible types of *Widerstreit*. In addition to the empirical contrast between appearing pictorial object and its physical base, a contrast between pictorial object and sujet (that the people in a photograph are gray, but not reality) is proposed, as well as a second contrast in pictorial context (hallucinations in the midst of the visual field, e.g.), and a third contrast with our knowledge about the type of object depicted so that the object depicted becomes self-contradictory, a so-called logically mediated skin. Husserl does not provide examples on the latter, but it must be taken to refer to fallacious perceptions like the well-known 'impossible objects' in psychology of perception. Husserl's idea here seems to be to refine the second contrast, that between pictorial object and sujet, be pointing out different possibilities of the structure of this contrast.
[385] 'Ist nicht bei aller auch reinen Phantasie Widerstreit? Mit Wahrnehmungen, mit Erinnerungen, mit Antizipationen. Ich inhibiere alle Weltsetzung. Aber hat nicht jede Phantasie irgend eine Stelle, irgend etwas, das sie bestreitet?' (593)
[386] As a matter of fact, most often, the object of a picture is not a particular depicted at an unextended point of time. Even a photograph depicts its object during a short but extended timespan, and paintings very often involves a generalization aimed at typical properties of its object over a certain period.
[387] See Chap. 6.
[388] In Ingarden's theory of literature four strata are distinguished, each possessing a formal level – the aesthetic intention may select either of them as its object, as well as any of their mutual interrelations. See Chap. 17 on Ingarden.
[389] '*Der Inhalt des Gegenstandes selbst ist nicht ästhetisch bedeutungslos*. Es ist nicht gleich, ob es ein Kaiser ist oder nicht, ob ein bedeutendes Schicksal oder ein alltägliches etc. Handelt es sich da um ain Anklingen von Gemütswirkungen (Ehrfurcht, Ergebenheit)? Aber auch Anderes: Jede Gegenständlichkeit, die existentiale Freude motiviert, oder phantasiert, *quasi*-Freude. In sich ist diese Freude nicht ästhetisch. Aber das ästhetische Gefallen, das an der Erscheinungsweise hängt, kann sich mit dieser Freude (als einer Aktualität) verbinden, und das Ganze hat den Charakter einer erhöhten ästhetischen Freude. Stilleben. Das Wechselspiel aktueller Freuden oder *quasi*-Freuden (an der Natur: Freude über die fruchttragenden Obstbäume, Felder, etc.) und Leiden und sonstigen aktuellen Stellungnahmen ist aber selbst ein Hauptstück wirklich ästhetischer Freude. Also auch das gehört zur 'Erscheinungsweise'.'
[390] As a matter of fact, these determinations must also hold for perception proper (and hence for natural beauty, beauty in architecture, etc.) which does not need pictorality nor fiction, but merely a certain weakening of the distinction between appearances and the sujet pole.
[391] The connection of such indeterminacy spots to the problem solving of diagrams may be indicated by the ideas of Otto Selz (1913) talking about the role of 'completion on the basis of a schema' in his treatise of the 'Laws of the ordered thought process'. Selz argues for the 'Komplexergänzung', the completion of complexes as against 'Konstellationsergänzung', the completion of associative constellations of though in James, the latter being judged too simple. Selz's laws of complex completion are relevant here: a part or a schema of a complex has a tendency to call forth the reproduction of the whole complex.
[392] Of course, the question must be asked whether the *Unbestimmtheitsstellen* do not possess a foundation in ordinary perception. When, in perception, we spontaneously endow the objects with back sides, cf. the analysis of perception of the *Ideen*, then we add *general* backsides to those objects (we do not know nor see

where the cracks are on the back side of a wooden box), and the single perception always possesses a certain granularity beyond which we know nothing further. We do assume, that is right, in the perceptual *Einstellung*, that all unclarities might in principle be made clear by a further observation of the object (wa walk around it, take a closer look, etc.) – but no real series of perceptions may ever exhaust the properties of the object. Thus, even in the most meticulous observation a degree of underdetermination, that is, ideality, remains. Peirce radicalizes this idea to the claim that the object *is* not in fact particular, but still possesses a certain generality (its future, e.g. is not in all details decided), and that, conversely, the complete determination in the individual object is an idea governing perception. We shall not go that far but merely stay by the claim that the existence of *Unbestimmtheitsstellen* in fiction and language are so to speak prepared by the existence of spots of indeterminacy already in ordinary perception which makes possible the strange fact that we do not find it strange at all to meet e.g. persons with vast dimensions of indeterminacy in fiction.

[393] 'Danach kann Phantasie eigentlich kein Individuum als solches wiedergeben, sie gibt 'etwas', das in der Form eines Individuellen auszugestalten ist und nur nach seinem hinsichtlich der Individualität unbestimmten Sinnesgehalt anschaulich wird.' (nr. 19, 1922/23, 552).

[394] 'Möglichkeiten, individuelle reine Möglichkeiten sind fliessend' (Beilage 553).

'Unendlich viele mögliche Wirklichkeiten, die miteinander unverträglich sind etc.' (Beilage LXII, c. 1918, 566)

'*Prädikat* ist Identisches, das in vielen, in 'Mannigfaltigem' identisch sein kann, das in wandelbaren und beliebigen Möglichkeiten identisch vorkommen und das Konkretum bestimmen kann.' (nr. 19, 1922/23, 564)

'Und ebenso kan ich irgendein Subjekt mir denken, das das ansetzt auf Grund seiner Tatsachen.' (570).

'. . . wenn ich exemplarisch den Umfang als reinen Umfang durchlaufe, fällt es mir nicht ein, auf die bezüglichen subjektiven individualisierenden Differenzen mich einzustellen. Vergegenwärtige ich mir einen einzelnen Zentauren, so meine ich 'ihn' nicht als gerade von mir vorgestellten. Sollen wir sagen, dass das Singuläre eines reinen Umfangs (eines Umfangs reiner individueller Möglichkeiten), das exemplarisch Einzelne selbst schon ein Allgemeines <ist>, ein Identisches, das seinerseits einen reinen Möglichkeitsumfang hat?' (Beilage LXIII, c. 1920/21, 568).

[395] 'In den Unbestimmtheitshorizont, den für jedermann die gegebene Welt und gegebene Zeit hat, stellt der Dichter, ihn bestimmend, ausfüllend, Gestalten hinein.

Leser, Dichter in einer Welt und Zeit. Zwei Extreme:

[a)] Die gegebene Welt und Zeit kann eine so voll bestimmte sein, als für uns jetzt unsere Umwelt ist (nicht die wirkliche Welt). Z.B. das heutige Berlin, so bestimmt, wie es für uns und selbst für die Berliner selbst ist.

[b)] Extremer Gegenfall. Es war einmal, irgendwo, in irgendeinem Fabelland, in irgendeiner Zeit, in irgendeiner Welt mit ganz anderen animalischen Wesen, sogar anderen Naturgesetze etc.

Zwischen diesen beiden Extremen bewegt sich alle Kunst.' (Beilage LIX, 1916 el. 1918, s540)

[396] 'Oder vielleicht doch noch ein wenig. Wir haben den Menschen in Deckung, die beiden überschieben sich, aber geben weit klaffende Unterscheide. Der Unterschied ist so gross, dass wir nicht mehr uns gestört fühlen durch das Bewusstsein der Verfälschung bei der Neigung zur Identifizierung, sondern es fehlt die Neigung zur Identifizierung und bei dem grossen Abstand finden wir die Darstellung lächerlich.' (143)

[397] '. . . one *flows* into the other and is yet again felt as different.' (our italics)

[398] 'Warum wirkt die Natur, eine Landschaft als 'B i l d'? Ein fernes Dorf. Die Häuser 'kleine Häuser'. Diese kleinen Häuser haben a) eine geänderte Grösse gegenüber den Häusern, wie wir sie gewöhnlich sehen, b) eine geringere Stereoskopie, geänderte Färbungen etc. Sie werden ähnlich als Bilder aufgefasst wie Spielzeughäuser. Ebenso Menschen: Puppen.' (144)

[399] As found for instance in Nelson Goodman, cf. Chap. 3. In a Danish context, this idea has been influentially inherited in Søren Kjørup (2000, originally 1971). The conventionality hypothesis may take the form that the ability to grasp similarities is acquired, or the form that similarities do not play any role and must be substituted by conventions which are then what is acquired. See the discussion in Chap. 2.

[400] The fact that exactly landscape (pictures) furnish the immediate example of this duplicity in perception is probably due to the fact that fundamental spatial orientation connects the sense of vision

with the possibility of bodily movements in space. That this is the case, is supported by the metaphor the positivist Otto Neurath uses in the foreword to his amazing political-pedagogical book of diagrams *Modern Man in the Making* (1939) where he argues that modern man may get at more direct understanding of the political relations of modenity by means of diagrams depicting basic social relations. Neurath developed a set of so-called 'isotype' diagrams (especially making use of maps, column diagrams and the cake diagrams which he himself invented), and he introduces them as follows: 'An attempt has been made to evolve for this purpose a special picture-text style which sould enable anybody to walk through the modern world that is beginning to appear about us and see it as he may see a landscape with its hills and its plains, woods and meadows.' (7) The immediate accessability of landscape – in both the concrete and abstract meanings of this word – are mapped onto the diagram wherein complicated relations may be viewed with the same directness.

[401] We do not have to wait for a *Abwehrvorgang* to appear such as Freud believed.

