References


Apostol, T. [1962], *Calculus*, Blaisdell Publ. Co., Waltham, MA.


Bartle, R. G. and L. M. Graves [1952], Mappings between function spaces, Transactions of the American Mathematical Society, 72, 400–413.
References


Dini, U. [1877/78], *Analisi infinitesimale*, Lezioni dettate nella R. Università di Pisa.


Eckart, C. and G. Young [1936], The approximation of one matrix by another of lower rank, Psychometrika, 1, 211–218.
References


Goursat, Ed. [1904], A course in mathematical analysis, English translation by E. R. Hedrick, Ginn Co., Boston.


Halkin, H. [1974], Implicit functions and optimization problems without continuous differentiability of the data, SIAM Journal on Control, 12, 229–236.


Hurwicz, L. and M. K. Richter [2003], Implicit functions and diffeomorphisms without $C^1$, Advances in mathematical economics, 5, 65–96.


Ioffe, A. D. [2003a], On robustness of the regularity property of maps, Control and Cybernetics, 32, 543–554.

References


Kummer, B. [1991], An implicit-function theorem for $C^{0,1}$-equations and parametric $C^{1,1}$-optimization, Journal of Mathematical Analysis and Applications, 158, 35–46.


Levitin, E. S. [1992], *Perturbation theory in mathematical programming and its applications*, Nauka, Moscow (Russian).


Lewis A. S. [2001], Ill-conditioned inclusions, Set-Valued Analysis, 9, 375–381.


Rockafellar, R. T. [1967], Monotone processes of convex and concave type, Memoirs of the American Mathematical Society, 77, Providence, RI.


Rockafellar, R. T. [1976a], Monotone operators and the proximal point algorithm, SIAM Journal on Control and Optimization, 14, 877–898.


Ursescu, C. [1975], Multifunctions with convex closed graph, Czechoslovak Mathematical Journal, 25 (100), 438–441.


Notation

2C(4): formula (4) in Section 2C
\(\mathbb{R}\): the real numbers
\(\mathbb{N}\): the natural numbers
\(\mathcal{N}\): the collection of all subsets \(N\) of \(\mathbb{N}\) such that \(\mathbb{N}\ \setminus N\) is finite
\(\mathcal{N}^\infty\): the collection of all infinite subsets of \(\mathbb{N}\)
\(\{x^k\}\): a sequence with elements \(x^k\)
\(\varepsilon_k \downarrow 0\): a sequence of positive numbers \(\varepsilon_k\) tending to 0
\(\limsup_k C^k\): outer limit
\(\liminf_k C^k\): inner limit
\(|x|\): Euclidean norm
\(\|x\|\): any norm
\((x,y)\): canonical inner product, bilinear form
\(|H|\): outer norm
\(|H|^\infty\): inner norm
\(\mathcal{B}_{a}(x)\): closed ball with center \(x\) and radius \(r\)
\(\mathcal{B}\): closed unit ball
\(\text{cl } C\): closure
\(\text{int } C\): interior
core \(C\): core
\(\text{rc } C\): recession cone
\(P_C\): projection mapping
\(T_C(x)\): tangent cone
\(N_C(x)\): normal cone
\(K^*\): polar to cone \(K\), mapping adjoint to \(K\), space dual to \(K\)
\(K_C(x,v)\): critical cone
\(A^\top\): transposition
\(\text{rank } A\): rank
\(\ker A\): kernel
\(\det A\): determinant
\(d_C(x), d(x,C)\): distance from \(x\) to \(C\)
\(e(C,D)\): the excess of \(C\) beyond \(D\)
\(h(C,D)\): Pompeiu-Hausdorff distance
\(\text{dom } F\): domain
\(\text{rge } F\): range
\(\text{gph } F\): graph
\(\nabla f(x)\): Jacobian
\(Df(x)\): derivative
$\mathcal{C}^k$: the space of $k$-times continuously differentiable functions
$DF(x|y)$: graphical derivative
$D^*F(x|y)$: coderivative
$\text{clm}(f;x), \text{clm}(S;y|x)$: calmness modulus
$\text{lip}(f;x), \text{lip}(S;y|x)$: Lipschitz modulus
$\text{clm}_p(f;p;x)$: partial calmness modulus
$\text{lip}_p(f;p;x)$: partial Lipschitz modulus
$\text{reg}(F;x|y)$: regularity modulus
$\text{subreg}(F;x|y)$: subregularity modulus
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