Hints and Solutions to Certain Exercises

Warning: This section may be bad for your academic health. In order to get any benefit from doing the exercises, do not look at the hints until you have given the problem an honest try and are stuck. More importantly, never look at the partial solutions below until you have finished that exercise yourself. Then use the answer only to check that you have not made a mistake. "Working backwards" from a solution will never teach you crucial problem-solving skills.

Chapter 1

1. **b.** $\bar{x} = 81.3008$.

2. **b.** $\hat{\mu}_n = \bar{x}_n = 0.01617$.
c. $\hat{b}_p = 0.00472$.

4. **Hint**: Average both sides of the equation $\mu_i = \mu + b_i$ over all observations. Using the hint, you obtain $\mu = \frac{1}{n}\sum_{i=1}^{k} n_i b_i$.

5. **Hint**: Solve for the last observation at each level; then $x_{in_i} = n_i\hat{\mu}_i - \sum_{j=1}^{n_i-1} x_{ij}$, so the last observation is determined.

6. **b.** $\hat{\mu} = 7.33$, $\hat{b}_d = 1.33$, $\hat{c}_{ex} = 1.83$.
c. $\hat{d}_{ex,d} = 0.25$.

7. $\frac{1}{n}\sum_{i=1}^{l}\sum_{j=1}^{m} n_{ij}\hat{b}_i = \frac{1}{n}\sum_{i=1}^{l} n_{i\bullet}(\bar{x}_{i\bullet} - \bar{x}) = \bar{x} - \bar{x} = 0$.

8. $\frac{1}{n_{i\bullet}}\sum_{j=1}^{m} n_{ij}\hat{d}_{ij} = \frac{1}{mr}\sum_{j=1}^{m} r(x_{ij} - \bar{x}_{i\bullet} - \bar{x}_{\bullet j} + \bar{x}) = \bar{x}_{i\bullet} - \bar{x}_{i\bullet} - \bar{x} + \bar{x} = 0$.

9. **b.** \$173.22.

10. **b.** 60.85 crimes per 1000 people.

11. expect 15.89 complaints in the first group.

13. **Hint**: When you sum expected counts over columns j, you should find that $x_{i\bullet}$ factors out.

14. The model is $\log(x_i) = \mu + b_i$; then, e.g. $3\mu = \log(43) + \log(35) + \log(22)$ because the b's sum to zero.
15. a. Imitate the argument in (7.3) using summation notation.
16. Do Exercise 15 first.
18. Do Exercise 17b, then interpolate.

Chapter 2

1. MSE = 1.56.
2. c. 0.14% bad peanuts.
3. The smaller side is 92% of the larger.
5. a. Sample variance = 0.00000069.
6. Hint: What does the principle of least squares say about a sample variance?
7. Hint: Write out the defining formula, then look at Exercise 4.
9. Hint: Express the orthogonality condition in summation notation, and try to simplify.
10. $F = 15.76$.
11. $F = 5.37$.
12. $K = 8.27$.
14. b. RMSE = 0.0394%.
17. b. F(inter) = 0.19; F(exer) = 3.94.

Chapter 3

2. Hint: P(different) = 1 − P(same).
3. Your list should have 70 items on it.
6. Hint: $50 \times 49 \times \cdots \times 39$.
7. P(5 spades) = 0.000495.
9. Hint: Let one marble in the urn be special. Now count separately the sets of k that include it and those that do not.
11. Hint: At each intersection on the way, I must choose either west or south, until I have taken each direction a certain number of times.
12. Hint: Section 3.4 has two suggestions.
14. b. Hint: Consider the three possible countries separately.
15. P(7|10) = 0.117.
16. P = 0.06.
19. P = 0.000558.
20. P(9 cameras) = 0.06.
21. P(7 cameras) = 0.317.
22. Hint: Noticing that $\binom{i}{1} = i$, use the result of Exercise 9 repeatedly, as in Pascal's triangle.
23. Hint: Use Exercise 9 repeatedly.

24. **Hint**: Use our inequality for the logarithm. But do not ignore the cases in which $x \leq -1$.

Chapter 4

3. **Hint**: What event is $A - (A - B)$?

5. **Hint**: A naive answer might be that one coin is certainly heads up, so the chance that the other is heads is $\frac{1}{2}$; this is wrong.

7. **Hint**: Look at the derivation of the multiplication rule for combinatorial probabilities (see 3.4.3) and translate it into our new notation.

8. **Hint**: if n outcomes are equally likely, what is the probability of each particular outcome?