[402] 'Ich kann wohl auch mich in das Bild 'hineinphantasieren'. Das kann aber nur sagen, dass ich den Bildraum über mich und meinen Umgebungsraum ausdehne und mich selbst unter Ausschluss der wirklichen Dinge, die ich sehe, mit ins Bild aufnehme, wodurch ich meine Aktualität ausschalte; ich werde dann selbst zum modifizierten Ich, zum setzungslosen. Dann ist meine Teilnahme die Teilnahme eines bildlichen Zuschauers (sie gehört zum Bildobjekt), nicht eines sympathisierenden vor dem Bild.'

[403] '. . . Neubildung von solchen Synthesen und Analysen, welche aus den einheitlichen Erscheinungen Teilerscheinungen herauslösen mit den Teilmeinungen in Synthesis gebracht usw. Im Expliziten treten also neue Stoffe (Kerne) auf. Also *Explikation des in vorgegebenen Erscheinungen verborgenen*.' (316).

[404] We thus receive a category to grasp the specificity of a project like the Flemish painter Coninxloo's landscape painting which is probably the first to block the access of the observer by orienting the gaze directly into brushes, bushes, and foliage and thus makes possible a distinct forest feeling for the observer lost in the wilderness.

[405] A further taxonomy of rather demanding character is constituted by the spectrum of imaginary action possibilities which the single picture offers the observer. It is well known that books and magazines exist which should, as Sade put it, be read with one hand only (the other hand presumably being elsewhere occupied): pornography is a pragmatic type of picture immediately suggesting which (more or less) imaginary action the observer may indulge in. It seems evident that enormous spectrum of possible actions open up which we can not make an even preliminary mapping of here – of how the single type of picture may be integrated into narratives, fictive or not. It is noteworthy, though, that these possibilities of action – thanks to the purposive structure of actions – mostly will be pertinent in figurative pictures. We have none or only marginally abstract pornography, and something similar holds for cookbooks, literary illustration, comics and a series of other picture genres correlated with actions possibilities, phantazised or real.

[406] 'Solche dargestellten Stimmungen, Gefühle etc. setzen nicht voraus eine Mitdarstellung des Beschauers, obschon er in eigener Weise in Aktion tritt. Genauer, sicher gehöre ich mit *dieser* Stimmung nicht ins Bild hinein.'

[407] Many other types of experiments may suit a given type of picture, cf. Chap. 13.

[408] Of course, the linguistic term has other prerequisites alongside with the visual type: knowledge of its use, bodily experience with that use, the object's relation to other objects, etc. – see the discussion of Eco's Cognitive Type and Nuclear Content in Chap. 3.

[409] Cf. Chap. 3.

[410] Translations from the *Traité* are my own. '. . . on peut supprimer certaines lignes pour ne laisser substister que celles jugées importantes. On obtient ainsi l'*esquisse*, qui fournit du sujet une vision synthetique, et livre souvent des tracés régulateurs utiles pour l'interpretation correcte de l'oeuvre finale.' (310).

[411] See Hintikka 1997; also Chap. 4.

[412] Peirce uses various other notions for abduction during his theoretical development: hypothesis, hypothetic inference, retroduction. Ab-, de-, and induction, being subspecies of arguments, are subsubspecies of symbols in Peirce's systematic sign doctrine.

[413] Michael Hoffmann (1999, 2002) has collected and discussed the various arguments pertaining to the concept of abduction in Peirce and his most important interpreters.

[414] As Hoffmann (2002, 257) comments, this forms an early version of Hempel's induction as the confirmation of a hypothesis by making it open for falsification.
[415] In that respect, it is involved in analogy. Peirce a few times mentions *analogy* as a further form of reasoning after Aristotle's "paradeigma", composed from ab- and induction ("History of science", 1896, 1.65), but he never develops a it thoroughly, and the supposed workings of analogy seems to be explainable by the abductive trial-and-error application of diagram structure on different materials.
[416] Hoffmann 2002 argues strongly and at length for this distinction between the logical form of abduction and its character of 'creative perception'.
[417] A basic hint lies, of course, in Peirce's possibility realism. Among all hypotheses which are possible in the weak sense of 'not known not to be true', hypotheses supported by known real possibilities (already recorded laws, tendencies, patterns, etc.) stand out as the hypotheses most promising to test.
[418] Peirce's discussions of the three different types of induction in the years after the turn of the century (in 'On the Logic of Drawing History from Ancient Documents,' (1901, EPII, 75; 7.164–231); in L75 (1902); in 'G,' (1905, 2.755–759); in L224 to William James, (1909, EPII, 492), etc., and their respective subtypes merit a whole separate discussion. Here, we shall stick to a short presentation aimed at the particular problem at issue.
[419] As always, other possible hypotheses abound: the appearance of death symbolism is an accident or coincidence, it owes it existence to the fact that the poem is pieced together of two autonomous parts, it owes it existence to the special location depicted, in which death signs naturally occur . . . etc.
[420] Cf. Freud 1948.
[421] In what sense are these observations deductions from the idea of the poem as a piece of *memento mori*? They are obviously not ordinary deductions; it does not follow with necessity that a poem about death contains these images. But they are deductions in the sense of 'probable deductions' which differ from induction as a probable inference. Probable deductions conclude with necessity about probabilities. Given this piece of knowledge of cultural history, an ideal type death poem could be said to exist, of which there is, with necessity, a certain probability that a death poem will instantiate certain features. If no necessary consequences flowed from the fact that the poem is a death poem, on the contrary, the claim would be empty.
[422] This is one of the implications of Ingarden's idea of a level of 'schematized aspects' of the literary text: the objectivities presented in the text are always seen as schemas, that is, with lots of 'Unbestimmtheitsstellen', that is, undetermined parts. The reader's filling-in of such parts prototypically takes place by 'crude induction' – if a man is introduced, he will be grasped as two-legged, until the contrary may be suggested – cf. next chapter.
[423] See Umberto Eco's discussion volume of the same title (1992). Eco here proposes a 'Popperian' principle which allows not to find the *best* interpretations, but to ascertaining which interpretations are 'bad' (Eco 1992, 52). It is an Augustinian idea: 'any interpretation given of a certain portion of the text can be accepted if it is confirmed by, and must be rejected if it is challenged by, another portion of the same text' (Eco 1992, 65). Eco picks the example of the British-Italian scholar Gabriele Rossetti's reading of Dante. He sets out to prove that Dante was a Rosicrucian (and, as a corollary, that the Rosicrucian symbol of a cross in a rose with a pelican ripping its own flesh from its breast underneath is present in his work). Rossetti finds a cross and a pelican at the same spot in Dante (not so strange as the self-sacrificing bird is an old symbol of Christ), he also finds a rose, but unfortunately not at the same place. His supporting hypothesis now goes that the parts of the symbol have been so dispersed in Dante's work for secrecy reasons. Eco, of course, points out, that this is a blatant case of overinterpretation, made possible only by this extra, unfounded hypothesis. In relation to the early existence of the Rosicrucians, Eco says that Rossetti adds the absurd argument of *Post hoc, ergo ante hoc*: because Rosicrucian symbols, dating after Dante, seemingly can be found in his text, the order must be even *older* than him. In our discussion, the theoretical assumption that the Rosicrucians are earlier than Dante governs the whole hypothesis in the symbol reading – instead of it being explicitly admitted as a hypothesis which may be confirmed or refuted depending on the findings of the Dante exegesis.
[424] Which is, of course, completely different from the deconstruction fad of the 80s assuming that all texts necessarily contain deep contradictions.

[425] As is evident, both deductions contain the word 'probably' – but this does not mean they are not necessary inferences, quite on the contrary. Probable inferences – inductions – build on observations, while deductions are based on idealized models. The idealized model of a catholic priest has a tonsure, just like the idealized model of a priest ties him to the city he lives in. This does not entail that all empirical priest are tonsured nor stick to their hometowns, but it implies that prototypical priests do so. Thus, the word 'probably' here refers to the relation between the ideal model and its empirical extension, before any empirical observation.

[426] Ingarden's whole œuvre thus forms a late flower of what is often called 'Göttingen' phenomenology from the period when Husserl lectured there. Among Ingarden's other chief influences thus count other early realist phenomenologists from the Göttingen circle or the München phenomenologists they were associated with: Adolf Reinach, Alexander Pfänder, Alexius Meinong, Johannes Daubert, etc.

[427] 'Idealism' here to be taken in the sense of the construal of the real world by the subject – not in the sense of claiming the real existence of ideal objects; in the former sense, Ingarden is an anti-idealist, in the latter, he – just like other phenomenologists, including Peirce – is an idealist. This turn has been the object of much discussion, ranging from the assertion that it took place somewhere between *Logische Untersuchungen* (1901) and *Ideen* (1913), with 1906–07 as peak candidate, to the claim that it never took place and that the internal continuity in Husserl's work remains far greater than the discontinuities. Ingarden was Husserl's pupil from around 1912–18 and obviously in this period found Husserl's philosophy as well as his teaching basically realist (in concert with the other Göttingen pupils), and he claims the internal tensions between realism and idealism in Husserl's thought only tipped to the side of idealism in the early twenties, only to be made public as late as in *Cartesianische Meditationen* and *Formale und transcendentale Logik*. In any case, Ingarden's experience of such a turn was what prompted his early publications 'Essentiale Fragen' (1925) and 'Bemerkungen zum Problem "Idealismus-Realismus" ' (1929) as well as *Das literarische Kunstwerk*. Indeed, Ingarden's enormous *Der Streit* (1965–74) forms the late culmination of this *Auseinandersetzung*. In Ingarden's letters to Husserl, the issue is discussed openly in detail (Husserl 1968).

