Comments

Chapter 1.

In this chapter we discuss three notions of optimality for infinite horizon problems. The notion of (f)-minimal solutions was introduced by Aubry and Le Daeron [6] in their study of the discrete Frenkel–Kontorova model related to dislocations in one-dimensional crystals. In [6] Aubry and Le Daeron established the existence of (f)-minimal solutions and obtained a full description of their structure. A minimal solution was called in [6] a minimal energy configuration. The theory developed in [6] is of great interest from the point of view of infinite horizon optimal control as well as from the point of view of the theory of dynamical systems. Leizarowitz and Mizel [44] used the notion of (f)-minimal solutions in their study of a class of variational problems arising in continuum mechanics. They established the existence of periodic (f)-minimal solutions under a certain technical assumption which was removed in [90].

The notions of overtaking optimal solutions and good solutions were introduced in the economics literature [4, 33, 81]. Usually the existence of overtaking optimal solutions is a difficult problem and we solve it for nonconvex variational problems only in a generic setting. But good solutions exist for all infinite horizon variational problems considered in the book. Note that for practical needs it is enough to obtain approximate solutions which are good functions. The results which we obtained for good solutions are an important tool in our existence theory of overtaking optimal solutions. In order to establish the existence of overtaking optimal solutions we usualy verify that all good functions have the same asymptotic behavior and then show that a good minimal function is overtaking optimal. Note that many results on infinite horizon optimal control problems are collected in [16, 26, 48, 60, 61, 67].

Chapter 2.

This chapter contains the turnpike results for nonautonomous nonconvex variational problems, the strongest and the most general results of this book. These results allow us to consider the turnpike property as a general phenomenon which holds for large classes of problems. We consider the complete metric space of integrands $\mathcal{M}$ and show that the turnpike property holds for most integrands of $\mathcal{M}$ in the sense of the Baire category. Such an approach is common in many areas of Mathematical Analysis [19, 21-23, 25, 35, 70, 106, 107].

The example of an integrand belonging to the space $\mathcal{M}$ which does not have the turnpike property given in Section 2.6 shows that the main results of the chapter cannot be improved. In [109] we obtained results which may help us to verify if a given integrand has the turnpike property.

The turnpike property is very important for applications. Suppose that our integrand has the turnpike property and we know a finite number of "approximate" solutions of the variational problems with this integrand. Then we know the turnpike X, or at least its approximation, and the constant τ which is an estimate for the time period required to reach the turnpike. This information can be useful if we need to find an "approximate" solution of a new variational problem with a new time interval $[T_1, T_2]$ and the new values y, z at the end points T_1 and T_2. Namely, instead of solving this new problem on the "large" interval $[T_1, T_2]$ we can find an "approximate" solution of the variational problem on the "small" interval $[T_1, T_1 + \tau]$ with the values $y, X(T_1 + \tau)$ at the end points and an "approximate" solution of the variational problem on the "small" interval $[T_2 - \tau, T_2]$ with the values $X(T_2 - \tau), z$ at the end points. Then the concatenation of the first solution, the function $X(t),\ t \in [T_1 + \tau, T_2 - \tau]$ and the second solution is an "approximate" solution of the variational problem on the interval $[T_1, T_2]$ with the values y, z at the end points. Numerical applications of the turnpike theory are discussed in [62, 63, 67].

Chapter 3.

The notion of a weakly optimal function was introduced in the economic literature by Brock [12]. The results of this chapter are obtained for the subspace $\mathcal{A}$ of the space of integrands $\mathcal{M}$ considered in Chapters 1 and 2. This subspace $\mathcal{A}$ consists of all time-independent integrands belonging to $\mathcal{M}$. Since the subspace $\mathcal{A}$ is a small set in $\mathcal{M}$ and the results of Chapter 2 are of generic nature, they cannot be applied for the subspace $\mathcal{A}$. The results which we obtain in this chapter are weaker than their analogs in Chapter 2. In Chapter 2 we establish a generic existence of an overtaking optimal function, while in this chapter we obtain a generic existence of a weakly optimal function. Also the convergence in the turnpike results for the autonomous case are weaker than the convergence in their analogs for the nonautonomous case. This fact is natural since in a large space we have more possibilities for perturbations. In Chapter 2 for a given integrand we use a perturbation which depends on t and obtain a new integrand which has the strong turnpike property. In Chapter 3 for a given integrand we may only use a perturbation which does not depend on t. As a result, for the space $\mathcal{A}$ we obtain a weaker turnpike property than for the space $\mathcal{M}$. Some nongeneric turnpike results for the space $\mathcal{A}$ were obtained in [110].

