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# List of Symbols and Abbreviations

|                     | Definition/Description  | Page  |
|---------------------|---|-------|
| $x \in D$           | $x$ is an element of $D$  | 1     |
| $\mathbb{N}$        | the set of all positive integers                                    | 1     |
| $\mathbb{Z}$        | the set of all integers   | 1     |
| $\mathbb{Q}$        | the set of all rational numbers                                     | 1     |
| $\mathbb{R}$        | the set of all real numbers   | 2     |
| $\sum$              | sum   | 3     |
| $\prod$             | product   | 3     |
| $A := B$            | $A$ is defined to be equal to $B$                                   | 3     |
| $\mathbb{R}^+$      | the set of all positive real numbers                                | 4     |
| $\emptyset$         | the empty set   | 4     |
| $\sup S$            | the supremum of a subset $S$ of $\mathbb{R}$                        | 5     |
| $\inf S$            | the infimum of a subset $S$ of $\mathbb{R}$                         | 5     |
| $\max S$            | the maximum of a subset $S$ of $\mathbb{R}$                         | 5     |
| $\min S$            | the minimum of a subset $S$ of $\mathbb{R}$                         | 5     |
| $[x]$               | the integer part of a real number $x$                               | 6     |
| $\lfloor x \rfloor$ | the integer part or the floor of a real number $x$                  | 6     |
| $\lceil x \rceil$   | the ceiling of a real number $x$                                    | 6     |
| $\sqrt[n]{a}$       | the $n$ th root of a nonnegative real number $a$                    | 7     |
| $\sqrt{a}$          | the square root of a nonnegative real number $a$                    | 7     |
| $m \mid n$          | $m$ divides $n$   | 8, 18 |
| $m \nmid n$         | $m$ does not divide $n$   | 8, 18 |
| $\pm$               | plus or minus   | 8     |
| $C \subseteq D$     | $C$ is a subset of $D$  | 9     |
| $\implies$          | implies   | 9     |
| $D \setminus C$     | the complement of $C$ in $D$ , namely, $\{x \in D : x \notin C\}$   | 9     |
| $(a, b)$            | the open interval $\{x \in \mathbb{R} : a < x < b\}$                | 9     |
| $[a, b]$            | the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$        | 9     |
| $[a, b)$            | the semiopen interval $\{x \in \mathbb{R} : a \leq x < b\}$         | 9     |
| $(a, b]$            | the semiopen interval $\{x \in \mathbb{R} : a < x \leq b\}$         | 9     |
| $\infty$            | the symbol $\infty$ or the fictional right endpoint of $\mathbb{R}$ | 9     |