[428] *Vom Erkennen* and *Der Streit* are preceded by Polish versions (from 1937 and 1947–48 and 1960–61, respectively). As I do not read Polish and as the German versions are Ingarden's own, revised, translations, I stick to them and the corresponding English translations.

[429] In the Austrian tradition, the notion of *Sachverhalte* often has this duplicity or ambiguity: referring both to the content of propositions, on the one hand, and to the objectivity referred to by those propositions, on the other.

[430] As always in the phenomenological tradition, 'object' here does not merely refer to physical things, but includes everything 'regardless of objectivity category and material essence. Thus it refers to things as well as persons, but also to all possible occurrences, states, acts performed by persons, etc.' (219)

[431] Thus, the beloved ambiguity and paradox of New Criticism, as well as the self-contradiction revered by deconstruction, are already grasped by Ingarden's theory – albeit not as indispensable properties of the literary work, but rather as possibilities among others.

[432] Ingarden gives a reference to Pfänder for the notion of 'quasi-judgment' as an intermediate with respect to Pfänder's distinction between two functions of the copula; assertive and predicative, respectively. Furthermore, the two extremes between which the *quasi* is localised are described with reference to Russell (the assertive judgment) and Husserl (neutrality modification), respectively (176). We may add that the very terminology (and the basic idea) also comes from Husserl (cf. Chap. 15) who uses 'quasi' intensely in the 1910s when Ingarden studied by him in Göttingen.

[433] In *Logik der Dichtung*, Käte Hamburger protests against the definition of the literary work by quasi-judgments. Quasi-judgments possess no special formal characteristics making it possible to isolate them – and Hamburger is on the search for such formal linguistic criteria (as the 'erlebte Rede' (style indirect libre), the historical present tense, and, especially, the epic past tense 'Tomorrow it was the first day . . .'). Hamburger's valuable work on these forms notwithstanding, they still do not constitute formal criteria of fiction – they are neither necessary nor sufficient, for there are many fictive works without any of these forms. So her use of these forms as weapon against Ingarden does not work – her argument that Ingarden's idea is circular because it only says a judgment is a quasi-judgment because it appears

in a novel would hit herself to the same degree if we were to take her formal studies as definitory for literariness or fiction. Maybe a misunderstanding is at stake here – Ingarden does not claim that what he finds is not circular, it is no attempt at *defining* the literary work by means of properties in its constituents, it is rather a description of the status of those constituents.

[434] Ingarden's dwelling with these metaphors indirectly refers to his idealism-realism agenda and his ongoing discussion with and of Husserl's gradual turn to idealism. Ingarden regards *Ideen* as ambiguous as regards idealism. The idealism-realism issue had not explicitly been the object of Husserl's published writings until the *Ideen*, and Husserl's Göttingen students obviously assumed – according to Ingarden – *Logische Untersuchungen* to be realist. They were thus surprised to see the much more explicitly idealist *Ideen I*, especially because Husserl's lectures was in a '. . . wenn man so sagen darf, realistischen Ton gehalten.' (Ingarden 1998 (1968), 407) – thus Husserl's lectures at the same time (summer semester 1913) on 'Natur und Geist' – being close to the more realist approach of the *Ideen II.* (ibid.)

In the *Ideen I,* on the one hand, the claim is repeated that the object is 'transcendent' in relation to the act, and the epistemology for ideal objects is not, unlike that of real objects, revised in the *Ideen*, so that Husserl still appears as a realist in these respects. On the other hand, the whole noesis-noema doctrine of the act is seen by Ingarden as idealist, reducing the object world of space-time to: '. . . ein Sein, das das Bewußtsein in seine Erfahrungen setzt, das prinzipiell nur als Identisches von motivierten Erscheinungsmannigfaltigkeiten anschaubar und bestimmbar – *darüber hinaus* aber ein Nichts ist.' (*Ideen I*, 93, quoted in Ingarden 1998, 211 (1956)) – a quote to which Ingarden returns.

This criticism of Husserl's position is constant in Ingarden's many writings about the subject. He writes – as early as in his 1918 letter to Husserl about the re-edition of the VI. investigation -: 'Wir kommen am Ende zu der Gleichung: Ding = ein eigentümlich gebautes Noema-Bewußtsein.' (Ingarden 1998, 5). So the relation between the single aspects-noemas and the thing-noema constituted by them is what recurs in Ingarden's idea of the states of affairs forming a 'net' around the represented objectivity which thus to some extent corresponds to the noema. Unlike Husserl's noema, this objectivity is, of course, distinct from the correlated real or ideal object itself (if it exists).

[435] Here, an important difference in relation to Peirce's metaphysics resides in the fact that Peirce would never allow particulars of any sort to be completely determined, inter alia because of the indeterminacy of their future states. To Peirce, thus, there are *real Unbestimmtheitsstellen* mirroring the case of semiotic representations. Indeterminacy, in any case, must be far more extensive in representations.

[436] Wellek says (1991, 379): 'I find myself in wide agreement with his views and acknowledge learning from him, on many of these questions, more than from anybody else.' – and still 'I find it difficult to isolate this stratum of schematic aspects from that of represented objects . . .' (383). The polemical dialogue between the two is recorded in later editions of *Das literarische Kunstwerk* and in Wellek 1991.

[437] 'All so-called sensory qualities – but not only these – are self-presenting' (196). Just like categorical entities in Husserl may also permit intuitive access, it is not only sensory qualities which satisfy self-presentation. We saw that occurrence states also did it to some extent (due to what – due to the schematic character of their event type?) – and we must assume that (certain simple?) ideal objects also display self-presentation (cf. Chap. 6). This becomes important for the stratum of schematized aspects.

[438] Ingarden's account for these qualities has hardly left the first, descriptive phase. It is evident that reading great literature, qualities like the mentioned are often experienced, but their status does not seem perfectly clear in Ingarden's theory. They seem to be objective qualities dependent on temporal situations depicted in the object level – but why is it that they are presented as extraordinary, almost quasi-religious breaks with everyday life (cf. the word 'revelation' used as common denominator for them)? It is true that literature often describes such qualities, but literature also has the power of redescribing aspects of everyday life with an intensity which may make us experience it as with new eyes. So their 'revelation' character ought to be described more meticulously in order to avoid identifying them rightaway with vitalist ecstasies or intensities. Recall, for a corrective point of view, that the doctrine of Russian formalism with its avant garde aesthetics applauded 'deviation' exactly for its potential for making us see the trivial things anew – to see again the stoneness of the stone, etc. If constructed properly, Ingarden's 'metaphysical qualities' should include this sort of aesthetic experience as well, far from the

vitalist shocks seeming to play center stage (in his examples, at least). What about the boring, the sick, the tedious, the ordinary, the vexatious, the brooding, the slowly threatening, the contingent . . . it takes a true master to make such metaphysical qualities appear with the same intensity as Ingarden's more vitalist examples.

[439] I owe many insights in these paragraphs to my discussions with Thomas Illum Hansen who also works with the ideas of integrating cogntive semantics insights into an Ingardenian framework (Hansen 2001, 2004).

[440] This would also throw new light on Ingarden's attempt at distinguishing the literary work of art from the scientific literary work. Of course his principal distinction goes between literary quasi-judgments and scientific judgments – where the latter serve the depitction of transcendent (real or ideal) objects. More debateable is his idea that in a scientific work one must focus upon the objects represented solely, and that the '. . . situation of the aspects which may be held in readiness in a scientific work is analogous, unless the aspects perform a particular cognitive role in the work.' (1968, 157). The addition is significant, for in the scientific work, the aspects held in readiness may perform a *cognitive* role akin to the aesthetic role played in the literary work – in both cases the function of making the objects represented *intuitive* unites the role of the aspects. Diagrams may directly play the role of such aspects. While we agree with Ingarden in the basic distinction (against all sorts of deconstructivism blurring fiction and reality), we find a greater degree of relatedness betweeen the two.

[441] Cf. Hayek 1978.

[442] With respect to this, a myth probably exists in phenomenology about the definite concreteness of what is perceptually given. But the concreteness of ordinary perceptual experience does not lie in it being constantly highly detailed, but rather in the fact that we know that we could always, in a given situation, choose to focus more attentively on this or that feature and thus bring forth further detail – which is not possible, of course, in purely textually or diagrammatically presented objectivities.

[443] This does not, however, imply that literary quality necessarily lies in the making explicit and thematic such thought experiments in themselves. As Nils Gunder Hansen (2004) argues, Iser's idea that the reader's explicit grasping of literary conventions and machinery involved therein constitutes the access to complicated literature and its aesthetic values, is wrong. The diagram experiments here discussed are undertaken primarily in order to co-constitute the level of represented objectivities, not to abstract from it and seeking the work's *raison-d'être* only in its ingenious use of literary devices.