In this chapter we show that the turnpike property holds for approximate solutions on finite intervals if the integrand has the so-called asymptotic turnpike property, which means that all good functions on the infinite interval $[0, \infty)$ have the same asymptotic behavior. An analogous result for one-dimensional second order variational problems arising in continuum mechanics was obtained in [55].

Chapter 4.

Chapter 4 is a continuation of Chapter 3. It contains a detailed analysis of the structure of optimal solutions on infinite horizon problems for an integrand which has the asymptotic turnpike property. In Chapter 3 we associate with any integrand the so-called long run average cost growth rate and a certain continuous function. In this chapter we show that this long run average cost growth rate and the continuous function depend on the integrand continuously if the integrand has the asymptotic turnpike property. This result may be useful if we try to apply some numerical procedures in order to calculate weakly optimal solutions. It implies stability of such procedures.

In this chapter we improve some turnpike results for optimal solutions of infinite horizon problems. For example, we show that all optimal solutions with initial points belonging to a given bounded set converge to the turnpike uniformly.

Chapter 5.

In Chapter 5 our goal is to improve the turnpike results obtained in Chapter 2 for approximate solutions of autonomous variational problems on finite intervals. In order to obtain this improvement we need to suppose additional assumptions on integrands. Namely, we assume that the integrands are smooth and their partial derivatives satisfy certain growth conditions. These assumptions imply, in particular, that minimizers of variational problems are solutions of the corresponding Euler–Lagrange equations. We establish the turnpike property for integrands which have the asymptotic turnpike property.

Chapter 6.

In this chapter we study the turnpike properties of a class of linear control problems arising in engineering. This class includes an infinite horizon problem of tracking of the periodic trajectory studied in [3]. An integrand is assumed to be periodic with respect to the time variable and strictly convex as the function of the state variable and the control variable. This assumption implies that the turnpike is a periodic trajectory. The strict convexity assumption implies the uniqueness of overtaking optimal solutions. As in the previous chapters we associate with our linear control problem a related discrete time optimal control problem which is in this case autonomous and strictly convex.

Chapter 7.

This chapter is devoted to the study of the turnpike properties for a class of linear control problems arising in engineering. This class includes the class of linear control problems discussed in Chapter 6 and, in particular, the infinite horizon problem of tracking of the periodic trajectory studied in [3]. An integrand is assumed to be strictly convex as the function of the state variable and the control variable, but we do not assume that it is periodic with respect to the time variable. The strict convexity assumption implies the uniqueness of overtaking optimal solutions which are not periodic in general.

Chapter 8.

Discrete-time control systems considered in this chapter appear in many areas of applied mathematics: in mathematical economics [47, 48, 56-61], continuum mechanics [20, 44] and in the theory of dynamical systems [6]. As we have already seen in the previous chapters these systems are also useful tools in the study of continuous-time optimal control problems. With any continuous-time control problem we associate a related discrete-time control problem. It turns out that there is a simple correspondence between solutions of the continuous-time problem and solutions of the related discrete-time problem. In Chapter 8 we study an autonomous problem with convex cost function on a bounded closed convex subset of a Banach space and a nonautonomous nonconvex problem on a complete metric space. Some results on infinity horizon autonomous nonconvex problems on a complete metric space were established in [108].

Chapter 9.

The primary area of appications of infinite-dimensional optimal continuous-time control problems concerns models of regional economic growth discussed in [36], cattle ranching models proposed in [24], and systems with distributed parameters and boundary controls related to engineering [8, 31] and to water resources problems [62, 63]. In this chapter we obtain the convergence to the turnpike in the weak topology. In order to establish the convergence to the turnpike in the strong topology we need to assume that the integrand is strictly convex.

Chapter 10.

This chapter is devoted to applications of the turnpike theory to mathematical economics. We consider a large class of nonlinear Leontjev type models of multisector economics. It should be mentioned that we obtain a sufficient condition for the turnpike property which can be verified if we know a generalized equilibrium state of the model. A version of this model which takes into account lag was considered in [87].

Chapter 11.

The general Neumann–Gale model has been studied in many publications. Many results on this model are collected in [48, 75, 77]. A stochastic version of the Neumann–Gale model was studied in [2].

Chapter 12.

This chapter is devoted to applications of the turnpike theory to game theory. It is based on the paper [102]. For other applications see [15, 17]. In [17] overtaking equilibria was studied for switching regulator and tracking games. Turnpike properties for infinite horizon open-loop competitive processes were discussed in [15]. It is not clear if it is possible to obtain turnpike results in game theory without convexity assumptions.

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