Appendix A
Answers to Problems

Chapter 1

1. $W = 2.28$ eV, sodium
2. 253.1nm
3. (a) The Lyman and Balmer lines are:

$(n \to m)$	$\Delta E_{n\to m}$ (eV)	Designation	λ (nm)
$4 \to 1$	12.75	L_γ	97.3
$3 \to 1$	9.06	L_β	136.9
$2 \to 1$	10.2	L_α	121.6
$5 \to 2$	2.86	H_γ	433.6
$4 \to 2$	2.55	H_β	486.3
$3 \to 2$	1.89	H_α	656.1

   (b) Lyman 91.2 nm; Balmer 364.7 nm
4. $a_{0\mu} = a_0/180$; αc; ≈ 2450 eV
7. (a) $\approx 1.2 \times 10^{-15}$m
   (b) 44×10^{-7}nm (γ rays)

Chapter 2

1. (a) $U(x) = \dfrac{\hbar^2\alpha^4}{2m}x^2$

   (b) $F = -\left(\hbar^2\alpha^4/m\right)x$

   (c) $A = \sqrt{\alpha/\sqrt{\pi}}$

2. (a) $P(x)\,dx = \dfrac{\alpha}{\sqrt{\pi}}e^{-\alpha^2x^2}dx$
   (b) 1

4. (a) $j(x,t) = \dfrac{\hbar k}{m}|A|^2$
   (b) amplitude increases to $\sqrt{2}A$

5. (a) Acceptable
   (b) Unacceptable
   (c) Acceptable
   (d) Acceptable
   (c) Acceptable
   (e) Unacceptable

6. (a)

$$\Psi(x,0) = \frac{1}{2}\psi_1(x) + \frac{1}{\sqrt{3}}\psi_2(x) + \sqrt{\frac{5}{12}}\psi_3(x)$$

(b)

$$\Psi(x,t) = \frac{1}{2}\psi_1(x)e^{-iE_1t/\hbar} + \frac{1}{\sqrt{3}}\psi_2(x)e^{-iE_2t/\hbar} + \sqrt{\frac{5}{12}}\psi_3(x)e^{-iE_3t/\hbar}$$

(c)

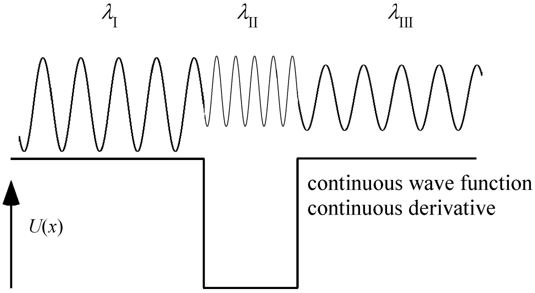
$$\langle E\rangle = \frac{1}{4}E_1 + \frac{1}{3}E_2 + \frac{5}{12}E_3$$

(d)

$$\langle E\rangle = \frac{1}{4}E_1 + \frac{1}{3}E_2 + \frac{5}{12}E_3$$

7.

$$\lambda_I = \frac{h}{\hbar k_I} = \frac{2\pi}{k_I} = k_{II}; \quad \lambda_{II} < \lambda_I \text{ because } k_{II} > k_I$$



Problem 7 of Chapter 2

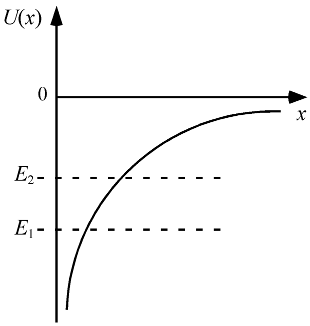
8. (a) $E_1 = -\frac{E_0}{1^2}; E_2 = -\frac{E_0}{2^2}; E_3 = -\frac{E_0}{3^2}$

(b) $\frac{2}{9}; \frac{1}{9}; \frac{6}{9}$ for states 1, 2, and 3

(c) $\langle E \rangle = -0.32E_0$

(d) $3Q_0$

9. (a) Yes, it can support bound states having TME< 0.



Problem 9 of Chapter 2

(b) $\psi_2(x)$

(c) $1/2$

(d) $3/2$ a.u.

Chapter 3

2. $A = \sqrt{2/L}$

3. $\left(1/2\sqrt{2}\right)\lambda_c$

6. (a) $\langle E \rangle = 3\frac{\pi^2\hbar^2}{2mL^2}$

(b) $\Psi(x,t) = \frac{1}{\sqrt{3}}\psi_1(x)e^{-\omega_1 t} + \sqrt{\frac{2}{3}}\psi_2(x)e^{-\omega_2 t}$

7. $\langle E \rangle = \frac{4^2\pi^2\hbar^2}{2mL^2} = E_4$

8. (a) $P_{cl}(x)\,\Delta x = \Delta x/L$

   (b) $\langle x^2\rangle_{\text{classical}} = L^2/3\,;\ \langle x^2\rangle_n = \dfrac{L^2}{3}\left[1 - \dfrac{3}{2\,(n\pi)^2}\right]$

9. $\Delta x_0 = 1/\sqrt{2}\alpha$

10. (a) zero

    (b) $\langle \hat{p}^2\rangle = \left(\dfrac{n\pi\hbar}{L}\right)^2$

11. $(\Delta x)^2 = L^2\left(\dfrac{1}{12} - \dfrac{1}{2\pi^2 n^2}\right)$

12. (a) ≈ 0.36

    (b) 0.5

13. (a) $K = \dfrac{\sqrt{210}}{2a^{3/2}}$

    (b) $P_n = \dfrac{24^2\cdot 105}{\pi^6}\cdot\dfrac{1}{n^6}$ $\quad n$ even; $P\,(n=1)=0$; $P\,(n=2)=0.983$

    (c) 1

    (d) $\langle E\rangle = \dfrac{42}{\left(2^2\pi^2\right)}E_2$

14. (a) ≈ 0.36
    (b) zero
    (c) The initial wave functions in the two problems are different. There is no symmetry in Problem 12 while the initial wave function in this problem is symmetric.

