# A

# Appendix

## A.1 Lemma: Transformation of Line Planning Game

**Lemma A.1** *Consider a line planning game* $\Gamma = (G, \mathcal{P}, f^{\min}, f^{\max}, c, N, M)$ *with cost functions* $c_e(f_e) \leq 0$ *for* $f_e \leq 0$*. Furthermore, we have* $f^{\min} \leq 0$ *and* $f_e^{\max} \leq 0, e \in E$*. We obtain a transformed line planning game* $\bar{\Gamma} = (G, \mathcal{P}, \bar{f}^{\min}, \bar{f}^{\max}, \bar{c}, N, M)$ *with* $\bar{c}_e(\bar{f}_e) \geq 0$ *for* $\bar{f}_e \geq 0$*,* $f^{\min} \geq 0$ *and* $f_e^{\max} \geq 0, e \in E$ *such that the set of equilibria in* $\Gamma$ *and* $\bar{\Gamma}$ *coincide, by the following transformation.*

*Set*

$$\begin{aligned}
\alpha_P &= -f^{\min}, \\
\beta_e &= -\min_{f^{\min} \leq f_e \leq 0} c_e(f_e), \\
\bar{f}_P &= f_P + \alpha_P, \\
\bar{c}_e(\bar{f}_e) &= c_e\left(\bar{f}_e - \sum_{P: e \in P} \alpha_P\right) + \beta_e, \\
\bar{f}^{\min} &= f^{\min} + \sum_{P \in \mathcal{P}} \alpha_P, \\
\bar{f}_e^{\max} &= f_e^{\max} + \sum_{P: e \in P} \alpha_P, \forall\ e \in E.
\end{aligned}$$

*It holds that* $\bar{f}$ *is an equilibrium in* $\bar{\Gamma}$ *if and only if* $f$ *is an equilibrium in* $\Gamma$*. The cost of player* $P$ *for feasible frequencies* $\bar{f}$ *changes by a transformation such as the following.*

$$\bar{c}_P(\bar{f}) = c_P(f) + \sum_{e \in P} \beta_e. \tag{A.1}$$

*Proof.* First note that $\beta_e$ exists for all $e \in E$, inasmuch as $c_e(f_e)$ is continuous and thus the minimum of a compact interval exists by the Weierstrass extreme value theorem. Furthermore, $\alpha_e$ and $\beta_e$ are nonnegative for $e \in E$ by definition.

The cost (A.1) of a line $P$ can be verified by inserting the definitions and using $f_e = \sum_{P:e\in P} f_P = \sum_{P:e\in P} \left(\bar{f}_P - \alpha_P\right) = \bar{f}_e - \sum_{P:e\in P} \alpha_P$:

$$\bar{c}_P(\bar{f}) = \sum_{e\in P} \bar{c}_e(\bar{f}_e) = \sum_{e\in P} \left( c_e \left( \bar{f}_e - \sum_{P:e\in P} \alpha_P \right) + \beta_e \right) = c_P(f) + \sum_{e\in P} \beta_e.$$

For the proof of the equivalence of equilibria, we check the three cases that appear in the payoff function.

**Part (a)**

$$\begin{aligned} \bar{b}_P(\bar{f}) = M &\Leftrightarrow \sum_{P\in\mathcal{P}} \bar{f}_P < \bar{f}^{\min} \\ &\Leftrightarrow \sum_{P\in\mathcal{P}} \left(\bar{f}_P - \alpha_P\right) < \bar{f}^{\min} - \sum_{P\in\mathcal{P}} \alpha_P \\ &\Leftrightarrow \sum_{P\in\mathcal{P}} f_P < f^{\min} \\ &\Leftrightarrow b_P(f) = M. \end{aligned}$$

