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Index of Notation

$\mathbb{B}$: closed unit ball
$\mathbb{R}$: the real numbers
$\overline{\mathbb{R}}$: the extended real numbers
$\mathbb{R}^n$: Euclidean n-dimensional space
$\|x\|$: the Euclidean norm of x
$(x)_+$: positive part of x
$\mathbb{R}^n_+$: the nonnegative orthant
$\mathbb{R}^n_{++}$: the positive orthant
$\mathbb{N}$: the natural numbers
$\operatorname{aff} C$: affine hull of set C
$\operatorname{ri} C$: relative interior of set C
$\operatorname{int} C$: interior of set C
$\operatorname{cl} C$: closure of set C
$\operatorname{bd} C$: boundary of set C
$\operatorname{conv} C$: convex hull of set C
$\operatorname{conv} f$: convex hull of function f
Δ_n: unit simplex in $\mathbb{R}^n$
$\operatorname{dom} f$: domain of function f
$\operatorname{ext} C$: extreme points of set C
extray: set of extreme rays
C_∞: asymptotic cone
f_∞: asymptotic function
$\mathcal{C}_f$: constancy space
$\mathcal{K}_f$: cone of aymptotic directions
L_f: lineality space
$U(x)$: neighborhood of x
$\operatorname{dist}(x, C)$: distance from C
$\operatorname{pos} C$: positive hull
$\operatorname{epi} f$: epigraph of function f
$\operatorname{lev}(f, \lambda)$: lower level set
σ_C: support function of C
$N_C(x)$: normal cone
$T_C(x)$: tangent cone
$\partial f(x)$: subdifferential set
$\ker A$: kernel of map A
$\operatorname{rge} A$: range of map A
$\operatorname{rank} A$: rank of A
$\operatorname{gph} S$: graph of S
$\nabla f(x)$: gradient of f
$\nabla^2 f(x)$: Hessian of f
$f'(\cdot; d)$: directional derivative
f^*, f^{**}: conjugate, biconjugate
K^*:polar cone
$M^\perp$: orthogonal complement
γ_C: gauge function
δ_C: indicator function
$f \Box g$: infimal convolution
S_n: $n \times n$ symmetric matrices
$\operatorname{tr} A$: trace of matrix A
$\lambda(A)$: eigenvalue of matrix A

diag(x): diagonal matrix
P_C: projection map onto C
Prox(f, λ): proximal map
lsc: lower semicontinuous
als: asymptotically lower stable
awb: asymptotically well behaved
psd: positive semidefinite

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