Appendix A
Sample Distribution Functions

This appendix gives the basic distributions used in the text. We provide their means and variances.

A.1 Discrete Random Variables

Uniform: U[1,n]

$$P(\boldsymbol{\xi} = i) = \frac{1}{n}, i = 1, \ldots, n, n \geq 1,$$

with $E[\boldsymbol{\xi}] = \frac{n+1}{2}$ and $Var[\boldsymbol{\xi}] = \frac{n^2-1}{12}$.

Binomial: Bi(n,p)

$$P(\boldsymbol{\xi} = i) = \binom{n}{i} p^i (1-p)^{n-i}, i = 0, 1, \ldots, n; 0 < p < 1,$$

with $E[\boldsymbol{\xi}] = np$ and $Var[\boldsymbol{\xi}] = np(1-p)$.

Poisson: P(λ)

$$P(\boldsymbol{\xi} = i) = e^{-\lambda} \frac{\lambda^i}{i!}, \lambda > 0, i = 0, 1, \ldots \ ,$$

with $E[\boldsymbol{\xi}] = \lambda$ and $Var[\boldsymbol{\xi}] = \lambda$.

A.2 Continuous Random Variables

Uniform: U[0,a]

$$f(\xi) = \frac{1}{a}, 0 \leq \xi \leq a, a > 0,$$

with $E[\boldsymbol{\xi}] = \frac{a}{2}$ and $Var[\boldsymbol{\xi}] = \frac{a^2}{12}$.

Exponential: exp(λ)

$$f(\xi) = \lambda e^{-\lambda\xi}, 0 \leq \xi, \lambda > 0,$$

with $E[\boldsymbol{\xi}] = \frac{1}{\lambda}$ and $Var[\boldsymbol{\xi}] = (\frac{1}{\lambda})^2$.

Normal: N(μ,σ²)

$$f(\xi) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\xi-\mu)^2}{2\sigma^2}}, \sigma > 0,$$

with $E[\boldsymbol{\xi}] = \mu$ and $Var[\boldsymbol{\xi}] = \sigma^2$.

Gamma: G(α, β)

$$f(\xi) = \frac{1}{\beta^2\Gamma(\alpha)} \xi^{\alpha-1} e^{\frac{-\xi}{\beta}}, \alpha > 0, \beta > 0,$$

where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx, \alpha > 0$, $E[\boldsymbol{\xi}] = \alpha\beta$ and $Var[\boldsymbol{\xi}] = \alpha\beta^2$.

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