[444] What about literary genres like poetry or essays, do they share the basic definition by quasi-judgments? Another of Ingarden's famous opponents, Käte Hamburger, has a counterargument along these lines. She famously claims that poetry is emphatically *not* fictive, but rather a special genre, to be classified along with the 'Ich-Erzählung', the 'I-tale', referring to the subjective experience reality of the lyrical ego. We shall not go into this discussion here, but Hamburger is obviously right that it seems questionable to define the notoriously fuzzy concept of literature as a whole on the basis of fictivity. Ruttkowski 1978, 291–92 details this argument: poetry may be fictive or not, just like epic literature (containing both adventure novels and autobiographies). Similarly, other counterexamples can be listed: fictional texts which are not literature (myth; scientific or other thought experiments). So most likely, it is not the case that literature and fictionality is coextensional, and Ingarden's theory does not hold for the whole wide field covered by the umbrella concept of literature but only for that large prototypical subset of it which develops fictive universes.

[445] A further point to be discussed is *formal* ontology. Ingarden mentions the idea in a short note (302) and says that as long as a literary work deals with *objects* at all, the laws of formal ontology must be observed. Given the broad phenomenological idea of objects (covering events, states, processes, qualities, being ideal, real, intentional; physical, psychic etc.) it may be more difficult than expected to transgress formal ontology, so assumably, there is hardly any specific literary truth effect connected to adherence to formal ontology.

[446] This taxonomy of iconicities in literature only pertains to the level of represented objectivities and thus does not prevent further, indirect iconicities to play a role in the literary work which are orthogonal to the types here mentioned and may freely combine with all of them: That of experienced events being seen through a narrator or a character; that of an indirect, symbolical or allegorical reference; that of letting linguistic devices function iconically (word order, etc.), that of adding illustrations (maps, graphs,

diagrams, etc.) to the text or directly shape the text after such icons as in figure poems, etc. Johansen (1996, 2003) chart some of these iconicity possibilities after Peircean mode of representation (image, diagram, metaphor, respectively) which is, of course, independent of the five types discussed here, so a combination will yield further subtypes to be investigated.

[447] Maybe these types of truth could ease the tension between Ingarden and Hamburger. The latter attacks fiction theories of literature with reference to the non-fictionality of poetry and first-person narration (and a related case could be made for the literary essay) – but even if 'quasi-judgments' are basically fiction, the two concepts are not completely coextensive. Quasi-judgments come in many types (they form a continuum between assertion and mere predication or assumption) and the most assertion-like quasi-judgments, so to speak, only marginally differ from full-fledged assertions. The personal truths experienced in poetry reading or the journalistic or philosophical truth in essayism could thus be seen as a mixture of assertion-like quasi-judgments with pseudo-truths of the types here listed.

To return to the discussion of interpretation in chapter 16, Ingarden's doctrine may also permit us to distinguish between the often mixed-up notions of reading, analysis and interpretation. Reading is prototypically Ingardenian concretization – it is the appropriation of the work which performs the different procedures of filling-in and of actualization of schematized aspects held-in-readiness. Analysis, then, is the operation which – typically on the basis of a reading – attempts to get an overview of the text's organic structure, which parts relate to what other parts in which way. Analysis, then, makes explicit a lot of things unthematically participating in reading – at the same time as it is, typically, ascetic regarding the filling-in which is artificially kept back. Interpretation, finally, is the appropriation of the text which is primarily aimed at the pseudo-truth of the work – in some or all of the five senses here mentioned. As is evident, many empirical examples of papers about specific literary texts indulge in combinations of all three procedures; probably the prototypical academic, informed treatment of literary texts will do so.

[448] Clausewitz's *Vom Kriege* is even remarkably sparse as to observations on the role of espionage in warfare; all is a 1-page chapter about 'Nachrichten im Krieg' containing little exceeding common sense: 'Ein grosser Teil der Nachrichten, die man im Kriege bekommet, ist widersprechend, ein noch größerer ist falsch und bei weitem der größte einer ziemlichen Ungewißheit unterworfen.' (48)

[449] In the case of espionage, we meet such structures in the recurrent, transhistorical claims about the nature, essence, principles, or problems of espionage in spy literature. In *Spys and Spymasters*, e.g., we read about twentieth century espionage that 'Though considerate advances had been made in technology, the basic principles and problems of intelligence remained unchanged.' (144). In the same vein, we are told that as to the human element of espionage '. . . nothing had changed since the days of Joshua.' (146). Such general ideas are subsequently applied in the analyses of specific subjects, as when the espionage satellites of the twentieth century are seen as evolutionary heirs to the balloons of the eighteenth century They, in turn, had the function '. . . to take one stage further the instruction Moses gave to his spies: "Go up into the mountain, and see the land!" ' (166).

[450] Smith has thus founded a center for philosophy and geography and conceives of political geography as an exemplary case for a priori studies, e.g., of border types. The idea of such a relation between reality and semantics remains, though, controversial. The present chapter has thus been turned down by several distinguished scholarly journals, not because of its quality (at least, so they claimed), but because of the fact that it included real-world issues in the discussion of a literary genre and thus was deemed unfit for literary studies.

[451] In an arbitrarily selected dictionary, *Nudansk Ordbog*, Copenhagen: Politiken 1977. This procedure in inspired by Greimas's investigation of the concepts 'challenge' and 'anger', in Greimas 1982.

[452] A prominent example is the allies' large-scale deception operation before D-day in order to make Hitler believe the Dunkirk area to be the invasion spot, including not only a planning of a feinted invasion there but also the planning of a feinted feint, a more northerly invasion supposed to take place from Scotland, thus adding further credibility to the Dunkirk possibility.

[453] I believe this is not generally acknowledged, and among many literary scholars, Greimas even counts as an especially malignant reductionist. This rests, however, upon a misinterpretation of Greimas' 'narrative schema' as an assumedly identical deep structure underlying all concrete texts. This idea

overlooks a crucial moment in all decent structuralisms: the concept of *transformation*, cf. Chap. 5. The schema is a prototype only which must be transformed in order to grasp the single text's specificity. The specific features of the single text is grasped only by understanding – not only the schema – but the specific transformation (and its motivations and implications) resulting in just that text. Moreover, the schema may develop with the addition of further assumptions which make new aspects of the fight appear. The schema is not a causal regularity, it is a teleological regularity, and hence it may bifurcate at every possible joint, failing to satisfy the telos in question. The simple phases of it are not causally determined by earlier phases; rather, later phases presuppose earlier phases.

[454] Here sociological criteria enter: espionage does not seem to have been anomalous in GDR, for instance, measured on what is known about the number of informants in the people employed by the Stasi, and generally espionage is considerably less controversial in pre-democratic or totalitarian states. But even here, the anomalous character is preserved in the secrecy of procedures.

[455] Carl Schmitt's personal career is highly controversial, involving extreme right wing positions and active support for the Nazi regime in the 30s. Despite Schmitt's dubious – to say the least – political positions, it is possible to discuss his more general philosophical and scientific points of view on a democratic basis. Schmitt's notion of sovereignty is explicitly copied from theological concepts; cf. the hypothesis of *Politische Theologie* that the concepts of modern political theory are constituted by secularized theology. A corollary is that fundamental political and politological issues inherit structures from theology; the political wars of the largely atheist twentieth century support this idea, cf. also Vögelin's notion of religious politics. It is easy to recognize the problems of incarnation and of theodicée in relation to espionage: how may democratic ideas become flesh? How may democracy be morally good when its own secret services are not?

[456] Schmitt does not explicitly claim this, but the idea clearly appears, e.g. in *Der Begriff des Politischen*, (34n) where the famous dictum of Clausewitz is interpreted with the conclusion that politics is determined by the Friend-or-Foe logic of war.

[457] The latter expression is, surprisingly, rather new and dates back only to American discussions in the beginning of the Cold War.

[458] We presuppose, of course, that the Prussians did not have the possibility of incarcerating the two and keep them as prisoners of war. We may note *en passant* that according to John Keegan, it was the Prussians' victory in the Franco-German war which made Clausewitz an international hero in military academies worldwide. This development formed part of the radical brutalization of war during the twentieth century supported by Clausewitz's idea of the war as tending to the utmost release of violence and aiming at the total defeat of the enemy.

[459] Brandt 1983, 129.

[460] Analogous cases occurred in USA during the same period – president Kennedy's affair with Judy Exner whom he shared with mafioso Sam Giancana, just like his affairs with the East German girl Ellen Rometsch and several upper class whores with connections to the Profumo case. These affairs were only made silent due to intensive emergency work by Robert Kennedy and J. Edgar Hoover (according to Hersh 1997).

[461] With Jens-Martin Eriksen, I wrote two books about the Balkan wars of the 90s and the actual predicaments of the region (2003, 2004).

[462] According to West, 296–97.

[463] Famous is the alleged refusal of the USA to perpetuate the services in the period between the World Wars, with reference to the fact that 'gentlemen do not read each other's mail'.

[464] The military function is the second function out of the three in Dumézil's theory of three main functions in Indo-European culture: *justice/magic, war, fertility.)*

[465] From *Ornicar!*, Vol. 19, Paris 1979.

[466] We may remark the British military historian John Keegan's empirical claim that the Napoleonic revolutionary armies with their general conscription played a decisive role in the democratization of Europe.

[467] Of course, military intelligence is subject to the same control, but the recurrent and delicate political tension between state security and the human rights of the same state's citizens is structurally relevant for domestic services primarily.