16. $\Delta x\Delta p = \dfrac{1}{2}\hbar\omega$

17. $P_{\text{out}}\,(n=1) = 0.111$

19. $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$ where $n = 1, 3, 5, \cdots$ and corresponding eigenfunctions

20. (a) $E_n = \left(n+\dfrac{1}{2}\right)\hbar\omega$ with $\omega = \sqrt{\dfrac{2k}{m}}$

    (b) ≈ 0.985
    (c) zero

22. (a) $P = \dfrac{2\alpha\beta}{\alpha^2 + \beta^2}$

23. (a) Eigenfunctions are the same with $x \to x + eF/\left(m_e\omega^2\right)$. Eigenvalues are $E_n = (n + 1/2)\,\hbar\omega - eF/\left(m_e\omega^2\right)$

    (b) $P = \exp\left[-\dfrac{1}{2}\dfrac{e^2F^2}{m_e\hbar\omega^3}\right]$

Chapter 4

9. (a) $\Psi(x, 0) = K\delta(x)$
   (b) Probability of measuring any odd state is zero. Normalization not recommended because the probabilities of measuring any even state are all the same.
   (c) No odd states. No even eigenfunction should contribute more than another.
   (d) $\Psi(x, t) = \dfrac{2}{a}\sum_{n\ \mathrm{odd}}^{\infty} e^{-iE_nt/\hbar}\sin\left(\dfrac{n\pi x}{L}\right)$

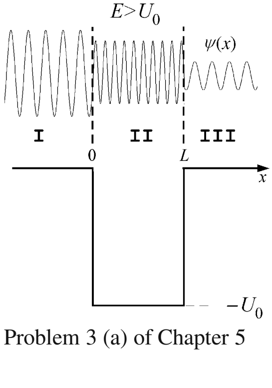
10. $\phi_n(p) = \sqrt{\frac{1}{2^n n!}}\left(\frac{1}{\pi\alpha^2\hbar^2}\right)^{1/4} H_n\left(\frac{1}{\sqrt{m\omega\hbar}}p\right)e^{-p^2/\left(2\alpha^2\hbar^2\right)}$

Chapter 5

1. $R = \dfrac{1}{1 + \left(\dfrac{\hbar^2k}{mU_0}\right)^2}$; $T = \dfrac{1}{1 + \left(\dfrac{mU_0}{\hbar^2k}\right)^2}$

2. $E = -\dfrac{mU_0^2}{2\hbar^2}$

3. (a)



Problem 3 (a) of Chapter 5

(b) $$T_{E>U_0} = \frac{4E(E+U_0)}{4E(E+U_0) + U_0^2 \sin^2\left(\frac{L}{\hbar}\sqrt{2m(E+U_0)}\right)}$$

5. The eigenfunctions are the odd eigenfunctions for the finite square well because they are the ones with a node at $x = 0$ which is demanded by the infinity in the potential.

7. $E = -\dfrac{2mU_0^2 a^2}{\hbar^2}$

8. (a) $\dfrac{P_{in}}{P_{out}} = \dfrac{e^{-2\kappa a}}{\sin^2 ka}\left(1 - \dfrac{\sin 2ka}{2ka}\right)\dfrac{\kappa a}{e^{-2\kappa a}}$

(b) A state just above $E = 0$, a continuum state.

11. $n + 1/2 = 20/\sqrt{2} = 14.14$

15. $E_n = \left(n + \dfrac{1}{2}\right)\hbar\omega$

16. $E_n = -\dfrac{m}{2\left(n+\frac{1}{2}\right)^2 \hbar^2} \cdot \left(\dfrac{e^2}{4\pi\epsilon_0}\right)^2$

17. $T = \exp\left[-\sqrt{\dfrac{2m}{E}}\dfrac{KZZ'e^2\pi}{\hbar}\right]$. As E increases the exponent decreases so T increases.

18. $T = \exp\left[-\dfrac{4W^{3/2}\sqrt{2m}}{3\hbar eF}\right]$

Chapter 6

6. (b) $e^{-iE_n t/\hbar}$

(c) No, because the operator $e^{-i\hat{H}t/\hbar}$ is not Hermitian.

13. $\left[\hat{H}, x\right] = -i\hbar\dfrac{\hat{p}}{m}$; $\left[\hat{H}, \hat{p}\right] = i\hbar\dfrac{dU(x)}{dx}$

17. $x(t) = \dfrac{F}{2m}t^2 + \dfrac{p_0}{m}t + x_0$; $\hat{p}(t) = Ft + p_0$

Chapter 7

5. (a) $|\Psi(x,t)\rangle = \frac{1}{\sqrt{3}}|1\rangle e^{-(3/2)i\omega t} + \sqrt{\frac{2}{3}}|2\rangle e^{-(5/2)i\omega t}$

   (b) $\langle E\rangle = \left(\frac{13}{6}\hbar\omega\right)$

   (c) $\langle\hat{x}(t)\rangle = \sqrt{\frac{\hbar}{m\omega}}\frac{\sqrt{2}}{3}2\cos\omega t$

11. $\langle n|\hat{x}^4|n\rangle = \frac{3}{4\alpha^4}\left(2n^2+2n+1\right)$

17. (b) $P_N = \left[\left(\frac{\alpha}{\sqrt{2}}x_0\right)^N \frac{e^{-\left|\alpha^2 x_0^2\right|/2}}{\sqrt{N!}}\right]^2$

Chapter 8

1. $\left[\hat{J}_x\hat{J}_y, \hat{J}_z\right] = i\hbar\left(\hat{J}_x^2 - \hat{J}_y^2\right)$