| Definition/Description                      | Page  |     |
|---|---|-----|
| $-\infty$                                   | the symbol $-\infty$ or the fictional left endpoint of $\mathbb{R}$ | 9   |
| $(a, \infty)$                               | the semi-infinite interval $\{x \in \mathbb{R} : x > a\}$           | 9   |
| $[a, \infty)$                               | the semi-infinite interval $\{x \in \mathbb{R} : x \geq a\}$        | 9   |
| $(-\infty, a)$                              | the semi-infinite interval $\{x \in \mathbb{R} : x < a\}$           | 9   |
| $(-\infty, a]$                              | the semi-infinite interval $\{x \in \mathbb{R} : x \leq a\}$        | 9   |
| $x \notin D$                                | $x$ is not an element of $D$  | 10  |
| $ a $                                       | the absolute value of a real number $a$                             | 10  |
| A.M.  | Arithmetic Mean   | 12  |
| G.M.  | Geometric Mean  | 12  |
| $D \times E$                                | the set $\{(x, y) : x \in D \text{ and } y \in E\}$                 | 14  |
| $\text{id}_D$                               | the identity function on the set $D$                                | 15  |
| $f _C$                                      | the restriction of $f : D \rightarrow E$ to a subset $C$ of $D$     | 16  |
| $g \circ f$                                 | the composite of $g$ with $f$                                       | 15  |
| $f^{-1}$                                    | the inverse of an injective function $f$                            | 16  |
| $\mathbb{R}[x]$                             | the set of all polynomials in $x$ with coefficients in $\mathbb{R}$ | 17  |
| $\deg p(x)$                                 | the degree of a nonzero polynomial $p(x)$                           | 17  |
| $\iff$                                      | if and only if  | 24  |
| IVP   | Intermediate Value Property   | 28  |
| $k!$  | the product of the first $k$ positive integers                      | 31  |
| H.M.  | Harmonic Mean   | 33  |
| GCD   | Greatest Common Divisor   | 37  |
| LCM   | Least Common Multiple   | 37  |
| $x + iy$                                    | the complex number $(x, y)$   | 38  |
| $\mathbb{C}$                                | the set of all complex numbers                                      | 38  |
| $\mathbb{C}[x]$                             | the set of all polynomials in $x$ with coefficients in $\mathbb{C}$ | 41  |
| $(a_n)$                                     | the sequence whose $n$ th term is $a_n$                             | 43  |
| $a_n \rightarrow a$                         | the sequence $(a_n)$ tends to a real number $a$                     | 44  |
| $\lim_{n \rightarrow \infty} a_n$           | the limit of the sequence $(a_n)$                                   | 45  |
| $a_n = O(b_n)$                              | $(a_n)$ is big-oh of $(b_n)$  | 53  |
| $a_n = o(b_n)$                              | $(a_n)$ is little-oh of $(b_n)$                                     | 53  |
| $a_n \sim b_n$                              | $(a_n)$ is asymptotically equivalent to $(b_n)$                     | 54  |
| $a_n \rightarrow \infty$                    | the sequence $(a_n)$ tends to $\infty$                              | 54  |
| $a_n \rightarrow -\infty$                   | the sequence $(a_n)$ tends to $-\infty$                             | 54  |
| $\nrightarrow$                              | does not tend to  | 55  |
| $\limsup_{n \rightarrow \infty} a_n$        | the limit superior of $(a_n)$                                       | 65  |
| $\liminf_{n \rightarrow \infty} a_n$        | the limit inferior of $(a_n)$                                       | 65  |
| $\lim_{x \rightarrow c} f(x)$               | the limit of the function $f$ as $x$ tends to $c$                   | 82  |
| $\lim_{x \rightarrow c^-} f(x)$             | the left (hand) limit of $f$ as $x$ tends to $c$                    | 88  |
| $\lim_{x \rightarrow c^+} f(x)$             | the right (hand) limit of $f$ as $x$ tends to $c$                   | 88  |
| $f(x) = O(g(x))$                            | $f(x)$ is big-oh of $g(x)$ as $x \rightarrow \infty$                | 90  |
| $f(x) = o(g(x))$                            | $f(x)$ is little-oh of $g(x)$ as $x \rightarrow \infty$             | 90  |
| $f(x) \sim g(x)$                            | $f(x)$ is asymptotically equivalent to $g(x)$                       | 90  |
| $f'(c), \left. \frac{df}{dx} \right _{x=c}$ | the derivative of $f$ at the point $c$                              | 104 |
| $f', \frac{df}{dx}$                         | the derivative (function) of $f$                                    | 105 |

|   |   |          |
|---|---|----------|
| $f''(c), \left. \frac{d^2 f}{dx^2} \right _{x=c}$     | the second derivative of $f$ at $c$                       | 112      |
| $f'''(c), \left. \frac{d^3 f}{dx^3} \right _{x=c}$    | the third derivative of $f$ at $c$                        | 112      |
| $f^{(n)}(c), \left. \frac{d^n f}{dx^n} \right _{x=c}$ | the $n$ th derivative of $f$ at $c$                       | 112      |
| $f'_-(c)$   | the left (hand) derivative of $f$ at the point $c$        | 113      |
| $f'_+(c)$   | the right (hand) derivative of $f$ at the point $c$       | 113      |
| MVT   | Mean Value Theorem  | 120      |
| $\approx$   | approximately equal                                       | 124      |
| L'HR  | L'Hôpital's Rule  | 133, 134 |
| $P_n$   | the partition of $[a, b]$ into $n$ equal parts            | 180      |
| $m(f)$  | the infimum of $\{f(x) : x \in [a, b]\}$                  | 180      |
| $M(f)$  | the supremum of $\{f(x) : x \in [a, b]\}$                 | 180      |
| $m_i(f)$  | the infimum of $\{f(x) : x \in [x_{i-1}, x_i]\}$          | 180      |
| $M_i(f)$  | the supremum of $\{f(x) : x \in [x_{i-1}, x_i]\}$         | 180      |
| $L(P, f)$   | the lower sum for $f$ with respect to $P$                 | 181      |
| $U(P, f)$   | the upper sum for $f$ with respect to $P$                 | 181      |
| $L(f)$  | the lower Riemann integral of $f$                         | 181      |
| $U(f)$  | the upper Riemann integral of $f$                         | 181      |
| $\int_a^b f(x) dx$                                    | the (Riemann) integral of $f$ on $[a, b]$                 | 182      |
| $f^+$   | the positive part of $f$                                  | 200      |
| $f^-$   | the negative part of $f$                                  | 200      |
| FTC   | Fundamental Theorem of Calculus                           | 202      |
| $\int f(x) dx$  | an indefinite integral of $f$                             | 204      |
| $[F(x)]_a^b, F(x) _a^b$                               | the difference $F(b) - F(a)$                              | 204      |
| $S(P, f)$   | a Riemann sum for $f$ corresponding to $P$                | 211      |
| $\mu(P)$  | the mesh of a partition $P$                               | 213      |
| $\ln$   | the (natural) logarithmic function                        | 228      |
| $e$   | the unique real number such that $\ln e = 1$              | 229      |
| $\exp$  | the exponential function                                  | 230      |
| $\arctan$   | the arctangent function                                   | 241      |
| $\pi$   | the real number $2 \sup\{\arctan x : x \in (0, \infty)\}$ | 241      |
| $\tan$  | the tangent function                                      | 244      |
| $\sin$  | the sine function   | 245, 246 |
| $\cos$  | the cosine function                                       | 245, 246 |
| $\csc$  | the cosecant function                                     | 250      |
| $\sec$  | the secant function                                       | 250      |
| $\cot$  | the cotangent function                                    | 250      |
| $\sin^{-1}$   | the inverse sine function                                 | 251      |
| $\cos^{-1}$   | the inverse cosine function                               | 251      |
| $\cot^{-1}$   | the inverse cotangent function                            | 252      |
| $\csc^{-1}$   | the inverse cosecant function                             | 252      |
| $\sec^{-1}$   | the inverse secant function                               | 253      |