[468] Cf. the Swedish conflict researcher Ola Tunander's work on the Palme case.
[469] When senator Moynihan (cf. below), after having served under Kennedy, Johnson, Nixon, and Ford, was elected to Senate, he got admission to his own FBI file of 561 pages, naming him as a communist.
[470] In one of the rare cases of principal reflection on these issues – the last chapter of former CIA-boss Allen Dulles's book *The Craft of Intelligence* (1963) – he claims that the president himself controls the services, that Dulles himself has supported a proposal for a civil control commission, that all his own knowledge of the services gives reason for trust: 'After more than a decade of service, I can testify that I have never known a group of men and women more devoted to the defense of our country and its way of life than those who are working in the Central Intelligence Agency.' (264). Apart from the fact that a natural scepticism easily awakens faced with claims like these from the leader's own lips, it remains correct that assumptions like the ones quoted is all that you have to rest your head on. On the other hand, Dulles adds immediately afterwards, as the very last two sentences of the book that 'The last thing we can afford to do today is to put our intelligence in chains. Its protective and informative role is indispensable in an era of unique and continuing danger.' (264). Dulles thus summarizes in a few lines all possible points of view: control, trust, limitation of control . . .
[471] A recent Danish example is the schoolmaster Kristian Kjær Nielsen who recently (in the Danish daily *Information* March 10, 1999) told about how, as a member of the Danish Neo-Nazi Party DNSB in the seventies, he spied on party comrades. The information he collected was delivered anonymously by P.O. Box in Copenhagen, and the spy had never any clear idea as to who his commissioners were. Obvious candidates included Israeli, West German, and Danish intelligence services, just like Jewish organizations for the tracing of World War II criminals are possible.
[472] Moynihan relates, not without comical effects, how general Butler, one of the main responsibles for the American atomic strategy, visited the Soviet Union for the first time in 1988 and got a shock. Everything is falling apart, and the chauffeur in the official limousine transporting him breaks off the gear stick. After all these years, Butler in a split second realizes that he has been dealing all along with a caricature (78–79). Moynihan himself tells about a parallel experience regarding the Sandinistas in Nicaragua where he, as an official guest, is witness to the Secretary of Interior trying in vain to order beans at a restaurant – all at the same time as the illegal Iran-Contra scandal develops on a CIA automat idea that the Sandinist state should be a strong and dangerous enemy (208–12). Instead, Moynihan's proposal would be that a '. . . reasonable American response to the new Communist government in Managua would have been a statement of condolence.' (207)
[473] Ed. Nigel West, London 1993.
[474] We may note that the postmodernist sceptic Jean Baudrillard took his most salient examples for his radical idea of the 'disappearance of the real' from the world of secret services. Who was responsible for the Italian terror bombs of the seventies and eighties? – many different groupings claimed responsibility, maybe it was instructed by one political wing in order to discredit the other, maybe by the police in order to discredit both, maybe by foreign interests in destabilizing Italy . . . reality vanishes behind such interpretations and their effects.
[475] I pick these informations from Haswell 1975, 48–49.
[476] The first organized use of this effect was probably Bismarck's famous espionage chief Wilhelm Stieber who was the father of many classical espionage inventions. He erected the so-called 'Greenhouse' in Berlin, an especially sophisticated and depraved whorehouse, with the intention of its use in blackmail of its customer circle involved in international politics.
[477] An explicit example supporting this idea being of course Graham Greene the Catholic.
[478] Maybe this fact gives part of an explanation of the often-noticed but relatively unexplained partition of the film and literature public into two segments: a masculine segment preferring the detectives, front pigs, spies, thrill, and objective action of B-movies, while a feminine segment wants doctors, artists, love, children, passion in melodramas with full possibilities of heavy psychological identification. Nobody could possibly overlook this, least of all Hollywood, still it seems anathema for academic investigation in most of literature and film departments.
[479] 'Metaphysics consist in the results of the absolute acceptance of logical principles not merely as regulatively valid, but as truths of being.' ('The Logic of Mathematics: An Attempt to Develop my Categories from Within', c. 1896, CP 1.487)

[480] Peirce presents several different systems of sciences, but the most developed ones stem from his mature period, the same period in which the concept of continuity is central. In Peirce's large application for the Carnegie foundation from 1902 (NEM IV, 17), the system is schematically presented; in Robin 693 (op.cit. 189 ff. undated), the classification is commented upon; in CP 1.203–83 (1902) it is the object of a thorough argumentation. Mathematics is 'engaged solely in tracing out the consequences of hypothesis' (CP 1.247) – Peirce's interpretation of his father Benjamin Peirce's definition of mathematics as the science which draws necessary conclusions. Its hypothetical character is important: '*Mathematics* is the study of what is or is not logically possible, without undertaking to ascertain what actually exists.' ('Reason's Conscience', undated, NEM IV, 191). Consequently, it is left to the more empirical sciences to ascertain which of its hypotheses may have which applications. Conversely, mathematics is the most general science, whose results (even if they may not be unambiguous, cf. the Non-Euclidean geometries) all the more specific sciences must apply. The philosophical sciences – in Peirce phenomenology (or phaneroscopy), normative sciences (including logic equaling semiotics) and finally metaphysics, in that order – hence depend on mathematics; it is, of course, their own decision which parts of mathematics they build on. It is decided by what may appear in experience (phenomenology), by which forms valid inferences may take (logic) and by which general properties reality possesses (metaphysics), the latter of these sciences specifying the former, not vice versa. Only after these sciences, the special sciences follow under the headline of 'idioscopy'. In the series of philosophical sciences, continuity has a central role, both in the sense of first philosophy and as a sort of connecting glue between the single domains.

[481] The very first philosopher on this point is probably Bolzano (*Paradoxien des Unendlichen*, 1850) to whom Peirce only refers in his late period. Peirce refers one of his main ideas (that the equivalence of sets with respect to size should be measured by a one-to-one pairing of elements), but at the same time he claims he has not read his work ('Bedrock', 1906, 6.175, and 'Some Amazing Mazes', 1908, 4.651).

[482] Peirce claims at several occasions that Dedekind's influential book – where the real numbers are defined by 'Dedekind cuts' – should be influenced by papers sent to him by Peirce. Cf. the following lecture note from 1903: 'My own first contribution to this branch was made in the autumn of 1867' (Robin 316a, NEM III, 129), and further 'My second contribution was 1881 [. . .] My third [. . .] That is all I have printed. My work has been, I believe, completely independent of Cantor. I never knew anything definite about him until 1884. I have seen it stated in some book that I modified the statements of Dedekind. But the truth is that Dedekind's *Was sind und was sollen die Zahlen* first appeared in [1888]. It contains not a single idea which was not in my paper of [1881?], of which an extra copy was sent to him and I do not doubt influenced his work.' (ibid. 130; the square brackets around the years are Eisele's). The unspecified reference to 1881 probably refers to 'On the Logic of Number' (CP 3.252–88), cf. what Peirce says a couple of years later in a footnote ('Pragmaticism, Prag. [4]' c. 1905, CP 5.526n): 'C.S. Peirce's paper *On the Logic of Number*, published in the spring of 1881 contained, though not in a perspicuous form, the leading results of Dedekind's classic of 1887.' We shall not take that text into accout here, as it – with respect to continuity – rests upon insuffucient definitions: 'A continuous system is one in which every quantity greater than another is also greater than some intermediate quantity greater than that other.' (3.256) – which is valid already for non-continuous objects, such as certain arrangements (after size, e.g.) of rational numbers.

[483] Cantor's decisive articles are the six-part 'Über unendlich lineare Punktmannigfaltigkeiten' (1879–84) which widely transgresses the subject indicated by the title to provide a philosophical argumentation for the basis of his theory of sets, as well as the two-part 'Beiträge zur Begründung der transfiniten Mengenlehre' (1895–97) which provides a rigorous mathematical presentation.

According to Parker 1992, Peirce only knew the first of these papers through a short excerpt in French translation (of part 6, §16), 'Sur divers théorèmes de la théorie des ensembles de points situés dans un espace continu a *n* dimensions' (1883), just like he only knew the second part of the second paper in addition to a final third paper 'Mitteilungen zur Lehre vom Transfiniten' (1887–88) which consists of a collection of letters discussing and polemizising with various authors. The background for these claims about Peirce's selective reading of Cantor is not provided in Parker 1992.

Most references to Cantor in Peirce's work are later than 1897, but e.g. CP 6.164 (The 'Continuity' entry from *Century Dictionary* 1889) or the reference in 1892 'The Law of Mind' points to the fact that Peirce at this point must have known considerably more about Cantor than §16 (1883a); in the notes of the CP a

reference points to §10 of (1879–84). In Peirce's draft for a letter to Cantor dating from 1900 (NEM III, 767) he says, 'Dear Sir: Before reading your wonderfully beautiful and profound studies (in fact, I have only just read your memoir in the Math. Annalen XLVI and XLIX, and have not had time to digest them, and have only looked a little at the memoir in Vol. XXI, because I could not catch the idea of your ordinal numbers) . . .' According to this passage, Peirce knows *both* parts of Cantor 1895–97, while the reference to a paper in the same journal's Vol. 21 refers to 'Grundlagen einer allgemeinen Mannigfaltigkeitslehre' (1883) which is 'Nr. 5' (by far the largest) of the six parts of Cantor 1879–84, containing among other things the first articulation of the continuum hypothesis. Thus it does not seem correct to claim that Peirce only knew the French version of Nr. 6, §16. When Peirce says about his knowledge of these texts that he has 'only looked a little', we may not be so sure he is speaking the whole truth; firstly, Peirce claims elsewhere that he has anticipated Cantor and hence it would not be clever of him to admit much too close an acquaintance with his early writings (even if Peirce's decisive argument for his autonomy 'On the Logic of Number' (CP 3.252–88) dates from 1881); secondly, he adds that he has not really understood ordinal numbers which is no doubt correct. The references to Cantor earlier than 1897, thus in 'The Law of Mind', seem to refer to a knowledge of part 5 of Cantor 1879–84.