5. $\left\langle\hat{J}_x\right\rangle = 0; \left\langle\hat{J}_x^2\right\rangle = \frac{\hbar^2}{2}\left[j(j+1) - m^2\right]$

14. (a) 0, $\sqrt{2}\hbar$, $\sqrt{6}\hbar$

    (b) zero

    (c) $\left\langle\hat{L}_z\right\rangle = 0; \left\langle\hat{L}^2\right\rangle = \frac{\hbar^2}{14}(8+54) \approx 4.43\hbar^2$

16. 12.5%

17. $\hat{L}_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}; \hat{L}^2 = 2\hbar^2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix};$

    $\hat{L}_+ = \hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}; \hat{L}_- = \hbar\begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix};$

    $\hat{L}_y = \frac{i\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

18. (a) $\hbar, 0, -\hbar$

(b) $|\uparrow\rangle_x = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$; $|\longrightarrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$; $|\downarrow\rangle_x = \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$

a. The probabilities of measuring $\hbar, 0, -\hbar$ for $\hat{L}_x$ are then $1/4, 1/2, 1/4$, respectively.

19. Two beams of equal intensity

Chapter 9

4. (a) It does have definite angular momentum. $\ell = 1$ and $m = 0$.

(b) $\frac{1}{2}\beta^2$; bound

(c) $U(r) = -\frac{5\beta}{2r} + \frac{7}{8r^2}$

5. $E_n = \frac{n^2\pi^2\hbar^2}{2m(b-a)^2}$; $\psi_{n00}(r,\theta,\phi) = \frac{1}{\sqrt{4\pi}}\sqrt{\frac{2}{(b-a)}}\left(\frac{1}{r}\right)\sin\left[\frac{(r-a)}{(b-a)}\pi\right]$

6. (c) ≈ 0.74

7. (a) $\psi(x,y,z) = \left(\sqrt{\frac{2}{L}}\right)^3 \sin\left(\frac{n_x\pi}{L}x\right)\sin\left(\frac{n_y\pi}{L}y\right)\sin\left(\frac{n_z\pi}{L}z\right)$

(b) $E_{n_x n_y n_z} = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$. Three times larger.

(c) $g_1 = 1$; $g_2 = 3$; $g_3 = 6$

15. $|\psi(\boldsymbol{r},t)\rangle = \sum_j a_k e^{-iE_n t/\hbar}|n\ell m\rangle$

Chapter 10

3. $r_c = n^2\left(1 \pm \sqrt{1 - \frac{\ell(\ell+1)}{n^2}}\right)$

4. $(\delta r_c)_{\max} = 2\left(n^2 a_0\right)$; $(\delta r_c)_{\min} = 2\left(n^2 a_0\right)\sqrt{\frac{1}{n}}$

6. ≈ 0.24

8. $\langle x\rangle_{n\ell m} = 0 = \langle y\rangle_{n\ell m} = \langle z\rangle_{n\ell m}$

11. $\ell = 0$
12. For $\ell = 0$: $\langle r\rangle = \frac{3}{2}\left(a_0 n^2\right)$; $(\Delta r) = \frac{a_0}{2} n\sqrt{n^2+2}$

    For $\ell = n-1$: $\langle r\rangle = a_0 n\left(n+\frac{1}{2}\right)$; $(\Delta r) = a_0 n\sqrt{\frac{1}{2}\left(n+\frac{1}{2}\right)}$
14. ≈ 0.70
17. (a) $|\psi(\boldsymbol{r},t)\rangle = \frac{1}{\sqrt{3}}e^{-iE_1t/\hbar}\,|100\rangle + \frac{1}{\sqrt{6}}e^{-iE_2t/\hbar}\,|210\rangle + \frac{1}{\sqrt{2}}e^{-iE_3t/\hbar}\,|320\rangle$

    (b) $\langle E\rangle = -\frac{\left(mc^2\right)\alpha^2}{2}\left(\frac{89}{144}\right)$; $\left\langle \hat{L}^2\right\rangle = \frac{13}{3}\hbar^2$; $\left\langle \hat{L}_z\right\rangle = 0$

    (c) As specified by Ehrenfest's theorem (see Section 6.3.3), in particular Equation 6.120)

Chapter 11

1. $\dot{\boldsymbol{A}} = \left[(\boldsymbol{p}\times\boldsymbol{L}) - \left(\frac{me^2}{4\pi\epsilon_0}\right)\hat{\boldsymbol{r}}\right]$

Chapter 12

1. (a) $E_n = \left(n+\frac{1}{2}\right)\hbar\omega - \frac{e^2F^2}{2m\omega^2}$

   (b) Second-order perturbation theory result is the same as the exact solution.

2. $E^{(1)}_{n\text{odd}} = \frac{2}{a}U_0$; $E^{(1)}_{n\text{even}} = 0$

3. $E^{(1)}_{n\text{odd}} = \frac{2}{a}U_0$; $E^{(1)}_{n\text{even}} = 0$. There is a node at the perturbation for n even.

   (a) $E^{(1)}_1 = \frac{eFL}{2}$

   (b) $\psi^{(1)}_1(x) \approx \frac{32}{27\pi^2}\frac{E^{(1)}_1}{E^{(0)}_1}\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$

   (c) $E^{(1)}_n = \frac{1}{2mc^2}\left(E^{(0)}_n\right)^2$

   (d) $E^{(\text{rel})}_n = E^{(0)}_n - \frac{1}{2}\left(\frac{1}{mc^2}\right)\left(E^{(0)}_n\right)^2 + \cdots$

4. $E_n^{(2)} = -\left(\frac{15}{8}\right)\left(\frac{1}{4D_e}\right)(\hbar\omega)^2\left[\left(n^2+n+11/30\right)\right]$

   (a) $E_1^{(1)} = \frac{1}{2}\left(\frac{\omega_1}{\omega}\right)^2\left(n+\frac{1}{2}\right)\hbar\omega$; $E_n^{(2)} = -\frac{1}{8}\frac{\omega_1^4}{\omega^4}\left(n+\frac{1}{2}\right)\hbar\omega$

   (b) The first three terms in the expansion of the exact result are $E_n^{(0)}$, $E_1^{(1)}$, and $E_n^{(2)}$.