**Part (b)**

$$\begin{aligned} \bar{b}_P(\bar{f}) = \bar{c}_P(\bar{f}) &\Leftrightarrow \underbrace{\sum_{P\in\mathcal{P}} \bar{f}_P \geq \bar{f}^{\min}}_{(*)} \ \wedge \ \underbrace{\forall\ e \in P : \bar{f}_e \leq \bar{f}_e^{\max}}_{(**)} \\ &\Leftrightarrow \ \sum_{P\in\mathcal{P}} f_P \geq f^{\min} \ \wedge \ \forall\ e \in P : f_e \leq f_e^{\max} \\ &\Leftrightarrow \ b_P(f) = c_P(f). \end{aligned}$$

$$\begin{aligned} (*) \quad \textstyle\sum_{P\in\mathcal{P}} \bar{f}_P \geq \bar{f}^{\min} \quad &\Leftrightarrow \quad \sum_{P\in\mathcal{P}} \bar{f}_P - \alpha_P \geq \bar{f}^{\min} - \sum_{P\in\mathcal{P}} \alpha_P \\ &\Leftrightarrow \quad \sum_{P\in\mathcal{P}} f_P \geq f^{\min}. \end{aligned}$$

$$\begin{aligned} (**)\ \forall\ e \in P : \bar{f}_e \leq \bar{f}_e^{\max} &\Leftrightarrow \ \bar{f}_e - \sum_{P:e\in P} \alpha_P \leq \bar{f}_e^{\max} - \sum_{P:e\in P} \alpha_P \\ &\Leftrightarrow \ f_e \leq f_e^{\max}. \end{aligned}$$

**Part (c)**

$$\begin{aligned}
\bar{b}_P(\bar{f}) = N &\Leftrightarrow \underbrace{\sum_{P\in\mathcal{P}} \bar{f}_P \geq \bar{f}^{\min}}_{(*)} \ \wedge\ \underbrace{\exists\, e \in P : \bar{f}_e > \bar{f}_e^{\max}}_{(***)} \\
&\Leftrightarrow \sum_{P\in\mathcal{P}} f_P \leq r \ \wedge\ \exists\, e \in P : f_e > f_e^{\max} \\
&\Leftrightarrow b_P(f) = N.
\end{aligned}$$

$$(*) \qquad \text{see } (ii)$$

$$\begin{aligned}
(***)\ \exists\, e \in P : \bar{f}_e > \bar{f}_e^{\max} &\Leftrightarrow \bar{f}_e - \sum_{P\in\mathcal{P}} \alpha_P > \bar{f}_e^{\max} - \sum_{P\in\mathcal{P}} \alpha_P \\
&\Leftrightarrow f_e > f_e^{\max}.
\end{aligned}$$

Consider any two flows $f_1, f_2$ and the corresponding transformations $\bar{f}_1, \bar{f}_2$. As the difference of $\bar{c}_P(\bar{f})$ and $c_P(f)$ is given by a constant, and as $M$ and $N$ are by definition chosen sufficiently large; that is, greater than $c_P(f) + \sum_{e\in P} \beta_e$ for all feasible $f$, it holds:

$$\bar{b}_P(\bar{f}_1) \leq \bar{b}_P(\bar{f}_2) \quad \Leftrightarrow \quad b_P(f_1) \leq b_P(f_2).$$

Thus, the set of equilibria coincides for $\bar{\Gamma}$ and $\Gamma$. □

## A.2 Polyhedron for Line Planning Game

The following matrix and vector describe the polyhedron in Example 4.31.

$$A = \left(\begin{array}{rrrrrrrrrr}
-1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\hline
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{array}\right), \quad b = \left(\begin{array}{r}
-1 \\ -1 \\ -1 \\ -1 \\
\hline
4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\
\hline
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{array}\right).$$

# References


[ABEA+06] E. Altman, T. Boulogne, R. El-Azouzi, T. Jiménez, and L. Wynter. A survey on networking games in telecommunications. *Computers and Operations Research*, 33:286–311, 2006.