[484] The centrality of time, however, in Peirce's continuity argument is evident as early as in the *Monist* papers from 1892 (cf. below) so Potter and Shields' idea of a fourth period based on the 1908 note may be subject to discussion.

[485] Thereafter Peirce adds that this ambitious and romantic philosophical program might have certain roots in his Concord childhood during the period in which Emersonian transcendentalism dominated the intellectual climate with ideas 'caught from Schelling, and Schelling from Plotinus, from Boehm, or from god knows what minds stricken with the monstrous mysticism of the East.' He is not, however, 'conscious of having contracted any of that virus', but he still holds it probable that 'some cultured bacilli, some benignant form of the disease was implanted in my soul, unawares, and that now, after long incubation, it comes to the surface, modified by mathematical conceptions and by training in physical investigations.' Thus Peirce perfectly realizes the apparently strange fact that his evolutionary cosmology introduces rather mystical ideas in the cool pragmatism. In spite of parallel formulas regarding mind and matter, Peirce in these lectures rejects monism (6.24) in favor of an idealism making matter a derivative phenomenon; later (6.73) he presents himself as monist, probably motivated by the continuity between mind and matter which is a recurring idea in him. Even if Peirce is thus no neutral monist, it seems fair to call his idealism a monism, because it refers to the metaphysical interpretation of the concept of continuity, due to which no absolute metaphysical distinctions are allowed.

[486] As Parker remarks (16, 52), the concept of continuity is here invoked at two epistemologically and metaphysically decisive points: the beginning point of science and the ground of being of reality; both of these solid and problematic entities are dissolved by means of continua establishing their connection to the actual level of knowledge at any time, backwards (knowledge) as well as forwards (being). This allows the young Peirce to cut down his category table in 'New List' (EPI, 6; 1.555) from five (minus Substance and Being) to the canonical three, at the same time as he gets rid of the Kantian problem of a *Ding an sich*.

[487] This argument remains constant in the mature Peirce, cf. 4.641 ('The Amazing Mazes. First Curiosity, 1908): 'The argument which seems to me to prove, not only that there is such a conception of continuity as I contend for, but that it is realized in the universe, is that if it was not so, nobody could have any memory.'

[488] The difference remains, of course, that Husserl's question refers to ideal objects only, defined by that the access to them is in principle always possible. Peirce's argument refers to memory in general where this principle does not hold.

[489] Ketner and Putnam's preface to *Reasoning and the Logic of Things* refers to this affinity between Peirce and non-standard analysis.

[490] The first occurrence of this intuition is probably Bernard Bolzano in his *Paradoxien des Unendlichen* from 1850 which Peirce only later realizes, cf. 'Notes on the Theory of Multitude', (NEM III, 1088) from 1903.

[491] Even if equivalent to Dedekind and Cantor's definitions, Peirce's 1881 description of infinite sets in terms of the syllogism of transposed quantities differs in being an intenstional description, pointing

to a seminal difference between Cantorian set theory and Peirce's approach. Peirce later highlighted the difference in a letter to *Science* in 1900 (3.563), and Randall Dipert (1997) convincingly argues that Peirce's criticism forms an 'extremely subtle' point. Peirce protests that Dedekind's notion of proper part ('echter Theil') remains undefined, and Dipert argues that this is because there is no procedure given to determine whether a given candidate is, in fact, a proper part of an infinte set. In order to tell whether a set A is a subset of another set B, we must have sufficient knowledge af that set in order to determine whether each of its elements are in fact elements of B as well. But we may only possess such knowledge of A if we know each of its individual elements by some procedure. Dipert acknowledges the constructivist leanings of this idea – but Peirce's obsession does not lie, as in the later constructivists after Brouwer, with the impossibility of admitting objects defined by infinite procedures, but rather with the distinctness of the individual elements. This is why Peirce insists on an intensional definition of subsets in the syllogism of transposed quantities, Dipert argues: 'One cannot *legislate* an ability to form proper subsets of a given purported collection. That is, of any purportedly collective entity, one cannot simply assert that one can, say, partition it. Rather, the ability to do so is a consequence of, and hence required for, its being a collection in the first place. A conception of the members of the collection as individuated entities comes first, and of the collections of such entities, the Axiom of Choice will in fact be *descriptively* true.' (65). Peirce only later, in the 1900 letter to Cantor, make explicit this idea, but its germ is present already in the definition of infinity by means of the syllogism of transposed quantities where the crucial subset (the subset of women in the French population) is intensionally defined. Dipert rightly notices the connection from Peirce's intensionality claim to his realism as to rules and laws.

[492] In 'The Logic of Quantity' (1893, 4.126) Peirce says (almost) the exact opposite: 'The multiplicity of points upon a surface must be admitted, as it seems to me, to be the square of the points of the line and so with higher dimensions.' If you do not add the rule of calculation claiming that Aleph-n times Aleph-n = Aleph-n, then this claim is fallacious. This only mentioned to indicate how vacillating Peirce may be in this field, and how difficult it may be to reconstruct his changing positions.

[493] Peirce later regrets his condemnation of Kant's continuity definitions and his adherence to Cantor's definition of the continuum as a set of points. In L77 (NEM III, 780), a letter to Paul Carus from 1899, he says that Kant in the first Critique does not claim that space is infinitely divisible but only that it may *not* be so divided in order to reach any least part. This definition is not, of course, more correct than the first one, but it is closer to the spirit of Peirce's idea that points are not parts of the line but limit values on it. Even later, Peirce returns to another and better idea in Kant: that continuity implies that all parts have parts of the same sort.

[494] Elsewhere, Peirce has a definition of the concept of 'perfect' deviating from Cantor's ('Notes on the Theory of Multitude', 1903?, NEM III, 1092). Peirce here comments upon Schönfliess' 'Die Entwicklung der Lehre von dem Punktmannigfaltigkeiten' in Heft VIII of the *Jahresbericht der Deutschen Mathematiker-Vereinigung* 1900, presenting Cantor's theory. In Peirce's note to page 32 he claims that 'A type (or a collection) is called secluded or concluded abgeschlossen if every fundamental series in it has its limit within it.

It is called *close* (in sich dicht) if every element is a capital element.

It is called *perfect* (perfekt) if it is both secluded and close. (That is, if you take away an element from a perfect type it will be *close* just the same but will cease to be *concluded*. But if you add an element it will still be *concluded* but will cease to be *close*. Hence we may translate *in sich dicht* by *dense* or *condensed* or *concise but as a whole*, and *abgeschlossen* by *complete*.)'

Here, perfect is defined as being close plus dense, where perfect earlier was only close, but had to be completed by density in order to yield continuity. The definitions of continuity are hence equivalent, even if the definition of perfection has changed.

[495] If you take e.g. a closed set of rationals on the number line, then the derivative set in this sense is the same closed set (because every rational is a limit element, cf. the Dedekind cut). A set of this kind is irreducible by this procedure, while on the other hand a convergence series is reducible to the number towards which it converges.

[496] The same Aristotelian idea plays a central role in René Thom's *Esquisse d'une sémiophysique*.

[497] If we go back to Cantor's definition of a perfect point, it is not far from Peirce's Aristotelicity. He defines (Cantor 1872) a 'limit point' as one of which it holds that in any neighborhood around the

point there will be an infinity of points belonging to the set, and he furthermore defines a derived set P' as that which consists of the limit points of a given set. If the derived set is identical to the set itself, then the set is perfect (Cantor 1879–84, 193), but this is only the case if it is not open (an open set's derived set is larger han itself, because it contains, in addition to the set itself, the points which might close it (which is what we today would call its limit points, NB), its end points. Already in his article about *Continuity* in *The Century Dictionary* (1889, CP 6.164), Peirce refers Cantor's definition, here with acclamation.

[498] Cf. Maddy 1990, 110, referring Cantor's definition of perfection as follows: 'A closed set of reals is one that contains all its accumulation points; a perfect set is closed, and every one of its points is an accumulation point', corresponding to Peirce's version: the former requirement grants Aristotelicity, the latter Kanticity.

[499] A truism, it must be said, which Peirce does not continue claiming. In his late philosophy with its finalist interpretation of continuity he no longer finds it strange that the future should be able to influence upon the present now (albeit only in general terms, in opposition to the past); this is the very definition of finality.

[500] It is from arguments like these that Peirce is later able to claim that his doctrine does not exclude the possibility of telepathy (even if it does not, on the other hand, positively imply nor argue for such a thing . . .).

[501] The infinitesimalist time theory very logically – but counterintuitively – leads Peirce to claim that the energy of a perceived idea in the present must be infinite, while an idea in the past has a finite and relative energy. This idea really claims a primacy of perception, cf. Peirce's theory of 'perceptual judgment' where generality is directly perceived.

[502] Cf. Holenstein 1972 on associationism in Husserl.

[503] An idea reminding of Kant's philosophy of history, according to which we can never know whether history has a goal – but we ought to act as if it had.

