Notes

Introduction

1. For a survey, see Rosen (2001).
2. Throughout the book we will use the term 'arithmetic', without further qualification, to refer to the theory of natural numbers.
3. See, for instance, Dummett (1978b).
4. In what follows we will use, unless otherwise specified, the adjective 'true' *tout court*, and its derivatives, implicitly assuming that statements considered as true are not vacuously such, or not only vacuously such (when appropriately interpreted).

1 The Origins

1. This change is all but explicit. However, we do not see how Plato's metaphor could be interpreted differently, especially in light of what has been said above concerning *Theaetetus*, *Meno* and *Phaedrus*.
2. Plato is here using the term '*logismos*'. A more literal translation would render it as 'calculation'. We prefer, however, to translate it as 'logistic' since it seems to us that Plato is here not merely referring to a mathematical procedure or practice, bur rather to a branch of mathematics, namely to that branch that he elsewhere calls '*hē logistikē*' (or '*hē logistikē technē*'), a term that we shall render as 'logistic' too. Logistic was the art of calculating with numbers, and was distinct from the science about them. We will come back to this point in what follows.
3. The terms '*dianoia*' and '*nous*' occur elsewhere in Plato's dialogues, and with different meanings. For example, they occur in the passage quoted above from the *Phaedrus* (247c–d), where they are rendered, respectively, as 'mind' and 'intelligence'.
4. Here too, Plato insists on the indispensability of earthly knowledge (342d–e): "For unless a man somehow or other grasps the four of these, he will never perfectly acquire knowledge of the fifth."
5. Plato uses '*nous*' here, but with a different meaning in respect to the meaning that the term has in the aforementioned metaphor of the divided line. See also *Timaeus*, 51d–52a.
6. This conception will be codified in definitions VII.1 and VII.2 of Euclid's *Elements*.
7. This second interpretation is close to the classic interpretation given by Jakob Klein (1934–6, pp. 17–25) who, however, translates the relevant passage differently from us, having "with reference to *how much* either happens to be" where we have "with the odd and the even, however many they are".
8. Technically, a progression is the structure common to every system of elements that are linearly ordered and include a first element relative to this

order, starting from which they can all be reached by progressing according to the order.

9. Aristotle is using here the plural neuter *ta mathēmatika*. Leaving aside the unorthodox literal translation of this expression as 'mathematical things', we remain with two other possible translations – 'mathematical objects' and 'mathematical entities' – among which we select the former, which squares nicely with modern terminology.
10. His confutation of option (*i*) hinges on various arguments presented at the beginning of the second chapter of Book XIII, which in turn take up those presented at the end of the second chapter of Book III.
11. In *Physics*, Aristotle is crystal clear that time – defined as "number of movement in respect of the before and after" (219b 2) – exists only in so far as movement (218b, 21) and the soul that enumerates it (223a, 21–9) exist.
12. This reading of Euclid's *Elements* is suggested in Panza (2012).
13. Proclus alludes to this argument in passing, however, already in the *Commentary*'s first Part (Chapter VI, 13, 9–11).
14. We use here on purpose the somewhat vague term 'thought'. A proper account of how Aristotle conceives of the higher faculties of the soul would take us too far afield.
15. With 'sensibility' we mean, here and in what follows, the faculty of having perceptual sensations.
16. Points would need a separate treatment, but this would not raise any further difficulty.
17. For a more detailed exposition of what is in this section, see Panza (1997). Among the many expositions of Kant's philosophy, see Cassirer (1918–21) and Strawson (1966). On Kant's philosophy of mathematics, see Friedman (1992).
18. As a consequence, knowledge is not, for Kant, true and justified opinion; it is rather a state of mind involving concepts and intuitions.
19. This does not seem to require any special explanation as long as geometrical schemas are concerned. As regards arithmetical ones, consider that, according to Kant, there is a concept for each number, and not only the concept of number, unit, even and odd. If there has to be multiplication of entities at all, this occurs at the level in which concepts, and not schemas, are generated.

2 From Frege to Gödel (Through Hilbert)

1. Where a and b are two real numbers, a complex number is a binomial $a + ib$, where i is the square root of –1 (namely, it is such that $i^2 = -1$). A suitable definition of reals yields thus a suitable definition of complex numbers.
2. For a comprehensive exposition of Frege's philosophy of mathematics, see Dummett (1991). General introductions to Frege's philosophy can be found in Dummett (1973), Kenny (1995).
3. The acronym 'FA' (for 'Frege arithmetic') is used today for a different theory, which will be discussed in § 5.1.
4. All we shall say on Frege in the present section is referred to the first part of *Grundgesetze*, §§ I.1–I.52.

5. A formal system is generally also called 'formal theory'. By the conventions presented in the *Preface*, however, a theory is a system of statements; it does therefore include neither a language nor deductive rules, even though its statements are obviously expressed in a certain language and can be derived by certain rules. A sentence of a given language (be this formal or not) provides a way for expressing a statement, but it is not necessary that it does. It can simply be taken as a concatenation of symbols or words in a given language. This is common in logic, where sentences of a formal system are studied as such and for their deductive relations, independently of any other function they might happen to have. Strictly speaking, by our terminological conventions, a formal system should not be taken as a theory, even though it includes (or might include) a (formal) theory.
6. Today, we have non-axiomatic formal systems in which the role of axioms is played by suitable inference rules, namely introduction and elimination rules. In our presentation we will only consider axiomatic systems, however.
7. Notice that interpreting a sentence is not *ipso facto* equivalent to turning it into a statement: it does not entail that the force of asserting something is attached to it, but only that it has been fixed what the sentence would assert were that force to be attached to it. On the other hand, a statement can be distinguished by the sentence that expresses it even if no interpretation has been fixed. Thus one might assert, for instance, that 2 + 3 = 5, by uttering the sentence '2 + 3 = 5' with assertoric force, without having established yet what the terms '2', '3', and '5' mean. If assertoric force is attached to a sentence, this turns it into a statement. If the sentence is interpreted, then also the statement counts as interpreted; it thus asserts something specific, and can be either true or false.
8. Things are by far less clear for the extension of this system presented in the second volume for the purpose of defining real numbers. We shall come back to this point in § 2.1.4.
9. As is well known, and as we shall see later, Frege's whole system is inconsistent and this is clearly enough to deny that is a logical system. The relevant question lies elsewhere, however, since different ways of emending Frege's system so as to free it from (apparent) inconsistency are known (and will be mentioned later). The relevant question is whether the system so emended can or cannot be taken as logic. And this is where the real controversy lies. We shall come back to this question in § 5.1, where we shall consider a way of recasting Frege's definition of natural numbers whose proponents (known as 'neo-logicists') take to remain within bounds of logic (or analyticity, at least).
10. We will use modern notation, in order to simplify the exposition. This allows us to simplify things significantly by avoiding considering the function —ξ, which plays a crucial role in Frege's system, since it is supposed to occur as a component in five of the seven fundamental functions we shall consider here (namely negation, implication, the two universal quantifies, and the value-range). Frege calls this function 'horizontal', and defines it by stating that its value is **T** for **T** as argument and **F** otherwise. Avoiding considering this function prevents us entering into some important details concerning the nature of Frege's formal language and the way he understands it.

On this and other matters concerned with Frege's formal language, we refer the reader to Landini (2012).

11. More precisely, the second-order quantifier should be denoted by '$\forall P[\mu_\xi(P(\xi))]$', where the subscript in 'μ_ξ' indicates that in $P(\xi)$ the symbol 'ξ' is not a placeholder for empty places for objects: the argument of the functions over which '*P*' ranges could for instance be held fixed, like in '$\forall P[f(P(\Gamma))]$', or bounded by another quantifier, like in '$\forall P \forall x[f(P(x))]$'.
12. Frege's use of the symbol '|—' should not be confused with the modern use of the '⊢', which indicates that the formula following the symbol has to be proved, or is a theorem.
13. Frege considers the symbol '|—' as composed by '|' and '—' and takes the former (which he calls 'judgement-stroke' to express the act of assertion and the latter to designate the function horizontal (see note 10 above). It follows that, for Frege, this function occurs in any statement.
14. So far, we have only talked about names of objects. Frege also believes, however, that functions have names, like '$f(\xi)$'. The difference between the two cases is crucial, but we cannot pause on similar aspects of Frege's philosophy of language, despite their relation to his philosophy of mathematics. The reader might find various texts treating these issues, among which we recommend Dummett (1973).
15. Following Quine, some believe that second-order logic is not properly logic, but rather set-theory in disguise, and are thus not disposed to concede that the fifth of Frege's fundamental functions presented above corresponds to a logical constant. The reason is that for Quine, second-order variables should be taken as ranging over sets (whose elements are individuals over which first-order variables range, or couples, etc. of individuals). Beside Quine's, there are several possible interpretations: see Quine (1970, pp. 66–8), Boolos (1975, 1984, 1985), Linnebo (2003, 2008), Shapiro (2005b).
16. For reasons that will be clear later, it is important to notice that Frege stresses the analogy between two of these axioms by denoting them with the same roman numeral. These are Laws II*a* and II*b*. The three others are Laws I, III, and IV (the latter of which is actually redundant).
17. By 'implicit definition' one generally means a definition that is obtained not by laying down what is the *definiendum*, or by introducing a convention that stipulates how some words or symbols should be understood, but rather by laying down certain conditions that *definiendum* must satisfy. When it is taken as a definition of the *Num.*(–) operator, HP is an implicit definition, since it just stipulates an identity condition for the values of this operator. An operator is just a suitable function: if it is applied to certain entities taken as its arguments, it delivers other entities as values. In § 5.1 we will explain why the operator implicitly defined by HP does not only deliver natural numbers, but cardinal numbers more generally. In a formal language, an operator is thus designated by a functional constant.
18. Kant has a similar definition of the extension of a concept also in the *Logik Jäsche* (§ 1.7). In both cases, he contrasts the extension of a concept to its "content", namely its characteristic marks. In *Logik Blomberg* (§ 204) he defines in a similar way the *sphaera* of a concept, and then appeals to the notion of extension in order to make is definition clearer: "The multitude of things . . . that I can think under the *conceptus communis* constitute the *sphaera conceptus*".

19. Notice that Frege agrees with Kant that geometrical truths are synthetic *a priori* (§ 89).
20. For more on what follows in this section, see Dummett (1991, pp. 201–8).
21. See note 16.
22. According to modern notation and conception of predicate logic, BLV should rather be written as follows: '($\grave{\varepsilon}[F(\varepsilon)] = \grave{\varepsilon}[G(\varepsilon)]) \Leftrightarrow \forall x[F(x) \Leftrightarrow G(x)]$'. Here '$F$' and '$G$' are (schematic) predicate letters, and so '$\grave{\varepsilon}[F(\varepsilon)] = \grave{\varepsilon}[G(\varepsilon)]$', '$F(x)$' and '$G(x)$' are sentences, rather than names of objects, which motivate the use of '$\Leftrightarrow$' instead of '='. This is due to the abandonment of Frege's idea that truth-values are objects, that is, items of the same type of those denoted by individual constants.
23. We are used today to use the term 'Peano axioms' to refer to different axiom systems, among which some are first-order and some are second-order (respectively called 'first-order Peano axioms' and 'second-order Peano axioms'). In the former case, these systems include an axiom schema, replacing the second-order induction axiom, and contain thus, properly speaking, an infinite number of axioms. Frege clearly proves them in a second-order version.
24. On the notion of logic in Kant and Frege, see MacFarlane (2002).
25. Just to give an example of how Frege is reasoning here, let us see how this proof works for the function $\xi = \zeta$. If we take both the arguments of these functions to be value-ranges of certain functions, then BLV allows us to determine whether these functions have the same value-range or not. If yes, the function takes **T** as value, if not it takes **F** as value. If we take an argument to be the value-range of a certain function, and the other a truth-value, then the functions take either **T** or **F** as value according to whether the latter has been supposed to be the same value-range as the former or not (that is, according to the foregoing supposition, whether the former is the same value-range as that of $\xi = (\xi = \xi)$, if the latter is **T**, or whether the former is the same value-range as that of $\xi = \neg\forall x[x = x]$, if the latter is **F**, or not). If we take an argument to be the value-range of a certain function and the other an object other than a truth-value, then the function takes **F** as value.
26. Today, we would not say that a name of function has a reference, unless we were willing to claim, *contra* Frege, that a function is an object. On the problems related to the extension of the notion of reference to constants different from singular terms, see for instance Dummett (1973, ch. 7) and Wright (1998c).
27. A recursive criterion is a criterion that applies to some cases if and only if it is established whether other cases pertaining (in some sense to be specified) to a lower level satisfy it. It thus applies only to cases satisfying a hierarchy of levels, and requires a starting point, or basis. The same can be said for any recursive rule or stipulation. A typical case of recursive definition is the following definition of natural numbers: 0 is a natural number; if n is a natural number, then $s(n)$ is a natural number; nothing else is a natural number. If we have introduced the function $s(–)$, called 'successor function', then it follows from the first and second clause that $s(0)$ is a natural number, and then that also $s(s(0))$ is a natural number, and so on, by the third clause, for all and only natural numbers.
28. On the source of the contradiction, see also Boolos (1993) and Dummett (1994).

29. Russell generally talks of classes, but he distinguishes between "class as many" and "class as one", a distinction that closely resembles the modern one between class and set. More on this issue can be found in any good introduction to set theory, such as Quine (1963), Suppes (1972), Potter (2004, especially Appendix C). Here, we will use 'set' in its current meaning.
30. But see note 15 above.
31. This idea echoes Frege's context principle, but it is radically different from that expressed by it: Frege maintained that the meaning of words should be found in the context of the statements in which they occur, but he did not claim that they have a meaning only in such contexts, or even worse that some words could contribute to the formation of meaningful statements despite their being meaningless.
32. What follows is merely a rough discussion of the rise of set theory. Many detailed texts are available though, like Hallet (1984) or Ferreirós (1999). Cavaillès (1938) is a classical exposition.
33. If we take a finite set, it is easy to see that if it has n members, the set of its subsets has 2^n. *Mutatis mutandis*, this holds for infinite sets too. If α is the cardinal of an infinite set, the cardinal of the set of its subsets is 2^α.
34. On the notion of progression, see Chapter 1, note 8. Here it is enough to notice that a set satisfies second-order Peano axioms if and only if it forms a progression under a successor relation which is appropriately defined on it (at least insofar as these axioms are not taken to include axioms for order, addition and multiplication, as it is sometimes the case).
35. Actually, the issue is far more complex: the precise relations between this model and Peano axioms depend on the way in which these latter are formulated. If they are formulated in a second-order language (thus being finite in number), they have a single model, up to isomorphism (namely, any two distinct models of these axioms are isomorphic to each other: there is a bijection among their elements preserving all the relations that make them count as models of the relevant axioms). When this happens, a theory is said to be categorical. If they are formulated in a first-order language (thus being infinite in number since they necessarily include an induction axiom schema: cf. note 23 above, not all of their models are isomorphic: some (called 'non-standard') include some additional elements. This is a consequence of the upward Löwenheim-Skolem theorem. Here are some logic manuals in which these issues can be explored further: Mendelson (1964), Kleene (1967), Shoenfield (1967), Boolos and Jeffrey (1989), Ebbinghaus, Flum and Thomas (2004). On model theory, see. Chang and Keisler (1973), Hodges (1993), Rothmaler (2000).
36. See § 2.5. For more on this, see Cohen (1966), Woodin (2001a, 2001b), Kanamori (2004).
37. We cannot but give a very general formulation of this tenet here, but we hope it will suffice for a sufficiently clear idea of the point at issue.
38. The presence of paradoxes will of course be unacceptable even from Carnap's point of view. The logical point then lies in the relation between impredicative definitions and the paradoxes. Poincaré (1906) saw in the former the main cause of the latter; we now know that many impredicative definitions are perfectly consistent (an instance is HP, used for a definition of natural numbers: see § 5.1). For more on this, see Chihara (1973), Heinzmann (1985), Feferman (2005).
39. But see § 4.2 and § 7.2.4.

40. Some of Hilbert's and Bernay's articles can be found in Hilbert (1932–5, vol. III). Translations in collections can be found in van Heijenoort (1967), Largeault (1992), Ewald (1996), Mancosu (1998). The most mature version of Hilbert's and Bernays' foundational proposal is in Hilbert and Bernays (1934–9). Many of the things we will say in what follows do not square with the ideas presented in this latter book, where finitary mathematics is seen as a formal theory. Our exposition will be closer to earlier versions of that proposal. The difference between the two approaches largely depends on a change in the conception of intuition supposedly involved in finitary artitmetic. On this, see Mancosu (1998), pp. 149–88). On Hilbert's programme, see Giaquinto (1983) and Detlefsen (1986).
41. Hilbert makes use of inverted commas neither around his number-signs nor around the signs he uses for the purpose of communication. For the former, this could be justified by the fact that these signs are not supposed to denote anything else but themselves, with the result that the object 1 + 1 is here the same as the object '1 + 1'. But for the latter, things are different. In this case, we prefer then to introduce inverted commas according to the conventions widely accepted today.
42. According to Parsons' terminology (see Parsons 1998, p. 252; 2008, pp. 241–2), this means that 'concrete' should here be taken as synonymous with 'quasi-concrete' (see § 5.4).
43. Today we know, thanks to Gödel's theorems themselves, that this prevents T from being a second-order theory satisfying other requisites that are generally required by a good mathematical theory.
44. Gödel actually proves a slightly weaker theorem than this. The stronger version we have stated has been proved by John Barkley Rosser (1936).
45. Brouwer (CW) contains Brouwer's original works (with English translations of Dutch texts). Translations can also be found in many collections (see also note 40). Among the many works on Brouwer, see Stigt (1990), van Dalen (1999–2005) and van Atten (2004).
46. Let us take variables to range over the relevant objects. If one wants to prove '$\exists x(F(x))$', for some property F, one proves that the negation of this statement, '$\neg\exists x(F(x))$'(which is equivalent to '$\forall x\neg F(x)$'), leads to contradiction, from which '$\neg\neg\exists x(F(x))$' (or '$\neg\forall x\neg F(x)$') follows by *modus tollens*. By double negation elimination, one then gets the statement to be proved.
47. On this, see Tieszen (2005) and van Atten (2006). On Husserl's philosophy of mathematics, see Tieszen (1995) and Haddock and Hill (2000).
48. Gödel provides a model of ZF (provided by the proper class L of constructible sets), and shows that it satisfies the Axiom of Choice and the Continuum Hypothesis. Clearly, all we will say here about CH is relative to classical mathematics.

3 Benacerraf's Arguments

1. Other accounts of the debate which will occupy us in the following chapters can be found in Engel (1995), Shapiro (2000a, pp. 281–9), Bostock (2009). For more detailed discussions, see the essays in Shapiro (2005a).
2. Benacerraf distinguishes between what he calls 'transitive' counting, by which objects are counted, and what he calls 'intransitive' counting, consisting in

uttering the names of natural numbers in their right order. According to Benacerraf, one cannot count transitively if one hasn't mastered intransitive counting, since to count transitively consists in associating objects to the names of natural numbers uttered in their correct order.

3. Benacerraf (1965, p. 51, note 3) acknowledges that not everyone concedes that the possibility of defining the usual arithmetical operations on the elements of a progression, and that of using these elements in counting objects, are necessary conditions for taking these elements to be the natural numbers. Many believe (Benacerraf mentions Quine, 1960, § 54) that natural numbers can be identified with the elements of any progression whatsoever. This point is relevant under many respects, but it does not affect the argument we are discussing, since for the latter it is enough that Zermelo's and von Neumann's progressions are considered as adequate. Benacerraf (1996a, Appendix; 1966b) will later admit that the decidability of the ordering relation on the elements of a progression is not essential, arguing that it is always possible to define a decidable ordering relation on a progression. This issue is not settled, however: see, for example, Halbach and Horsten (2005), who maintain that the decidability condition is essential.
4. See note 3.
5. On the various theories of truth, see for instance Künne (2003); Burgess and Burgess (2011).
6. For general discussions on the epistemological challenges to platonism raised by Benacerraf's dilemma, see for instance Wright (1983, §§ 11–12); Hale (1987, ch. 4); Lewis (1986, pp. 108–15); Field (1989a, pp. 25–30), (1989b, § 2); Burgess and Rosen (1997, pp. 35–60); Divers and Miller (1999), Cheyne (2001), Liggins (2006, 2010b), Linnebo (2006), Callard (2007), Azzouni (2008), Kasa (2010).