[AD54] J. Arrow and G. Debreu. Existence of an equilibrium for a competitive economy. *Econometrica*, 22(3):265–290, 1954.

[ADK+04] E. Anshelevich, A. Dasgupta, J. Kleinberg, É. Tardos, T. Wexler, and T. Roughgarden. The price of stability for network design with fair cost allocation. In *Proceedings 45th IEEE Symposium on Foundations of Computer Science (FOCS)*, pages 295–304. 2004.

[AEAP02] E. Altman, R. El-Azouzi, and O. Pourtallier. Properties of equilibria in competitive routing with several users types. In *Proceedings of 41th IEEE Conference on Decision and Control*, pages 3646–3651. 2002.

[AK05] E. Altman and H. Kameda. Equilibria for multiclass routing in multi-agent networks. *Advances in Dynamic Games*, 7:343–367, 2005.

[AK07] B. Awerbuch and R. Khandekar. On cost sharing mechanisms in the network design game. In *Proceedings of the 26th annual ACM symposium on Principles of distributed computing (PODC07)*, pages 364–365, 2007.

[arr] ARRIVAL: *Algorithms for robust and online railway optimization: improving the validity and reliability of large scale systems.* Funded by the "Future and Emerging Technologies" Unit of the EC within the 6th Framework Programme of the European Commission, `http://arrival.cti.gr/`.

[AT02] A. Archer and É. Tardos. Frugal path mechanisms. In *Proceedings of the 13th Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 991–999. ACM Press, New York, 2002.

[BCKV06] R. Beier, A. Czumaj, P. Krysta, and B. Vöcking. Computing equilibria for a service provider game with (im)perfect information. *ACM Transactions on Algorithms*, 2(4):679–706, 2006.

[BDHS02] R. Baron, J. Durieu, H. Haller, and P. Solal. Control costs and potential functions for spatial games. *International Journal of Game Theory*, 31:541–561, 2002.

[BE05] U. Brandes and T. Erlebach, editors. *Network Analysis*, volume 3418 of *Lecture Notes in Computer Science*. Springer, Berlin, 2005.

[BGP04a] R. Borndörfer, M. Grötschel, and M. E. Pfetsch. Models for line planning in public transport. Technical Report 04-10, ZIP Berlin, 2004.

[BGP04b] R. Borndörfer, M. Grötschel, and M.E. Pfetsch. A path-based model for line planning in public transport. Technical Report 05-18, ZIP Berlin, 2004.

[Bil98] J.-M. Bilbao. Values and potential of games with cooperation structure. *International Journal of Game Theory*, 27:131–145, 1998.

[Bir76] C.G. Bird. On cost allocation for a spanning tree: A game theoretic approach. *Networks*, 6(4):335–350, 1976.

[BKZ96] M.R. Bussieck, P. Kreuzer, and U.T. Zimmermann. Optimal lines for railway systems. *European Journal of Operational Research*, 96(1):54–63, 1996.

[BLPGVR04] Y. Bramoullé, D. López-Pintado, S. Goyal, and F. Vega-Redondo. Network formation and anti-coordination games. *International Journal of Game Theory*, 33(1):1–19, 2004.

[BMI06] C. Busch and M. Magdon-Ismail. Atomic routing games on maximum congestion. In *Algorithmic Aspects in Information and Management*, volume 4041 of *Lecture Notes in Computer Science*, pages 79–91. Springer, Berlin, 2006.

[BS79] M. Bazaraa and C.M. Shetty. *Nonlinear Programming*. Wiley, New York, 1979.

[BS91] I.N. Bronstein and K.A. Semendjajew. *Taschenbuch der Mathematik*. Nauka, Teubner, Moskau, Leipzig, 1991.

[BS02] T. Başar and R. Srikant. A stackelberg network game with a large number of followers. *Journal of Optimization Theory and Applications*, 155(3):479–490, 2002.