| Definition/Description   | Page     |
|--|----------|
| $\angle(OP_1, OP_2)$ the angle between $OP_1$ and $OP_2$                             | 264      |
| $L_1 \parallel L_2$ the lines $L_1$ and $L_2$ are parallel                           | 266      |
| $L_1 \not\parallel L_2$ the lines $L_1$ and $L_2$ are not parallel                   | 266      |
| $\angle(L_1, L_2)$ the (acute) angle between $L_1$ and $L_2$                         | 266      |
| $L_1 \perp L_2$ the lines $L_1$ and $L_2$ are perpendicular                          | 267      |
| $L_1 \not\perp L_2$ the lines $L_1$ and $L_2$ are not perpendicular                  | 267      |
| $\angle(C_1, C_2; P)$ the angle at $P$ between $C_1$ and $C_2$                       | 268      |
| Area ( $R$ ) the area of a region $R$  | 292      |
| Vol ( $D$ ) the volume of a solid body $D$   | 299, 303 |
| $\ell(C)$ the length of a curve $C$  | 311      |
| Area ( $S$ ) the area of a surface $S$   | 321      |
| Av( $f$ ) the average of a function $f$  | 325      |
| Av( $f; w$ ) the weighted average of $f$ with respect to $w$                         | 325      |
| $(\bar{x}, \bar{y})$ the centroid of a curve or a planar region                      | 326, 329 |
| $(\bar{x}, \bar{y}, \bar{z})$ the centroid of a surface or a solid body              | 327, 330 |
| $Q(f)$ a Quadrature Rule for $f$   | 336      |
| $R(f)$ Rectangular Rule for $f$  | 337      |
| $M(f)$ Midpoint Rule for $f$   | 337      |
| $T(f)$ Trapezoidal Rule for $f$  | 337      |
| $S(f)$ Simpson's Rule for $f$  | 337      |
| $R_n(f)$ Compound Rectangular Rule for $f$   | 338      |
| $M_n(f)$ Compound Midpoint Rule for $f$  | 339      |
| $T_n(f)$ Compound Trapezoidal Rule for $f$   | 339      |
| $S_n(f)$ Compound Simpson's Rule for $f$   | 339      |
| $\sum_{k=1}^{\infty} a_k$ the series whose sequence of terms is $(a_k)$              | 362      |
| $\int_a^{\infty} f(t)dt$ the improper integral of $f$ on $[a, \infty)$               | 384      |
| $\int_{-\infty}^b f(t)dt$ the improper integral of $f$ on $(-\infty, b]$             | 398      |
| $\int_{-\infty}^{\infty} f(t)dt$ the improper integral of $f$ on $(-\infty, \infty)$ | 398      |
| $\int_{a^+}^b f(t)dt$ the improper integral of $f$ on $(a, b]$                       | 399      |
| $\int_a^{b^-} f(t)dt$ the improper integral of $f$ on $[a, b)$                       | 400      |
| $\int_{a^+}^{b^-} f(t)dt$ the improper integral of $f$ on $(a, b)$                   | 400      |
| $\beta(p, q)$ the beta function for $p > 0$ and $q > 0$                              | 405      |
| $\gamma(s)$ the gamma function for $s > 0$   | 407      |

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