4 Non-conservative Responses to Benacerraf's Dilemma

1. As a matter of fact, Field (1984) suggests that it is still possible to speak of mathematical truth even without denying that knowledge requires truth. However, mathematical knowledge is not given, according to him, by the theorems of mathematics (as complying with clause (*i*)); it is, rather, either knowledge that certain theorems have some (modal) logical properties, or knowledge of the fact that some mathematicians accept certain theories or assumptions.
2. Granted that a nominalistic statement is a statement formulated in a nominalistic language, nothing prevents it being necessarily true. If '*t*' is a term of such a language, the statement 'For any *x*, if *x* is identical to *t*, then it is identical to *t*' is a case in point, and also is a consequence (given any minimal logic) of any theory some of whose axioms are statements in which '*t*' occurs. If this happens with a bridge-law of a nominalistically impure mathematical theory, the statement is a consequence of this theory.
3. One could doubt that it is possible, given this condition, to have nominalistically impure mathematical theories that have nominalistic consequences non-necessarily true. Still, Field (1982, pp. 48–9) gives an example of such a theory. He shows how to obtain a (consistent) version of set-theory, allowing for "sets with non-sets as members" and for the existence of "a set

of all non-sets", where the axiom schema of separation and replacement admit of instances involving "the empirical vocabulary that is ordinarily used to describe non-sets", and in which the axiom of infinity is dropped and replaced by its negation. This theory would have "consequences about the physical world that conflict with most current physical theories", and are then, obviously, non-necessarily true. But notice that the "most current physical theories" to which Field refers here are theories implying – or so understood as to imply – the "existence of an infinite discrete linear ordering of non-mathematical entities". His point is, indeed, that in such a theory, together with the described version of set-theory, one could "define a function symbol mapping some *x* in the ordering into 0, its immediate successor into 1, the successor of that into 2 and so on", which, together with the existence of a set of all non-sets and the replacement axioms-schema extended in the mentioned way, entails "the existence of a set containing 0, 1, 2 and so on, in violation of the negation of the axiom of infinity". One could have, however, good reasons for arguing that such a "physical theory" – or at least such a theory so understood – is not genuinely physical, after all.

4. This would be the case, for example, with the version of set-theory mentioned in note 3, if it were taken to be a pure mathematical theory, namely if the non-sets that are allowed to be members of sets were taken to be some sort of mathematical entities (under the supposition, of course, that some mathematical entities could be non-sets).
5. Intuitively speaking, an uninteresting (mathematical) theory is certainly not good, whatever condition it might meet. Still, Field's limitation to interesting mathematical theories has also a technical aim: we shall come back to this in § 6.3.
6. In order to understand why this condition is equivalent to the preceding one, one can reason as follows. Suppose that ¬φ is a nominalistic, non-contradictory, statement, and that M+φ is inconsistent. Then, ¬φ would be a consequence of M that is not necessarily true (since φ is not contradictory), and thus M would have nominalistic consequences that are not necessarily true. It follows that ¬φ would be a consistent nominalistic statement, whereas M+¬φ would be inconsistent.
7. In Field (1980) the conservativeness of mathematics is not defined as we have defined it earlier following Field (1982), but rather directly through the condition just stated. Field (1980, pp. 12–13) thus has a slightly modified version of the first part of the argument given earlier to show that "our mathematical theories" are conservative, and presents (*ibid.*, p. 16–19) two other more technical arguments, specifically addressing the case of set theory (which is considered as a mathematical theory *par excellence*).
8. Notice that the issue here is whether these terms and variables can be eliminated from a not purely mathematical language. The easy solution adopted here cannot thus serve to define natural numbers as such and to give an extra-mathematical foundation for arithmetic.
9. Field (1980, p. 21) defines these numerical existential quantifier on the basis of other quantifiers: $\exists_{\geq i}x[F(x)]$, to be read as 'There are at least *i* *x*s that are *Fs*'. It is easy to pass from one definition to the other; we give only one of these for the sake of simplicity.

10. Field (1985) contains replies to Shapiro's objections (see also Field, 1984, for more on the second-order issue), but these do not have the last word on the matter.
11. In the sense that is relevant here, the language of mereology is based on a fundamental relation, that of (being) part of, that gets applied to concrete objects. If these objects do not have parts, they are called 'atoms'. A concrete object that has parts is said to be the 'mereological sum' of its parts. These latter can be either atoms or other mereological sums. A mereological sum is thus a concrete object constituted by its parts (atoms or sums), these being concrete in their turn. These parts need not necessarily be contiguous in space (we can thus have the mereological sum of Mount Everest and Big Ben). This language clearly involves quantification over concrete objects only (atoms or sums). For an introduction, see Varzi (2009).
12. Burgess and Rosen (1997, § II.A) show how this argument could be carried out in detail.
13. The distinction was originally suggested by Burgess (1983), refined in Burgess and Rosen (1997, pp. 6–7). For a more detailed exposition, see Burgess, Rosen (2005, pp. 515–18). For discussion, see Chihara (2005, pp. 483–9) and Hellman (1998).
14. Burgess and Rosen interchangeably use expressions like 'content', 'what a statement says' and 'meaning'. In order to avoid undue qualifications, we will only use the term 'content'.
15. We say 'propositional attitude' in order to make clear that the term 'attitude' is used so as to denote a propositional mental state. But we do not, nor do the authors considered here, want to imply that attributing a particular propositional attitude entails the existence of propositions as abstract objects.
16. Hellman (1998, pp. 342–4) has argued against Burgess' and Rosen's distinction, claiming that it is not generally among the aims of nominalists, not even those qualifying as revolutionary, to suggest that nominalistic versions of mathematical and scientific theories should replace traditional ones in practice. Nominalists would rather be putting forward theories on the epistemology of mathematics fully compatible with the actual use of traditional theories.
17. Yablo seems to have modified his views lately, elaborating suggestions presented in Yablo (2006, 2009). We present them in the form in which they have been discussed until recently.
18. See Rayo, Yablo (2001).
19. See, on this, McGee (1997, pp. 56–62).
20. Though Yablo (2002, pp. 179–85) suggests how to extend his view concerning the real content of mathematical statements to set-theory, it is not clear whether he can claim that the real content of set-theoretical statements is given by logical truths, nor if this can be claimed for any mathematical theory.
21. Yablo has suggested in unpublished work that a statement like 'There are infinite prime numbers' should be interpreted as 'There are natural numbers, and they are such that there is an infinite number of them which are prime'. 'There are infinite prime numbers' should then be considered analogously to a physical law like 'Objects with no net force on them move with constant

velocity' (to be rendered as 'There are bodies with no net force on them and they move with constant velocity'), and its real content would express a condition that natural numbers would satisfy, if they existed. But what is this condition, if it is to be external to arithmetic?

22. For a defence of Field's programme on similar lines, see also Liggins (2006).
23. Leng (2010) defends a fictionalist view that closely resemble Yablo's. Leng also has detailed discussions of many issues related to recent debates on mathematical fictionalism.
24. The first component is called 'hypothetical' since the translation scheme transforms any statement of a given mathematical theory in a modal conditional statement. By 'hypothetical component' one can mean, depending on the context, both the component of the overall interpretation procedure, or, for any specific modalized version of a theory, the collection of statements translated in accordance with that procedure.
25. To these assumptions the axioms of second-order logic must be added (where the comprehension schema is appropriately modified in modal terms, see Hellman, 1989, p. 24; 2005, pp. 553–4), together with those of the S5 system of quantified modal logic, without the Barcan formula (for details, see any good introduction to modal logic, such as Hughes and Cresswell, 1996).
26. To be precise: neither of actual objects, nor of possible but non-actual objects, the so-called *possibilia*. These latter are avoided thanks to a particular restriction of the second-order comprehension schema (Hellman, 2005, pp. 553–4).
27. Parsons is commenting on a suggestion by Putnam (1967) on the possibility of interpreting set theory nominalistically over concrete domains, a suggestion that Hellman (1989, p. 71) recalls, before offering a modal structuralist interpretation that is not nominalist (1989, ch. 2). Parsons' objection can however be addressed to Hellman's modal structuralism.
28. This seems to go against Zalta's idea (see § 5.2) that an abstract object is an object that is not concrete in any possible world, an object that could not exist for a nominalist.
29. For discussions on the various kinds of modality, see Plantinga (1974), Kripke (1980), Lewis (1986), Hale (1999), Fine (2005), Mackie (2006). For various uses of modality in mathematics, see Putnam (1967), Field (1984, 1989b), Parsons (1983b), Hale (1996b), Chihara (2008), Leng (2010, § 3.2), Friedman (2005).
30. For more on this point and its consequences for the interaction between cognitive sciences and the philosophy of mathematics, see Doridot and Panza (2004). For a similar objection, see Parsons (2008, p. 211).
31. Maddy has subsequently rejected the indispensability argument (see §§ 7.3.1 and 7.4.3). In her 1997 work (p. 152, note 30), Maddy still takes it that set theory has cognitive grounds, but claims for set-theoretical platonism no more on the basis of an argument as that just presented, but on the basis of her conception of naturalism (see § 7.3.1). Recently, Maddy has also argued for a form of mathematical realism that leaves unsettled whether platonism about sets is true (see § 7.3.1.1); according to her latest views, her argument for set-theoretical platonism on cognitive grounds could turn out to be "unnecessary" (Maddy 2011, p. 72, cf. also p. 118).
32. Two simple examples might help appreciating the difference: both the notion of empty set and the distinction between a set *E* and the sets

$\{E\}$ and $E \cup \{E\}$, which are crucial in set-theory, have no clear analogue in any system of concepts suitable for an appropriate account of perceptual experiences.

5 Conservative Responses to Benacerraf's Dilemma

1. Several essays by Hale and Wright are collected in their (2001) work. See also Hale (1987); Hale and Wright (2002); Cook (2007). For a survey, see MacBride (2003).
2. This name is common today, and comes from a passage in Wright (1983, p. 51). For a more detailed formulation, see MacBride (2003, p. 108).
3. A relation R is an equivalence relation if it is reflective, symmetric, and transitive, namely if for every x, y, and z, respectively: $R(x,x)$; if $R(x,y)$, then $R(y,x)$; if $R(x,y)$ and $R(y,z)$ then $R(x,z)$.
4. Zalta (PM) presents a version of the theory updated to 1999.
5. Using a typed language not only ensures greater rigour, but also allows that what we will later say about mathematical objects can be said about higher-order entities, such as monadic properties, relations, etc.
6. It is important to understand that the sheaf of properties of which Nessy is a reification is not given by the two properties of being a monster and living in Loch Ness, but only by the property of being a monster living in Loch Ness and by all properties, if any, that an object has if it has that property. Taken separately, the two properties of being a monster and of living in Loch Ness do not satisfy the formula '$\forall y[P(y) \Leftrightarrow [Mst(y) \wedge LcNs(y)]]$', indeed.
7. In the philosophy of language, by 'proposition' one usually means the content of a statement or what a statement expresses. In mathematics the same term is used instead to denote theorems or, more generally, statements, and this usage is for the sake of simplicity often adopted also in the philosophy of mathematics as well. Here the term is used with the former meaning.
8. The term 'second-order Peano arithmetic' is used to denote different variants of second-order arithmetic, all involving a finite number of axioms suggested by those advanced by Peano himself in his seminal work (1889). In the variant considered by Zalta, these axioms are five and include a monadic predicate constant designating the property of being a natural number (this is necessary, since the theory has to be immersed within Object Theory, and the objects the former characterizes must then be distinguished from other possible objects of the latter; in other presentations, this constant can be avoided, since no individuals distinct from natural numbers are involved) and no dyadic functional or predicate constant used to designate addition, multiplication and order (which makes this variant merely characterize the structure of progression; see Chapter 1, note 8).

6 The Indispensability Argument: Structure and Basic Notions

1. On the applicability of mathematics, see Steiner (1989, 1998, 2005), Pincock (2012).

2. We mentioned Gödel (1947) in § 2.5. For the others, see Frege (1893–1903, § 91), on which Garavaso (2005); Carnap (1939, p. 64), on which Psillos (1999, pp. 10–11); von Neumann (1947, p. 6).
3. See Quine (1948, pp. 16–19; 1951a, pp. 44–6; 1954, pp. 121–2; 1966b, p. 244; 1969c, p. 97–8; 1981c, pp. 149–50).
4. Though typically endorsed under the assumption that mathematical objects are abstract, IA is independent of the nature of mathematical objects, and then in principle acceptable by nominalists who believes that there are mathematical objects, but they are concrete. For the sake of generality, we will merely speak of mathematical objects.
5. Those who believe the first premise to be too harsh can still accept a weaker formulation in which that premise is discharged and the conclusion is conditional in form: if there are true theories / if S is true, then . . .
6. On this issue, see also Pincock (2004), Paseu (2007).
7. Mark Steiner (1978, pp. 19–20) argues, however, for a different interpretation of Quine's views, suggesting a transcendental version of IA.
8. While Putnam's formulation suggests an epistemic reading, his overall discussion suggests a non-epistemic one.
9. A close, but distinct, reading of 'realism' is the one famously introduced by Dummett, according to whom the term refers to the thesis "that statements of [a] disputed class possess an objective truth-value, independently of our means of knowing it: they are true or false in virtue of a reality existing independently of us" (1978b, p. 146).
10. In §§ 6.2.2, 7.3.2 and 7.4.2 we shall pause on another related view, usually called 'scientific realism'.
11. Notice that not all of Quine's views that might be relevant to mathematics are always considered as relevant, or even compatible for IA. Bueno (2003) has argued, for instance, that Quine's views on ontological relativity (Quine, 1968) may clash with his alleged endorsement of IA. We will here consider only those aspects of Quine's views that bear relevance to IA.
12. But see Busch and Sereni (2012).
13. Baker's argument has been offered in the context of a discussion concerning the role of mathematical explanation in science and its relations to IA. Here we can only point to the relevant works, among which are Baker (2001, 2005), Colyvan (2002), Melia (2002), Batterman (2010), Pincock (2011), Saatsi (2011), Bueno and French (2012). On the partly related issue of explanations in mathematics, see Mancosu (2008b, 2011).
14. Here and in what follows we call 'vocabulary' the set of non-logical constants (namely, singular terms and predicate or functional constants) of a language.
15. We use the adjective 'putative' in order to stress that the indispensability to S of either quantification over a certain domain of entities, or these very entities, or the reference to them, should not be taken to entail by itself that there is a certain relation between S and some existent objects, for this would of course make IA question-begging.
16. Option (*ii*) is suggested by argument [3] and is particularly faithful to Quine's view. Notice that, if only quantification has to be considered, and not also singular terms, then the vocabulary fixed by T is to be restricted to predicate constants alone. As suggested by Quine (1948) himself, this could be done by introducing for any singular term a corresponding predicate which is

intended to apply only to the referent of the original term, and substitute all occurrences of the former accordingly: for instance, 'Pegasus flies' would become 'There is one and only one *x* such that *x* pegazises and *x* flies'.

17. See Chapter 5, note 3.
18. See Quine (1960, ch. 7, note 5). Quine's doubts on the nominalist programme endorsed in Goodman and Quine (1947) goes back as far as 1948; see Mancosu (2008c).
19. There are various reasons telling against the availability of a sharp separation of the vocabularies of scientific theories into observational and theoretical sub-vocabularies. Acknowledging this possibility was crucial for logical positivists, but has been subject to several objections (see Putnam, 1962; Suppe, 1977). We can set these issues aside here, since as far as the discussion on IA is concerned, it is usually granted that the required separation is available for the relevant scientific theories, supposing that the theoretical vocabulary is (or includes) a mathematical vocabulary.
20. A theory is recursively enumerable if its theorems can be effectively enumerated, that is arranged in a succession $\tau_1, \tau_2, \ldots$ whose elements can be effectively obtained one after the other (possibly in an infinite time). Notice that 'recursive' and 'recursively', here and in the next note have a different meaning than that suggested in Chapter 3, note 27. Here, they indicate that some effective decision procedure is available.
21. A theory is recursively axiomatizable if, given any well-formed formula of its language, it is possible to determine whether it is one of the axioms of the theory. The set of the theory's axioms is then said to be recursive.
22. Craig himself implicitly warned against this philosophical use of his theorem, observing that, in his proof, V_Φ is such that its theorems are obtained in a "mechanical and artificial way" and are not "psychologically or mathematically . . . more perspicuous" than those of V (1956, p. 49).
23. An example of the necessity of distinguishing between the empirical content of a theory and the content of its observational statements is offered in Melia (2000), on which see § 7.1.
24. Colyvan's discussion of "the role of confirmation theory" in his (2001, pp. 78–81) hints at the relative character of the notion of "preferability". We take our clarification of (in)dispensability to improve on that suggestion.
25. IA apart, this criterion plays a crucial role in ontology: see, among others, Carnap (1950a), Alston (1958), Church (1958), Jackson (1980), Hodes (1990), Melia (1995, 2000), Azzouni (1998), Yablo (1998, 2001), Alspector-Kelly (2001), Eklund (2005), Hofweber (2005a, 2005b, 2007), Chalmers *et al.* (2009).
26. See note 16 above.
27. Clearly, our choices about which objects to countenance will not be wholly arbitrary, and criteria for this can be thought of. One minimal criterion adopted by Quine is that we should not accept the existence of entities for which no clear identity conditions can be stated: hence his slogan, "no entity without identity" (Quine, 1958, p. 20).
28. Notice that the 'if' direction of this double implication is much less innocent than it appears at first glance, since it depends on admitting the *prima facie* reading of a statement of the form '$\exists x[\mathcal{A}(x)]$', according to which such a statement just asserts that there exists an x such that $\mathcal{A}(x)$: this reading of the quantifiers, sometimes also called 'objectual', might be questioned by alternative conceptions of quantification.

29. Notice, however, that in order to avoid circularity, some basic statements – like those used in reporting results of empirical tests, or, generally, those which Quine calls ‘observational statements’ – must be to a certain extent independent of the whole theory. See, for details, Quine (1960, § 2.10; 1969b; 1981e, ch. 2; 1993); see also Quine and Ullian (1975, ch. 1).
30. See Haack (2009, ch. 6), on Quine’s ambiguity in the use of the term ‘science’.
31. This leaves open to be decided what makes a statement part of a theory, and what makes a theory scientific. These are rather complex questions, and different answers might come with different conceptions of naturalism.
32. Stronger conceptions are available. Some see naturalism as the thesis that only non-abstract entities exist. Both the Eleatic Principle discussed by Colyvan (2001, ch. 3), and the naturalism endorsed by Weir (2005) and Armstrong (1997, p. 5) are cases in point (for a discussion, see Cheyne, 2001, ch. 7). Under these readings, naturalism is almost indistinguishable from nominalism. Hence, as argued by Colyvan, this form of naturalism is in principle incompatible with IA.
33. Confirmational holism should be kept distinct from semantic holism, which is roughly the thesis that the meaning of words or statements cannot be determined in isolation, but depends on the role they play in the whole language they belong to (see Fodor, Lepore, 1991). Both theses were suggested by Quine as consequences of his objections to the “dogmas” of logical empiricism (Quine, 1951a), though the former was originally advanced in Duhem (1906, especially p. 225), and is also known as the Duhem-Quine thesis.
34. On occasions, Quine states his view in more moderate terms, claiming for instance that “it is an uninteresting legalism . . . to think of our scientific system of the world as involved *en bloc* in every prediction. More modest chunks suffice” (1981b, p. 71).
35. A noticeable exception, besides Putnam, is Azzouni (2009). As mentioned in § 6.2.2, Colyvan himself (2001, p. 12) observes that the ‘only’ implication in premise (*i*) of argument [3] is not required, although he includes it in his argument “for the sake of completeness and also to help highlight the important role naturalism plays in questions of ontology”. Whatever this role may be, acknowledging that this implication is not required suggests that argument [3] can be freed by naturalism.
36. Some have accepted this unpalatable conclusion nonetheless (see Colyvan, 2000; 2001, pp. 134–40).
37. Quine accepted this conclusion. He claimed that the objects of unapplied mathematical theories are “without ontological rights” and considered those theories as “mathematical recreation” (1986, p. 400; in Quine 1990, pp. 94–5 and 1995, pp. 55–6 this position is partially revised but the general attitude remains the same). This has engendered a wide debate: see § 7.3.1, and Parsons (1983b), Maddy (1992), Leng (2002), Colyvan (2007).