5. $E_0^{(1)} = \frac{1}{4\alpha^2}\left(\frac{3C}{\alpha^2} - m\omega^2\right)$

   (a) $\hat{H}_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$; $\hat{H}_1 = \begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon & 0 & 0 \\ 0 & 0 & -\epsilon \end{pmatrix}$

   (b) $E_1^{(1)} = 0$; $E_2^{(1)} = 0$; $E_3^{(1)} = -\epsilon$; $E_1^{(2)} = -\epsilon^2$; $E_2^{(2)} = \epsilon^2$; $E_3^{(2)} = 0$

   (c) The exact result reduces to the second-order perturbation theory result.

11. (a) $E_{000}^{(2)} = -\frac{1}{2}\frac{e^2F^2}{m_e\omega^2}$

    (b) $E_{000}^{(2)} = -\frac{1}{2}\frac{e^2F^2}{m_e\omega^2}$

12. (a) $\left|1^{(s)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|1^{(0)}\right\rangle + \left|2^{(0)}\right\rangle\right)$; $\left|2^{(s)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|1^{(0)}\right\rangle - \left|2^{(0)}\right\rangle\right)$; $\left|3^{(s)}\right\rangle = \left|3^{(0)}\right\rangle$

13. (a) $E_{\text{variational}} = \frac{3}{8}\left(6\frac{C\hbar^4}{m^2}\right)^{1/3}$

15. $E_{\text{variational}} = \sqrt{3}\hbar\omega$

16. (a) $E_{\text{variational}} = \frac{3^{5/3}}{2^{4/3}}\left(\frac{e^2F^2\hbar^2}{2m}\right)^{1/3}$

Chapter 13

5. (a) $E_{SO}^{(1)}\left(j=\ell+\frac{1}{2}\right) = \frac{(\hbar\omega)^2}{4mc^2}\left(\frac{\ell}{2}\right)$;

   $E_{SO}^{(1)}\left(j=\ell-\frac{1}{2}\right) = -\frac{(\hbar\omega)^2}{4mc^2}\left(\frac{\ell+1}{2}\right)$

   (b) zero

8. 2.6 eV

9. $E^{(1)}_{1s2\ell} = J_{1s2\ell} \pm K_{1s2\ell}; |\psi_\pm\rangle = \frac{1}{\sqrt{2}}\{|1s\rangle_1 |n\ell\rangle_2 \pm |1s\rangle_2 |2\ell\rangle_1\}$

11. (a) $^2P_{1/2}$ and at higher energy $^2P_{3/2}$

(b) $^2S_{1/2}$; $^2P^2_{3/2,1/2}$; $D_{5/2,3/2}$

12. (a) $^1S;^1D;\,^1G;\,^3P;\,^3F$

(b) ^3F

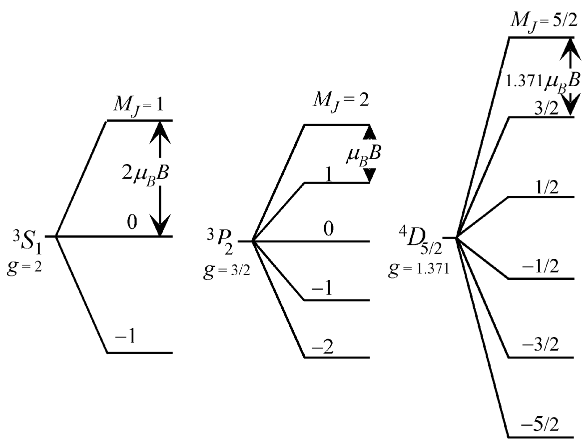
Chapter 14

3. $E^{(0)}_m = \frac{(m\hbar)^2}{2\mu a^2}; E^{(2)}_m = \frac{\mu q^2 a^4 F^2}{\hbar^2}\left(\frac{1}{4m^2-1}\right)$

6. (a) $\hat{H} = \hat{\boldsymbol{\mu}}_S \bullet \boldsymbol{B} + \hat{\boldsymbol{\mu}}_p \bullet \boldsymbol{B} + \frac{2\kappa}{\hbar^2}\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$

(b) $E_\pm = -\frac{\kappa}{2} \pm \left(\kappa^2 + \mu_B B\right)^{1/2}$; The proton magnetic moment is tiny compared with that of the electron.

8.



Problem 8 of Chapter 14

Chapter 15

3. $P^{(1)}_{0\to 1} = \dfrac{(eF_0)^2}{2m\hbar\omega}\pi e^{-\omega^2\tau^2/2}\tau^2$

4. Transitions to all states vanish except $n = 1$ and 3.

$$P^{(1)}_{0\to 1} = |A|^2\,\frac{9}{8}\frac{\hbar}{m^3\omega^3}\frac{1}{\left(\omega^2 + 1/\tau^2\right)}$$
$$P^{(1)}_{0\to 3} = |A|^2\,\frac{3}{4}\frac{\hbar}{m^3\omega^3}\frac{1}{\left(9\omega^2 + 1/\tau^2\right)}$$

5. $P^{(1)}_{1\to 2} = \frac{e^2F_0^2}{\hbar^2}\left(\dfrac{2^{15}}{3^{10}}a_0^2\right)\left(\dfrac{1}{\omega_{21}^2 + 1/\tau^2}\right)$

Appendix B
Useful Constants

Table B.1 A few physical constants listed to a limited number of significant figures