[BS06] N. Baumann and M. Skutella. Solving evacuation problems efficiently–earliest arrival flows with multiple sources. In *Proceedings of the 47th Annual IEEE Symposium on Foundations of Computer Science (FOCS06)*, pages 399–410, 2006.

[CC01] R. Cominetti and J. Correa. Common-lines and passenger assignment in congested transit networks. *Transportation Science*, 35(3): 250–267, 2001.

[CCLE+06] C.Chekuri, J. Chuzhoy, L. Lewin-Eytan, J. Naor, and A. Orda. Non-cooperative multicast and facility location games. In *Proceedings of the 7th ACM conference on Electronic commerce (EC06)*, pages 72–81, 2006.

[CCSM06] R. Cominetti, J.R. Correa, and N.E. Stier-Moses. Network games with atomic players. In *Automata, Languages and Programming*, volume 4051 of *Lecture Notes in Computer Science*, pages 525–536. Springer, Berlin, 2006.

[CDR03] R. Cole, Y. Dodis, and T. Roughgarden. Pricing network edges for heterogeneous selfish users. In *Proceedings of the 35th Annual ACM Symposium on Theory of Computing (STOC)*, pages 521–530. ACM Press, New York, 2003.

[CGY99] G.Y. Chen, C.J. Goh, and X.Q. Yang. Vector network equilibrium problems and nonlinear scalarization methods. *Mathematical Methods of Operations Research*, 49:239–253, 1999.

[CKV02] A. Czumaj, P. Krysta, and B. Vöcking. Selfish traffic allocation for server farms. In *Proceedings of the 34th ACM Symposium on Theory of Computing (STOC)*, pages 287–296. 2002.

[CLPU04] L. Castelli, G. Longo, R. Pesenti, and W. Ukovich. Two-player noncooperative games over a freight transportation network. *Transportation Science*, 38(2):149–159, 2004.

[CP82] D. Chan and J.-S. Pang. The generalized quasi-variational inequality problem. *Mathematics of Operations Research*, 7(2):211–220, 1982.

[CR06] H.L. Chen and T. Roughgarden. Network design with weighted players. In *Proceedings of the 18th Annual ACM Symposium on Parallelism in Algorithms and Architectures (SPAA06)*, pages 29–38. 2006.

[CRV08] H.-L. Chen, T. Roughgarden, and G. Valiant. Designing networks with good equilibria. In *Proceedings of the 19th annual ACM-SIAM symposium on Discrete algorithms (SODA '08)*, pages 854–863, 2008.

[CSS04a] J.R. Correa, A.S. Schulz, and N.E. Stier Moses. Computational complexity, fairness, and the price of anarchy of the maximal latency problem. In *Integer Programming and Combinatorial Optimization, 10th International IPCO Conference*, pages 59–73. 2004.

[CSS04b] J.R. Correa, A.S. Schulz, and N.E. Stier Moses. Selfish routing in capacitated networks. *Mathematics of Operations Research*, 29(4): 961–976, 2004.

[CV07] A. Czumaj and B. Vöcking. Tight bounds for worst case equilibria. *ACM Transactions on Algorithms*, 3(1):4, 2007.

[Deb52] G. Debreu. A social equilibrium existence theorem. *Proceedings of the National Academy of Sciences*, 38(10):886–893, 1952.

[DGK+05] N. Devanur, N. Garg, R. Khandekar, V. Pandit, A. Saberi, and V.V. Vazirani. Price of anarchy, locality gap, and a network service provider game. In *Proceedings of the First Workshop on Internet and Network Economics (WINE)*, pages 1046–1055. 2005.

[DH81] D.Granot and G. Huberman. Minimum cost spanning tree games. *Mathematical Programming*, 21(1):1–18, 1981.

[DR02] T.S.H. Driessen and T. Radzika. A weighted pseudo-potential approach to values for tu-games. *International Transactions in Operational Research*, 9:303–320, 2002.