7 The Indispensability Argument: The Debate

1. Since the set of theorems of a theory could itself be taken as a system of axioms.

2. Notice that if this is so, the presence, in these theories, of statements that are neither nominalistic nor mathematical does not prevent the conclusion that, supposing that it is conservative over consistent bodies of non-mathematical statements, acceptable mathematics is dispensable from these theories according to a specification of [*(In)dispensability*] in terms of *o*-equivalence (and of some appropriate criterion of virtuosity). Let S be a consistent, axiomatized, deductively closed, first-order scientific theory, the set of whose axioms is divided into three complementary sets A_N, A_M and A_H, formed, respectively, by nominalistic axioms, by M-loaded axioms for some acceptable mathematical theory M, and by axioms that are neither nominalist nor mathematical. From the conservativeness of acceptable mathematics over any consistent body of non-mathematical statements it follows that a nominalistic statement φ is a theorem of S if and only if it is derivable from A_{N+H}. Thus the sub-theory S_{N+H}, whose axioms are those belonging to A_{N+H}, includes all the nominalistic consequences of S and no other nominalistic consequence. It is then *o*-equivalent to S (granted that *o*-equivalence is defined as said).
3. Pincock (2004) offers a criticism to IA for platonism that somehow recalls Field's use of representation theorems. He suggests what he calls a 'mapping account', according to which applicability of mathematics in science is due to the possibility of correlating (mapping) physical with mathematical models. This would allow him to claim that even if mathematics is indispensable, it does not follow that mathematical objects exist (for discussion, see Bueno and Colyvan, 2011; Batterman, 2010; Pincock, 2011).
4. Representation theorems that must be proved in order to specify the second among the mentioned equivalence relations will have to be necessarily proved in a platonist metatheory. We have already touched upon this issue in § 4.1.
5. The piecemeal character of this argument is criticised in Dummett, 1993, p. 435.
6. We slightly modify Melia's notation for the sake of exposition.
7. Being concrete, a mereological sum is usually identified with the region of space it occupies.
8. These assumptions are not mutually equivalent, not even if one assumes, as Quine does, that statements of the form '$\exists x[\mathcal{A}(x)]$' of a first-order predicate language render the unique sense in which it can be said of an object that it exists. First, it might be that some of the statements considered in (*b*) are resistant to a reduction into a first-order predicate language. Second, one might think that expressions like 'exists', 'there are', etc. occurring in those statements are just idiomatic, and do not indicate the existence of objects at all, or at least do not indicate the existence of objects satisfying the first-order open formula '$\mathcal{A}(x)$'.
9. And leaving aside issues of ontological relativity discussed in Quine (1968). As already mentioned (see note 11, Chapter 6) we abstract here from some of Quine's theses, giving a somewhat simplified picture of Quine's ideas on the relations between language and ontology, which is, however, faithful enough to the way in which Quine's views are usually treated in connection to IA.
10. Wiles' proof is not internal to Peano arithmetic (either first- or second-order), since it employs various other mathematical theories, and nothing

guarantees, to the best of our knowledge, that an internal proof is possible, that is, that the axioms of Peano arithmetic are strong enough to make such a proof possible. Quite to the contrary, in order to prove ¬*Fermat* it would be enough to prove, within Peano arithmetic or any other appropriate version of arithmetic, a formula of the form $ap + bp = cp$ where a, b, c, and p are given natural numbers.

11. One may see also assumption (*a*) as called into question by the claim that all that the acknowledgement that there are numbers allows one to say is that some modal facts concerning progressions hold. Analogously, one could think that the idea that statements of the form '$\exists x[\mathcal{A}(x)]$' of a first-order predicate language should be interpreted in terms of objectual quantification (see note 28, Chapter 6) is contradicted by the idea that ¬*Fermat* and '$\Box[AX^* \Rightarrow \neg Fermat^*]$' have the same 'mathematical content'. The way Putnam understands the expressions 'for mathematics' and 'mathematical content' seems however to leave room for different conclusions. In any case, what seems to be beyond discussion (given the overall content of Putnam, 1967, and also Putnam, 2004, ch. 2) is rather that his argument puts pressure on assumption (*b*).
12. For more on Putnam's notion of equivalence involved here, see Putnam (1983). Criticism of Putnam's view of equivalent descriptions is in Burgess and Rosen 1997, pp. 200–1. A recent discussion of equivalent descriptions is in Field (Forthcoming). Maddy (2011) advances a view on the ontology of mathematics that, despite important differences, echoes Putnam's idea of equivalent descriptions.
13. On the evolution of Putnam's thought on the issue, see also Liggins (2007).
14. Of course, (s)he can do that, but it is not mandated. This and related issues for fictionalists are discussed in Sainsbury (2010); see also Azzouni (2010).
15. The two would still be different, since Field gives nominalist reformulations for scientific theories using mathematics, but not for mathematical theories themselves, whereas Yablo would offer them for mathematical theories too.
16. We shall suppose, for the sake of the argument, that the statement 'The average family has 2.3 children' is not understood as meaning that, given a certain particular number of families, and a certain particular number of children, the average family has n children average. Rather, it should be understood as a statement concerning what happens generally, independently of the particular number of families and children under consideration.
17. For reference on the relevance of Quine's criterion in ontological debates, see Chapter 6, note 25.
18. See Chapter 4, note 31.
19. For objections to this argument, see Chihara (2006).
20. Maddy (2005, 2011) examines the differences between her and Burgess' and Rosen's position. These differences do not seem to us to be as clear cut as Maddy seems to suggest.
21. This view echoes Putnam's conception of equivalent descriptions; see § 7.2.2.
22. The debate on constructive empiricism is huge: see Churchland and Hooker (1985) and Monton (2007), for example.
23. It is controversial whether it is allowed, or even mandated, to van Fraassen to reject IBE only when it would lead to justifying the existence of either

unobservable entities or mathematical objects, or whether he should rather reject it in general as a non-valid inference. See Psillos (1999, pp. 211–15).

24. It is unclear what Sober is meaning when he speaks of background assumptions. It is natural to assume that they include bridge-laws (see § 4.1). Should this be the case, however, it would be implausible to think that two scientific theories to which two different mathematical theories are indispensable could be confronted with experience relative to the same background assumptions. In order to oppose Lobacewski, who thought hyperbolic geometry could be experimentally verified, Poincaré (1902) gives an argument showing that no experimental evidence can favour this geometry over Euclid's. His argument rests on the very idea that those geometries cannot be confronted with experience if not through different bridge-laws.
25. For other objections to Sober's view, see Resnik (1995), Hellman (1999), Colyvan (1999; 2001, pp. 126–34). Another objection to IA, somehow related to Sober's views, is raised by Vineberg (1996). For other discussions of IBE as related to IA, see Saatsi (2007, 2011), Busch (2011).
26. Leng (2010) argues against confirmational holism on similar lines, and combines her argument with a fictionalist interpretation of idealizations in empirical sciences.
27. See also § 6.7.
28. Quite independently of IA, Resnik (1997, ch. 7) defends confirmational holism from numerous objections (on this, see Peressini, 2008).
29. The argument is presented in slightly different terms in Resnik (1995, pp. 169–71) and in (1997, pp. 46–8). We stick to the original formulation.
30. Though it has similarities with Burgess' and Rosen's naturalistic argument; see § 7.3.1.1.
31. Resnik's argument in (1997, pp. 46–8) stops with conclusion 7(*vi*); however, conclusion 7(*vii*) is present in the formulation of the argument in (1995, pp. 169–71).
32. See also Leng, 2010, pp. 255–8.

References

Where not otherwise indicated, references in the main text are to the original edition; otherwise, we have indicated with an '*' the edition to which references in the main text refer. As far as texts in languages other than English, and especially Greek texts, are concerned, translations have been occasionally modified with respect to the English translations referenced, through examination of the original texts and confrontation of other available translations in English and other languages.

Alspector-Kelly, M. (2001), "On Quine on Carnap on Ontology", *Philosophical Studies*, 102, 93–122.

Alston, W. (1958), "Ontological Commitments", *Philosophical Studies*, 9, 8–16.

Anderson, R. L. (2008), "Comments on Wayne Martin, *Theories of Judgment*", *Philosophical Studies*, 137, 91–108.

Andrews, P. (2002), *An Introduction to Mathematical Logic and Type Theory: To Truth Through Proof*, 2nd edition (1st edition 1986), Applied Logic Series, 27, Kluwer Academic Publishers, Dordrecht.

Annas, J. (1976), *Aristotle's Metaphysics: Books M And N*, Clarendon Press, Oxford and New York.

Aristotle, (AO), *Aristotelis opera ex recensione I. Bekker*, 11 vols, Oxford, 1837 (original edition *Aristoteles Grecae ex recensione I. Bekker*, vols. I and II of *Aristotelis Opera*, edidit Academia Regia Borussica, 5 vols, Berolini 1831–70).

—— (Metaph), *Metaphysica*, in Aristotle (AO), vol. VIII, (English Translation: *Metaphysica*, by W.D. Ross, 2 vols, Clarendon Press, Oxford, 1924, rev. 1954, in W.D. Ross (ed.), *The Works of Aristotle*, Clarendon Press, Oxford, vol. 8, 1954).

—— (Phys), *Naturalis Auscultationis Libri VIII,* in Aristotle (AO), vol. II, pp. 1–210 (English translation: *Physica*, by R.P. Hardie and R.K. Gaye, in W.D. Ross, ed., *The Works of Aristotle*, Clarendon Press, Oxford, vol. 2, 1930).

Armstrong, D. (1997), *A World of States of Affairs*, Cambridge University Press, Cambridge. Atten, M. van (2004), *On Brouwer*, Thomson/Wadsworth, Belmont (CA).

—— (2006), *Brouwer Meets Husserl. On the Phenomenology of Choice Sequences*, Springer, Dordrecht.

Atten, M. van and Kennedy, J. (2003), "On the Philosophical Development of Gödel", *The Bulletin of Symbolic Logic*, 9, 425–72.

Ayer, A.J. (1946), *Language, Truth and Logic*, Dover, New York.

Azzouni, J. (1997a), "Applied Mathematics, Existential Commitment and the Quine–Putnam Indispensability Thesis", *Philosophia Mathematica* (III), 5, 193–209.

—— (1997b), "Thick Epistemic Access: Distinguishing the Mathematical from the Empirical", *The Journal of Philosophy*, 94, 472–84.

—— (1998), "On 'On What There Is'", *Pacific Philosophical Quarterly*, 79, 1–18.

—— (2004), *Deflating Existential Consequence. A Case for Nominalism*, Oxford University Press, Oxford, New York.

—— (2008), "A Cause for Concern: Standard Abstracta and Causation", *Philosophia Mathematica* (III), 16, 397–401.

—— (2009), "Evading Truth Commitments: The Problem Reanalyzed", *Logique & Analyse*, 206, 139–76.

—— (2010), *Talking About Nothing*, Oxford University Press, Oxford, New York.

Baker, A. (2001), "Mathematics, Indispensability and Scientific Progress", *Erkenntnis*, 55, 85–116.

—— (2003a), "The Indispensability Argument and Multiple Foundations for Mathematics", *The Philosophical Quarterly*, 53, 49–67.

—— (2003b), "Does the Existence of Mathematical Objects Make a Difference?", *Australasian Journal of Philosophy*, 81, 246–64.

—— (2005), "Are there Genuine Mathematical Explanations of Physical Phenomena?", *Mind*, 114, 223–38.

—— (2009), "Mathematical Explanation in Science", *The British Journal for the Philosophy of Science*, 60, 611–33.

Balaguer, M. (1996a), "Towards a Nominalization of Quantum Mechanics", *Mind*, 105, 209–26.

—— (1996b), "A Fictionalist Account of the Indispensable Applications of Mathematics", *Philosophical Studies* 83, 291–314.

—— (1998), *Platonism and Anti-Platonism in the Philosophy of Mathematics*, Oxford University Press, Oxford, New York.

—— (2011), "Fictionalism in the Philosophy of Mathematics", *The Stanford Encyclopedia of Philosophy (*Fall 2011 Edition*)*, Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/fall2011/entries/fictionalism–mathematics/>

—— (2009), "Fictionalism, Theft, and the Story of Mathematics", *Philosophia Mathematica* (III), 17, 131–62.

Barkley Rosser, J. (1936), "Extensions of Some Theorems of Gödel and Church", *Journal of Symbolic Logic*, 1, 87–91.

Baron-Cohen S. and Leslie A.M., Frith U. (1985), "Does the Autistic Child Have a 'Theory of Mind'?", *Cognition*, 21, 37–46.

Batterman, R.W. (2010), "On the Explanatory Role of Mathematics in Empirical Science", *British Journal for the Philosophy of Science*, 61, 1–25.

Benacerraf, P. (1965), "What Numbers Could not Be", *The Philosophical Review*, 74, 47–73*; also in Benacerraf, Putnam (1964), 272–94.

—— (1973), "Mathematical Truth", *The Journal of Philosophy* 70, 661–79*; also in Benacerraf, Putnam (1964), 403–20.

—— (1996a), "What Mathematical Truth Could Not Be – I" in A. Morton and S. P. Stich (eds), *Benacerraf and His Critics*, Blackwell, Oxford, 9–59.

—— (1996b), "Recantation or Any Old ω–Sequence Would Do After All", *Philosophia Mathematica* (III), 4, 184–89.

Benacerraf, P. and Putnam, P. (1964) (eds), *Philosophy of Mathematics. Selected Readings*, Prentice-Hall, Englewood Cliffs (NJ), 1964; 2nd edition 1983*, Cambridge University Press, Cambridge.

Berto, F. (2009), *There's something about Gödel!*, Wiley Blackwell, Oxford.

Blanchette, P.A. (1996), "Frege and Hilbert on Consistency", *The Journal of Philosophy*, 93, 317–36.

Bolzano, B. (1851), *Paradoxien des Unendlichen*, Meiner & Reclam, Leipzig.

Boolos, G. (1975), "On Second-order Logic", *The Journal of Philosophy*, 72, 509–27; also in Boolos (1998), Chapter 3.

—— (1984), "To Be Is To Be a Value of a Variable (or to Be Some Values of Some Variables)", *The Journal of Philosophy*, 81, 430–50; also in Boolos (1998), Chapter 4.

—— (1985), "Nominalist Platonism", *The Philosophical Review*, 94, 327–44; also in Boolos (1998), Chapter 5.

—— (1986–7), "Saving Frege from Contradiction", *Proceedings of the Aristotelian Society*, 87, 137–51; also in Boolos (1998), 171–82.

—— (1987), "The Consistency of Frege's *Foundations of Arithmetic*", in J. Thomson (ed.), *On Being and Saying: Essays in Honor of Richard Cartwright*, MIT Press, Cambridge (MA), 1987, 3–20; also in Boolos (1998), Chapter 12.

—— (1990), "The Standard of Equality of Numbers", in Boolos, G. (ed.), *Meaning and Method: Essays in Honour of Hilary Putnam*, Cambridge, Cambridge University Press, 261–77; also in Boolos (1998), Chapter 13.

—— (1993), "Whence the Contradiction?", *Aristotelian Society Supplementary Volume*, 67, 213–33; also in Boolos (1998), Chapter 14.

—— (1997), "Is Hume's Principle Analytic?", in Heck, R. Jnr. (ed.), *Language, Thought and Logic*, Oxford University Press, Oxford, New York, 1997, 245–62; also in Boolos 1998, Chapter 19.

—— (1998), *Logic, Logic, and Logic*, Harvard University Press, Cambridge (MA), 1998.

Boolos, G. and Heck, R. Jnr. (1998), "*Die Grundlagen der Arithmetik* §§ 82–3", in Schirn (1998), 407–28; also in Boolos (1998), 315–38.

Boolos, G. and Jeffrey, R. (1989), C*omputability and Logic*, Cambridge University Press, Cambridge; 1st edition, 1974; 5th edition, Boolos, G., Jeffrey, R., Burgess, J. (eds), 2007.

Bostock, D. (2009), *Philosophy of Mathematics: An Introduction*, Oxford University Press, Oxford, New York.

Bourbaki, N. (EM), *Éléments de mathématiques*. This collective work was written in 1939 by a group of several mathematicians under the pseudonym of 'Nicolas Bourbaki'. At present, it consists of ten volumes, published by Dunod, Paris; some of these have been translated in English and published by Springer-Verlag, Heildelberg. More details can be found at the website of the Association de Collaborateurs de Nicolas Bourbaki (by the École Normale Supérieure of Paris): www.bourbaki.ens.fr/

Boyd, R. (1991) (ed.), *The Philosophy of Science*, MIT Press, Cambridge (MA).

—— (2002), "Scientific Realism", *The Stanford Encyclopedia of Philosophy* (Fall 2008 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/fall2008/entries/scientific–realism/>

Brouwer, J.E.J. (1912), *Intuitionisme en formalisme*, Noordhoff, Groningen, 1912; also in Brouwer J.E.J. (1913), *Wiskunde, waarheid, werkelijkheid*; English translation "Intuitionism and formalism", *Bulletin of the American Mathematical Society*, 20; also in Brouwer (CW)*, vol. I, 123–38; also in Benacerraf, Putnam (1964), 77–89.

—— (CW), (edited by A. Heyting), *L. E. J. Brouwer: Collected Works*, vol. I, *Philosophy and Foundations of Mathematics*, 1975; vol. II, *Geometry, Analysis, Topology and Mechanics*, (1976), North Holland, Amsterdam/American Elsevier, New York.

Brown, J.R. (1999), *Philosophy of Mathematics: An Introduction to a World of Proofs and Pictures*, Routledge, London and New York.

Bueno, O. (2003), "Quine's Double Standard: Undermining the Indispensability Argument via the Indeterminacy of Reference", *Principia*, 7, 17–39.

Bueno, O. and Colyvan, M. (2011), "An Inferential Conception of the Application of Mathematics", *Noûs,* 45, 345–74.
Bueno, O. and French, S. (2012), "Can Mathematics Explain Physical Phenomena?", *British Journal for the Philosophy of Science*, 63, 85–113.
Burgess, A. and Burgess, P. (2011), *Truth*, Princeton University Press, Princeton (NJ), 2011.
Burgess, J. (1983), "Why I Am Not a Nominalist", *Notre Dame Journal of Formal Logic*, 24, 93–105.
—— (2004), "Mathematics and *Bleak House*", *Philosophia Mathematica* (III), vol. 12, 18–36.
—— (2005), *Fixing Frege*, Princeton University Press, Princeton (NJ), 2005.
—— (2008), "Review of Charles Parsons, Mathematical Thought and its Objects", *Philosophia Mathematica* (III), 16, 402–20.
Burgess, J.P., Hazen, A. and Lewis, D. (1991), "Appendix on Pairing", in D. Lewis, *Parts of Classes*, Blackwell, Oxford, 121–49.
Burgess, J. and Rosen, G. (1997), *A Subject With No Object. Strategies for Nominalistic Interpretation of Mathematics*, Oxford University Press, Oxford, New York.
—— (2005), "Nominalism Reconsidered", in Shapiro (2005), 515–36.
Burnyeat, M.F. (1987), "Platonism and Mathematics: A Prelude to Discussion", in A. Graeser (ed.), *Mathematics and Metaphysics in Aristotle*, Haupt, Bern and Stuttgart, 213–40.
Busch, J. (2011), "Scientific Realism and the Indispensability Argument for Mathematical Realism: A Marriage Made in Hell", *International Studies in the Philosophy of Science*, 25, 307–25.
Busch, J. and Sereni, A. (2012), "Indispensability Arguments and Their Quinean Heritage", *Disputatio*, 4, 343–60.
Callard, B. (2007), "The Conceivability of Platonism", *Philosophia Mathematica* (III), 15, 347–56.
Cantor, G. (1872), "Über die Ausdehnung eines Satzes aus der Theorie der trigonometrichen Reihen", *Matematische Annalen*, 5, 123–32.
—— (1874), "Über eine Eigenschaft des Inbegriffs aller reellen algebraischen Zahlen", *Journal für die reine und angewandte Mathematik*, 77, 1874, 258–62; also in *Gesammelte Abhandlungen*, 115–18; English translation "On a property of the set of real algebraic numbers", in Ewald (1966), vol. 2, 839–43.
—— (1879–84), "Über unendliche, lineare Punktmannigfaltigkeiten", *Mathematische Annalen* 15(1879), 1–7; 17(1880), 355–58; 20(1882), 113–21; 21(1883), 51–8 and 545–86; 23(1884), 453–88; also in Cantor (GA), 139–145, 145–148, 149–157, 157–164, 165–208, 210–244, reprinted in *Über unendliche, lineare Punktmannigfaltigkeiten. Arbeiten zur Mengenlehre aus den Jahren 1872–84,* edited and with a foreword and commentary by G. Asser, *Teubner–Archiv zur Mathematik*, 2, Teubner, Leipzig, 1984.
—— (1883) *Grundlagen einer allgemeinen Mannigfaltigkeitslehre. Ein mathematisch–philosophischer Versuch in der Lehre des Unendlichen*, Teubner, Leipzig, 1883; also in Cantor (GA), 165–208; English translation "Foundation of the theory of manifolds", *The Campaigner* (The Theoretical Journal of the National Caucus of Labor Committees), 9, 1976, 69–96.
—— (1895–1897), "Beiträge zur Begründung der transfiniten Mengenlehre", *Mathematische Annalen*, 46(1895), 481–512; 49(1897), 207–246; also in Cantor

(GA), 282–311; English translation in *Contributions to the Founding of the Theory of Transfinite Numbers*, Dover, New York, 1952.