Constant	Symbol	Value
speed of light	c	$3 \times 10^{8}\mathrm{m/s}$
elementary charge	e	$1.6 \times 10^{-19}\mathrm{C}$
electronic mass	m_e	$9.1 \times 10^{-31}\mathrm{kg}$
Bohr magneton	μ_B	$9.27 \times 10^{-24}\mathrm{J/T}$
nuclear magneton	μ_N	$5.05 \times 10^{-27}\mathrm{J/T}$
fine structure constant	α	$1/137$
Rydberg constant	R_∞	$1.09 \times 10^{7}\mathrm{m}^{-1}$
Bohr radius	a_0	0.053nm

Appendix C
Energy Units

Because of the wide variation of the magnitudes of energies encountered in quantum physics, energies are often specified in convenient units that are tailored to a given physical problem. For example, while electron volts (eV) are convenient units for dealing with electronic levels of atoms and molecules, they are not particularly suited to describe fine structure intervals. Below is a table that gives the relationship between the electron-volt and some commonly used units. Only a few significant figures are included because the table is meant to demonstrate the orders of magnitude of these relationships.

Table C.1 The relationship between electron-volts and other commonly used units of energy

$1\text{eV} = 1 \times 10^{-6}\text{MeV}$	$1\text{MeV} = 1 \times 10^{6}\text{eV}$
$1\text{eV} = 1.6 \times 10^{-19}\text{J}$	$1\text{J} = 6.24 \times 10^{18}\text{eV}$
$1\text{eV} = 8065.6\ \text{cm}^{-1}$	$1\ \text{cm}^{-1} = 1.24 \times 10^{-4}\text{eV}$
$1\text{eV} = 2.42 \times 10^{8}\text{MHz}$	$1\text{MHz} = 4.12 \times 10^{-9}\text{eV}$
$1\text{eV} = 3.68 \times 10^{-2}$ hartree	1 hartree $= 27.21\text{eV}$

Appendix D
Useful Formulas

Table D.1 Some key formulas in SI units and atomic units

Quantity	Symbol	SI Units	a.u.
Fine structure constant	α	$\left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{\hbar c}$	$\frac{1}{c}$
Bohr energy	E_n	$-\frac{1}{2}\frac{\alpha^2 m_e c^2}{n^2}$	$\frac{-1}{2n^2}$
Bohr radius	a_0	$\frac{1}{\alpha}\left(\frac{\hbar}{m_e c}\right)$	1
Compton wavelength—electron	$\lambda\!\!\!^{-}_c$	$\frac{\hbar}{m_e c} = \alpha a_0$	$\sim \frac{1}{c}$
Classical radius—electron	R_e	$\sim\alpha^2 a_0$	$\sim \frac{1}{c^2}$
Bohr magneton	μ_B	$\frac{e\hbar}{2m_e}$	$\frac{1}{2}$

Appendix E
Greek Alphabet

Table E.1 The letters of the Greek alphabet. Where appropriate, their primary usage in this book is indicated. U. C. refers to uppercase

L. C.	U. C.	Name	Usage in this book
α	A	Alpha	fine structure constant, harmonic oscillator, spin up
β	B	Beta	general parameter, spin down
γ	Γ	Gamma	Γ function (U. C.), square root of transmission coefficient T (U. C.)
δ	Δ	Delta	Dirac δ-function, small increment, difference (U. C.)
ϵ, ε	E	Epsilon	unitless energy parameter, small quantity
ζ	Z	Zeta	general parameter
η	H	Eta	general parameter
θ, ϑ	Θ	Theta	polar angle, function (U. C.)
ι	I	Iota	—
κ	K	Kappa	real exponent, hyperfine energy
λ	Λ	Lambda	wavelength
μ	M	Mu	general parameter, mass
ν	N	Nu	frequency (radians/s)
ξ	Ξ	Xi	unitless length harmonic oscillator
o	O	Omicron	—
π	Π	Pi	—
ρ	P	Rho	parameter, unitless length hydrogen
σ	Σ	Sigma	Pauli matrices, summation
τ	T	Tau	increment of time
υ	Υ	Upsilon	—
ϕ	Φ	Phi	azimuthal angle, function (U. C.)
χ	X	Chi	spin state
ψ	Ψ	Psi	wave function (L. C. and U. C.)
ω	Ω	Omega	frequency (radians/s) (L. C.), Bohr frequency $\omega_{nm} = (E_n - E_m)/\hbar$ (not just hydrogen)
	F	Digamma	function

Appendix F
Acronyms

Table F.1 Some acronyms used in this book

Acronym	Meaning
RWA	rotating wave approximation
TDSE	time-dependent Schrödinger equation
TISE	time-independent Schrödinger equation
TME	total mechanical energy (the “energy”)
WKB	Wentzel, Kramers, Brillouin approximation

Appendix G
Γ-Functions

G.1 Integral Γ-Functions

$$\Gamma(n+1) = n\Gamma(n) = n! \quad n = 1, 2, 3, \ldots \tag{G.1}$$

$$\Gamma(1) = 1 \tag{G.2}$$

$$\Gamma(2) = 1 \tag{G.3}$$

$$\Gamma(3) = 2 \tag{G.4}$$

$$\Gamma(4) = 3! = 6 \tag{G.5}$$

$$\Gamma(5) = 4! = 24 \tag{G.6}$$

G.2 Half-Integral Γ-Functions

$$\Gamma(m+1) = \frac{1 \cdot 3 \cdot 5 \cdots (2m-1)}{2^m}\sqrt{\pi} \quad m = 1, 2, 3, \ldots \tag{G.7}$$

$$\Gamma(1/2) = \sqrt{\pi} \tag{G.8}$$

$$\Gamma(3/2) = \frac{\sqrt{\pi}}{2} \tag{G.9}$$

$$\Gamma(5/2) = \frac{3\sqrt{\pi}}{4} \tag{G.10}$$

$$\Gamma(7/2) = \frac{15\sqrt{\pi}}{8} \tag{G.11}$$

Appendix H
Useful Integrals

$$\int x^2 e^{-ax} dx = \frac{e^{-ax}}{-a}\left(x^2 + \frac{2x}{a} + \frac{2}{a^2}\right) \tag{H.1}$$