[EBKS07] E. Even-Bar, M. Kearns, and S. Suri. A network formation game for bipartite exchange economies. In *Proceedings of the 18th annual ACM-SIAM symposium on Discrete algorithms (SODA)*, pages 697–706, 2007.

[Ehr05] M. Ehrgott. *Multicriteria Optimization*. Springer, Berlin, 2005.

[ESS04] E. Elkind, A. Sahai, and K. Steiglitz. Frugality in path auctions. In *Proceedings of the 15th Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 701–709. ACM Press, New York, 2004.

[FKK+02] D. Fotakis, S.C. Kontogiannis, E. Koutsoupias, M. Mavronicolas, and P.G. Spirakis. The structure and complexity of Nash equilibria for a selfish routing game. In *Proceedings of the 29th International Colloquium on Automata, Languages and Programming (ICALP)*, pages 123–134. 2002.

[Fle04] L. Fleischer. Linear tolls suffice: New bounds and algorithms for tolls in single source networks. In *Automata, Languages and Programming:*

*31st International Colloquium, (ICALP)*, pages 544–554. Springer, New York, 2004.

[FLM+03] A. Fabrikant, A. Luthra, E. Maneva, C.H. Papadimitriou, and S. Shenker. On a network creation game. In *Proceedings of the 22nd Annual Symposium on Principles of Distributed Computing*, pages 347–351. 2003.

[FMBT97] G. Facchini, F.v. Megen, P. Borm, and S. Tijs. Congestion models and weighted Bayesian potential games. *Theory and Decision*, 42:193–206, 1997.

[FP05] M. Fukushima and J.-S. Pang. Quasi-variational inequalities, generalized Nash equilibria, and multi-leader-follower games. *Computational Management Science*, 2(1):21–56, 2005.

[FP06] F. Facchinei and J.-S. Pang. Exact penalty functions for generalized Nash problems. In G. Di Pillo and M. Roma, editors, *Large-scale Nonlinear Optimization*, pages 115–126. Springer, New York, 2006.

[FT91] D. Fudenberg and J. Tirole. *Game Theory.* MIT Press, Cambridge, MA, 1991.

[FV04] S. Fischer and B. Vöcking. On the evolution of selfish routing. In *Proceedings of the 12th Annual European Symposium on Algorithms (ESA)*, pages 323–334. 2004.

[FV05] S. Fischer and B. Vöcking. On the structure and complexity of worst-case equilibria. In *Proceedings of the 1st Workshop on Internet and Network Economics (WINE)*, pages 151–160. 2005.

[GC05] D. Grosu and A.T. Chronopoulos. Noncooperative load balancing in distributed systems. *Journal of Parallel and Distributed Computing*, 65(9):1022–1034, 2005.

[GGAM+03] D. Gómez, E. González-Arangüena, C. Manuel, G. Owen, M. del Pozo, and J. Tejada. Centrality and power in social networks: A game theoretic approach. *Mathematical Social Sciences*, 46:27–54, 2003.

[GKR03] A. Gupta, A. Kumar, and T. Roughgarden. Simpler and better approximation algorithms for network design. In *Proceedings of the 35th Annual ACM Symposium on Theory of Computing*, pages 365–372. 2003.

[GO82] B. Grofman and G. Owen. A game theoretic approach to measuring centrality in social networks. *Social Networks*, 4(3):213–224, 1982.

[GS04] M.X. Goemans and M. Skutella. Cooperative facility location games. *Journal of Algorithms*, 50(2):194–214, 2004.

[GVR05] S. Goyal and F. Vega-Redondo. Network formation and social coordination. *Games and Economic Behavior*, 50:178–207, 2005.

[GY99] C.J. Goh and X.Q. Yang. Vector equilibrium problem and vector optimization. *European Journal of Operational Research*, 116: 615–628, 1999.