—— (GA), *Gesammelte Abhandlungen*, (ed.) by Ernst Zermelo, Springer, Berlin, 1932.

Carnap, R. (1931), "Die logizistische Grundlegung der Mathematik", *Erkenntnis*, 2, 91–105; English translation "The Logicist Foundations of Mathematics", in Benacerraf and Putnam (1964), 41–52.

—— (1939), "Foundation of Logic and Mathematics", in *International Encyclopedia of Unified Science*, vol. I/3, The University of Chicago Press, Chicago.

—— (1950a), "Empiricism, Semantics, and Ontology", *Revue International de Philosophie*, 4, 20–40; revised version in *Meaning and Necessity*, The University of Chicago Press, Chicago, 1956, 205–21; this latter version reprinted in Benacerraf, Putnam (1964)*, 241–57.

—— (1950b), *Logical Foundations of Probability*, University of Chicago Press, 1950, 2nd edition 1965.

—— (1952), *The Continnum of Inductive Methods*, University of Chicago Press, Chicago, 1952.

Cassirer, E. (1918–21), *Kant Leben und Lehre*, Cassirer, 1918 and 1921; English translation *Kant's Life and Thought*, Yale University Press, 1981.

Cassou–Noguès (2005), "Gödel and 'The Objective Existence' of Mathematical Objects", *History and Philosophy of Logic*, 26, 211–28.

Cavaillès, J. (1938), *Remarques sur la formation de la théorie abstraite des ensembles*, Hermann, Paris, 1938, 2nd edition in Cavaillès, J., *Philosophie mathématique*, Herman, Paris, 1962, 23–174, republished by PUF, Paris, 1994; also in Cavaillès, J. *Œuvres complètes de philosophie des sciences*, Hermann, Patis, 1994, 205–374.

Chakravartty, A. (2011), "Scientific Realism", *The Stanford Encyclopedia of Philosophy (*Summer 2011 Edition*)*, Edward N. Zalta (ed.), URL: <http://plato.stanford.edu/archives/sum2011/entries/scientific–realism/>.

Chalmers, D., Manley D. and Wasserman R. (eds) (2009), *Metametaphysics,* Oxford University Press, Oxford,– New York.

Chang C.C. and Keisler, H.J. (1973), *Model Theory*, Elsevier, Amsterdam, 1973, 3rd rev. edition 1990.

Changeux, J.P. and Connes, A. (1989), *Matière à pensée*, Odile Jacob, Paris, 1989; English translation *Conversations on Mind, Matter, and Mathematics*, Princeton University Press (NJ), 1998.

Cheyne, C. (2001), *Knowledge, Cause, and Abstract Objects: Causal Objections to Platonism*, Kluwer, Dordrecht.

Chihara, C.S. (1973), *Ontology and the Vicious-Circle Principle*, Cornell University Press, Ithaca (NY).

—— (1981), "Quine and the Confirmational Paradoxes", in P.A. French, T. Uehling, H. Wettstein (eds), *The Foundations of Analytic Philosophy*, University of Minnesota Press, Minneapolis, 425–52.

—— (1990), *Constructibility and Mathematical Existence,* Clarendon Press, Oxford.

—— (2004), *A Structural Account of Mathematics*, Oxford University Press, Oxford, New York.

—— (2005), "Nominalism", in Shapiro (2005), 483–514.

—— (2006), “Burgess’s ‘Scientific’ Arguments for the Existence of Mathematical Objects”, *Philosophia Mathematica (III)*, 14, 318–37.
—— (2008), *The Worlds of Possibility: Modal Realism and the Semantics of Modal Logic*, Clarendon Press, Oxford.
Church, A. (1958), “Ontological Commitment”, *The Journal of Philosophy*, 55, 1008–14.
Churchland, P. and Hooker, C. (eds) (1985), *Images of Science: Essays on Realism and Empiricism. With a reply from Bas C. van Fraassen*, University of Chicago Press, Chicago.
Coffa, A.J. (1991), *The Semantic Tradition from Kant to Carnap. To the Vienna Station*, Cambridge University Press, Cambridge, 1991.
Cohen, P.J. (1963), “The Independence of the Continuum Hypothesis”, *Proceedings of the National Academy of Sciences of the United States of America*, 50, 1143–48.
—— (1966), *Set Theory and the Continuum Hypothesis,* W.A. Benjamin, New York.
Colyvan, M. (1998), “In Defence of Indispensability”, *Philosophia Mathematica* (III), 6, 39–62.
—— (1999), “Contrastive Empiricism and Indispensability”, *Erkenntnis*, 51, 323–32.
—— (2000), “Conceptual Contingency and Abstract Existence”, *Philosophical Quarterly*, 50, 87–91.
—— (2001), *The Indispensability of Mathematics*, Oxford University Press, Oxford, New York.
—— (2002), “Mathematics and Aesthetic Considerations in Science”, *Mind*, 111, 69–74.
—— (2007), “Mathematical Recreation versus Mathematical Knowledge”, in Leng *et al*. (2007), 109–22.
—— (2010), “There’s No Easy Road to Nominalism”, *Mind*, 119, 285–306.
—— (2012), *An Introduction to the Philosophy of Mathematics*, Cambridge University Press, Cambridge.
—— (*Forthcoming*), “Road Work Ahead: Heavy Machinery on the Easy Road”, *Mind*.
Colyvan, M. and Zalta, E.N. (1999), “Mathematics: Truth and Fiction? Review of Mark Balaguer *Platonism and Anti-Platonism in Mathematics*”, *Philosophia Mathematica* 7, 336–49.
Cook, R. (ed.) (2007), *The Arché Papers on the Mathematics of Abstraction*, Springer, Dordrecht.
Coquand, T. (2010) “Type Theory”, *The Stanford Encyclopedia of Philosophy (*Spring 2010 Edition*)*, Edward N. Zalta (ed.), URL: <http://plato.stanford.edu/archives/spr2010/entries/type–theory/>.
Craig, W. (1953), “On Axiomatizability within a System”, *Journal of Symbolic Logic*, 18, 30–32.
—— (1956), “Replacement of Auxiliary Expressions”, *Philosophical Review*, 65, 38–55.
Dalen, D. van (1999–2005), *Mystic, Geometer, and Intuitionist. The Life of L.E.J. Brouwer*, Clarendon Press, Oxford, 2 vols., Vol. I 1999, Vol. II 2005.
De Caro, M. and Macarthur, D. (eds) (2004), *Naturalism in Question*, Harvard University Press, Cambridge (MA).
—— (eds) (2010), *Normativity and Naturalism*, Columbia University Press, New York.

Dedekind, R. (1872), *Stetigkeit und irrationale Zahlen*, Vieweg, Brunswick, 1872; also in Dedekind, R. (GMW); English translation "Continuity and Irrational Numbers", in Dedekind (2001), 1–27.

—— (1888), *Was sind und was sollen die Zahlen?*, Vieweg, Brunswick, 1888; also in Dedekind, R. (GMW), English translation "The Nature and Meaning of Numbers", in Dedekind (1901), 31–115.

—— (1901), *Essays on the Theory of Numbers*, Open Court, Chicago, 1901, reprinted by Dover, New York, 1963.

—— (GMW), *Gesammelte mathematische Werke*, 3 vols, (ed.) R. Fricke, E. Noether, Ø. Ore, Vieweg, Brunswick, vol. I 1930, vol. II 1931, vol. III 1932.

Dehaene, S. (1997), *The Number Sense: How the Mind Creates Mathematics*, Oxford University Press, New York, 1997; revised and updated edition (2011).

Demopoulos, W. (2005), "Our Knowledge of Numbers as Self–Subsistent Objects", *Dialectica*, 59, 2005, 141–59.

—— (*Forthcoming*), "Generality and Objectivity in Frege's *Foundations of Arithmetic*", in A. Miller (ed.), *Logic, Language and Mathematics: Essays in Honour of Crispin Wright*, Oxford Unversity Press, Oxford.

Detlefsen, M. (1986), *Hilbert's Program*, Reidel, Dordrecht.

Dieveney, P. (2007), "Dispensability in the Indispensability Argument", *Synthese*, 157, 105–28.

Divers, J. and Miller, A. (1999), "Arithmetical Platonism: Reliability and Judgement–Dependence", *Philosophical Studies*, 95, 277–310.

Doridot, F. and Panza, M. (2004), "À propos des apports des sciences cognitives à la philosophie des mathématiques", *Intellectica*, 39, 263–87.

Duhem, P. (1906), *La theorie physique. Son object, sa structure*, Chevalier & Rivière, Paris, 1906; 12th revised edition, Marcel Riviere et Cie, 1914; English translation *The Aim and Structure of Physical Theory*, Princeton University Press, Princeton (NJ), 1991.

—— (1908), *SOZEIN TA PHAINOMENA. Essai sur la notion de theorie physique*, A. Hermann, Paris, 1908; English translation *To Save the Phenomena*, University Of Chicago Press, Chicago, 1969.

Dummett, M. (1956), "Nominalism", *Philosophical Review*, 65, 491–505; also in Dummett (1978a), 38–49.

—— (1973), *Frege: Philosophy of Language*, Duckworth, London, 1973; 2nd edition 1981*.

—— (1978a), *Truth and Other Enigmas*, Duckworth, London.

—— (1978b), "Realism", in Dummett (1978a), 145–65.

—— (1991), *Frege: Philosophy of Mathematics*, Harvard University Press, Cambridge (MA).

—— (1993), "What is Mathematics About?", in *The Seas of Language*, Oxford University Press, Oxford, New York, 1993, 429–45.

—— (1994), "Chairman's Address: Basic Law V", *Proceedings of the Aristotelian Society*, 94, 43–251.

Earman, J. and Salmon, W. (1999), "The Confirmation of Scientific Hypothesis", in Salmon, H.M. (ed.), *Introduction to the Philosophy of Science*, Hackett Publishing Co., 1st edition 1992, Chapter 2.

Ebbinghaus, H.-D., Flum, J. and Thomas, W. (2004), *Mathematical Logic,* Springer-Verlag, New York, 2004, 2nd edition; 1st edition, 1994.

Eisler, R. (1930), *Kant Lexikon*, E.S. Mittler, Berlin.

Eklund, M. (2005), "Fiction, Indifference, and Ontology", *Philosophy and Phenomenological Research*, 71, 557–79.

—— (2011), "Fictionalism", The Stanford Encyclopedia of Philosophy (Fall 2011 Edition), Edward N. Zalta (ed.), URL: <http://plato.stanford.edu/archives/fall2011/entries/fictionalism/>.

Engel, P. (1995), "Platonisme mathématique et antiréalisme", in M. Panza and J.M. Salanskis (eds), *L'objectivité mathématique. Platonismes et structures formelles*, Masson, Paris, 133–46.

Errera, A. (1953), "Le problème du continu", *Atti dell'Accademia Ligure di Scienze e Lettere*, 9, 176–83.

Ewald, W.B. (1996), (ed.), *From Kant to Hilbert. A Source Book in the Foundations of Mathematics*, Oxford University Press, Oxford, New York, 2 vols.

Feferman, S. (2005), "Predicativity", in Shapiro (2005), 590–624.

Ferreirós, J. (1999), *Labyrinth of Thought: A History of Set Theory and Its Role in Modern Mathematics*, Birkhäuser, Basel, Boston, Berlin, 2nd edition, 2007.

Field, H. (1974), "Quine and the Correspondence Theory" Philosophical Review, 83, 200–28, reprinted with a Postscript in H. Field, *Truth and The Absence of Fact*, Oxford University Press, Oxford, New York, 2001, 199–221.

—— (1980), *Science Without Numbers*, Blackwell, Oxford.

—— (1982), "Realism and Anti-realism about Mathematics", *Philosophical Topics*, 13, 45–69; also in Field (1989), 53–78.

—— (1984), "Is Mathematical Knowledge just Logical Knowledge", *Philosophical Review*, 93, 509–52; also in Field (1989), 79–124.

—— (1985), "On Conservativeness and Incompleteness", *The Journal of Philosophy*, 82, 239–60; also in Field (1989), 125–46.

—— (1989a), *Realism, Mathematics and Modality*, Blackwell, Oxford.

—— (1989b), "Realism, Mathematics and Modality", in Field (1989a), 227–81 (largely modified version of the same-titled original in *Philosophical Topics*, 19, 1988, 57–107).

—— (1998), "Mathematical Objectivity and Mathematical Objects" in C. MacDonald and S. Laurence (eds), *Contemporary Readings in the Foundations of Metaphysics*, Blackwell, Oxford, 387–403.

—— (*Forthcoming*), "Mathematical Undecidables, Metaphysical Realism and Equivalent Descriptions", in R.E. Auxier (ed.), *The Philosophy of Hilary Putnam*, Library of Living Philosophers, Open Court Press, Chicago (IL).

Fine, K. (2002),*The Limits Of Abstraction*, Oxford University Press, Oxford, New York.

—— (2005), "The Varieties of Necessity", in K. Fine, *Modality and Tense. Philosophical Papers*, Clarendon Press, Oxford, 2005, 235–60.

Fodor, J. and Lepore, E. (1991), *Holism, A Shopper's Guide*, Wiley Blackwell, Oxford.

Føllesdal, D. (1995), "Gödel and Husserl", in J. Hintikka (ed.), *From Dedekind to Gödel*, Kluwer, Dordrecht, Boston, 1995, 427–46.

Fraassen, B. van (1980), *The Scientific Image*, Oxford University Press, Oxford, New York.

Frege, G. (1879), *Begriffsschrift, eine der Arithmetischen Nachgebildete Formalsprache de reinen Denkens*, Verlag Von L. Nebert, Halle, 1879; English translation "Begriffsschrift, a Formula Language, Modeled upon that of Arithmetic, for Pure Thought", in van Heijenoort (1967), 5–82.

—— (1884), *Die Grundlagen der Arithmetik: eine logische mathematische Untersuchung über den Begriff der Zahl,* Koebner, Breslau, 1884; English translation by J. Austin, in G. Frege, *The Foundations of Arithmetic. A Logico–Mathematical Enquiry into the Concept of Number*, Blackwell, Oxford, 1974.

—— (1893–1903), *Grundgesetze der Arithmetik*, H. Pohle, Jena, 1893–1903 (2 vols.); partial English translation in G. Frege, *The Basic Laws of Arithmetic*, (ed.) M. Furth, University of California Press, Berkeley, 1967.

—— 1919, *Aufzeichnungen für Ludwig Darmstaedter,* in *Nachgelassene Schriften und wissenschaftlicher Briefwechsel,* vol. I, (ed.) H. Hermes *et al.,* Felix Meiner Verlag, Hamburg 1969, pp. 273–7; English translation *Notes for Ludwig Darmstaedter,* in G. Frege, *Posthumous Writings,* Blackwell, Oxford 1979, pp. 253–7*.

—— (WB), *Wissenschaftlicher Briefwechsel*, Felix Meiner, Amburgo, 1976; English translation in Frege (PMC).

—— (PMC), *Philosophical and Mathematical Correspondence*, (ed.) G. Gabriel *et al.*, Blackwell, Oxford, 1980.

Friedman, J. (2005), "Modal Platonism: An Easy Way to Avoid Ontological Commitment to Abstract Entities", *Journal of Philosophical Logic*, 34, 227–73.

Friedman, M. (1992), *Kant and the Exact Sciences*, Harvard University Press, Cambridge (MA), London.

Garavaso, P. (2005), "On Frege's Alleged Indispensability Argument", *Philosophia Mathematica* (III), 13, 160–73.

Gentzen, G. (1936), "Die Widerspruchfreiheit der reinen Zahlentheorie", *Mathematische Annalen*, 112, 493–565; English translation "The Consistency of Elementary Number Theory", in Gentzen (CW), 132–213.

—— (1938), "Neue Fassung des Widerspruchsfreiheitsbeweises fuer die reine Zahlentheorie", *Forschungen zur logik und zur Grundlegung der exacten Wissenschaften, neue Folge*, 4, 19–44; English translation "New version of the consistency proof for elementary number theory", in Gentzen (CW), 252–86.

—— (CW), *The Collected Works of Gerhard Gentzen,* (ed.) M.E Szabo*,* North-Holland, Amsterdam, 1969.

George, A. and Velleman, D.J. (2001), *Philosophies of Mathematics*, Wiley-Blackwell, Oxford.

Gettier, E. (1963), "Is Justified True Belief Knowledge?", *Analysis*, 23, 121–23.

Giaquinto, M. (1983), "Hilbert's Philosophy of Mathematics", *British Journal for the Philosophy of Science*, 34, 119–32.

—— (2002), *The Search for Certainty: A Philosophical Account of Foundations of Mathematics*, Clarendon Press, Oxford.

Glymour, C. (1980), *Theory and Evidence*, Princeton University Press, Princeton (NJ).

Gödel, K. (CW1), *Collected Works, Vol. I, Publications 1929–1936*, (ed.) S. Feferman, Oxford University Press, Oxford, New York, 1986.

—— (CW2), *Collected Works, Vol. II, Publications 1938–1974*, (ed.) S. Feferman, Oxford University Press, Oxford,New York, 1990.

—— (CW3), *Collected Works, Vol. III*, Unpublished Essays and Lectures, (ed.) S. Feferman *et al.*, Oxford University Press, Oxford,New York, 1995.

—— (UPE), Unpublished Philosophical Essays, (ed.) F.A. Rodriíguez-Consuegra, Birkäuser Verlag, Basel-Boston-Berlin.