[504] In this, Peirce continues Kant, for whom the finitude of man prevents us from reaching the *noumenon*, which is why we must be satisfied with *phenomena*. But this delimitation so to speak incarcerates us in generalia (not in particulars, as one might think); all we can do is to predicate a substance with general predicates (but maybe even only the simpler general predicates . . .). Elsewhere, Peirce says: '. . . I had long before declared that absolute individuals were entia rationis, and not realities. A concept determinate in all respects is as fictitious as a concept definite in all respects.' (A letter to Signor Calderoni, 1905, 8.208); the idea of an object determinate through and through is rather a regulative idea which requires of us not to remain satisfied with any partial knowledge of the object. But we can not even reach any thoroughgoing knowledge of *realized* generals, Peirce here adds: they are necessarily vague.

Towards the end of the article, Peirce's speculations speed up, and in the course of the next few paragraphs we learn that the association of ideas, just like personality, has a teleological aim, because the future reference is a crucial element in both (just like in Peirce's symbol category in general). Consequently, the idea of an evolutionary universe is coextensive with the idea of a personal Creator, Peirce all of a sudden concludes – as against determinism which is always unable to make sense of any personal contact with the deity. The question why we have not long since received full clarity about this personal God is answered with reference to our unability to grasp facts right before our noses (6.162). These surprising teleological and even theological speculations are brief and do not influence the preceding overall argument. We include them here in order to show the range of Peirce's trust in the continuity concept. It may be added, that Peirce's famous personal epiphany probably took place around this time (cf. the mystic musings in the last lines of 'Man's Glassy Essence', EPI, 350–51; 6.271); theological reflections connected to the continuity concept are not generic in Peirce's later writings.

[505] The beginning of the text in question with its rich use of these terms is so sudden because it constitutes page 23 of a manuscript with its first pages missing.

[506] Later, around 1909, Peirce realizes that not only CH but also its generalized version, the general continuum hypothesis (GCH) may not hold: '. . . it has never been exactly proved that there are no multitudes between two successive abnumerable multitudes, nor, which is more important, that there is no multitude greater than all the abnumerable multitudes.' ('Some Amazing Mazes, Fourth Curiosity', 4.656).

[507] Discussing Peirce's different ideas about CH, it is important to keep in mind that he omits Cantor's doctrine of ordinal numbers. In fact, Cantor's Aleph series is constructed not as a power set series, but as the series of powers of the successive sets of ordinal transfinite numbers. These two, in Cantor, are different procedures for constructing larger cardinal numbers. Thus, Cantor's CH basically asks the question whether the cardinal series of ordinal number set powers is the same series as the series obtained by power set constructions taking Aleph-0 as its point of departure. As far as I can judge, Peirce never understood Cantor's concept of ordinal transfinite numbers and hence did not grasp this aspect of CH. This is probably why he – until late in his life – immediately identifies the Aleph series with the series obtained by successive power set constructions from Aleph-0. But this identification, in Cantorian set theory, is the whole issue of CH.

[508] While Peirce insists on the existence of infinitesimals, that is, numbers smaller than any finite real number but larger than zero, then for the infinitesimal x we may take its inverse $1/x$. This number must be larger than any finite real number, but still smaller than infinity. Since infinitesimals may assume any infinitely small size, some of these must make $1/x$ an integer larger than any given finite integer but still smaller than Aleph-0.

[509] Cantor has already found that Aleph-2 must be the power of the set of all real functions (if you take the concept of functions as referring to the formation of n-tuples between the elements of a number of sets, then the set of real functions will of course be of the same power as the set of subsets of the reals). Elsewhere, Peirce does not refrain from giving examples of Aleph-2 or even Aleph-3 sets.

[510] Peirce here writes 'Exp γ, $(\text{Exp})^2\gamma$, $(\text{Exp}\gamma)^3\gamma$'; supposedly, he intends the version given.

[511] What may have lead Peirce to believe in this proof may be found in the alternative version found in R&LOT. Here a decisive step in the argument is the following 'But now let us consider a collection containing an individual for every individual of a collection of collections comprising af collection for every abnumeral multitude. That is, this collection shall consist of all finite multitudes together with all possible collections of those multitudes, together with all possible collections of collections of those multitudes, together with all possible collections of collections of collections of those multitudes, and so on *ad infinitum*. This collection is evidently of a multitude as great as that of all possible collections of its members. But we have just seen that this cannot be true of any collection whose individuals are distinct…' (159) The way of constructing the pseudo-set S here differs from the way just presented. When a set of power Aleph-n + 1 is made a part of S, then at the same time it is supposed to give an inventory of the subsets of Aleph-n. Aleph-1, e.g., constitutes a set of subsets of Aleph-0, and it is of course correct that this is in a certain sense the case (because the decimal expansion of every single real number constitutes a set of figures which corresponds to a subset of the natural numbers). This trick now allows Peirce to claim that just like any Aleph is constituted by the subsets of the former Aleph, then by the simple sum of all Aleph-n's you get the subsets of S which will then have the same power as all of its subsets. But here Peirce does two things at the same time: he conceives of the elements of an Aleph as simple entities and as sets. For all these sets are naturally subsets of S but do they *exhaust* its subsets? Some subsets 'lack', namely those that appears by the pairing of e.g. each of the elements in Aleph-0 with each of the elements of Aleph-1. But they *are* included, Peirce will say (we must assume), because Aleph-0 can make no additions to Aleph-1. But by saying so, he no longer regards the elements as sets, but merely as elements. If we pair, e.g. 1 and $\sqrt{2}$ (1.4142…) then Peirce may say that we may just construct the element 1.14142… which is already a member of Aleph-1. That is correct, but not for the set $\{1, \sqrt{2}\}$. In that way he conceives of S's elements as sets, when that is called for, and as elements when that is needed, and this is why he can abstain from taking the exponent of the whole sum of Alephs: he so to speak divides the whole series with 2^n – and so it is no wonder he may obtain a result like $n = 2^n$.

[512] Cf. Thom's mirror idea that the concept of 'point' might yet contain secrets able to solve the continuum riddle.

[513] This lecture, as it appears, is a construct on the part of the editors; it is composed by Robin 751 (as a sort of preface) plus Robin 439 which probably is the lecture proper, because its contents correspond to what is referred to in the other lectures. In our context, this is of little importance.

[514] These signs give, by the embedding of the former in the latter, rise to the three classes of signs which Peirce in his third tricotomy calls 'rhema', 'dicisign', and 'argument', respectively, – also 'seme', 'pheme', and 'delome'. Closest to ordinary language is the names 'term', 'proposition', and 'argument'.

This analysis, moreover, constitutes the background for Peirce's own construction of existential graphs, providing the verbal, predicative, iconic part of the graphs; the demonstrative-pronoun, indexical parts; and the logical connective parts, respectively. ('Detached Parts Continued', 1898, NEM IV, 339). The first form the diagram icon proper, the second form propositions connecting that icon with indices; the third connect such propositions into inference chains and provide the possibility of controlled experiment with the diagram icon.

[515] This idea refers to Putnam's classic paper 'Mathematics without foundations' (1967, reprinted in Benacerraf and Putnam 1983) where he claims the idea that mathematics has a modal logic character so that every proposition about mathematical objects may be read as a proposition about a possible construction: 'In short, if one fastens on the first picture, (the "object" picture), then mathematics is wholly extensional, but presupposes a vast totality of eternal objects; while if one fastens on the second picture (the "modal" picture), then mathematics has *no* special object on its own, but simply tells us what follows from what.' (300).

[516] In contrast to Peirce's metaphysics, this interpretation is too constructivist. Peirce follows Cantor all the way as regards actual infinity; this is indeed the very metaphysical motivation for his position. Continuity is merely potential, but it may, in turn, be actualized as 'real possibilities' in the general phenomena of reality (that is, without any finitistic restriction is imposed on its actualized parts, which, on the other hand, seems to be Putnam's classical position).

[517] It is interesting here to note that also John von Neumann's version of set theory allows for the construction of a multitude which is the union set of all transfinite numbers and which hence exceeds their powers (his 'proper class'). In his case this class lies beyond not only the series of Aleph-n's, but beyond any ensuing series of Aleph-Alephs etc. of classical set theory.

[518] This is obviously no strong argument for Peirce's particular doctrine of a supermultitudinous set *S* (which, interpreted in standard set theory, would have the cardinality of Aleph-omega-0; all other cardinalities might as well be invoked here). But this undecidability could, more generally, be taken as an indication of the possibility that no formalism may exhaust the continuum and hence that the continuum might be 'larger' than any formalization attempt – thus supporting Peirce's modal interpretation of it.

[519] Unfortunately many of his notes on Husserl are written in the vanishing brand of Austrian stenography known as Gabelsberger shorthand and has not yet been transcribed and published.

[520] The intuition necessary for symbolic calculi – assumed to be infallible – should amount to no more than the ability of distinguishing 0 from 1 on the line, to count the length of finite symbol strings in number of units, to manipulate symbols and symbol groups on the line according to certain rules, etc. In short, this presupposes the intuition of the line, segmented in distinct places, and of the characterization of those places measured in (at least) two different possibilities. It presupposes, moreover, the parsing of the row of symbols into meaningful groups (dictated by the given symbol language), and, most complicated, the substitution for certain symbol or symbol groups by others (determined by the axioms, expressed in the same symbol language).

[521] This argument is very often stated in a strangely naive manner: because physics is unthinkable without mathematics, the two sciences are taken to be inseperable (this argument is also found in the structuralist Resnik, cf. Resnik 1997) without considering the equally possible conclusion that a unilateral dependency is at stake (so that mathematics is independent of physics even if physics depend on mathematics). This would, of course, be Peirce's position.