[Har91] P.T. Harker. Generalized Nash games and quasi-variational inequalities. *European Journal of Operational Research*, 54:81–94, 1991.

[HK00] H.W. Hamacher and K. Klamroth. *Linear and Network Optimization.* Vieweg, Braunschweig, Wiesbaden, 2000.

[HMC89] S. Hart and A. Mas-Colell. Potential, value, and consistency. *Econometrica*, 57:589–614, 1989.

[HS88] J.C. Harsanyi and R. Selten. *A General Theory of Equilibrium Selection in Games.* MIT Press, Cambridge, MA, 1988.

[HSK02] D. Helbing, M. Schönhof, and D. Kern. Volatile decision dynamics: Experiments, stochastic description, intermittency control and traffic optimization. *New Journal of Physics*, 4(33):1–16, 2002.

[HT01] H.W. Hamacher and S.A. Tjandra. Mathematical modelling of evacuation problems: A state of art. Berichte des Fraunhofer ITWM 24, Fraunhofer ITWM, Kaiserslautern, 2001.

[HTW05] A. Hayrapetyan, É. Tardos, and T. Wexler. A network pricing game for selfish traffic. In *Proceedings of the 24th Annual ACM SIGACT-SIGOPS Symposium on Principles of Distributed Computing (PODC)*, pages 284–289. ACM Press, New York, 2005.

[Jac05] M.O. Jackson. Allocation rules for network games. *Games and Economic Behavior*, 51:128–154, 2005.

[JMSSM05] O. Jahn, R.H. Möhring, A.S. Schulz, and N.E. Stier-Moses. System-optimal routing of traffic flows with user constraints in networks with congestion. *Operations Research*, 53(4):600–616, 2005.

[JS08] B.H. Junker and F. Schreiber, editors. *Analysis of Biological Networks.* Wiley & Sons, New York, 2008.

[JT04] R. Johari and J.N. Tsitsiklis. Efficiency loss in a network resource allocation game. *Mathematics of Operations Research*, 29(3): 407–435, 2004.

[Kel97] F. Kelly. Charging and rate control for elastic traffic. *European Transactions on Telecommunications*, 8:33–37, 1997.

[KK03] G. Karakostas and S.G. Kolliopoulos. Selfish routing in the presence of side constraints. Technical Report CAS-03-13-GK, Department of Computing and Software, McMaster University, 2003.

[KLO97] Y.A. Korilis, A.A. Lazar, and A. Orda. Achieving network optima using stackelberg routing strategies. *IEEE/ACM Transactions on Networking*, 5(1):161–173, 1997.

[KLS05] J. Könemann, S. Leonardi, and G. Schäfer. A group-strategyproof mechanism for steiner forests. In *Proceedings of the 16th annual ACM-SIAM symposium on Discrete algorithms (SODA05)*, pages 612–619, 2005.

[KP99] E. Koutsoupias and C. Papadimitriou. Worst-case equilibria. In *Proceedings of the 16th Annual Symposium on Theoretical Aspects of Computer Science (STACS'99)*, pages 404–413. Springer, New York, 1999.

[Kra05] W. Krabs. *Spieltheorie.* Teubner, Wiesbaden, 2005.

[KS06] A.C. Kaporis and P.G. Spirakis. The price of optimum in stackelberg games on arbitrary single commodity networks and latency functions. In *Proceedings of the 18th annual ACM symposium on Parallelism in algorithms and architectures (SPAA06)*, pages 19–28, 2006.

[Kuk99] N.S. Kukushkin. Potential games: A purely ordinal approach. *Economics Letters*, 64:279–283, 1999.

[Kuk02] N.S. Kukushkin. Perfect information and potential games. *Games and Economic Behavior*, 38:306–317, 2002.

[LMMR04] T. Lücking, M. Mavronicolas, B. Monien, and M. Rode. A new model for selfish routing. In *Proceedings of the 21st Annual Symposium on Theoretical Aspects of Computer Science*, pages 547–558. 2004.