—— (1930), "Die Vollständigkeit der Axiome des logischen Funktionenkalküls", *Monatshefte für Mathematik und Physik*, 37; English translation "The completeness

of the axioms of the functional calculus of logic", in Gödel (CW1), 103–23 and in van Heijenoort (1967), 582–91.
—— (1931), "Über formal unentscheidbare Sëtze der *Principia Mathematica* un verwandter Systeme I", *Monatshefte Für Mathematick un Physik*, 38, 173–98; English translation "On formally undecidable propositions of *Principia Mathematica* and related systems I", in Gödel (CW1), 145–95.
—— (1939a), "Consistency Proof for the Generalized Continuum Hypothesis", *Proceedings of the National Academy of Sciences*, 25, 220–24; also in Gödel (CW2), 28–32.
—— (1939b), "The Consistency of the Generalized Continuum Hypothesis", *Bulletin of the American Mathematical Society*, 45, 93; also in Gödel (CW2), 27.
—— (1940), "The Consistency of the Axiom of Choice and of the Generalized Continuum Hypothesis with the Axioms of Set Theory", *Annals of Mathematics Studies*, 3, Princeton University Press, Princeton (notes of Gödel's lectures at Princeton in fall 1938, by G.W. Brown); also in Gödel (CW2), 33–101.
—— (1944), "Russell's Mathematical Logic", in Schlipp P.A. (ed.), *The Philosophy of Bertrand Russell*, Northwestern University Press, Evanston (IL), 1944, 125–53; also in Benacerraf, Putnam (1964), 447–69 (1st ed.) and 447–69 (2nd ed. 1983)*, and in Gödel (CW2), 119–41.
—— (1947), "What is Cantor's Continuum Problem?", *The American Mathematical Monthly*, 54, 515–25, and in Gödel (CW2), 176–87.
—— (1964), "What is Cantor's Continuum Problem?", revised and expanded version of Gödel (1947), in Benacerraf, Putnam (1964), 258–73 (1st ed.) and 470–85 (2nd 1983)*; also in Gödel (CW2), 254–70.
Goodman, N. (1951), *The Structure of Appearance,* Harvard University Press, Cambridge (MA); 2nd edition, Bobbs-Merrill, Indianapolis, 1966; 3rd edition Reidel, Boston, 1977.
—— (1954), "The New Riddle of Induction", in N. Goodman, *Fact, Fiction and Forecast*, Harvard University Press, Cambridge (MA); 4th edition, 1979.
Goodman, N. and Quine, W.V. (1947), "Steps Toward a Constructive Nominalism", *Journal of Symbolic Logic*, 12, 105–22.
Haack, S. (2009), *Evidence and Inquiry*, Prometheus Books, Amherst (NY), 1st edition Blackwell, Oxford, 2003.
Haddock, G.R. and Hill, C.O. (2000), *Husserl or Frege?*, Open Court, La Salle (IL).
Halbach, V. and Horsten, L. (2005), "Computational Structuralism", *Philosophia Mathematica* (III), 13, 174–86.
Hale, B. (1987), *Abstract Objects*, Blackwell, Oxford.
—— (1994a), "Dummett's Critique of Wright's Attempt to Resuscitate Frege", *Philosopia Mathematica* (III), 2, 122–47; also in Hale and Wright (2001), 189–213.
—— (1994b), "Singular Terms (2)", in B. McGuinnes and G.L. Oliveri (eds), *The Philosophy of Michael Dummett*, Kluwer, Dordrecht, Boston, London, 1994, 17–44; also in Hale, Wright (2001), 48–71.
—— (1996a), "Singular Terms (1)", in M. Schirn (ed.), *Frege: Importance and Legacy*, Walter de Gruyter, Berlin, New York, 1996, 438–57; also in Hale, Wright (2001), 31–47.
—— (1996b), "Structuralism's Unpaid Epistemological Debts", *Philosophia Mathematica* (III), 4, 124–47.
—— (1999), "Modality", in B. Hale and C. Wright (eds), *A Companion to the Philosophy of Language*, Blackwell, Oxford, 487–514.

—— (2000), "Reals by Abstraction", *Philosophia Mathematica* (III), 8, 100–24; also in Hale and Wright (2001), 399–420.

Hale, B. and Wright, C. (2000), "Implicit Definition and the A Priori", in P. Boghossian and C. Peacocke (eds), *New Essays on the A Priori*, Clarendon Press, Oxford, 286–319; in Hale, Wright (2001), 117–50.

—— (2001), *The Reason's Proper Study. Essays towards a Neo–Fregean Philosophy of Mathematics*, Clarendon Press, Oxford.

—— (2002), "Benacerraf's Dilemma Revisited", *European Journal of Philosophy*, 10, 101–29.

Hallet, M. (1984), *Cantorian Set Theory and Limitation of Size*, Clarendon Press, Oxford.

Hardy, G.H. (1940), *A Mathematician's Apology*, Cambridge University Press, Cambridge.

Hart, W.D. (1996), *The Philosophy of Mathematics*, Oxford University Press, Oxford, New York.

Hebb, D. (1949), *The Organisation of Behavior*, John Wiley and Sons, New York.

—— (1980), *Essay on Mind*, Lawrence Erlbaum Associates, Hillsdale.

Heck, R. Jr. (1993), "The Development of Arithmetic in Frege's *Grundgesetze der Arithmetic*", *Journal of Symbolic Logic*, 58, 579–601; also in W. Demopoulos (ed.), *Frege's Philosophy of Mathematics*, Harvard University Press, Cambdridge (MA), 257–94.

—— (2011), *Frege's Theorem*, Oxford University Press, Oxford, New York.

Heijenoort, J. van, (ed.), (1967), *From Frege to Gödel: A Source Book in Mathematics, 1879–1931*, Harvard University Press, Cambridge (MA).

Heinzmann, G. (1985), *Entre intuition et analyse: Poincaré et le concept de prédicativité,* A. Blanchard, Paris.

Hellman, G. (1989), *Mathematics without numbers*, Oxford University Press, Oxford, New York.

—— (1990), "Modal-Structural Mathematics", in E. Irvine (ed.), *Physicalism in Mathematics*, Dordrecht, Kluwer, 1990, 307–29.

—— (1994), "Real Analysis without Classes", *Philosophica Mathematica* (III), 2, 228–50.

—— (1996), "Structuralism without Structures", *Philosophia Mathematica* (III), 4, 100–23.

—— (1998), "Maoist Mathematics? Review of John P. Burgess and Gideon Rosen, *A Subject With No Object*", *Philosophia Mathematica* (III), 6, 334–45.

—— (1999), "Some Ins and Outs of Indispensability: A Modal Structural Perspective", in A. Cantini, E. Casari, and P. Minari (eds), *Logic and Foundations of Mathematics*, Kluwer Academic Press, Dordrecht, 1999, 25–39.

—— (2005), "Structuralism", in Shapiro (2005), 535–62.

Hempel, G. (1945), "Studies in the Logic of Confirmation", *Mind*, 54, 1–26 and 97–121.

Hermite, C. and Stieltjes, J.T. (1905) *Correspondance d'Hermite et de Stieltjes*, (ed.) B. Baillaud and H. Bouget, with a preface by Émile Picard, Gauthier–Villars, Paris.

Heyting, A. (1930) "Die formalen Regeln der intuitionistischen Logik", *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, 1930, 42–71 and 158–69; partial English translation in Mancosu (1998), 311–27.

—— (1956) *Intuitionism, an Introduction*, North-Holland, Amsterdam.

Hilbert, D. (1899), *Grundlagen der Geometrie*, Teubner, Leipzig; English translation *The Foundations of Geometry*, Open Court, Chicago, 1902.
—— (1922), "Neubegründung der Mathematik: Erste Mitteilung", *Abhandlungen aus dem Seminar der Hamburgischen Universität*, 1, 1922, 157–77; also in Hilbert (GA), vol. 3, 157–77; English translation in Ewald (1996), 1115–34 and Mancosu (1998), 198–214.
—— (1926), "Über das Unendliche", *Mathematische Annalen*, 95, 161–90; also in the 7th edition of Hilbert (1899), 1930; English translation *On the Infinite*, in van Heijenoort (1967), 367–92.
—— (GA), *Gesammelte Abhandlungen*, Springer, Berlin, 1932–35, 3 vols., I vol. 1932, II vol. 1933, III vol. 1935.
Hilbert, D. and Bernays, P. (1934–39), *Grundlagen der Mathematik*. Springer, Berlin, 2 vols., vol. I 1934, vol. II 1939.
Hintikka, J. (1968), "The Varieties of Information and Scientific Explanation", in B. van RootSelaar and J.F. Staal (eds), *Logic, Methodology and Philosophy of Science,* III, North Holland Publishing Company, Amsterdam, 311–31.
—— (1996), *Lingua Universalis vs. Calculus Ratiocinator: An Ultimate Presupposition of Twentieth–century Philosophy*, Springer, Berlin.
Hodes, H.T. (1990), "Ontological Commitments: Thick and Thin", in G. Boolos (ed.), *Meaning and Method: Essays in Honor of Hilary Putnam*, Cambridge University Press, Cambridge, 347–407.
Hodges, W. (1993), *Model Theory,* Cambridge University Press, Cambridge, 1993, abridged version*: A Shorter Model Theory*, Cambridge University Press, Cambridge, 1997.
Hofweber, T. (2005a), "A Puzzle About Ontology", *Noûs*, 39, 256–83.
—— (2005b), "Number Determiners, Numbers, and Arithmetic", *The Philosophical Review*, 114, 179–225.
—— (2007), "Innocent Statements and Their Metaphysically Loaded Counterparts", *The Philosophers' Imprint*, 7, 1–33.
Horwich, P. (1991), "On the Nature and Norms of Theoretical Commitment", *Philosophy of Science*, 58, 1–14.
Howson, C. and Urbach, P. (1989), *Scientific Reasoning. The Bayesian Approach*, Open Court, La Salle.
Hughes, G. and Cresswell, M. (1996), *A New Introduction to Modal Logic*, Routledge, London.
Hume, D. (1739–40), *A Treatise of Human Nature*, (ed.) L.A. Selby-Bigge, 2nd revised edition by P.H. Nidditch, Clarendon Press, Oxford, 1975.
Hussey, E. (1991), "Aristotle on Mathematical Objects", *Apeiron*, 24, 105–34.
Irvine, A. (1989), *Epistemic Logicism and Russell's Regressive Method,* in "Philosophical Studies", 55, 3, pp. 303–27.
—— (1990), (ed.), *Physicalism in Mathematics*, Kluwer A.P., Dordrecht, Boston, London.
Jackson, F. (1980), "Ontological Commitment and Paraphrase," *Philosophy*, 55, 303–15.
Jacob, P. (1980), *L'Empirisme logique*, Editions de Minuit, Paris.
Kalderon, M.E. (2005) (ed.), *Fictionalism in Metaphysics*, Oxford University Press, Oxford, New York.
Kanamori, A. (2004), *The Higher Infinite: Large Cardinals in Set Theory from Their Beginnings,* Springer, Berlin (2nd enlarged and corrected edition; 1st edition 1994).

Kant, I. (AK), *Gesammelte Schriften*, hrgb. von der Königlich Preussischen (Deutschen) Akademie der Wissenschaften, Berlin, Leipzig, 1900–sgg., 29 vols.

—— (KRV), *Kritik der reinen Vernunft*, 1st edition [A] 1781 in Kant (AK), vol. IV, 2nd edition [B] 1787 in Kant (AK), vol. III; also in *Kritik der reinen Vernunft* [editions A and B], hrsg. von Ingeborg Heidemann, Reclam, Stuttgart 1995; English translation *Critique of Pure Reason*, (ed.) P. Guyer and A. Wood, Cambridge University Press, Cambridge, 1997.

—— (LB), *Logik Blomberg*, in Kant (AK), vol. XXIV (1–2), *Vorlesungen über Logik*; English translation *The Logic Blomberg,* in Kant (LL), Part I.

—— (LJ), *Logik. Ein Handbuch zu Vorlesungen* (*Logik Jäsche*), in Kant (AK), vol. IX, *Logik*; English translation *The Jäsche Logic,* in Kant (LL), Part IV.

—— (WL), *Wiener Logik*, in Kant (AK), vol. XXIV (1–2), *Vorlesungen über Logik*; English translation *The Vienna Logic*, in Kant (LL), Part II.

—— (LL), *Lectures on Logic*, (ed.) J.M. Young, Cambridge University Press, Cambridge, 1992.

Kasa, I. (2010), "On Field's Epistemological Argument Against Platonism", *Studia Logica*, 96, 141–47.

Kenny, A. (1995), *Frege*, Penguin Books, London.

Keränen, J. (2001), "The Identity Problem of Realist Structuralism", *Philosophia Mathematica* (III), 9, 308–30.

Kitcher, P. (1983), *The Nature of Mathematical Knowledge*, Oxford University Press, Oxford, New York.

Kitcher, P. and Salmon, W. (eds) (1989), *Scientific Explanation, Minnesota Studies in the Philosophy of Science, Vol. 13*, University of Minnesota Press, Minneapolis.

Kleene, S.C. (1967), *Mathematical Logic*, John Wiley and Sons, Inc., New York, reprinted by Dover, New York, 2002.

Klein, J. (1934–6), "Die griechischste Logistik und die Entstehung der Algebra", *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abteilung B, Studien, 3, 1934, 18–105, and 3, 1936, 122–235; modified English translation by E. Brann, *Greek Mathematical Thought and the Origin of Algebra*, MIT Press, Cambridge (MA), 1967*.

Kripke, S. (1980), *Naming and Necessity*. Harvard University Press, Cambridge (MA).

Kuipers, T. (1997), "The Carnap-Hintikka Programme in Inductive Logic," in M. Sintonen (ed.), *Knowledge and Inquiry: Essays on Jaakko Hintikka's Epistemology and Philosophy of Science*, Poznan Studies, vol. 51, Rodopi, Amsterdam, 1997, 87–99.

Künne, W. (2003), *Conceptions of Truth*, Oxford University Press, Oxford, New York.

Lakatos, I. (1968), "Changes in the Problem of Inductive Logic", in I. Lakatos (ed.), *The Problem of Inductive Logic*, North-Holland, Amsterdam.

—— (1970), "Falsification and the Methodology of Scientific Research Programmes", in I. Lakatos, A. Musgrave (eds), *Criticism and the Growth of Knowledge*, Cambridge University Press, Cambridge, reprinted with corrections 1976, 91–196.

—— (1976), "A Renaissance of Empiricism in the Recent Philosophy of Mathematics", *The British Journal for the Philosophy of Science*, 27, 201–23.

Landini, G. (2011), *Russell*, Routledge, London.

—— (2012), *Frege's Notations. What they are and how they mean*, Palgrave Macmillan, Basingstoke.

Largeault, J. (ed.) (1992), *Intuitionnisme et théorie de la démonstration*, Vrin, Paris.
Laudan, L. (1981), "A Confutation of Convergent Realism", *Philosophy of Science*, 48, 19–49.
Lear, J. (1982), "Aristotle's Philosophy of Mathematics", *Philosophical Review*, 91, 161–92.
Leitgeb, H. and Ladyman, J. (2008), "Criteria of Identity and Structuralist Ontology", *Philosophia Mathematica* (III), 16, 388–96.
Leng, M. (2002), "What's Wrong With Indispensability? (Or, The Case for Recreational Mathematics)", *Synthese*, 131, 395–417.
—— (2005a), "Revolutionary Fictionalism: A Call to Arms", *Philosophia Mathematica* (III), 13, 277–93.
—— (2005b), "Mathematical Explanation", in C. Cellucci and D. Gillies (eds), *Mathematical Reasoning, Heuristics and the Development of Mathematics*, King's College Publications, London, 167–89.
—— (2010), *Mathematics and Reality*, Oxford University Press, Oxford, New York.
Leng M., Paseu A. and Potter M. (2007) (eds), *Mathematical Knowledge*, Oxford University Press, Oxford, New York.
Leslie, A. (1987), "Pretence and Representation: The Origins of the 'Theory of Mind'", *Psychological Review*, 94, 412–26.
—— (1994), "Pretending and Believing: Issues in the ToMM", *Cognition*, 50, 211–38.
Lewis, D. (1986), *On the Plurality of Worlds*, Blackwell Press, Oxford.
Liggins, D. (2006), "Is There a Good Epistemological Argument Against Platonism?", *Analysis*, 66, 135–41.
—— (2007), "Quine, Putnam, and the 'Quine-Putnam' Indispensability Argument", *Erkenntnis*, 68, 113–27.
—— (2010a), "The Autism Objection to Pretence Theories", *Philosophical Quarterly*, 60, 764–82.
—— (2010b), "Epistemological Objections to Platonism", *Philosophy Compass*, 5, 67–77.
Linnebo, Ø. (2003), "Plural Quantification Exposed", *Noûs*, 37, 71–92.
—— (2006), "Epistemological Challenges to Mathematical Platonism", *Philosophical Studies*, 129, 545–74.
—— (2008), "Plural Quantification", *The Stanford Encyclopedia of Philosophy* (Spring 2009 Edition), Edward N. Zalta (ed.), URL: <http://plato.stanford.edu/archives/spr2009/entries/plural–quant/>
Linsky, B. and Zalta, E. (1995), "Naturalized Platonism versus Platonized Naturalism", *The Journal of Philosophy*, 92, 525–55.
—— (2006), "What is Neo-logicism?", *The Bulletin of Symbolic Logic*, 12, 60–99.
Lipton, P. (2002), *Inference to the Best Explanation,* Routledge, London, 2nd revised edition; 1st edition 1991.
MacBride, F. (1999), "Listening to Fictions: A Study of Fieldian Nominalism", *British Journal for the Philosophy of Science*, 50, 431–55.
MacBride, F. (2003), "Speaking with Shadows: A Study of Neo-Logicism", *British Journal for the Philosophy of Science*, 54, 103–63.
MacFarlane, J. (2002), "Frege, Kant, and the Logic in Logicism", *The Philosophical Review*, 111, 25–65.
Mackie, J.L. (1977), *Ethics: Inventing Right and Wrong,* Penguin.

—— (2006), *How Things Might Have Been: Individuals, Kinds, and Essential Properties*, Oxford University Press, Oxford, New York.

MacLane, S. (1997), *Categories for the Working Mathematician*, Springer, Dordrecht.

Maddy, P. (1980), "Perception and Mathematical Intuition", *Philosophical Review,* 89, 163–96.

—— (1989), "The Roots of Contemporary Platonism", *The Journal of Symbolic Logic*, 54, 1121–44.

—— (1990a), *Realism in Mathematics*, Oxford University Press, Oxford, New York.

—— (1990b), "Physicalistic Platonism", in A. Irvine (ed.), *Physicalism in Mathematics*, Kluwer Academic Publishers, 259–89.

—— (1992), "Indispensability and Practice", *The Journal of Philosophy*, 89, 275–89.

—— (1994), "Taking Naturalism Seriously", in D. Prawitz, B. Skyrms, and D. Westerståhl (eds), *Logic, Methodology and Philosophy of Science*, IX, 383–407.

—— (1997), *Naturalism in Mathematics,* Clarendon Press, Oxford.

—— (1998), "How to be a Naturalist about Mathematics", in H.G. Dales and G. Oliveri (eds), *Truth in Mathematics,* Clarendon Press, Oxford, 161–80.

—— (2005), "Three Forms of Naturalism", in S. Shapiro (2005), 437–60.

—— (2007), *Second Philosophy*, Oxford University Press, Oxford, New York.

—— (2011), *Defending the Axioms*, Oxford University Press, Oxford, New York.

Malament, D. (1982), "Review of Hartry Field's *Science Without Numbers*", *The Journal of Philosophy*, 79, 523–34.

Mancosu, P. (1998) (ed.), *From Brouwer to Hilbert. The Debate on the Foundations of Mathematics in the 1920s*, Oxford University Press, Oxford, New York.

—— (2001), "Mathematical Explanation: Problems and Prospects", *Topoi*, 20, 97–117.

—— (2008a) (ed.), *The Philosophy of Mathematical Practice*, Oxford University Press, Oxford, New York.

—— (2008b), "Mathematical Explanation: Why it Matters", in P. Mancosu (ed.), 2008, *The Philosophy of Mathematical Practice*, Oxford University Press, Oxford, New York.

—— (2008c), "Quine and Tarski on Nominalism", *Oxford Studies in Metaphysics*, vol. IV, 2008, 22–55.

—— (2011) "Explanation in Mathematics", *The Stanford Encyclopedia of Philosophy* (Summer 2011 Edition), Edward N. Zalta (ed.), URL: <http://plato.stanford.edu/archives/sum2011/entries/mathematics–explanation/>.

Martin, W. (2006), *Theories of Judgment: Psychology, Logic, Phenomenology*, Cambridge University Press, Cambridge.

McGee, V. (1997), "How We Learn Mathematical Language", *The Philosophical Review*, 106, 35–68.

McGrath, M. (2007), "Propositions", *The Stanford Encyclopedia of Philosophy* (Fall 2008 Edition), Edward N. Zalta (ed.), URL: <http://plato.stanford.edu/archives/fall2008/entries/propositions/>.

Melia, J. (1995), "On What There's Not", *Analysis*, 55, 223–29.