$$\int_0^\infty x^m e^{-ax} dx = \frac{\Gamma\left[(m+1)\right]}{a^{m+1}} = \frac{m!}{a^{m+1}} \tag{H.2}$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \tag{H.3}$$

$$\int_{-\infty}^{\infty} e^{-\left(ax^2+bx+c\right)} dx = \sqrt{\frac{\pi}{a}} e^{\left(b^2-4ac\right)/4a} \tag{H.4}$$

$$\int_0^\infty x^m e^{-ax^2} dx = \frac{\Gamma\left[(m+1)/2\right]}{2a^{(m+1)/2}} \tag{H.5}$$

$$\int_0^1 \sqrt{\frac{1}{x} - 1}\, dx = \frac{\pi}{2} \tag{H.6}$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} \tag{H.7}$$

$$\int \frac{dx}{\left(x^2 + a^2\right)^2} dx = \frac{x}{2a^2\left(x^2 + a^2\right)} + \frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right) \tag{H.8}$$

$$\int \frac{dx}{\left(x^2+a^2\right)^n}dx = \frac{x}{2\left(n-1\right)a^2\left(x^2+a^2\right)^{n-1}} + \frac{2n-3}{\left(2n-2\right)a^2}\int \frac{dx}{\left(x^2+a^2\right)^{n-1}}dx \tag{H.9}$$

$$\int \frac{x^m dx}{\left(x^2+a^2\right)^n}dx = \int \frac{x^{m-2}dx}{\left(x^2+a^2\right)^{n-1}}dx - a^2\int \frac{x^{m-2}dx}{\left(x^2+a^2\right)^n}dx \tag{H.10}$$

Appendix I
Useful Series

I.1 Taylor Series

A Taylor series expansion of a function $f(x)$ about a point $x = a$ is

$$\begin{aligned} f(x) &= f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \cdots \\ &= \sum_{n=0}^{\infty} \frac{(x-a)^n}{n!} f^{(n)}(a) \end{aligned} \tag{I.1}$$

where the primes signify differentiation with respect to x. Therefore, for example, $f''(a)$ is the second derivative of the function $f(x)$ with respect to x evaluated at $x = a$.

There are at least four Taylor series that every physics student should have at their command:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \tag{I.2}$$

$$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \tag{I.3}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \tag{I.4}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x3}{3} - \frac{x^4}{4} + \cdots \tag{I.5}$$

I.2 Binomial Expansion

Binomial series are special cases of Taylor series for $f(x) = (1+x)^m$ and $a = 0$. The exponent m may be positive or negative and is not restricted to being an integer. The general form of the binomial expansion is, in three equivalent forms,

$$\begin{aligned}(1+x)^m &= 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \cdots \\ &= \sum_{n=0}^{\infty} \frac{m!}{n!(m-n)!}x^n \\ &= \sum_{n=0}^{\infty} \binom{m}{n} x^n \end{aligned} \tag{I.6}$$

where

$$\binom{m}{n} \equiv \frac{m!}{n!(m-n)!} \tag{I.7}$$

is called the binomial coefficient. A few of the most common binomial expansions are listed below:

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots \tag{I.8}$$

$$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots \tag{I.9}$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \frac{1}{2 \cdot 4 \cdot 6}x^3 + \cdots \tag{I.10}$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \cdots \tag{I.11}$$

I.3 Gauss' Trick

Because Gauss' trick is useful in quantum physics, we show a simple method of deriving an expression for the sum of the first M integers. We write this sum in two different ways. First,

$$\sum_{n=0}^{M} n = 1 + 2 + 3 + \cdots + (M-2) + (M-1) + M \tag{I.12}$$

and, second, by simply reversing the order of summation:

$$\sum_{n=0}^{M} n = M + (M-1) + (M-2) + \cdots + 3 + 2 + 1 \tag{I.13}$$

Now add these sums term by term and obtain

$$2\sum_{n=0}^{M} n = (M+1) + (M-1+2) + (M-2+3) + \cdots + (M+1) \tag{I.14}$$

Examination of Equation I.14 reveals that the right-hand side comprises $(M+1)$ added to itself M times, the sum of which is $M\,(M+1)$. Solving for $\sum_{n=0}^{M} n$ we obtain Gauss' trick, Equation 8.157:

$$\sum_{n=0}^{M} n = \frac{M\,(M+1)}{2} \tag{I.15}$$

Appendix J
Fourier Integrals

A function $f(x)$ that is periodic on an interval $-L \leq x \leq L$ can be expanded in a Fourier series

$$f(x) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=0}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) \tag{J.1}$$

where a_n and b_n are constants, but functions of n. This representation of $f(x)$ can be put in more concise form by substituting the Euler forms of the sine and cosine:

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}; \quad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} \tag{J.2}$$

Combining the constants and renaming them B_n and C_n we have

$$f(x) = \sum_{n=0}^{\infty} B_n e^{in\pi x/L} + \sum_{n=0}^{\infty} C_n e^{-in\pi x/L} \tag{J.3}$$

By extending the summation to $n = -\infty$ we can write Equation J.3 as a single term

$$f(x) = \sum_{n=-\infty}^{\infty} D_n e^{in\pi x/L} \tag{J.4}$$

where D_n is the new constant.