[Mal07] L. Mallozzi. Noncooperative facility location games. *Operations Research Letters*, 35(2):151–154, 2007.

[Mil96] I. Milchtaich. Congestion games with player-specific payoff functions. *Games and Economic Behavior*, 13:111–124, 1996.

[Mon07] D. Monderer. Multipotential games. In *Proceedings of 20th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 1422–1427. 2007.

[MS96] D. Monderer and L. S. Shapley. Potential games. *Games and Economic Behavior*, 14:124–143, 1996.

[MTV00] L. Mallozzi, S. Tijs, and M. Voorneveld. Infinite hierarchical potential games. *Journal of Optimization Theory and Applications*, 107(2):287–296, 2000.

[MU05] S. Morris and T. Ui. Generalized potentials and robust sets of equilibria. *Journal of Economic Theory*, 124:45–78, 2005.

[Nag00] A. Nagurney. A multiclass, multicriteria traffic network equilibrium model. *Mathematical and Computer Modelling*, 32:393–411, 2000.

[Nas50] J. Nash. Equilibrium points in $n$-person games. *Proceedings of the National Academy of Sciences*, 36:48–49, 1950.

[Nis99] N. Nisan. Algorithms for selfish agents. In *Proceedings of the 16th Annual Symposium on Theoretical Aspects of Computer Science (STACS99)*, volume 1563 of *Lecture Notes in Computer Science*, pages 1–15. Springer, Berlin, 1999.

[NIS05] E. Nikolova N. Immorlica, D. Karger and R. Sami. First-price path auctions. In *Proceedings of 6th ACM Conference on Electronic Commerce (EC05)*, pages 203–212, 2005.

[NR01] N. Nisan and A. Ronen. Algorithmic mechanism design. *Games and Economic Behavior*, 35:166–196, 2001.

[NT98] H. Norde and S. Tijs. Determinateness of strategic games with a potential. *Mathematical Methods of Operations Research*, 46: 377–385, 1998.

[Owe95] G. Owen. *Game Theory.* Academic Press, San Diego, third edition, 1995.

[Pap01] C.H. Papadimitriou. Algorithms, games, and the internet. In *Proceedings of the 33th ACM Symposium on Theory of Computing (STOC)*, pages 749–753. 2001.

[Per04] G. Perakis. The price of anarchy when costs are non-separable and asymetric. In *Integer Programming and Combinatorial Optimization, 10th International IPCO Conference*, pages 46–58. Springer, New York, 2004.

[PPT07] F. Patrone, L. Pusillo, and S. Tijs. Multicriteria games and potentials. *TOP*, 15(1):138–145, 2007.

[PSS] J. Puerto, A. Schöbel, and S. Schwarze. Games on polyhedra. Working paper.

[PSS06] J. Puerto, A. Schöbel, and S. Schwarze. A class of infinite potential games. Technical Report 2006-15, Institut für Numerische und Angewandte Mathematik, Universität Göttingen, 2006. Preprint Reihe.

[PSS08] J. Puerto, A. Schöbel, and S. Schwarze. The path player game: A network game from the point of view of the network providers. *Mathematical Methods of Operations Research*, 2008. to appear.

[Ros65] J.B. Rosen. Existence and uniqueness of equilibrium points for concave $n$-person games. *Econometrica*, 33(3):520–534, 1965.

[Ros73] R.W. Rosenthal. A class of games possessing pure-strategy Nash equilibria. *International Journal of Game Theory*, 2:65–67, 1973.

[Rou03] T. Roughgarden. The price of anarchy is independent of the network topology. *Journal of Computer and System Sciences*, 67(2):341–364, 2003.

[Rou04] T. Roughgarden. Stackelberg scheduling strategies. *SIAM Journal on Computing*, 33(2):332–350, 2004.