—— (2000), "Weaseling Away the Indispensability Argument", *Mind*, 109, 455–79.

—— (2002), "Response to Colyvan", *Mind*, 111, 75–9.

—— (2006), "The Conservativeness of Mathematics", *Analysis*, 66, 202–08.

Mendelson, E. (1964), *Introduction to Mathematical Logic*, Van Nostrand Reinhold, New York, 4th edition, 1997.

Meyer, G. (2009), "Extending Hartry Field's Instrumental Account of Applied Mathematics to Statistical Mechanics", *Philosophia Mathematica* (III), 17, 273–312.

Mill, J.S. (1843), *A System of Logic, Ratiocinative and Inductive*. Longmans, London.

Monton, B. (ed.) (2007), *Images of Empiricism: Essays on Science and Stances. With a Reply from Bas C. van Fraassen*, Oxford University Press, Oxford, New York.

Mueller, I. (1970), "Aristotle on Geometrical Objects", *Archiv für Geschichte der Philosophie*, 52, 156– 71.

Nagel, E. and Newman, J.R. (1958), *Gödel's Proof*, NYU Press, New York.

Neumann, von J. (1923), "Zur Einführung der transfiniten Zahlen", *Acta Litterarum ac Scientiarum: Sectio Scientiarum Mathematicarum*, 1, 199–208; English translation "On the Introduction of Transfinite Numbers", in van Heijenoort (1967), 346–54.

—— (1947), "The Mathematician", in R.B. Heywood (ed.), *The Works of the Mind*, University of Chicago Press, Chicago, 1947, 180–96; also in *John von Neumann. Collected Works*, (ed.) A.H. Taub, Pergamon Press, Oxford, 1961*, vol. 1, 1–9.

Panza, M. (1997), "Mathematical Acts of Reasoning as Synthetic *a priori*", in M. Otte M., M. Panza (eds), *Analysis and sinthesis in Mathematics*, Kluwer, Dordrecht, Boston, London, 261–326.

—— (2012), "The Twofold Role of Diagrams in Euclid's Plane Geometry", in M., J. Mumma, M. Panza and G. Sandu (eds), *Diagrammatic Reasoning in Mathematics*, special issue of *Synthese*, 186, n. 1, 2012, 55–102.

Papineau, D. (1988), "Mathematical Fictionalism", *International Studies in the Philosophy of Science*, 2, 151–73.

—— (1993), *Philosophical Naturalism*, Basil Blackwell, Oxford.

—— (2007), "Naturalism", *The Stanford Encyclopedia of Philosophy* (Spring 2009 Edition), Edward N. Zalta (ed.), URL: <http://plato.stanford.edu/archives/spr2009/entries/naturalism/>.

Parsons, C. (1964), "Frege's Theory of Number", in M. Black (ed.), *Philosophy in America*, Allen & Unwin, London, 180–203; reprinted in W. Demopoulos (ed.), *Frege's Philosophy of Mathematics*, Harvard University Press, Cambridge (MA), 1995, 182–210.

—— (1979–80), "Mathematical Intuition", *Proceedings of the Aristotelian Society*, New Series, 80, 145–68, in Hart (1996), 95–113.

—— (1982), "Objects and Logic", *The Monist*, 65, 91–516.

—— (1983a), *Mathematics in Philosophy: Selected Essays,* Cornell University Press, Ithaca.

—— (1983b), "Quine on the Philosophy of Mathematics", in Parsons (1983a), Chapter 7; also in L. Hahn, P. Schilpp (eds), *The Philosophy of W.V. Quine,* Open Court, La Salle, (Ill.), 369–95.

—— (1990), "The Structuralist View of Mathematical Objects", *Synthese*, 84, 303–46.

—— (1998), "Finitism and Intuitive Knowledge", in Schirn (1998), 251–70.

—— (2000), "Reason and Intuition", *Synthese*, 125, 299–315.

—— (2004), "Structuralism and Metaphysics", *The Philosophical Quarterly*, 54, 56–77.

—— (2008), *Mathematical Thought and Its Objects*, Cambridge University Press, Cambridge.

Paseu, A. (2007), "Scientific Realism", in Leng *et al.* (2007), 123–49.

Peano (1889), *Arithmetices Principia, Nova Methodo Exposita*, Bocca, Augustæ Taurinorum, 1998; also in G. Peano, *Opere scelte*, Cremonese, Roma, 1957–59, vol. II, 20–55.

Peressini, A. (2008), "Confirmational Holism and its Mathematical (W)holes", *Studies in History and Philosophy of Science*, 39, 102–11.

Petitot, J. (1995), "Pour un platonism transcendantal", in M. Panza and J.M. Salanskis (eds), *L'objectivité mathématique. Platonismes et structures formelles*, Masson, Paris, 147–78.

Pettigrew, R. (2009), "Aristotle on the Subject Matter of Geometry", *Phronesis* 54, 239–60.

Pincock, C. (2004), "A Revealing Flaw in Colyvan's Indispensability Argument", *Philosophy of Science*, 71, pp. 61–79.

—— (2011), "On Batterman's 'On the Explanatory Role of Mathematics in Empirical Science'", *British Journal for the Philosophy of Science*, 62, 211–17.

—— (2012), *Mathematics and Scientific Representation*, Oxford University Press, Oxford, New York.

Plantinga, A. (1974), *The Nature of Necessity*, Clarendon Press, Oxford.

Plato (PO), *Platonis opera, recognovit brevique adnotatione critica instruxit Ioannis Burnet*, 5 Vols., Oxford 1900–07. References in the text follow numeration in Plato, *Platonis opera quae extant omnia, excudebat H. Stephanus*, 3 vols., Geneva 1578, which is indicated in brackets in the following lemmas, together with the corresponding volume).

—— (Epist.), *Epistulae*, in Platone (PO), vol. V (St. III, 309a–363e) (English translation: *Letters, in Plato in Twelve Volumes,* vol. 7, translated by R.G. Bury. Cambridge (MA), Harvard University Press; London, William Heinemann Ltd, 1966).

—— (Gorg.), *Gorgias*, in Platone (PO), vol. III (St. I, 447a–527e) (English translation: *Letters, in Plato* (PTW), Vol. 7, by R.G. Bury, 1966).

—— (Hipp. Maj.), *Hippias Maior*, in Platone (PO), vol. III (St. III, 281a–304e) (English translation: *Gorgias*, in Plato (PTW), vol. 3, by W.R.M. Lamb., 1967).

—— (Men.), *Meno*, in Platone (PO), vol. III (St. II, 70a–100b) (English translation: *Meno*, in Plato (PTW), Vol. 3, by W.R.M. Lamb, 1967).

—— (Phaed.), *Phaedrus*, in Platone (PO), vol. II (St. III, 227a–279c) (English translation: *Phaedrus*, in Plato (PTW), vol. 9, by Harold N. Fowler, 1966).

—— (Rep.), *Respublica*, in Platone (PO), vol. IV (St. II, 327a–621d) (English translation: *The Republic*, in Plato (PTW), vols. 5 and 6, by Paul Shorey, 1969).

—— (Theaet.), *Thaetetus*, in Platone (PO), vol. I (St. I, 142a–210d) (English translation: *Theaetetus*, in Plato (PTW), vol. 12, by Harold N. Fowler, 1921).

—— (PTW), *Plato in Twelve Volumes*, 12 Vols., Harvard University Press, Cambridge (MA), William Heinemann Ltd., London.

Poincaré, H. (1900), *La valeur de la science*, Flammarion, Paris; English translation *The Value of Science*, in Poincaré (1913), 201–55.

—— (1902), *La science et l'hypothèse*, Flammarion, Paris; English translation *Science and Hypothesis*, in Poincaré (1913), 9–197.

—— (1906), *Science et méthode*, Flammarion, Paris; English translation *Science and Method*, in Poincaré (1913), 359–546.

—— (1913), *The Foundations of Science*, The Science Press, Lancaster (PA).

Popper, K. (1934), *Logik der Forschung,* Mohr Siebeck, Tübingen; English translation *The Logic of Scientific Discovery*, Hutchinson, London, 1959.

Potter, M. (2002), *Reason's Nearest Kin: Philosophies of Arithmetic from Kant to Carnap*, Oxford University Press, Oxford, New York.
—— (2004) *Set Theory and Its Philosophy. A Critical Introduction*, Oxford University Press, Oxford, New York.
Proclus (CEM) (1992), *A Commentary on the First Book of Euclid's* Elements, translated and annotated by G.R. Morrow, with a Preface by I. Mueller, Princeton University Press, Princeton NJ).
Psillos, S. (1999), *Scientific Realism: How Science Tracks Truth*, Routledge, London.
Putnam, H. (1962), "What Theories Are Not", in E. Nagel, P. Suppes and P. A. Tarski (eds), *Logic, Methodology and Philosophy of Science*; *Proceedings of the 1960 International Congress*, Stanford University Press, Stanford (CA), 240–51; also in Putnam (1975b), Chapter 13.
—— (1965), "Craig's Theorem", *The Journal of Philosophy*, 62, 251–60; also in Putnam (1975b), Chapter 14.
—— (1967), "Mathematics without Foundations", *The Journal of Philosophy*, 64, 5–22*; also in Putnam (1975b), Chapter 3; also in Benacerraf, Putnam (1973), 295–311.
—— (1971), *Philosophy of Logic*, Harper & Row, New York; also in Putnam (1975b), Chapter 20*.
—— (1975a), "What is Mathematical Truth?", *Historia Mathematica*, 2, 529–43; also in Putnam (1975b), Chapter 6.
—— (1975b), *Mathematics Matter and Method: Philosophical Papers Vol. 1,* Cambridge University Press, Cambridge, 2nd edition, 1985.
—— (1983), "Equivalence", originally published in Italian as lemma 'Equivalenza' in Enciclopedia Einaudi, Giulio Einaudi Editore, 5, 547–64; also in H. Putnam, *Realism, and Reason*, *Philosophical Papers*, vol. III, Cambridge University Press, Cambridge, Chapter 2.
—— (2004), *Ethics Without Ontology*, Harvard University Press, Cambridge (MA).
—— (2012), "Indispensability Arguments in the Philosophy of Mathematics", in H. Putnam, *Philosophy in an Age of Science*, (ed.) M. De Caro and D. Macarthur, Cambridge (MA), Harvard University Press, 2012, 181–201.
Quine, W.V.O. (1936), "Truth by Convention", (ed.) O.H. Lee, *Philosophical Essays for A.N. Whitehead,* Longmans, New York, 90–124; also in Quine (1966a), Chapter 1.
—— (1939), "Designation and Existence", *The Journal of Philosophy*, 36, 701–709; also in F. Feigl and W. Sellars (eds), *Readings in Philosophical Analysis*, Appleton-Century-Crofts, Inc., New York.
—— (1948), "On What There Is", *Review of Metaphysics*, 2, 21–38; also in Quine (1953a)*, Chapter 1.
—— (1950), *Methods of Logic*, Harvard University Press, Cambridge (MA), 4th edition 1982.
—— (1951a), "Two Dogmas of Empiricism", *Philosophical Review*, 60, 20–43; also in Quine (1953a)*, Chapter 2.
—— (1951b), "On Carnap's Views on Ontology", *Philosophical Studies*, 2, 65–72; also in Quine (1966a), Chapter 19.
—— (1953a), *From a Logical Point of View*, Harper & Row, New York, 2nd edition 1961.
—— (1953b), "Logic and the Reification of Universals", in Quine (1953a), Chapter 6.

—— (1954), "Carnap and Logical Truth", first published in *Synthese*, 12, 1960, 350–74; also in Quine, 1966a*, Chapter 12.

—— (1955), "Posits and Reality", in S. Uyeda (ed.), *Basis of Contemporary Philosophy,* Waseda University Press, Tokyo; also in Quine (1966a), Chapter 23.

—— (1958), "Speaking of Objects", *Proceedings and Addresses of the American Philosophical Association*, 31, 5–22; also in Quine, 1969a, Chapter 1.

—— (1963), *Set Theory and its Logic*, Harvard University Press, Cambridge (MA), second edition 1969.

—— (1960), *Word and Object*, MIT Press, Cambridge (MA).

—— (1966a), *The Ways of Paradox and Other Essays*, Columbia University Press, New York, 2nd revised edition, 1976.

—— (1966b), "The Scope and Language of Science", in Quine (1966a)*, Chapter 22.

—— (1968), "Ontological Relativity", *Journal of Philosophy*, 65, 185–212 in Quine (1969a), Chapter 2.

—— (1969a), *Ontological Relativity and Other Essays*, Columbia University Press, New York and London.

—— (1969b), "Epistemology Naturalized", in Quine (1969a), Chapter 3.

—— (1969c), "Existence and Quantification", *L'Âge de la Science*, 1, 151–64; also in Quine, 1969a*, Chapter 4.

—— (1970), *Philosophy of Logic*, Harvard University Press, Cambridge (MA), 2nd edition 1986.

—— (1975), "On Empirically Equivalent Systems of the World", *Erkenntins*, 9, 313–28.

—— (1981a), "Things and Their Place in Theories", in Quine (1981e), 1–23.

—— (1981b), "Five Milestones of Empiricism", in Quine (1981e), 67–72.

—— (1981c), "Success and Limits of Mathematization", in Quine (1981e)*, 148–55.

—— (1981d), "Reply to Chihara", in P.A. French, T. Uehling and H. Wettstein (eds), *The Foundations of Analytic Philosophy*, University of Minnesota Press, Minneapolis, 453–54.

—— (1981e), *Theories and Things,* Harvard University Press, Cambridge (MA).

—— (1986), "Reply to Charles Parsons", in L. Hahn and P. Schilpp (eds), *The Philosophy of W.V. Quine,* Open Court, La Salle, (IL), 396–403.

—— (1990), *Pursuit of Truth*, Harvard University Press, Cambridge (MA), 2nd revised edition 1992.

—— (1993), "In Praise of Observation Sentences", *The Journal of Philosophy*, 90, 107–16.

—— (1995), *From Stimulus to Science*. Harvard University Press, Cambridge (MA).

Quine, W.V.O. and Ullian, J.S. (1978), *The Web of Belief*, McGraw-Hill, New York, 2nd edition 1970.

Ramsey, F.P. (1926), "The Foundations of Mathematics", *Proceedings of the London Mathematical Society*, II, 338–84; also in F.P. Ramsey, *Philosophical Papers*, Cambridge University Press, Cambridge, 164–224.

Rayo, A., Yablo, S. (2001), "Nominalism through De-nominalization", *Noûs*, 35, 74–92.

Reichenbach, H. (1951), *The Rise of Scientific Philosophy*, University of California Press, Berkeley, Los Angeles, London.

Resnik, M.D. (1981), "Mathematics as a Science of Patterns: Ontology and Reference", *Noûs*, 15, 529–50.

—— (1983), "Review of Hartry Field's *Science without Numbers*", *Noûs*, 17, 514–19.
—— (1985), "How Nominalist is Hartry Field's Nominalism", *Philosophical Studies* 47, 163–81.
—— (1992), "Applying Mathematics and the Indispensability Argument", in J.Echeverria, A. Ibarra, and T. Morman (eds), *The Space of Mathematics*, Walter de Gruyter, Berlin, 115–31.
—— (1995), "Scientific vs Mathematical Realism: The Indispensability Argument", *Philosophia Mathematica* 3, 166–74.
—— (1997), *Mathematics as a Science of Patterns*, Clarendon Press, Oxford.
—— (2005), "Quine and the Web of Belief', in Shapiro (2005), 412–37.
Richardson, A. and Übel, T. (eds) (2007), *The Cambridge Companion to Logical Empiricism*, Cambridge University Press, Cambridge, New York.
Ritchie, J. (2009), *Understanding Naturalism*, Acumen Publishing, Durham.
Rosen, G. (2001), "Abstract Objects", *The Stanford Encyclopedia of Philosophy* (Fall 2009 Edition), Edward N. Zalta (ed.), URL: <http://plato.stanford.edu/archives/fall2009/entries/abstract–objects/>.
Ross, S. (1979), *A First Course in Probability Theory*, Prentice Hall, Upper Saddle River (NJ), 8th edition 2008.
Rothmaler, P. (2000), *Introduction to Model Theory*, Gordon and Breach, Amsterdam.
Russell, B. (1903), *The Principles of Mathematics*, Allen & Unwin, London, 2nd edition 1938.
—— (1905), "On Denoting", *Mind*, 14, 479–93.
—— (1906), *Les paradoxes de la logique,* in "Revue de Métaphysique et de Morale", XIV, pp. 627–50 (English translation *On "insolubilia" and Their Solution by Symbolic Logic,* in Russell, 1973, Chapter 9).
—— (1907), "The Regressive Method of Discovering the Premises of Mathematics", in Russell (1973, Chapter 13).
—— (1908), "Mathematical Logic as Based on the Theory of Types", *American Journal of Mathematics*, 30, 222–62.
—— (1918–19), "The Philosophy of Logical Atomism", *The Monist*, 28 (1918)–29 (1919); also in B. Russell, *The Collected Papers of Bertrand Russell*, vol. III, *The Philosophy of Logical Atomism and Other Essays*, (ed.) J.G. Slater, Allen & Unwin, London, 1914–19.
—— (1973), *Essays in Analysis*, Allen & Unwin, London.
Russell, B. and Whitehead, A.N. (l910–13), *Principia Mathematica*, 3 vols, Cambridge, Cambridge University Press, 2nd edition 1925 (vol. 1), 1927 (vols 2, 3).
Saatsi, J. (2007), "Living in Harmony: Nominalism and the Explanationist Argument for Realism", *International Studies in the Philosophy of Science*, 21, 19–33.
—— (2011), "The Enhanced Indispensability Argument: Representational versus Explanatory Role of Mathematics in Science", *British Journal for the Philosophy of Science*, 62, 143–54.
Sainsbury, R.M. (2010), *Fiction and Fictionalism*, Routledge, London, New York.
Salmon, W. (1989), *Four Decades of Scientific Explanation,* University of Minnesota Press, Minneapolis.
Schirn, M. (ed.) (1998), *Philosophy of Mathematic Today,* Clarendon Press, Oxford.
Shapiro, S. (1983), "Conservativeness and Incompleteness", *The Journal of Philosophy*, 80, 521–31.

—— (1997), *Philosophy of Mathematics. Structure and Ontology*, Oxford University Press, Oxford, New York.

—— (2000a), *Thinking about Mathematics: The Philosophy of Mathematics*, Oxford University Press, Oxford, New York.

—— (2000b), "Frege Meets Dedekind: A Neo-logicist Treatment of Real Analysis", *Notre Dame Journal of Formal Logic*, 41, 335–64.

—— (2005a) (ed.), *Oxford Handbook of Philosophy of Mathematics and Logic*, Oxford University Press, Oxford, New York.

—— (2005b), "Higher-order Logic", in S. Shapiro (2005), Chapter 25.

—— (2006a), *Structure and Identity,* in F. MacBride (ed.), *Identity and Modality,* Oxford University Press, Oxford, New York, 109–45.

—— (2006b), "The Governance of Identity", in F. MacBride (ed.), *Identity and Modality*, Oxford University Press, Oxford, New York, 164–73.

—— (2008), "Identity, Indiscernibility, and *ante rem* Structuralims: The Tale of *i* and *–i*", *Philosophia Mathematica* (III), 16, 285–309.

—— (2009) "A Scientific Enterprise? Penelope Maddy's Second Philosophy", *Philosophia Mathematica (III)*, 71, 247–71.

Shapiro, S. and Weir, A. (2000), "'Neo-logicist' Logic is not Epistemologically Innocent", *Philosophia Mathematica* (III), 8, 160–89.

Shoenfield, J.R. (1967) *Mathematical Logic*, Addison-Wesley Publishing Co., Reading (MA), London, Don Mills, Ont., 1967, reprinted by AK Peters, Ltd, Natick (MA), 2001.

Skolem, T. (1923), "Begründung der elementaren Arithmetik durch die rekurrierende Denkweise ohne Anwendung scheinbare Veränderlichen mit unendlichem Ausdehnungsbereich", *Videnskapsselkapet skrifter Kristiania, I. Mathematisk – naturvidenskabelig klasse*, 6; English translation in van Heijenoort (1967), 302–33.