11. P(H|N) $= 0.00105$.

12. **b.** P(New|Out) $= 0.571$.

13. **a.** **Hint**: If she has talked to more than six people, then the first six were all right-handed.

14. **Hint**: There are two reasons why it might be for sale on Sunday. It might have just arrived, or it might have been there Saturday but did not sell.

16. **Hint**: if x is any number in $(0, 1]$, then $y = 1/x$ is ≥ 1. What determines the value of the integer part of y?

17. **Hint**: Figure out what event is described by the expression $\cup_j A_j - \cup_i(\cup_j A_j - A_i)$.

20. **Hint**: Piece events together out of finite rectangles.

Chapter 5

1. **b.** A typical entry is $p(1) = 0.214$.

3. P(5 older|2 Sophs) $= 0.1107$.

4. **Hint**: Generalize the idea of a negative hypergeometric variable to more than two categories (three kinds of trees).

5. P(3 or more) $= 0.097$.

9. p-value $= 0.0354$.

10. p-value $= 0.0403$.

13. **Hint**: You want the probability of a vertical strip whose left edge is at $x = -1$.

15. **a.** **Hint**: $F(x) = \int_0^x f(X)\,dX$.

16. **Hint**: The two calculations should be quite different but have exactly the same answer, illustrating positive–negative duality.

19. **Hint**: The calculations will be different, but the answers should be the same, illustrating black–white symmetry.

20. **Hint**: The fact that F(8|N(32, 8, 5)) $= 0.0574$ should reduce (but not eliminate) your arithmetic.

21. E(X) $= 2.3929$.

26. **a.** 179 sheep.
b. **Hint**: Start with P(so many if the total is 80) $= 0.1016$ and P(if the total is 50) $= 0.0147$.

Chapter 6

1. **b.** $P \approx 0.076$.
2. $\mathrm{P} = 0.0493$.
3. **a.** **Hint**: You need not use any formula you may happen to remember for the sum of a finite or infinite geometric series; use only the definition of F and the reasoning in (5.4.1).
5. **b.** $\mathrm{P} \approx 0.334$.
6. **b.** **Hint**: What is past is past. Consider only future races and wrecks.
9. **Hint**: Reread the derivation of the birthday inequality in (3.5.3).
10. **b.** $\mathrm{P} \approx 0.712$.
13. $\mathrm{P} = 0.136$.
15. **Hint**: The equality between your answers illustrates positive–negative duality.
16. **Hint**: The equality illustrates black–white symmetry for the negative binomial family.
17. **b.** **Hint**: Treat each day's work as an independent experiment.
18. **b.** **Hint**: Notice that failing is a rare event.
20. **Hint**: There is a simple duality principle that drastically reduces your work.
22. **Hint**: You will need the expectation of a Geometric(p) variable.
23. **a.** $\mathrm{E}([x - 3]^2) = 1.3375$.
25. **Hint**: Use the inductive method.
27. **b.** $\hat{p} = 0.585$.
28. **Hint**: In Exercise 3 you derived a helpful formula.
29. **Hint**: For a balanced die, $p(39) = 0.0145$ and $p(38) = 0.0108$.

Chapter 7

1. **c.** $\mathrm{P} = 0.0125$.
2. **b.** $\mathrm{P} = 0.286$.
5. **b.** **Hint**: It may be easier to reason it out than to compute with mass functions.
8. One entry is $\mathrm{F}(5, 5) = 0.55$.
10. **Hint**: Our formula for the probability of a rectangle should inspire you.
14. $\mathrm{Cov}(X, Y) = 0.6$.
17. $\sigma = 11.14$.
18. $\sigma = \$377.20$.
20. $\rho_{XY} = -0.041$.
22. $\mathrm{Var}(\bar{x}) = 3$.

23. **Hint**: After a particular failure, how many successes will there be before the *next* failure?

25. **b.** $\sigma = \$316.66$.

28. **b.** $P(X \geq 81|B(96, 0.75)) = 0.0189$; $P(X \geq 81|B(97, 0.75)) = 0.0306$.

29. **a.** **Hint**: The posterior distribution of the number of bears is $48 + Z$, where Z is a Poisson random variable with mean 105.

Chapter 8

1. R = 8.7.

2. R = 2.826.

3. $\hat{p} = 0.897$.

4. **Hint**: Do not use calculus. Find the ratio between probabilities for successive larger integers W (or B), and note when it stops increasing and starts decreasing.

6. **Hint**: In order to force your estimates to sum to 1, substitute $p_k = 1 - \sum_{i=1}^{k-1} p_i$.

7. **Hint**: Move x to the denominator.

