

List of Symbols

- \forall : Logical sign, meaning “for all.”
- \exists : Logical sign, meaning “there exists.”
- \mathbb{R} : The real numbers.
- \mathbb{R}^+ : The strictly positive real numbers.
- \mathbb{N} : The natural numbers: 1,2,3,....
- \mathbb{N}_0 : The nonnegative integers: 0,1,2,3,....
- \mathbb{Z} : The integers.
- \mathbb{Q} : The rational numbers.
- \mathbb{C} : The complex numbers.
- \bar{x} : The complex conjugate of $x \in \mathbb{C}$.
- X, Y : Banach spaces.
- \mathcal{H}, \mathcal{K} : Hilbert spaces.
- \oplus : Direct sum.
- $\prod_{k=1}^{\infty}$: Infinite product.
- $L^p(\mathbb{R})$: For $p \in [1, \infty[$, the space of (measurable) functions $f : \mathbb{R} \mapsto \mathbb{C}$ for which $\int_{\mathbb{R}} |f(x)|^p dx < \infty$.
- $L^\infty(\mathbb{R})$: The set of bounded functions on \mathbb{R} .
- $C^k(\mathbb{R})$: The space of k times differentiable functions with a continuous k th derivative.
- $C[a, b]$: The space of continuous functions $f : [a, b] \rightarrow \mathbb{C}$.
- $C_0(\mathbb{R})$: The space of continuous functions $f : \mathbb{R} \rightarrow \mathbb{C}$ for which $f(x) \rightarrow 0$ as $x \rightarrow \pm\infty$.
- $C_c(\mathbb{R})$: The space of continuous functions $f : \mathbb{R} \rightarrow \mathbb{C}$ with compact support.
- $\mathcal{F}f(\gamma) = \hat{f}(\gamma)$: The Fourier transform, for $f \in L^1(\mathbb{R})$ given by $\hat{f}(\gamma) = \int_{\mathbb{R}} f(x)e^{-2\pi ix\gamma} dx$.
- $\ell^p(\mathbb{N})$: For $p \in [1, \infty[$, the space of p -summable sequences, indexed by \mathbb{N} .
- $\ell^\infty(\mathbb{N})$: The set of bounded sequences, indexed by \mathbb{N} .

- χ_A : The characteristic function for a set A ,
 $\chi_A(x) = 1$ if $x \in A$, otherwise 0.
 \bar{A} : The closure of a set A .
 $A \cap B$: The set of elements belonging to A and B .
 $A \cup B$: The set of elements belonging to at least one of the sets
 A and B .
 $A \setminus B$: The set of elements belonging to A but not to B .
 A^c : The complement of a set A .
 A^\perp : The orthogonal complement of a subset A in a
Hilbert space.
 $\text{supp } f$: The support of the function f : $\text{supp } f = \overline{\{x \in \mathbb{R} : f(x) \neq 0\}}$.
 $\delta_{k,j}$: The Kronecker delta: $\delta_{k,j} = 1$ if $k = j$, $\delta_{k,j} = 0$ if $k \neq j$.
 T_a : The translation operator $(T_a f)(x) = f(x - a)$.
 E_b : The modulation operator $(E_b f)(x) = e^{2\pi i b x} f(x)$.
 D_a : The dilation operator $(D_a f)(x) = \frac{1}{\sqrt{a}} f(\frac{x}{a})$, $a > 0$.
 D : The dilation operator $(Df)(x) = 2^{1/2} f(2x)$.
 $\psi_{j,k}$: $\psi_{j,k}(x) = D^j T_k \psi(x) = 2^{j/2} \psi(2^j x - k)$.
 N_m : B-spline of order m , supported on $[0, m]$.
 B_m : Centered B-spline of order m , supported on $[-m/2, m/2]$.
 P_ℓ : Legendre polynomial of order ℓ .
 $P_{\ell,n}$: Associated Legendre functions.
 Q_ℓ : Laguerre polynomial of order ℓ .
 H_ℓ : Hermite polynomial of order ℓ .

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