Smullyan, R. (1992), *Gödel's Incompleteness Theorems*, Oxford University Press, Oxford.

Smith, P. (2007), *An Introduction to Gödel's Theorem*, Cambridge University Press, Cambridge, 2007, reprinted with corrections (2008).

Sober, E. (1993), "Mathematics and Indispensability", *The Philosophical Review*, 102, 35–57.

Stadler, F. (1997), *Studien zum Wiener Kreis. Ursprung, Entwicklung und Wirkung des Logischen Empirismus im Kontext*, Suhrkamp, Frankfurt a. M.; English translation *The Vienna Circle. Studies in the Origins, Development and Influence of logical Empiricism*, Springer, Vienna, New York, 2001.

Stanley, J. (2001), "Hermeneutic Fictionalism", *Midwest Studies in Philosophy*, XXV, 2001, 36–71.

Steiner, M. (1978), "Mathematics, Explanation, and Scientific Knowledge", *Noûs*, 12, 17–28.

—— (1989), "The Application of Mathematics to Natural Science", *The Journal of Philosophy*, 86, 449–80.

—— (1998), *The Applicability of Mathematics as a Philosophical Problem,* Harvard University Press, Cambridge (MA).

—— (2005), "Mathematics: Application and Applicability", in Shapiro (2005), 625–50.

Stigt, W.P. van (1990), *Brouwer's Intuitionism*, Studies in the History and Philosophy of Mathematics, vol. 2, North Holland.

Stolz, O. (1885), *Vorlesungen über Allgemeine Arithmetik*, Teubner, Leipzig.
Strawson, P. (1966), *The Bounds of Sense: An Essay on Kant's Critique of Pure Reason*, Methuen, London.
Suppe, F. (1977), "The Search for Philosophical Understanding of Scientific Theories", in F. Suppe (ed.), *The Structure of Scientific Theories*, University of Illinois Press, 2nd edition revised and enlarged, 3–241, 1st edition 1974.
Suppes, P. (1972), *Axiomatic Set Theory*, Dover, New York.
Tait, W.W. (1986), "Truth and Proof: The Platonism of Mathematics", *Synthese*, 69, 341–70; also in Tait W.W., *The Provenance of Pure Reason. Essays in Philosophy of Mathematics and its History*, Oxford University Press, Oxford, New York, 2005, 61–88.
Thomas, R. (2000), "Mathematics and Fiction I: Identification", *Logique et Analyse*, 43, 171–2, 301–40.
—— (2002), "Mathematics and Fction II: Analogy", *Logique et Analyse*, 45, 177–8, 185–228.
Tieszen, R. (1992), "Kurt Gödel and Phenomenology", *Philosophy of Science*, 59, 176–94.
—— (1995), "Mathematics", in B. Smith and D.W. Smith (eds), *The Cambridge Companion to Husserl*, Cambridge University Press, Cambridge.
—— (2005), *Phenomenology, Logic, and the Philosophy of Mathematics*, Cambridge University Press, Cambridge.
Varzi, A. (2009), "Mereology", *The Stanford Encyclopedia of Philosophy* (Summer 2009 Edition), Edward N. Zalta (ed.), URL: <http://plato.stanford.edu/archives/sum2009/entries/mereology/>.
Vineberg, S. (1996), "Confirmation and the Indispensability of Mathematics to Science", *Philosophy of Science*, 63: Supplement, S256–S263.
Walton, K. (1990), *Mimesis As Make-believe: On the Foundations of the Representational Arts*, Harvard University Press, Harvard.
—— (1993), "Metaphor and Prop Oriented Make-believe" *European Journal of Philosophy* 1, 39–57; also in Kalderon, 2005, 65–87.
Watkins, J. (1958), "Confirmable and Influential Metaphysics", *Mind*, 67, 344– 65.
Weir, A. (2005), "Naturalism Reconsidered", in S. Shapiro (2005), 460–82.
White, N. (1974), "What Numbers Are", *Synthese*, 27, 111–24; also in D. Giaquette (ed.), *Philosophy of Mathematics, An Anthology*, Wiley Blackwell, Oxford, 2001, 91–8.
Wiles, A. (1995), "Modular Elliptic Curves and Fermat's Last Theorem", *Annals of Mathematics*, 141, 443–551.
Woodin, H. (2001a), "The Continuum Hypothesis, Part I", *Notices of the AMS*, 48, 567–76.
—— (2001b), "The Continuum Hypothesis, Part II", *Notices of the AMS*, 48, 681–90.
Woodward, J. (2009), "Scientific Explanation", *The Stanford Encyclopedia of Philosophy* (Spring 2009 Edition), Edward N. Zalta (ed.), URL: <http://plato.stanford.edu/archives/spr2009/entries/scientific–explanation/>.
Wright, C. (1983), *Frege's Conception of Numbers as Objects*, Aberdeen University Press, Aberdeen.
—— (1990), "Field and Fregean Platonism", in A. Irvine (1990), 73–93; also in Hale and Wright (2001), 153–68.

—— (1997), "On the Philosophical Significance of Frege's Theorem", in R. Heck Jr. (ed.), *Language, Truth and Logic: Essays in Honour of Michael Dummett,* Clarendon Press, Oxford, 201–44; also in Hale and Wright (2001), 272–306.

—— (1998a), "Response to Dummett", in Schirn (1998), 389–405; also in Hale and Wright (2001), 256–71.

—— (1998b), "On the Harmless Impredicativity of $N^{=}$", in Schirn (1998), 339–68; also in Hale and Wright (2001), 229–55.

—— (1998c), "Why Frege Does Not Deserve his Grain of Salt: A Note on the Paradox of "The Concept Horse" and the Ascription of *Bedeutungen* to Predicates", in *Grazer Philosophische Studien*, 55, *New Essays on the Philosophy of Michael Dummett*, (ed.) J. Brandl and P. Sullivan, Rodopi, Amsterdam, Atlanta, 1998, 239–63; also in Hale and Wright (2001), 72–90.

—— (1999), "Is Hume's Principle Analytic?", *Notre Dame Journal of Formal Logic*, 40, 6–30; also in Hale and Wright (2001), 307–32.

—— (2000), "Neo-Fregean Foundations for Real Analysis", *Notre Dame Journal of Formal Logic*, 41, 317–34.

Yablo, S. (1998), "Does Ontology Rest on a Mistake?", *Proceedings of the Aristotelian Society*, Supp. vol. 72, 229–61; also in Yablo (2010), 117–44.

—— (2001), "Go Figure: A Path Through Fictionalism", *Midwest Studies in* Philosophy, XXV, 72–102; also in Yablo (2010), 177–99.

—— (2002), "Abstract Objects: A Case Study", in A. Bottani, M. Carrara and P. Giaretta (eds) *Individuals, Essence and Identity*. *Themes of Analytic Metaphysics*, Topoi Library, vol. 4, Kluwer, Dordrecht, 2002*, 163–88; also in E. Sosa and E. Villanueva (eds), *Philosophical Issues 12: Realism and Relativism*, Blackwell, Oxford, 220–40; also in Yablo (2010), 200–20.

—— (2005), "The Myth of the Seventh", in Kalderon (2005), 88–115; also in Yablo (2010), 221–45.

—— (2006), "Non-catastrophic Presupposition Failure", in J.J. Thomson and A. Byrne (eds), *Content and Modality*, Oxford University Press, Oxford, New York, 164–90; also in Yablo (2010), 269–95.

—— (2009), "Must Existence-questions Have Answers?", in D. Chalmers, D. Manley and R. Wasserman (eds), *Metametaphysics*, Oxford University Press, Oxford, New York, 507–25; also in Yablo (2010), 296–314.

—— (2010), *Things*, Oxford University Press, Oxford, New York.

Zalta, E. (1983), *Abstract Objects: An Introduction to Axiomatic Metaphysics*, D. Reidel, Dordrecht.

—— (1999), "Natural Numbers and Natural Cardinals as Abstract Objects: A Partial Reconstruction of Frege's *Grundgesetze* in Object Theory*", Journal of Philosophical Logic*, 28, 619–60.

—— (2000), "Neo-logicism? An Ontological Reduction of Mathematics to Metaphysics", *Erkenntnis*, 53, 219–65.

—— (PM), *Principia Metaphysica*, draft available online at http://mally.stanford.edu/principia.pdf.

Zermelo, E. (1908), "Untersuchungen über die Grundlagen der Mengenlhere I", *Mathematische Annalen*, 65, 261–81; also in van Heijenoort (1967), 199–215.

Index

Notes: This index only points to the most salient passages where the relevant arguments of the listed authors are discussed. This is not meant, therefore, as an exhaustive index of names or of the occurrences of listed lemmas.

abstract structure of progression, 104
abstraction
  abstraction principles 157–8, 164–5 (*see also* Hume's Principle)
  in Aristotle, 29, 31
  of Dedekind, 192–3
  linguistic, 185
analysis (real and complex), 45–7 (*see also* real numbers, definition)
analytic philosophy, x–xi
Annas, J., 27–9
applicability of mathematics, ix, 113, 117–18, 121, note 1 ch. 6, note 3 ch. 7
  and conservativeness, 112–25
argument from pessimistic meta-induction, 241
Aristotle, 27–32, 33–4, 38
  on ideas, 27–8
  Metaphysics, 27–30, note 10, ch. 1
  Physics, 29, note 11 ch. 1
arithmetic
  applied, 130, 133
  finitary, 81–6 (*see also* mathematics, finitary)
  first-order, 139–40, 193, 227–8, 245, note 23 ch. 2, note 10 ch. 7
  Frege arithmetic (FA) *see* Frege
  primitive recursive (PRA), 193–4
  pure, 25
  second-order, 139, 158, note 33 ch. 2, note 10 ch. 7
arithmetization of analysis, 45–7
assert, xiii
atoms
  atomic theory, *see* theory
  mereological, 140–3, 222–3, note 11 ch. 4 (*see also* mereology)
autism objection, *see* Stanley
automorphism, 186–7
axioms, xv, 48 (*see also* Frege, Peano)
Azzouni, J., 230, 247, note 6 ch. 3, note 35 ch. 6, note 14 ch. 7

Baker, A., 199, 203, 209, note 13 ch. 6 (*see also* indispensability argument; explanation)
Balaguer, M., 135
  objection to Field, 127
  on easy road, *see* easy road
  on fictionalism, 125
Bedeutung, *see* reference
Benacerraf, P., viii, ch. 3, 112, 136, 162, 164, 176, 180, 181, 228, notes 2, 3, 6 ch. 3
  Benacerraf 's (1965) argument, 101–7
  Benacerraf's (1973) argument, *see* Benacerraf's dilemma
Benacerraf's dilemma, chs 3, 4, 5
Bernays, P., 81, note 40 ch. 2.
Bolzano, B., 70
Boolos, G., 65, 141, 157, 160, 162, note 15 ch. 2, note 28 ch. 2
Bourbaki, N., 184–5
Brouwer, L. E. J., 87–90, 90–2, 190, 225, note 45 ch. 2 (*see also* intuitionism; pure two-oneness)
Bueno, O., 230, notes 11, 13 ch. 6, note 3. ch 7
Burgess, J., 122, 137, 141–2, 161, 192, 200, 230, 237–8, notes 12, 13, 14, 16 ch. 4, note 12, 20, 30 ch. 7
  objections to fictionalism, 135
  hermeneutic/revolutionary distinction, 128–9
Burnyeat M. F., 22–3, 29

calculus of individuals, *see* mereology
canonical notation, 210
Cantor, G., 63, 68–9, 94, 98 (*see also* transfinite cardinals; sets; Continuum Hypothesis; real numbers; transfinite ordinals)
  Cantor's paradox, 68
  definition of real numbers, 63
Carnap, R., x, 73–5, 80–1, 87, 91, 134–5, 196, 225–6, 230, 234, 243, note 38 ch. 2 (*see also* existence, internal and external; linguistic *framework*)
Cauchy, A.L., 46 (*see also* arithmetization of analysis)
characteristic marks (of a concept), note 17 ch. 2
Chihara, C., 69, 121, 137, 215, 225, 242, note 38 ch. 2, note 13 ch. 4, note 19 ch. 7
class
  equivalence class, xiii, 79,
  class *L* of constructible sets, note 48 ch. 2
  no-class theory, 69, 92
  "class as many" vs "class as one", note 20 ch. 2
Cohen, P., 71, 93, note 36 ch. 2
Colyvan, M., 198, 201, 202–3, 204, 207–8, 215, 230, 247, note 13, 24, 32, 35, 36, ch. 6, note 3, 25 ch. 7 (*see also* indispensability argument; naturalism; holism)
concepts (*see also* Frege; Kant)
  concept ⌜(being a) natural number⌝, 62, 232
  conditions of application and identity (for objects supposed to fall under a concept), 57, 59, 155
  of level *n*, 51
  equinumerous, 55–8, 101, 154, 157
  numbers of concepts, *see* numbers
  pure, 42–3
  sortal, 5, 52, 57, 60, 62, 100–1, 152, 155, 163, 174
consistency proof in Hilbert, *see* formalism, Hilbert's
constraint
  application constraint or Frege's constraint, 163–5
  of conservativeness, 85
conditions of identity,
  and HP, 57
  for quasi-concrete objects, 190–1
  for sortal concepts, *see* concepts, sortal
confirmation, 229–30, 241–9, note 24 ch. 6,
  probabilistic conception, 243
  confirmational holism, 214–15, 215–16, 243, note 33 ch. 6, notes 26, 28 ch. 7
  hypothetico-deductive model, 241–2
consequence of a theory, xv
  false nominalistic consequences, 114
  necessarily true nominalistic consequences, 114–15, note 3 ch. 4
conservativeness, 112–25, 217–24, note 2 ch. 7 (*see also* theory, conservative)
  Field's definition, 115–16, note 7 ch. 4
  according to Hilbert, 85
  Shapiro's objection, 121–2
  semantics vs syntactic, 121
consistent body of nominalistic / non-mathematical statements, 116–17, 217, 221, note 2 ch. 7
constants
  functional, 159, note 17 ch. 2, notes 8, 14 ch. 6
  individual, 12, 105, 192, 222–3, note 22 ch 6
  logical / non-logical, 48–9, 52–3, 61, 67, 77, 94, 174, 190, 211, note 15 ch. 2, note 14 ch. 6 (*see also* Frege, fundamental functions)
  predicate, 77, 106, 131, 166–8, 171, 174, 204, 222–3, note 8 ch. 5, note 16 ch. 6
content
  literal, 129–30, 135, 232, 234
  mathematical, 228–9, note 11 ch. 7
  real, 128, 130–4, 233–5, notes 20, 21 ch. 4

Continuum Hypothesis (CH), 71–2, 93–5, 98, 127, note 48 ch. 2
contradiction in Frege's formal system, 64–6
Craig, W., *see* theorem, Craig's
criterion of ontological commitment, 210–12, 224–35
criterion of virtuosity for a theory, 208–10, 212, 218, note 2 ch. 7

Dedekind, R., 63, 79–80, 137, 192–3 (*see also* abstraction, of Dedekind)
  definition of real numbers, 63
  Dedekind-infinite, 70
deductive rules, xiv, 47–8, 50, 83, note 5 ch. 2
  in GA, 53–4, 65
definite description symbol, 60, 168
definitions
  contextual, 101, 189, 206–7, 232
  creative, 75–6
  explicit, 48, 54–5, 59–60, 101, 152, 171–2, 180, 206
  impredicative, 69, 74, 91–2, note 38 ch. 2
  implicit, 56, 81–2, 120, 155, 161, 167, 180, 185, note 17 ch. 2
Demopoulos, W., 164
Detlefsen, M., 87, note 40 ch. 2
diagrams, 23, 32, 190
dianoia, 21–2, 33–6, 38, 194, note 3 ch. 1
  objects of (*dianoēta*), 34
discourse, figurative or metaphorical, 233–4
  figurative or metaphorical language, 129, 134
double negation elimination, 89, note 46 ch. 2
doxa, 22
Dummett, M., 24, 61, 149, 163, note 2, 14, 20, 26, ch. 1
  definition of realism, 24, note 3 Introduction, note 9 ch. 6, note 5 ch.7
  on Russell's paradox, 64, 66, note 28 ch. 1

easy road for rejecting IA, 230
ε-equivalence, 209, 218–19
eikasia, 21–2
empirical sciences, ix, xiv, 142–3, 235, note 26 ch. 7
empiricism, 10–11
  contrastive, 242–5
  constructive, 242–3, note 22 ch. 7
  logical, 73–90, note 33 ch. 6
encoding, 166–72, 174, 176
entities
  saturated / unsaturated, 50
  theoretical, vii–viii, 118, 203, 235
epistēmē, 17
equinumerosity, *see* concepts, equinumerous
equivalence
  observational (*o*-equivalence), 207–9
  observational as different from empirical equivalence, 208–9
  and paraphrase (*p*-equivalence), 205–7
  equivalence relation, note 3 ch. 5
equivalent descriptions, 227–30
ermēneia, see interpretation
Errera, A., 93
existence
  internal and external, 73–90, 134, 234
  of abstract objects, *see* objects, abstract
explanation (*see also* indispensability; inference to the best explanation)
  intrinsic and extrinsic, 221
extension (*Umfang*), 50, 57–61, 63, 65–7, 107, 156–8, note 18 ch. 2
Euclid, 32, 37, 45, 76, 83, 94–5, 190, note 6 ch. 1, note 24 ch. 7
  Elements, 11, 23, 26, 32, 37, 45, notes 6, 12 ch. 1
  (non-)Euclidean geometry, 63, 76, 78–9, 81, 94, 120, 123–4, 187, 190, 245
  Euclid's fifth postulate, 94
Euler, L., 46, 50

façon de parler, 10, 57, 206
False (as an object), 51

fictionalism, 13–14, 125–35
  Field's fictionalism, 125–9
  hermeneutic vs revolutionary, 129, 135
  meta-fictionalism, 130
  as an answer to IA, 231–5
  object, 130–1
  Yablo's, 129–35
Field, H., 11–14, 106, 110, 112–25, 125–35 (*see also* conservativeness; fictionalism; nominalism; space-time points)
  on Benacerraf's dilemma, 109–10
  objections to the indispensability premise in IA, 217–24
field of reals, 81, 120
formal system, 47–54, 67, 90, note 5 ch. 2
formalism, Hilbert's, 81–6
  ideal / finitary distinction, 206, 85–7, 89
  metamathematics, 81–6, 191
  consistency proofs, 78–9, 81, 84–7, 90
formula (of a language), xii
formulation (of a statement), xiii
Fraassen, B. Van, 203, 240, 242–3, note 23 ch. 7 (*see also* constructive empiricism; scientific realism, objections; inference to the best explanation)
Fraenkel, A., 71 (*see also* sets)
Frege, G., 45–66, 149–65 (*see also* assertion; *Bedeutung*; definitions; False; Russell's paradox; Peano, axioms; reference; context principle; formal system; True)
  Frege arithmetic (FA), 161–3, 206, 232, note 3 ch. 2.
  axioms of GA, 52–4 (*see also* law, of *Grundgesetze*)
  Begriffsschrift, 47
  on concepts, 51
  on definitions against Hilbert, 78–80
  definition of natural numbers, 54–62
  definition of real numbers, 62–4
  concept/object distinction, 54
  psychological/logical distinction, 54
  fundamental functions, 52–3
  GA (Grundgesetze Arithmetic), 47, 47–54
  Grundgesetze der Arithmetik, 47, 49, 59–63, 65, 75–6, 78, 163–4, note 4 ch. 2
  Grundlagen der Arithmetik, 47, 54, 59–60, 62, 76, 78, 81, 101, 119, 153, 156, 158
  formal system, 47–54
fruitfulness (of new axioms), 95, 98 (*see also* successful applications)

games of make-believe, 134, 234
Gauss, C. F., 46 (*see also* arithmetization of analysis)
geometry (*see also* Plato, Aristotle, Proclus):
  Cartesian, 37, 45
  Euclidean, *see* Euclid
  Hilbert axiomatic foundation, 75–80
  hyperbolic, note 24 ch. 7
  non-Euclidean, 94, 245
Gentzen, G., 87
Gettier, E., 17
Gödel, K., x, 71, 86–7, 90–8, 109, 121, 145, 194–6, note 43, 44, 48 ch. 2, note 2 ch. 3 (*see also* Continuum Hypothesis; understanding; intuition; platonism; theorem, Gödel's incompleteness)
Goodman, N., 132, 207, 244, note 18 ch. 6 (*see also* calculus of individuals)