The functions $e^{in\pi x/L}$ and $e^{im\pi x/L}$ are orthogonal, but not orthonormal, as may be seen from the integral

$$\int_{-L}^{L} e^{i(n-m)\pi x/L} dx = 2L\delta_{nm} \tag{J.5}$$

where δ_{nm} is the Kronecker delta. By taking advantage of the orthogonality relation we may determine the constant A_n if $f(x)$ is known. Multiplying both sides of Equation J.4 by $e^{-im\pi x/L}$ and integrating over the interval we have

$$D_m = \frac{1}{2L} \int_{-L}^{L} f(x)\, e^{-im\pi x/L} dx \tag{J.6}$$

Our goal is to extend the infinite series that represents a periodic function to an expression that represents a nonperiodic function. In preparation for this extension we make the substitution

$$k_n = \frac{n\pi}{L} \tag{J.7}$$

from which we see that

$$\Delta k = \frac{\pi}{L} \Delta n \tag{J.8}$$

If Δn is the difference between successive integers, then it is unity so it may be inserted in Equation J.4 without affecting the equation:

$$\begin{aligned} f(x) &= \sum_{n=-\infty}^{\infty} D_n e^{in\pi x/L} \Delta n \\ &= \sum_{n=-\infty}^{\infty} D_n e^{in\pi x/L} \left(\frac{L}{\pi}\right) \Delta k \end{aligned} \tag{J.9}$$

The conversion from periodic to nonperiodic function can be effected by letting $L \to \infty$, in which case k becomes a continuous variable, $\Delta k \to dk$, $D_n \to D(k)$, and $\sum_{n=-\infty}^{\infty} \to \int_{-\infty}^{\infty}$. Also, to conform with the common notation of quantum physics we make the change $f(x) \to \psi(x)$. With these substitutions Equation J.9 becomes

$$\psi(x) = \frac{L}{\pi} \int_{-\infty}^{\infty} D(k)\, e^{ikx} dk \tag{J.10}$$

Now we rescale $D(k)$ according to

$$D(k) = \sqrt{\frac{\pi}{2}} \frac{A(k)}{L} \tag{J.11}$$

and obtain

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k)\, e^{ikx} dk \tag{J.12}$$

Now let us return to Equation J.6 for the Fourier coefficients using Equation J.11:

$$A_n = \frac{1}{\sqrt{2\pi}} \int_{-L}^{L} \psi(x)\, e^{-ik_n x/L} dx \tag{J.13}$$

Again letting $L \to \infty$ we have

$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-ikx/L} dx \tag{J.14}$$

In summary, the equations

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) e^{ikx} dk \tag{J.15}$$

$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-ikx/L} dx \tag{J.16}$$

are the Fourier transforms. That is, $\psi(x)$ is the Fourier transform of $A(k)$, and $A(k)$ is the Fourier transform of $\psi(x)$. In quantum mechanics the variable x is space and k is the wave number. Thus, the wave function in coordinate space is $\psi(x)$ and the wave function in k-space is $A(k)$. Often it is desirable to have the wave function in terms of the momentum $p = \hbar k$. That is,

$$\begin{aligned} \phi(p) &\equiv \frac{1}{\sqrt{\hbar}} A(k) \\ &= \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x) e^{-ipx/\hbar} dx \end{aligned} \tag{J.17}$$

Appendix K
Commutator Identities

K.1 General Identities

$$\left[\hat{A}, \hat{B}\right] \equiv \hat{A}\hat{B} - \hat{B}\hat{A} \tag{K.1}$$

$$\left[\hat{A}, \hat{A}\right] = 0 \tag{K.2}$$

$$\left[\hat{A}, \hat{B}\hat{C}\right] \equiv \left[\hat{A}, \hat{B}\right]\hat{C} + \hat{B}\left[\hat{A}, \hat{C}\right] \tag{K.3}$$

$$\left[\hat{A}\hat{B}, \hat{C}\right] \equiv \left[\hat{A}, \hat{C}\right]\hat{B} + \hat{A}\left[\hat{B}, \hat{C}\right] \tag{K.4}$$

$$\begin{aligned}\left[\hat{A}\hat{B}, \hat{C}\hat{D}\right] \equiv{}& \left[\hat{A}, \hat{C}\right]\hat{B}\hat{D} + \hat{A}\left[\hat{B}, \hat{C}\right]\hat{D} \\ &+ \hat{C}\left[\hat{A}, \hat{D}\right]\hat{B} + \hat{C}\hat{A}\left[\hat{B}, \hat{D}\right]\end{aligned} \tag{K.5}$$

K.2 Quantum Mechanical Identities

$$[x, \hat{p}] = i\hbar \tag{K.6}$$

$$\left[x, \hat{p}_x^n\right] = i\hbar n\hat{p}_x^{n-1} \tag{K.7}$$

$$\left[\hat{J}_i, \hat{J}_j\right] = i\hbar\hat{J}_k\epsilon_{ijk} \tag{K.8}$$

Appendix L
Miscellaneous Operator Relations

L.1 Baker–Campbell–Hausdorff (BCH) Formula

Depending upon the reference source consulted, there are two formulas that are referred to in the literature as the Baker–Campbell–Hausdorff formula, theorem, or lemma. Both are used in this book, so we will discuss them in this appendix. The proof of the first is somewhat involved and is of limited pedagogical value in a course on quantum physics, so it will not be presented. The proof of the second is considerably simpler and offers a good exercise in exponentiated operators, so we give this proof below.

First, let us state the one that we will not prove. Given that $\hat{A}$, $\hat{B}$, and $\hat{C}$ are operators, then the product of two exponentiated operators is

$$e^{\hat{A}}e^{\hat{B}} = e^{\hat{C}} \tag{L.1}$$

where

$$\hat{C} = \hat{A} + \hat{B} + \frac{1}{2}\left[\hat{A}, \hat{B}\right] + \frac{1}{12}\left\{\left[\left[\hat{A}, \hat{B}\right], \hat{B}\right] + \left[\hat{A}, \left[\hat{A}, \hat{B}\right]\right]\right\} + \cdots \tag{L.2}$$

where we have truncated the series after the terms that are necessary for the work described in this book.