9. **Hint**: It helps to replace $p_{r\bullet} = 1 - p_{1\bullet} - \cdots - p_{(r-1)\bullet}$ where there are r rows; and similarly for columns.

10. **a.** G-squared = 41.4.

b. Chi-squared = 42.3.

14. For example, the expected count of male, urban nonsmokers is 124.49.

16. $\lambda = 1.678$.

18. **c.** P(six|25 mg) = 0.353.

Chapter 9

1. **b.** **Hint**: Be careful about what happens at $X = 0$.

2. The answer starts out 0, 0, 1, 4, 5, ...

3. **b.** P = 0.713.

5. **Hint**: Use the results in Chapter 6.8.

9. **b.** mode $= b\left(\frac{a-1}{a}\right)^{1/a}$.

10. **Hint**: As often happens, it is easier to find the maximum of the log of f.

14. P = 0.577.

16. P = 0.474.

17. **Hint**: A statistics program that calculates cumulative Poisson probabilities will help here.

18. **a.** **Hint**: The best size you can get turns out to be about 0.035.

Chapter 10

1. **Hint**: Your answer should be in the form of a table.
2. The 25th percentile is 0.82.
5. **Hint**: See Exercise 9.6
7. **b.** P = 0.612.
12. **Hint**: $E(1 + X)$ and $[(1 + X)^2]$ are easy to find.
14. $\text{Cov}(X, Y) = -690$.
17. $0.4028 \le \log(1.5) \le 0.4167$.
18. **Hint**: Move the exponent to the denominator.
19. **Hint**: Write them as integrals.
20. Your answer will be within one part in 300 of the exact value.
21. Exact probability is 0.1254 and approximate probability is 0.1262.
22. The seventh and eighth terms of the series in Section 5.6 should give your bounds.
24. **Hint**: This involves summing many terms, but if you sum them in a sensible order, you will find that the terms quickly become negligible and you have an answer accurate to 2 significant figures.
27. **c.** $P(39) \approx 0.0518$.
28. Coverage probability = 0.918.

Chapter 11

4. P = 0.278.
9. **a.** P = 0.1792.
11. **Hint**: Review addition formulas from trigonometry.
13. **b.** The lower bound is 12.34
15. The upper limit of the interval is 81.30117.
16. **b.** **Hint**: Remember Beta–Binomial duality (see 9.5.3). P = 0.182.
17. **b.** $P \approx 0.158$.
18. **a.** **Hint**: A computer would help here. P = 0.219.
22. **Hint**: Try to rearrange it so it looks like a bivariate normal density.

Chapter 12

2. **Hint**: These look like $z = x \cos\theta + y \sin\theta$ and $w = -x \sin\theta + y \cos\theta$ for a rotation through any angle θ.
3. **b.** **Hint**: This involves evaluating a very easy integral.
8. **Hint**: The column for the one degree of freedom for interaction should be proportional to the product of the two centered columns for the factors.
9. **a.** **Hint**: You may have to discard some redundant columns so that $X^T X$ becomes nonsingular.

11. standard error of prediction = 0.00744.
13. **a.** $I_\beta = \frac{n}{\beta^2}$.
18. **Hint**: treat σ^2 as a parameter itself, not the square of σ.

Chapter 13

1. $\pi(q) = \dfrac{q(1-q^W)}{(W+1)(1-q)}$.
2. **Hint**: Does Exercise 1 help?
3. P = 0.028.
7. P = 0.297.
8. P = 0.0593.
10. Var = 24.
13. **Hint**: Does Exercise 12 help?
14. $\mathrm{E}(X) = 15$ months.
19. P ≈ 0.584.
25. **b.** P ≈ 0.0117.
27. **b.** P ≈ 0.000305.

Index

Springer Texts in Statistics *(continued from page ii)*

Noether: Introduction to Statistics: The Nonparametric Way
Peters: Counting for Something: Statistical Principles and Personalities
Pfeiffer: Probability for Applications
Pitman: Probability
Rawlings, Pantula and Dickey: Applied Regression Analysis
Robert: The Bayesian Choice: A Decision-Theoretic Motivation
Santner and Duffy: The Statistical Analysis of Discrete Data
Saville and Wood: Statistical Methods: The Geometric Approach
Sen and Srivastava: Regression Analysis: Theory, Methods, and Applications
Shao: Mathematical Statistics
Terrell: Mathematical Statistics: A Unified Introduction
Whittle: Probability via Expectation, Third Edition
Zacks: Introduction to Reliability Analysis: Probability Models and Statistical Methods