Hale, B., 110–11, 149–65 (*see also* neo-logicism; responses to Benacerraf's dilemma)
  definition of real numbers, 164–5
Hazen, A., 141
Hebb, D., 146
Hellman, G., 110, 112, 138–43, 200, 225, 229, 242, note s13, 25, 26, 27 ch. 4, note 25 ch. 7 (*see also* modal eliminative structuralism)
  on the hermeneutic / revolutionary distinction, note 16 ch. 4

Hempel, C. G., 242
Heyting, A., 90
Hilbert, D.
  on definitions, 78–80
  axiomatic foundation of geometry (*see* geometry)
  Grundlagen der Geometrie, 76, 78
  formalist programme, *see* formalism, Hilbert's
holism
  confirmational, *see* confirmation, confirmational holism
  delivers sufficient conditions for the existence of mathematical objects / for the truth of mathematical theories in IA, 215–16
  semantic, note 33, ch. 6
Horwich, P., 135
Husserl, E., 5, 90–1, 191, note 47 ch. 2
Hussey, E., 31
hierarchy of types, *see* type-theory
hyperouranios, 19
hypothetico-deductive model, *see* confirmation

identity (*see also* Frege, fundamental functions; conditions of identity)
  of indiscernibles, 124, 186–8
  problem of identity for structuralism, 186–7
  tout court, concrete, and among monadic properties, 168
imagination (*phantasia*), 34–6, 38–9, 43–4, 169, 252
imperscrutability of reference, 107
implication (*see also* Frege, fundamental functions)
  expressed by "if" and "only if", 202 (*see also* Colyvan's IA, confirmational holism, naturalism)
(in)dispensability, 203–10
  arguments against the indispensability premise in IA, 217–24
  general clarification, 209–10
  and observational equivalence, 207–9
  for explanation, 209
  and paraphrase, 205–7
  s make indispensable use of T, 204–5
indispensability argument (IA), chs 6, 7
  IA as a transcendental argument, note 7 ch. 6
  IA enhanced (Baker version), 203
  IA Colyvan's version, 202
  IA epistemic for platonism, 198
  IA epistemic for semantic realism, 199–200
  IA non-epistemic for platonism, 197
  IA non-epistemic for semantic realism, 199
  objections to IA, ch. 7
  IA pragmatic, 248
  IA Quine-Putnam, 201
indispensability premise, 217–24, 231–2, note 4 ch. 7
ineliminable metaphors, 233–4
inference to the best explanation (IBE), 147, 203, 203, 240, 243, notes 23, 25 ch. 7
integral, Lebesgue's, Radon's, Stieltjes', 6
interpretation
  of axioms, 80–1
  ermēneia, 18
  of modality, *see* modality
  of a statement, 48–9
intuition, 37–43, 76, 78, 80, 82–3, 87–91, 110, note 18 ch. 1, note 40 ch. 2 (*see also* intuitionism)
  mathematics, 90–1, 94–8, 145 (*see also* Gödel)
  for Parsons, 190–5
  pure, 42–4, 83, 87–8, 91, 191
intuitionism, 87–90, 225 (*see also* Brouwer; Heyting)
  pure two-oneness, 88

judgement
  analytic / synthetic , 40–1
  a priori / *a posteriori*, 40–1
  for Kant, 38–9
  mathematical as synthetic *a priori*, 38
  judgement-stroke ("|—"), 53, note 13 ch. 2

Kant I., 36–44 (*see also* concepts; judgement; intellect; intuition; schema; sensibility)
Critique of Pure Reason, 39, 57
Logik Blomberg, note 18 ch. 2
Logik Jäsche, note 18 ch. 2
sphaera conceptus (or extension of a concept), 57, note 18 ch. 2
space as form of outer sense, 42, 87
time as form of inner sense, 42
Vienna Logic, 57
Keränen, J., 186
Kitcher, P., 148, 203
knowledge
a posteriori, 39–41, 43, 125
a priori, 39
hypothetical, 21, 44
as justified true belief/opinion, 17–18
propositional / non-propositional, 18

Ladyman, J., 187–8
Lagrange, J.-L., 46, 50
Lakatos, I., 11, 240, 243
language
formal, 47–50
nominalistic, xv, 119, 125, 220–2
predicate, 77, 131, 174, 225, note 8, 11 ch 7
first-order predicative language and canonical notation, 210–13
law
of excluded middle, 89
Leibniz's law *see* identity of indiscernibles
bridge-laws, 114, 117, 222, note 2 ch. 4, note 24 ch. 7
law (in *Grundgesetze*), 53–4, 59–60, note 16 ch. 2
V (o *Basic Law* V, BLV), 59, 61, 64–5, 156, 158, notes 22, 25 ch. 2
VI, 60, 156
Leitgeb, H., 187–8
Leng, M., 135, notes 23, 29 ch. 4, note 37 ch. 6, note 26 ch. 7
linguistic conventions, 10 (*see also* linguistic *framework*)
linguistic *framework*, 73–4, 91, 134, 135, 225
Linsky, B., 149, 165–77 (*see also* Object Theory)
logistic, 21, 25–6, note 2 ch. 1

MacBride, F., 121, note 1, 2 ch. 5
Mackie, J. L., 125
Maddy, P., 11, 111–12, 212, 225, note 31 ch. 4, note 37 ch. 6, notes 12, 20 ch. 7
on atomic theory, *see* atomic theory
cognitive basis of set theory, 144–8
compromise platonism, 145
against Quine's naturalism, 235–9
scientific practice objection to IA, 245–7
Malament, D., 119, 123, 222
Mancosu, P., ix, 92, note 40 ch. 2, notes 13, 18 ch. 6
mathematics
current mathematics, 85
contentual and ideal, *see* formalism, Hilbert's
finitary, 82–7, 190–1, 193, note 40 ch. 2
good, according to Field, 113–14, 116–18, 217
as modal logic, 227–9
pre-modern, 183
recreational, *see* mathematical recreation
as set theory, 227–9
structuralist, 188–90, 192–3, 194–5
Melia, J., 117, 222–4, 230, note 23 ch. 6, note 6 ch. 7
mereological sums, *see* mereology
mereology, 141, 222, note 11 ch. 4
meta-fictionalism, *see* fictionalism
metamathematics, 81, 84, 95, 191
method of paraphrase, 9–10, 205–7 (*see also* paraphrase)
Mill, J. S., 11
modality, 140, 143–4, note 29 ch. 4
Mueller, I., 31

naturalism, 214–15, 215–16
delivers necessary conditions for the existence of mathematical objects / for the truth of mathematical recreation, 235, note 37 ch. 6

mathematical theories in IA, 215–16
  Maddy's, 235–9
  ontological, 213, 237, 245
  semantic, 213, 237, 239
negation, *see* Frege, fundamental functions
neo-logicism, 149–65
  definition of natural numbers, 158–60
  definition of real numbers, 164–5
  application constraint, *see* constraint, Frege's
Neumann, J. von, *see* sets, von Neumann's
Newton, I., 37, 69, 120
  Principia (*Philosophiae naturalis principia mathematica*), 37, 69
  nominalistic reformulation of Gravitational Theory, 119–21, 123–4, 205, 220–1
noēsis, 21–2
nominalism, 9
  Field's, 112–25
  global, 9
  hermeneutic vs revolutionary, 128–9
nous, 22, 33–6, 38, 97, notes 3, 5 ch. 1
numbers
  of concepts, 155, 159, 162–3, 181
  naturals, definition, *see* Frege; neo-logicism; Russell
  reals, definition, *see* Cantor; Dedekind; Frege; neo-logicism
numerical quantifiers, 119, 130, 206, 233, 252, note 9 ch. 4

objectivity
  of mathematics, 7–8, 127, 200, 227–30
  of mathematical objects, 16, 92, 97, 188–9
objects
  abstract, 2–3
  copies of an idea, 18, 20, 28, 44
  intermediate, 27–8, 33–4
  mathematical, 3–4
  mathematical as characters in a fiction, 13–14, 126, 129
  ordinary, 167–8
  "quasi-concrete", 190–3, 195, note 42 ch. 2
  of a theory T ($O_{[T]}$), 197
Object Theory, 165–77
o-equivalence (*see* equivalence)
one-one correspondence, 55, 70, 102, 157
ontological commitment, *see* criterion of ontological commitment
ontological relativity, note 11 ch. 2, note 9 ch. 7
Ω, 79, 81

paraphrase, xv, 9–10, 136, 205–7, 220, 233, 247, 252 (*see also* p-equivalence)
Parsons, C., 11, 106, 111, 136–8, 142–3, 147, 149, 156, 177–8, 181, 183, 186–96, note 42 p. 2, notes 27, 30 ch. 4, note 37 ch. 6 (*see also* eliminative structuralism; intuition; Reason)
parts or portions of a theory, xiv
pattern recognition, 185
Peano, G.,
  Peano axioms, 60, 72, 81, 139, 156, 174, 228, notes 23, 34, 35 ch. 2
  derivation of Peano axioms in Frege's formal system, 60
p-equivalence, *see* equivalence
Peressini, A., note 28 ch. 7
Pettigrew, R., 32
pistis, 21–2
Plato, 17–26 (*see also eikasia*; *epistēmē*; *ermēneia*; geometry; hyperuranios; logistic; *nous*; *pistis*; recollection)
  Gorgias, 25–6
  Hippias Major, 55
  on ideas, 18, 22, 27–8
  Meno, 18–20, 23, 25, note 1 ch. 1
  Phaedrus, 19–20, notes 1, 3 ch. 1
  Philebus, 25
  Republic, 20, 44
  divided line, in *Republic*, 20–2, note 5 ch. 1
  Seventh Letter, 20, 24, 26, 44
  Theaetetus, 17–20, 23, 25, note 1 ch. 1

platonism
arithmetical, 4
arrogant, 107–8, 161–2
compromise platonism, *see* Maddy
empiricist, 11
physicalist, 11
sober, 107
Poincaré, H., x, 89, 91, 246, note 38 ch. 2, note 24 ch. 7
Popper, K., 240
posit, 213, 240, 242
PRA, *see* arithmetic
principle
Vicious Circle principle 69, 91
context principle, 54, 61–2, 149, 167, note 31 ch. 2
Eleatic principle, note 32 ch. 6
Hume's Principle (HP), 55–6, 60–1, 152–65, 177, note 17 ch. 2
Likelihood principle, 244–5
problem
of access, 17, 110, 125, 149, 154, 162, 250
Caesar problem, 56–8, 101, 155, 162–3, 164, 180
of identity, *see* identity
Plato's problem, vii–ix, 1, 26–8, 37, 39, 74, 99, 101–2, 105, 110, 181–2, 195, 254
neo-logicist solution to Caesar problem, 162–4
Proclus, 32–6 (*see also* geometry, *phantasia*, projections)
Commentary to the First Book of Euclid's Elements, 32–3, note 13 ch. 1
on ideas, 32–5
logoi, 34–6, 38, 44, 97
progression, note 8 ch. 1
projections (*probolai*), 35–6, 44
proposition, xiii, note 7 ch. 5 (*see also* encoding)
propositional attitude, 128, 134–5, 234, note 15 ch. 4
prospective places-are-objects, 179
prospective places-are-roles, 179
Psillos, S., 239, 241, note 23 ch. 7 (*see also* scientific realism)
Putnam, H, 142, 196, 197–201, 202, 215, 227–30, note 27 ch. 4, notes 8, 19, 35 ch. 6, notes 11, 12, 13, 21 ch. 7

quantification
and IA, 204–5
and ontological commitment, 210–12, 224–35
objectual, note 28 ch. 6, note 11 ch. 7 (*see also* existential statements *and* ontological commitment)
quasi-assertions, 130
quasi-concrete, *see* objects
Quine, W.V.O, viii, 107, 109, 134 (*see also* criterion of ontological commitment; IA; imperscrutability of reference; indeterminacy of translation; objectual interpretation of existential statements; second-order logic; naturalism; holism; quantification and ontological commitment, ontological relativity; canonical reformulation of a theory)

Ramsey, F.P., 92
realism
ontological, 8, 201
scientific, 8, 203, 235–41, 248, note 10 ch. 6
objections to scientific realism, 239–41
semantic, 8, 113, 199, 201, 229, 250 (*see also* indispensability argument)
Reason
distinct from reasons, 194
recollection, 19–20
reconceptualization, 152, 160
recursive, note 27 ch. 2, (*see also* recursively enumerable sets; recursively axiomatizable theories)
reference
sense/reference distinction in Frege, 53
of names of functions, 61

reformulation
  of statements, xiii, xv, 9–10
  canonical and ontologically minimal of a theory, 211–13, 230, 233–4
relation
  ancestral, 56, 159
  of equivalence, *see* equivalence
  of set membership, 67–8
responses to Benacerraf's dilemma, 110–11
  conservative, ch. 5
  non-conservative, ch. 4
  intellectual and intuitional, 110
Resnik, M., 105, 123–4, 136–7, 187, 215, notes 25, 28, 29, 31 ch. 7 (*see also* pragmatic IA)
Rosen, G., 122, 128–9, 135, 200, 230, 237–8, note 1 Introduction, notes 12, 13, 14, 16 ch. 4, notes 12, 20, 30 ch. 7
  hermeneutic / revolutionary distinction, 128–9
Russell, B., 66–9, 71, 73–4, 81, 87, 91–3, 99, note 29 ch. 2 (*see also* Russell's paradox; Vicious Circle principle; type theory)
  no-class theory, *see* class
  definition of natural numbers, 67–8
  Principia Mathematica, 69, 74, 91
  The Principles of Mathematics, 66–7, 69, 99
  Russell's paradox, 64–6
  incomplete symbols, 69

schema
  of axioms, 157–8, 222–4, notes 23, 35 ch. 2, note 3 ch. 4, notes 25, 26 ch. 4
  comprehension schema, 140–3, 157–60, 167–8, 222–3
  in Kant, 39, 43–4, note 19 ch. 1
  of statements, 131, 228
  of theorems, 83
  translation schema, 139–40, note 24 ch. 4
scientific practice objection, *see* Maddy
second order logic, 47–8, 50, 54, 61, 66, 72, 131, 160
  nominalistic interpretation of, 140–2
  modal, 139–40, note 25 ch. 4 (*see also* mathematics as modal logic)
  as set theory or mathematics, 131, 140–1, 160, note 15 ch. 2
semantics, 48–9
sense of a name, *see* reference
sensibility, 34, 38, 42–3, 58, note 15 ch. 1
sentence, xiii–xiv, 47–8.
separation conditions between nominalistic and mathematical statements, 115
sets
  Cantor and the development of set theory, *see* Cantor
  equinumerous, 67–9
  and the foundation of arithmetic, *see* Benacerraf's (1965) argument
  natural numbers as sets of equinumerous sets, 67–8
  recursively enumerable, 208, note 20 ch. 6
  set theory, 69–72
  Von Neumann's, 71, 81, 102–3, 178, note 3 ch 2
  Zermelo-Fraenkel's, 71–2
  ZF, 71–2, 93–5, 98, 127, 222–4, note 48 ch. 2
Shapiro, S., 30, 74–5, 80, 85–6, 111, 115, 121–3, 137, 143–4, 149, 160, 164, 177–87, 188–9, 192, 217–8, 239, 252, note 15 ch. 2, note 1 ch. 3, note 10 ch. 4 (*see also* structuralism; conservativeness; structure theory)
sheaf of properties, 168–9, note 6 ch. 5
simulation, 134–5, 234
Skolem, T., 193 (*see also* Löwenheim-Skolem theorem)
Sobe,r E., 242–5 (*see also* contrastive empiricism)
Socrates, 17–20, 25
space-time points, 11, 120, 122–4, 142
Stanley, J., 129
  autism objection, 135
statement, xiii–xv, 48

existential statement and ontological commitment, 210–12, 224–5
mathematical / of mathematics, xiv–xv
nominalistic, xv
observational, 207–9, 219, 239, note 29 ch. 6
M-loaded, 217, 219, 232, 235, note 2 ch. 7
T-loaded, 205–9 (*see also* M-loaded)
vacuously true, note 4 Introduction, 12–13, 112, 117, 138, 140
Steiner, M., notes 1, 4 ch. 6
stipulations, in a formal system, 48–9
Stolz, O., 75, 78
subtheories, xiv
structuralism
ante rem, 111, 149, 177–87, 188
eliminative, 136–44
modal eliminative, Hellman's, 138–44
modal eliminative as a form of neutralism, 143
non-eliminative, 149, 177–87, 187–95
non-eliminative, Parsons', 149, 187–95
suggested by Benacerraf (1965), *see* abstract structure of progression
structure
grammatical, 6–7, 12, 110, 131
semantical, 7, 12, 106
successful applications (o mathematics), 145 (*see also* fruitfulness)
successor, 56, 139, 159, 187, 192–3, 259–60, note 3 ch. 4
syntax, 48–9
systems or bodies of statements, xiv

terms
natural numerical, 161
role of representational aids, 133–4
role of things represented, 134
empty singular, 13
theorem
Craig's, 208
Frege's, 157, 160–1, 164
Gödel's incompleteness theorem, 86–7, 121–2, notes 43, 44 ch. 2
Löwenheim-Skolem theorem, note 35 ch. 2
last theorem of Fermat, 75, 227–9, notes 10, 11 ch. 7
representation theorems, 120, 220–1, notes 3, 4 ch. 7
theory
atomic, 246
category theory, 199
conservative, 85, 116–23, 217–18, 220, 221, note 7 ch. 4, note 2 ch. 7 (*see also* conservativeness)
as a deductively close set of statements, xiv–xv
empirically adequate, 240, 242
error theory, 125
of functions, or analysis, *see* analysis
indispensable, *see* indispensability
impure mathematics, xv
nominalistically impure mathematics, 114–17, note 2 ch. 4
mature scientific, 240–1
model theory, 260
natural numbers theory, note 2 Introduction
Newton's Gravitational Theory, *see* Newton
nominalist, *see* nominalistic language
platonic theory of ideas, *see* Plato
preferable, 208
pure mathematics, xv
Q-justified, 246–7
reasonably attractive, 208
recursively axiomatizable, 208, note 21 ch. 6
scientific, xiv–xv
scientific theory under-determined by empirical evidence, 240
set-theory, *see* sets
set-theory in disguise, *see* second-order logic
structure theory, 183, 185
type theory, 69

thesis
Frege's, 150–2, 154, 162, 177, 189
Hilbert's, 193
syntactic priority thesis, 151–2, 154–5, 159, 161, 162
thought
in Frege, 53
as understanding in Gödel, *see* Gödel, understanding
token / *type*, 83, 190
transfinite cardinals, 70
transfinite ordinals, 70
True (as object), 51
true-in-a-fiction/-in-the-story, 126–7, 129–33, 135, 231
truth *tout court*, 14, note 4 Introduction

understanding
for Gödel, 96–7
for Kant, 38, 42, 97

value-range of a function, 59–62, 64–5, notes 10, 25 ch. 2
Vineberg, S., note 25 ch. 7
vocabulary, 204–7, 217, 220, 223, 228, 233, 263, notes 14, 16, 19 ch. 6
observational vs theoretical, 207–9

Walton, K., 134 (*see also* games of make-believe)
Weir, A., 160, note 32 ch. 6
White, N., 106, 137
Whitehead, A. N., 69, 91–2
Principia Mathematica, see Russell
Wiles, A., 75, 227, note 10 ch. 7
Wright, C., 107, 110–12, 144, 149–65, note 26 ch. 2, notes 1, 2 ch. 5 (*see also* neo-logicism; responses to Benacerraf's dilemma)

Yablo, S., 14, 111–12, 125, 129–35, 230, 232–4, 240, notes 17, 18, 20, 21, 23, ch. 4, note 15 ch. 7 (*see also* fictionalism)

Zalta, E., 111, 149, 165–77 note 28 ch. 4, notes 4, 8 ch. 5 (*see also* Object Theory)
Zermelo, E., 71, 81, 102–3, 163, 178, note 3 ch. 3
ZF e ZFC, *see* set theory