[522] A fine example of this is the application of (parts of) mathematical group theory in everyday perception, such as Cassirer 1944 argues. Here it is shown how a series of arch problems of the psychology of perception can be explained by reference to the group of transformations characterizing a group in group theory. Object constancy (why do we see objects having the same shape when it changes contour in different perspectives and distances?), color constancy (why do we see the object having the same color under different light conditions?) can be interpreted as referring to an invariance not affected by transformations. As is known from the Erlanger program, different geometries can be defined by each their group of transformations, so that the more tranformations are allowed, the more 'empty' and general the corresponding invariant object will become – culminating in topology, as both Peirce and Cassirer realize (cf. Chap. 5). But the variety of geometries is not – as it is often taken to be – an argument for the lack of objectivity of geometry (and, consequently, for the subjectivity of perception), but quite on the contrary an argument for the strength of the mind, possessing a differentiated series of objective

tools for the conception of space, some of which are already in play in ordinary perception. In this sense, it becomes understandable how already everyday perception is ripe with sophisticated knowledge – as well as how it is possible to isolate and sophisticate the evidence of mathematical basic objects on the basis of everyday perceptions without assuming Benacerafferian causal-reference access to such objects. In this manner, this train of thought – parallel to Peirce – runs counter to the materialist-empiristic idea that perception 'first' consists of an uncoordinated aggregate of sensory stimuli which 'later' is put in order by the subject. Corresponding to Peirce's claim for the existence of infinitesimals, perception in this perspective always treats continuous, organized phenomena – and the idea of an uncoordinated set of pure sense impressions will be relegated to a limit case never actually realized: it is an idea, not reality. Quite on the contrary, the object is always constituted as an invariance measured on different sets of transformations, and perception is never limited to a present now without any extension. This is also equivalent to Peirce to whom pure secondness, pure thisness, pure haeccity, remains a mere limit case – any event is only minimally graspable to the extent to which it is referred to a domain in some qualitative continuum, to which it is embedded in a more extended temporal process, to which it forms part of a general, lawlike space.

[523] The resume in the following paragraph of developments in the CH discussion from Cohen's 1964 till now is indebted to Chap. 4 in Maddy 1990.

[524] Gödel, in the period from 1938 to 1947, also changes his viewpoint in this direction (Maddy 134–35) and in the same way Moschovakis 1980 (610), as quoted by Maddy, takes it as an unsuitable constraint on the concept of '*arbitrary* set', because there is no a priori reason to suppose that every subset should be definable (definability by first order formulas is equivalent to being a finite union of open intervals with named endpoints).

[525] '. . . donc, elle est un fait "objectivement" scientifique, et à ce titre, elle doit être "objectivement" expliquée. Si l'on n'a pas à sa disposition une théorie immanentiste du continu dans notre physiologie neuronale, qu'on m'explique la genèse de l'illusion!' (Thom 1992, 140)

[526] Harthong, in fact, proposes four new antinomies for modern mathematical rationality (not unrelated to Kant's): randomness/determinism, continuity/discontinuity, simplicity/complexity, and finally a finite/infinite number of basic rational principles. He argues that all of these antinomies spring from two basic phenomena: 1) knowledge being approximative (corresponding to Kant's idea of human finitude, and 2) the existence of several different size scales. The latter he views as a mathematical constraint (not only a property in empirical nature, 354), because a scalar shift may let a discrete and complicated description give way for a continuous and simple description (the arch example of course being thermodynamics in Boltzmann). Naturally it is the fact that nature does present us with vast differences in size which makes us aware of this antinomy: '. . . les antinomies de la rationalité mathématique se manifestent sur de très grandes échelles de grandeurs; elles n'existeraient pas si les grandeurs existant dans la nature étaient toutes du même ordre.' (363) To Harthong, the possibility of ever making CH decidable vanishes for exactly this reason: 'Le débat reste métaphysique tant que la plus petite des deux échelles de grandeurs reste *inaccessible* á la perception (. . .) Les lois "utiles" sont des lois phénoménologiques qui marchent à l'échelle où nous observons.' – he pragmatically concludes, polemically aimed against reductionist brain science and its assumptions that the final neuronal explanation of the mind is close at hand.

[527] In Salanskis' Continuity volume, Robinson's non-standard analysis plays a constant role because it reintroduces infinitesimals – even if it takes place on the base of orthodox set theory, as non-standard points in a halo around the standard points of **R**, appearing in external and internal sets, respectively. Yet, this 'galaxy' of non-standard points have, as Salanskis says (199) the effect of expressing the 'richness' of the continuum which is not present in standard analysis – this especially so in the so-called Harthong-Reeb version of non-standard analysis.

[528] '. . . ce qui apparaît ainsi dans l'ensemble de ces travaux de non standardistes reprenant la tradition borélienne, c'est que le continu est quelque chose comme un "effet" qui résulte d'un *excès*. Si l'approche de l'aspect "désensemblisation" du destin du continu de Cantor-Dedekind met en vedette l'*incommensurabilité*, la prise en compte du regain d'attention envers le constructif et sa place dans le monde des entités mathématiques nous fait rencontrer le thème philosophique de l'*excès*. Le lien entre les deux termes, cela dit, nous semble clair: l'excès engendre de l'incommensurabilité, et la distribution de l'incommensurabilité sur la droite entière lui ajoute la *qualité* constitutive du continu (. . .)' (210)

[529] Petitot's contribution to Salanskis 1992 is complemented by the programmatical paper 'Pour un platonisme transcendantal' (1994).
[530] Namely this: if anything exists as a psychological primitive, this phenomenon is eo ipso also natural (pace naturalism in psychology) and has reality which is not merely psychological, se the paragraph on Maddy.
[531] 'Le platonisme transcendantal est "négatif", et non pas "positif" comme le platonisme ontologique vulgaire. Il permet *d'inverser* les affirmations philosophiques de l'antiplatonisme *et de transformer les phénomènes d'incomplétude et d'indécidabilité en arguments en faveur du platonisme.'* (254)
[532] The axiom of determinacy states that if there is a game between two players, each in turn selecting successive nested subsets of the unit interval, then there is a strategy for one of the players to let the selecting process terminate in a number belonging to a given subset of the unit interval.
[533] 'Evidemment, demeure entière l'énigme de l'intuition pure, c'est-à-dire de la façon dont nous sommes affectés par l'exteriorité. Ce problème n'est pas d'abord mathématique mais *physique et cognitif*.' (261)
[534] Among many possible quotes from his work see e.g. *Physique du sens* from 1992 where the introduction claims that the morpho-mathematical foundation of structuralist models necessitates a critique of the formalist position regarding language and meaning: 'Ce dernier point pourra paraître irrecevable aux linguistes et sémioticiens qui croient encore, suite aux dogmes du positivisme logique, que la conception métamathématique hilbertienne (c'est-à-dire le rapport entre syntaxe et sémantique que l'on trouve en théorie des modèles peut être transférée à l'analyse *des rapports de dépendance* constitutifs des structures.' (XXII). The fact that this critique does not aim at the semiotic domain only is evident from, e.g., 53 where, following Cavaillès, it is claimed that in any doctrine of objectivity, formal logic must be connected to an ontology: 'Ses concepts doivent acquérir un contenu transcendantal – c'est-à-dire concertant l'*objet* de la connaissance et non pas simplement sa *forme discursive* – et, selon Kant, ils le peuvent si les catégories prennent pour "matière" le divers pur (spatio-temporel) de l'Esthétique transcendantale (schématisme).' Also mathematics has a content to be schematized (cf. the schema 55).
[535] For a prototypical articulation of this point of view, see the influential American philosopher of mathematics Saunders MacLane's (the father of category theory) antiplatonist doctrine *Mathematics. Form and Function* (1986), 385: 'These results [the proof of the independence of CH, AC, and other axioms from set theory], and others too numerous to mention, show that many interesting Mathematical questions cannot be settled on the basis of the Zermelo-Fraenkel axioms for set theory. (. . .) This variety and the undecideability results indicate that set theory is indeterminate in principle: There is no unique and definitive list of axioms for sets; the intuitive idea of a set as a collection can lead to wildly different and mutually inconsistent formulations. On the elementary level, there are options such as ZFC, ZC, ZBQC or intuitionistic set theory; on the higher level, the method of forcing provides many alternative models with divergent properties. The platonic notion that there is somewhere *the* ideal realm of sets, not yet fully described is a glorious illusion.' MacLane later turns the argumentation directly against Gödel and his 'realist view' (449).
[536] This implies, of course, problems to which we shall later return: the metaphysics (1) which operate on the threshold between the many and the one Platonic world, how is that to be situated in relation to that metaphysics (2) which constructs the four-part scheme above and consequently must be more extensive than mathematics?
[537] I think an account in a Peircean spirit would claim that the hierarchy of prescission and abstraction (Chap. 11) allows us to reason with infinite procedures, because they are here taken as objects and subjected to imaginary completions.
[538] An example: 'Pure Mathematics is the study of pure hypothesis regardless of any analogies they may have in our universe.' ('Lectures on Pragmatism' 1903, NEM IV, 149)

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(E. Husserl and C. Peirce are not included)

SUBJECT INDEX

(much-used terms such as diagram, phenomenology, ontology, semiotics, etc. are not included)