[Rou05a] T. Roughgarden. *Selfish Routing and the Price of Anarchy.* MIT Press, Cambridge, MA, 2005.

[Rou05b] T. Roughgarden. Selfish routing with atomic players. In *Proceedings of the 16th annual ACM-SIAM symposium on Discrete algorithms (SODA05)*, pages 1184–1185, 2005.

[RT02] T. Roughgarden and É. Tardos. How bad is selfish routing? *Journal of the ACM*, 49(2):236–259, 2002.

[San01] W.S. Sandholm. Potential games with continuous player sets. *Journal of Economic Theory*, 97:81–108, 2001.

[Sch05] S. Scholl. *Customer-oriented line planning.* PhD thesis, Technische Universität Kaiserslautern, 2005.

[SDNT00] M. Slikker, B. Dutta, A.v.d. Nouweland, and S. Tijs. Potential maximizers and network formation. *Mathematical Social Sciences*, 39: 55–70, 2000.

[SK95] D. Skorin-Kapov. On the core of the minimum cost Steiner tree game in networks. *Annals of Operations Research*, 57:233–249, 1995.

[Sli01] M. Slikker. Coalition formation and potential games. *Games and Economic Behavior*, 37:436–448, 2001.

[SS02] M. Schreckenberg and S.D. Sharma, editors. *Pedestrian and Evacuation Dynamics.* Springer, Berlin, 2002.

[SS05] A. Schöbel and S. Scholl. Line planning with minimal transfers. In *Proceedings of 5th Workshop on Algorithmic Methods and Models for Optimization of Railways (ATMOS).* 2005.

[SS06a] A. Schöbel and S. Schwarze. Dominance and equilibria in the path player game. In *Operations Research Proceedings 2005*, pages 489–494. 2006.

[SS06b] A. Schöbel and S. Schwarze. A game-theoretic approach to line planning. In *ATMOS 2006-6th Workshop on Algorithmic Methods and Models for Optimization of Railways*, number 06002 in Dagstuhl Seminar Proceedings, 2006. `http://drops.dagstuhl.de/opus/volltexte/2006/688`.

[STZ04] S. Suri, C.D. Tóth, and Y. Zhou. Selfish load balancing and atomic congestion games. In *Proceedings of the 16th annual ACM symposium on Parallelism in algorithms and architectures (SPAA04)*, pages 188–195. 2004.

[Swa07] C. Swamy. The effectiveness of stackelberg strategies and tolls for network congestion games. In *Proceedings of the 18th annual ACM-SIAM symposium on Discrete algorithms (SODA07)*, pages 1133–1142, 2007.

[Ui00] T. Ui. A Shapley value representation of potential games. *Games and Economic Behavior*, 31:121–135, 2000.

[Ui01] T. Ui. Robust equilibria of potential games. *Econometrica*, 69(5):1373–1380, 2001.

[Vet02] A. Vetta. Nash equilibria in competitive societies, with applications to facility location, traffic routing and auctions. In *Proceedings of the 43rd Symposium on Foundations of Computer Science (FOCS)*, pages 416–425. 2002.

[VN97] M. Voorneveld and H. Norde. A characterization of ordinal potential games. *Games and Economic Behavior*, 19:235–242, 1997.

[Voo97] M. Voorneveld. Equilibria and approximate equilibria in infinite potential games. *Economics Letters*, 56:163–169, 1997.

[Voo00] M. Voorneveld. Best-response potential games. *Economics Letters*, 66:289–295, 2000.

[War52] J.G. Wardrop. Some theoretical aspects of road traffic research. *Proceedings of the Institute of Civil Engineers, Pt. II*, 1:325–378, 1952.

[Yan07] Q. Yan. On the price of truthfulness in path auctions. In *Internet and Network Economics*, volume 4858 of *Lecture Notes in Computer Science*, pages 584–589. Springer, Berlin, 2007.

# Index