For the second formula we assume two operators, $\hat{A}$ and $\hat{B}$, that do not commute and write

$$e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{A} + \hat{B} + \left[\hat{A}, \hat{B}\right] + \frac{1}{2!}\left[\hat{A}, \left[\hat{A}, \hat{B}\right]\right] + \frac{1}{3!}\left[\hat{A}, \left[\hat{A}, \left[\hat{A}, \hat{B}\right]\right]\right] + \cdots \tag{L.3}$$

Proof:

Define the product on the left-hand side in terms of an arbitrary parameter λ.

$$f(\lambda) = e^{\lambda\hat{A}}\hat{B}e^{-\lambda\hat{A}} \tag{L.4}$$

Now we strive to write a Taylor expansion of $f(\lambda)$ about $\lambda = 0$. To do so will require the derivatives of $f(\lambda)$, the first few of which we tabulate bearing in mind that $\left[\hat{A}, e^{\pm\lambda\hat{A}}\right] = 0$:

$$\begin{aligned}
\frac{df(\lambda)}{d\lambda} &= e^{\lambda\hat{A}}\left[\hat{A}, \hat{B}\right]e^{-\lambda\hat{A}} \\
\frac{d^2 f(\lambda)}{d\lambda^2} &= e^{\lambda\hat{A}}\left[\hat{A}, \left[\hat{A}, \hat{B}\right]\right]e^{-\lambda\hat{A}} \\
&\vdots \\
\frac{d^n f(\lambda)}{d\lambda^n} &= e^{\lambda\hat{A}}\left[\hat{A}, \cdots\left[\hat{A}, \left[\hat{A}, \hat{B}\right]\right]\cdots\right]e^{-\lambda\hat{A}}
\end{aligned} \tag{L.5}$$

Using these derivatives to write the Taylor expansion we have

$$e^{\lambda\hat{A}}\hat{B}e^{-\lambda\hat{A}} = \hat{A} + \hat{B} + \left[\hat{A}, \hat{B}\right]\frac{\lambda}{1!} + \left[\hat{A}, \left[\hat{A}, \hat{B}\right]\right]\frac{\lambda^2}{2!} + \left[\hat{A}, \left[\hat{A}, \left[\hat{A}, \hat{B}\right]\right]\right]\frac{\lambda^3}{3!} + \cdots \tag{L.6}$$

Because λ is arbitrary, we may set it equal to unity and obtain Equation L.3.

L.2 Translation Operator

We will derive the form of the translation operator, also known as the displacement operator, or the propagator. This latter term is often used for a Green's function, so it is advisable to use one of the first two appellations. We begin by writing the Taylor expansion for a function $f(x)$ about the point $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \left.\frac{d^n f(x)}{dx^n}\right|_{x=a} \frac{(x-a)^n}{n!} \tag{L.7}$$

Then, letting $x \to x + x_0$, and continuing to expand about $x = a$ we have

$$\begin{aligned}
f(x + x_0) &= \sum_{n=0}^{\infty} \left.\frac{d^n f(x + x_0)}{d(x + x_0)^n}\right|_{(x+x_0)=a} \frac{[(x + x_0) - a]^n}{n!} \\
&= \sum_{n=0}^{\infty} \left.\frac{d^n f(x)}{dx^n}\right|_{x=a} \frac{[(x + x_0) - a]^n}{n!}
\end{aligned} \tag{L.8}$$

because

$$\left.\frac{d^n f(x + x_0)}{d(x + x_0)^n}\right|_{(x+x_0)=a} = \left.\frac{d^n f(x)}{dx^n}\right|_{x=a} \tag{L.9}$$

Now let $a \to x$ in Equation L.8 and obtain

$$\begin{aligned} f(x+x_0) &= \sum_{n=0}^{\infty}\left[\frac{x_0^n}{n!}\frac{d^n}{dx^n}\right]f(x) \\ &= \left[1+\frac{x_0^1}{1!}\frac{d}{dx}+\frac{x_0^2}{2!}\frac{d^2}{dx^2}+\frac{x_0^3}{3!}\frac{d^3}{dx^3}+\cdots\right]f(x) \qquad \text{(L.10)} \\ &= \exp\left[x_0\frac{d}{dx}\right]f(x) \qquad \text{(L.11)} \end{aligned}$$

which shows that application of the operator, $\exp\left[x_0\frac{d}{dx}\right]$, to an arbitrary function $f(x)$, effects a translation $x \to x+x_0$.

We can cast the translation operator in terms of the momentum operator using Equation 2.28 to make the identification

$$\frac{d}{dx} \to \frac{i\hat{p}}{\hbar} \qquad \text{(L.12)}$$

we see that

$$\begin{aligned} f(x+x_0) &= \left\{\sum_{n=0}^{\infty}\frac{1}{n!}\left(x_0\frac{i\hat{p}}{\hbar}\right)^n\right\}f(x) \\ &= e^{ipx_0/\hbar}f(x) \qquad \text{(L.13)} \end{aligned}$$

We can also derive the action of the translation operator by beginning with the Fourier transforms, Equations 4.33 and 4.31. This will provide the action of the translation operator on $\psi(x)$ and $\phi(x)$. Multiplying Equations 4.33 by $e^{ipx_0/\hbar}$ we have

$$\begin{aligned} e^{ipx_0/\hbar}\psi(x) &= \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\phi(p)\,e^{ip(x+x_0)/\hbar}dp \\ &= \psi(x+x_0) \qquad \text{(L.14)} \end{aligned}$$

Multiplying Equations 4.31 by $e^{ip_0x/\hbar}$ we find that

$$\begin{aligned} e^{ip_0x/\hbar}\phi(p) &= \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\psi(x)\,e^{-i(p+p_0)x/\hbar}dx \\ &= \phi(p+p_0) \qquad \text{(L.15)} \end{aligned}$$

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