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# Index of Notation

$\mathbb{B}$ : closed unit ball	$U(x)$ : neighborhood of $x$
$\mathbb{R}$ : the real numbers	$\text{dist}(x, C)$ : distance from $C$
$\overline{\mathbb{R}}$ : the extended real numbers	$\text{pos } C$ : positive hull
$\mathbb{R}^n$ : Euclidean $n$ -dimensional space	$\text{epi } f$ : epigraph of function $f$
$\ x\ $ : the Euclidean norm of $x$	$\text{lev}(f, \lambda)$ : lower level set
$(x)_+$ : positive part of $x$	$\sigma_C$ : support function of $C$
$\mathbb{R}_+^n$ : the nonnegative orthant	$N_C(x)$ : normal cone
$\mathbb{R}_{++}^n$ : the positive orthant	$T_C(x)$ : tangent cone
$\mathbb{N}$ : the natural numbers	$\partial f(x)$ : subdifferential set
$\text{aff } C$ : affine hull of set $C$	$\ker A$ : kernel of map $A$
$\text{ri } C$ : relative interior of set $C$	$\text{rge } A$ : range of map $A$
$\text{int } C$ : interior of set $C$	$\text{rank } A$ : rank of $A$
$\text{cl } C$ : closure of set $C$	$\text{gph } S$ : graph of $S$
$\text{bd } C$ : boundary of set $C$	$\nabla f(x)$ : gradient of $f$
$\text{conv } C$ : convex hull of set $C$	$\nabla^2 f(x)$ : Hessian of $f$
$\text{conv } f$ : convex hull of function $f$	$f'(\cdot; d)$ : directional derivative
$\Delta_n$ : unit simplex in $\mathbb{R}^n$	$f^*, f^{**}$ : conjugate, biconjugate
$\text{dom } f$ : domain of function $f$	$K^*$ : polar cone
$\text{ext } C$ : extreme points of set $C$	$M^\perp$ : orthogonal complement
$\text{extray}$ : set of extreme rays	$\gamma_C$ : gauge function
$C_\infty$ : asymptotic cone	$\delta_C$ : indicator function
$f_\infty$ : asymptotic function	$f \square g$ : infimal convolution
$C_f$ : constancy space	$S_n$ : $n \times n$ symmetric matrices
$\mathcal{K}_f$ : cone of asymptotic directions	$\text{tr } A$ : trace of matrix $A$
$L_f$ : lineality space	$\lambda(A)$ : eigenvalue of matrix $A$

$\text{diag}(x)$ : diagonal matrix  
 $P_C$ : projection map onto  $C$   
 $\text{Prox}(f, \lambda)$ : proximal map  
lsc: lower semicontinuous  
als: asymptotically lower stable  
awb: asymptotically well behaved  
psd: positive semidefinite

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